Introduction to Fluid Mechanics
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Lecture – 13
Surface Tension – Part- II

Last time we were discussing as some of the fundamental concepts of surface tension and we will continue with that. Let us say that we want to see some application of surface tension as a very simple case.

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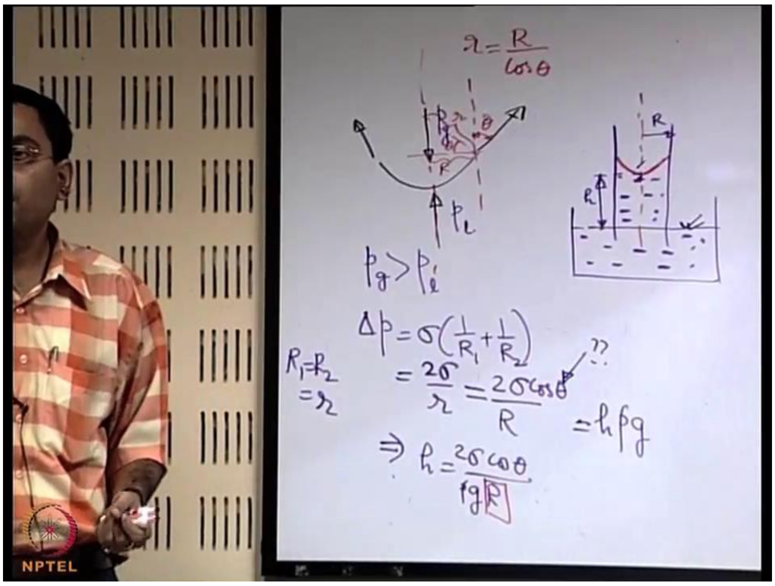

To do that we will take an example when there is a capillary tube which is small in radius. It is dipped into larger volume of fluid, and let us say that this was the initial level of the fluid. And now once this capillary is introduced this fluid is expected to either rise or fall and we will see that what dictates that it should rise or fall.

To understand that let us assume that the fluid is rising over the capillary. If it rises over the capillary then what dictates its rising and to what extent it should rise, let us try to make an estimate of that. First question should come that why it should rise or why it should fall. When there is a fluid it is a collection of molecules after all and these molecules are subjected to various interactions.

One type of interaction is the inter-molecular forces of attraction between the similar types of molecules which are like so called cohesive forces. The other one is the, inter molecular force of interaction between the molecules of the fluid and the molecules at the boundary. So, when you have that type of interaction that is known as adhesion, so it is between two different types of entities.

Now, if the adhesion wins over the cohesion then that means, the surface at the end has a net attraction towards the fluid and it likes the fluid. Earlier we used a terminology called as hydrophilic material. So, such types of substrates which like water are called as hydrophilic ones. A better terminology would be wetting because when we say hydro it means water, but it may be any other fluid also. So, better we say that those are wetting surfaces.

Let us say that there is a wetting fluid and it has formed a meniscus like this. This meniscus can be of a very complicated shape, but let us just for sake of simplicity. Assume that it is a part of a hemisphere or like it is a hemispherical cap so to say. Now, if you try to identify that what are the various forces which are related to the development of this meniscus.

We have discussed about this earlier, so just if we reiterate that if we have a meniscus like this loosely speaking there will be a force because of surface tension which acting over the periphery there is a pressure acting from this side, there is a pressure acting from the other side the bottom side. And it is basically filled with the same fluid which is also there in the reservoir. So, if we say that at the top there is some gas or vapour, and at the bottom there is a liquid then let us say this is P_l and let us say this is P_g. So, which one will be greater here P_l or P_g in this configuration?

P_g to be more, because it has to balance the p liquid and the component of the surface tension in that direction. So, if you have P_g greater than P_l that means, there is a pressure differential across the meniscus. So, if you have the pressure in the gas as a atmospheric pressure; obviously, the pressure in the liquid is not atmospheric pressure across the meniscus. There is a difference in pressure and that difference $\Delta p = \sigma(\frac{1}{R_1} + \frac{1}{R_2})$. So, if we consider it a part of the sphere $R_1 = R_2 = R$.

