

Concepts of Thermodynamics
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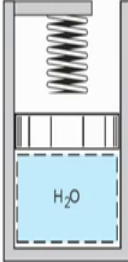
Lecture – 06
Properties of Pure Substances: Example Problems (Contd.)

In the previous lecture, we were discussing about some problems on Properties of Pure Substances and in this lecture, we will continue with some more problems.

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Problem 1.3: A cylinder/piston arrangement contains water at 105 °C, 85% quality, with a volume of 1 litre. The system is heated causing the piston to rise and encounter a linear spring as shown in the figure below. At this point the volume is 1.5 litre, the piston diameter is 150 mm, and the spring constant is 100 N/mm. The heating continues, so the piston compresses the spring. What is the cylinder pressure when the temperature reaches 600 °C?

Ans: pressure = 193 kPa

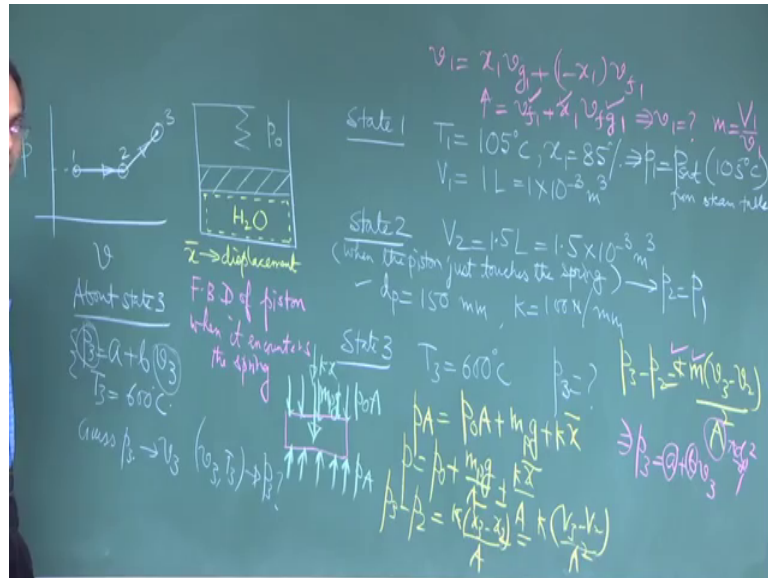


The diagram shows a vertical cylinder with a piston at the bottom. The piston is in contact with a linear spring. The cylinder contains water, labeled as H₂O. The piston is shown in a position where it has just made contact with the spring, and the spring is compressed.

So, let us look into this problem. Problem 1.3: A cylinder piston arrangement contains water at 105 degree centigrade and 85 percent quality, with a volume of 1 litre. The system is heated causing the piston to rise and encounter a linear spring as shown in the figure. At this point the volume is 1.5 litre and the given data is that the piston diameter is 150 millimeter and spring constant is 100 Newton per millimeter. The heating continues so that the piston compresses the spring. What is the cylinder pressure when the temperature reaches 600 degree centigrade?

So, let us try to look into this problem carefully as per custom let us first draw semantics of this.

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So, state 1: I am just copying the property data given in the problem. Then, the other data are diameter of the piston is equal to 150 millimeter spring constant is 100 Newton per millimeter and state 3, is T_3 is equal to 600 degree centigrade. What is the corresponding pressure at state 3? State 2 is the state when the piston just touches the spring.

So, when the piston just touches the spring, can you tell what is the pressure? Whatever is the pressure at state one at which this is floating it should be same as that pressure. So, this means that p_2 is equal to p_1 because the equilibrium pressure will change only after it compresses the spring. Before it compresses the spring it moves in a quasi-equilibrium process because the resistance does not change the internal pressure also does not change,.

So, then what we have to do is we. So, sorry this is p_3 what is p_3 . So, we have to draw or we have to semantically represent these processes in many $p-v$ and $T-v$ diagram just to get a good visualization of what is happening. Sometimes without plotting the processes it is possible to get this answer, but I would prefer as a teacher of this subject with some experience that if you can draw this diagrams $P-v$ diagram $T-v$ diagram later on when entropy comes into the picture then $T-s$ diagram, these types of diagrams will give you a good visualization of physically what is the process happening.