So, we are approximating the shape of the meniscus in that way. Otherwise it may have different radii of curvature at like different planes, but this simplistic assumption. So, if,

$R_1 = R_2 = r$, r is not same as the radius of the capillary. So, this is different because r is the radius of curvature of the meniscus which is not same as the radius of the capillary.

So, we have $\Delta p = \sigma(\frac{1}{R_1} + \frac{1}{R_2}) = \frac{2\sigma}{r}$. obviously, we need to relate r with R. And to do that we can say have a small geometrical construction let us say that this angle is θ. So, when you have this angle as θ this is the angle made by the tangent to the interface with the vertical. So, that should be same as the angle between the normal to the interface and the horizontal. So, let us say that, that is θ this normal is in the direction of the radius of curvature of the interface. So, this length is what small r. And what is this length? This is capital R because this is say the centre line of the tube. So, this is capital R. So, how you can relate r with R?

So, $r = \frac{R}{\cos\theta}$. So, we can simplify this $\frac{2\sigma}{r} = \frac{2\sigma \cos g\theta}{R}$. Δp is the difference in pressure between the outside atmosphere and what is there just inside. The pressure at the outside atmosphere is same as what is the pressure at this level, right. So, if you know, what is the pressure at this level then from that you can find out the relationship between the pressure difference of these two and that pressure difference is nothing but because of the presence of this much of liquid column.

So, if this is let us say that it has average height of h, so $\Delta p = \frac{2\sigma\cos\theta}{R} = h\rho g$. So, $h = \frac{2\sigma\cos\theta}{\rho g R}$

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This expression is very simple, and this expression is valid only if this meniscus is under a static equilibrium condition it is not moving. But still it gives a lot of good insight because it tells you that if there is a capillary rise say in this kind of a situation we call it a capillary rise because as if the fluid is rising from the reservoir towards the capillary.

So, that capillary rise is dictated by many things. One of the features of course, is the angle θ which is the contact angle and which depends on the combination of the fluid. So, if we say that for a glass water combination glass water air combination say this θ is close to zero maybe; that means, that it requires three different phases so to say. I mean glass is a solid phase which forms the surface of the capillary then you have water here and maybe air here, but if you replace water with say mercury it may be something different and it may be possible that instead of a rise it has a fall. That is dictated by the competition between the adhesion and the cohesion force, and eventually it is manifested by what is the value of the contact angle.

So, in that case say if the contact angle say is 140 degree which is like if it is mercury glass air that may be like one of the possibilities. So, when you have this one; that means, you will get if you have theta in that range obviously, h is negative; that means, instead of rise it is a fall the meniscus shape also will be just curved oppositely. The other important factor which is one of the decisive factors is, what is R because if R is large then no matter whatever is the contact angle this effect will be small and will virtually be imperceptible.

At the same time if R is very small, then h can be very large. We get such examples in nature very nicely that is if you have trees this tress absorb water from the ground there is no pump which is existing in the nature as an artificial mean of pumping the fluid from the root to the top most leaves and branches, but you will see tall trees also get nutrition from the ground.

So, when they are transported by the fluid medium which is the so-called assent of sap in terms of the biological terminology. It goes vertically such a large distance it covers such a huge height just with the consideration that this R is small. So, those are really very narrow capillaries and then the capillary acts like a pump. So, just because of the surface tension it can attain in a large height of transport, and that kind of will be beautiful mechanism prevails in nature and that makes the plants at least the tall trees sustain their lives.

Now, just to have a slight variation from this, let us consider an example when you do not have just one fluid as a liquid and another fluid as gas, but maybe the both of the fluids are liquids and they are immiscible ones. If they are miscible liquids; obviously, when you when you put them one with the other, they may mix with each other and the clear meniscus may not be formed. But if they are not miscible with each other there may be a clear meniscus that is formed.

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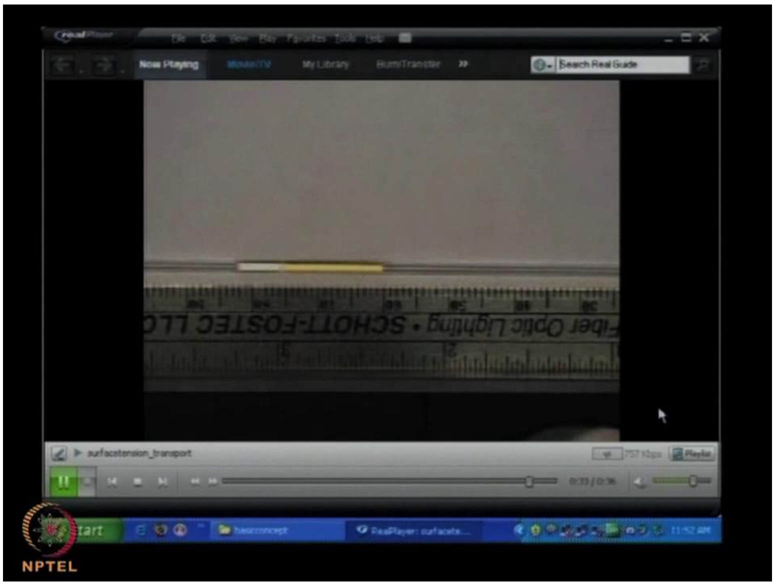

So, in this example what we will try to see is that if there are two different liquids which are there side by side then when these two different liquids are put in a capillary tube sometimes magically because of the resultant surface tension driven flow transport that is created the entire column may move from one end to the other end and we see one such example here.

So, there will be two different fluids those liquids which have already been put and see that it is just like moving magically. It is not that there is a pump that is being put or there is no other driving force that has been created to induce the motion. Here we will not first be bothered about the motion we are first considering about the equilibrium. So, let us consider such a case, but not a moving case, but two different liquids which are keeping meniscus in equilibrium.

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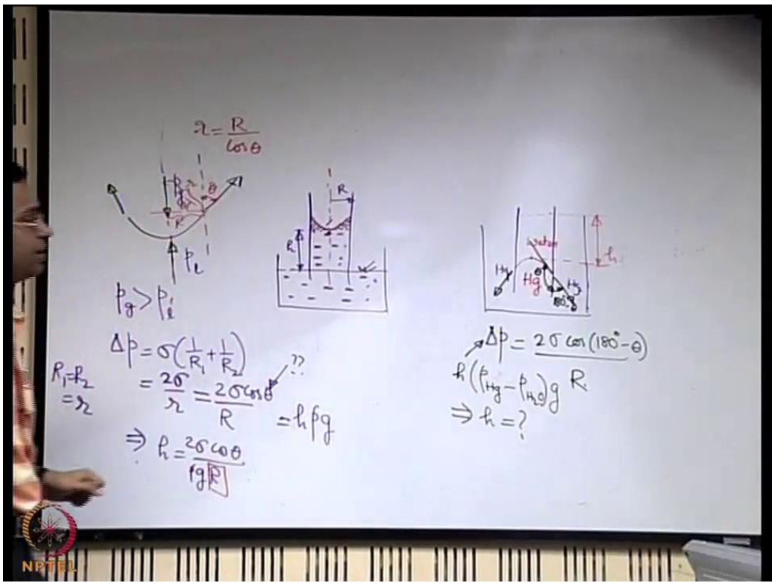

So, let us say that you have a capillary tube, in the capillary tube now you have two liquids on the top you have water and in the bottom you have mercury; with a given tube materials say it is glass it assumes this particular meniscus shape. The outside is filled up with mercury, so there is mercury in the outside and water is being poured from inside.