So, let us try to draw our P v diagram of this. So, P v diagram state 1. So, what is state 1, it is in the 2 phase region because quality is 85 percent. So, it is qualities between 0 to 100 percent. So, it is in the two-phase region. So, what is p 1? p 1 is the saturation pressure corresponding to 105 degree centigrade from steam table because the substance is water otherwise you have to look into the table of that substance,.

So, I am requesting please you know stretch the camera little bit because it is not visible. Yes, now it is visible fine. So, p 1 is p sat corresponding to 105 degree centigrade then what is that. So, it will be a value calculated from the obtained from the steam table. So, from 1 to 2 the volume increases, but the pressure remains constant. So, it is a constant pressure process. From 2 to 3 it encounters a linear spring. So, because it encounters a linear spring the press from 2 to 3 it will be a straight line like this with pressure increasing because the resistance pressure also increasing ok. Question is: what is the equation of this line 2 to 3. So, for that all these details are required. So, let us draw the free body diagram of the piston when it encounters the spring.

So, now I will not spend much time in drawing the free body diagram, we have already understood how needs to be these needs to be done. So, m g, p A, then there is p 0 A, the atmosphere equation p 0 acting from here and there is a Kx; Kx upwards or downwards? So, when this is moving up it is pressing the spring in the upward direction. So, the spring will apply a force in the downward direction on it.

So, Kx right therefore, for equilibrium you have p A. So, p is equal to p 0 plus m g by A plus Kx by A. So, p 3 minus p 2 is equal to K into x 3 minus x 2 by A. x 2, right x 3 minus x 2 by A.

Student: Sir, justify how to use the symbol x.

So.

Student: By conflict with quality.

Ok. So, interesting thing is that this x is not same as quality. So, let us put x bar at the top to indicate that it is displacement. So, x is quality and x bar is displacement. So, this one. Now, what is the change in x that is the change in volume divided by A. So, K into V 3 minus V 2 by A square.

So, to sum it up we can write $p_3 - p_2$ is equal to $K \cdot m \cdot (v_3 - v_2)$ by A square because capital V the total volume is specific volume into mass. How do you get the mass? See at state point 1, you look at the state point 1; so, what is v_1 ? $x_1 v_{g1} + (1 - x_1) v_{f1}$, right. Sometimes in terms of the steam table notion this is $v_{f1} + x_1 (v_{g1} - v_{f1})$ which is also called as v_{fg1} .

So, this is this you get from the table of water at one 105 degree centigrade. At 105 degree centigrade calculate what is V_1 , what is V_{fg1} or V_{f1} whatever, any one of these two. x_1 is also given. So, this will tell you what is v_1 . So, what is m , the mass that does not change during the process that is V_1 by v_1 , ok. So, that mass you can substitute here. So, and this K you know already area of the piston is πd^2 by 4. So, from here you can write an equation p_3 is equal to some $a + b \cdot v_3$ where a and b are numerical constants, ok.

So, about state 3 what you know? p_3 is equal to $a + b \cdot v_3$ and T_3 is equal to 600 degree centigrade, right. So, you do not know p_3 and v_3 independently you know a relationship between p_3 and v_3 , this is a very unique problem. So, from this combination you have to identify what is p_3 and T_3 . So, what you can do is if you are doing it manually if you are doing it by a computerized system this combination of equations will give you a unique property if you are doing it manually you have make a guess of v_3 and you have to or make a guess of p_3 , that is better.

Make a guess of p_3 that p_3 will give you what is v_3 and then with these v_3 comma T_3 see whether this will give back to the same p_3 or not. In this way you iterate till you make guess of p_3 which is such that that convergence to the same p_3 once you substitute these two, ok. So, the answer to this is p_3 is equal to 193 kilo Pascal, ok. So, we will go to the next problem 1.4.