Now, if you see let us concentrate or focus on the meniscus. Let us say that the water is being poured from this height and there has been a so called depression of the meniscus from the top level. So, let us say that this is h. Again when you see such a simple analysis you should keep in mind that this has many approximations. When I say that this height is h obviously, it has no sanctity because we are disregarding the variation in height from one end to the other when we calculate the pressure difference.

See whenever we calculated the pressure difference here $\Delta p = h\rho g$ that h we assume some average height maybe the centre one we took as a reference, but it is not actually a uniform height, the entire meniscus has a variation in height and one of the approximations may be that not to use this as the height, but use the centre line one instead just like what we did in the other example. So, maybe take this as h.

What is the error in this? When you are considering this as the h if you refer to such a figure you are neglecting the shaded volume. So, the shaded volume has a contribution. Effectively if you see it is the weight of the volume of the liquid that is being sustained to the surface tension and the pressure differential, and there these shaded volumes also have their own roles. It is only an approximation by which we are neglecting it. There may be cases when this gives rise to some significant error and fortunately in most cases it does not give rise to that much error. So, it is fine as a practical approximation.

Now, if you consider this surface and try to see that, what are the different types of forces which are acting on the surface, you may evaluate the surface tension force just like what you did in the previous example. So, when you are considering the contact angle say we are interested about the angle that is made by the mercury with the glass. So, that is measured with respect to this one. So, this is basically the theta. So, it is important to see what is the sign convention for the definition.

So, when we are referring to the mercury it should be from the solid within the mercury domain that is how the angle is there, otherwise with a different notation maybe $180^0 - \theta$ also we taken as an equivalent representation of the effect on the contact angle. So, this angle will be $180^0 - \theta$ and for writing the force balance that angle will be useful. So, the same formula will be applicable here that is if you have $\Delta p = \frac{2\sigma\cos(180^0 - \theta)}{R}$. So, now on which side the pressure will be more mercury or water?

Student: Mercury

And how can you calculate the differential of that pressure? When you consider the water side it is basically $h\rho_{water}g$ that is the height of the column of water. When you consider the mercury side it is $h\rho_{mercury}g$. $\Delta p = h(\rho_{Hg-}\rho_{H_2O})g$. h is clearly a depression in this case that has been induced by this theta which is roughly 140^0 as an example. So, if you know the surface tension coefficients then it is possible to put that in this expression and find out what should be the h under these conditions. Again this has many approximations, but it gives some kind of idea that what should be the estimation for capillary rise or capillary depression.

With surface tension one may also have a dynamic nature of the meniscus and when you have a dynamic nature of the meniscus it is not that you just have to consider this type of equilibrium at the interface, you may have to consider overall dynamical nature of one fluid as it is displacing the other and moving in the capillary. In the process there may be many things, in the static condition we have a contact angle, in the dynamic condition this contact angle may change and the change of this contact angle may be because of the dynamic nature of the forces which are acting on the system. One of such forces is the viscous force.

Then you also have a dynamically evolving maybe Van Der Waal’s force of interaction. So, it is possible to have all the forces of interaction which are not just like constants, but those are evolving dynamically as the shape of the meniscus is changing because as the shape of the meniscus is changing you have different arrangements of the molecules close to the wall and that may dynamically give rise to different contact angles. So, the contact angle that we are referring to in these cases is commonly known as a static contact angle.

But when dynamics is evolving one may have a dynamic contact angle which depends on the relative interplay of various forces which are acting and since the two important forces dictating this type of capillary advancement in a dynamic condition are the surface tension and the viscous forces. So, their relative interplay has a strong role to play in determining the contact angle that evolves dynamically.

Surface roughness also has a strong role to play because that dictates the proper intermolecular interaction close to the surface. So, there is a very rich physics that takes place close to the interface in a dynamic condition and this elemental study does not focus on that, it gives just a road idea of if it is a static condition what can be the consequence of surface tension. But at least it gives us an idea that surface tension may be a very important force in a small scale and as the radius becomes smaller and smaller its effect becomes more and more prominent.