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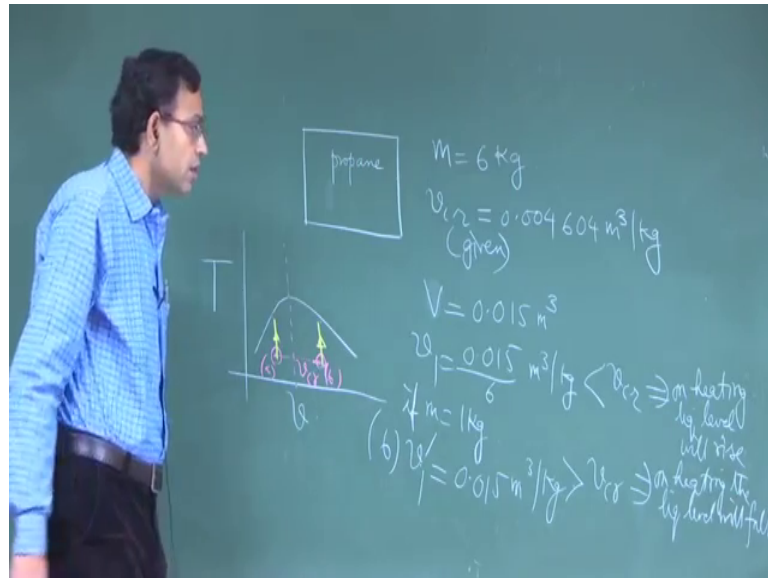
Problem 1.4: A steel tank contains 6 kg of propane (liquid + vapor) at 20°C with a volume of 0.015 m³. The tank is now slowly heated. (a) Will the liquid level inside eventually rise to the top or drop to the bottom of the tank? (b) What if the initial mass is 1 kg instead of 6 kg?

Ans: (a) Liquid level will rise (b) Liquid level will fall

A steel tank contains 6 kg of propane ok, liquid plus vapor at 20 degree centigrade with a volume of 0.015 meter cube. The tank is now heated. So, this tank is closed. Will the liquid level rise or fall and what the initial mass is 1 kg instead of 6 kg? So, this is the very interesting problem you have a liquid plus vapor in a tank and you are heating it. So, what you expect? You expect that the liquid will become vaporized and water or whatever propane the liquid here; the liquid level will fall and the vapor volume will increase, right. That is your intuition.

This intuition always may not work because your working in a close phase. Because you are working in a close phase when the liquid is getting converted into vapor that vapor increases the vapor pressure within the system, it is closed system and that might dynamically alter state of the system till until an equilibrium state is achieved. The equilibrium state may be dictated by the fact that the liquid level despite heating may not fall, but actually rise. So, we will look into this very non-intuitive problem now through the property tables. So, let us work out this problem.

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So, let us call it whatever T v diagram now what. So, I will illustrate the principal of solving this problem before getting into the actual problem. So, there is a very critical data point in the property consideration that is the critical point. So, these volumes specific volume is the critical specific volume.

Now, we have two possibilities. Of course, three possibilities one is v equal to v critical, but other than that we have one possibilities v less than v critical another possibilities v greater than v critical. So, this is possibility one or possibility a this is possibility b ok. Now, if you are heating the propane in the closed tank what is remaining constant total volume is remaining constant and mass also is remaining constant because the total volume is remaining constant and mass is remaining constant the specific volume which is volume per unit mass is also remaining constant.

So, the process here is a constant specific volume process. So, if now you heat this; heat means it is temperature will increase it will move along this path, but if you heat here it move to along this path. What is the difference? See, as you are heating here it is going more closer to the saturated liquid line, and once you are heating here it is going closer to the saturated vapor line. For example; here quality could be say may be 70 percent and here it is close to quality equal to 100 percent. That means, here the vapor fraction is increasing here similarly the liquid fraction is increasing.

So, now, let us look into the problem data. So, in the problem data you have m equal to 6 kg it is given that thus critical specific volume of. So, this is very important v critical for propane is 0.004604 meter cube per kg this is given. So, as per the given a and b actually this is a where v is. So, let us see if m is 6 kg and what is the volume of the tank is 0.015 meter cube. So, what is v_1 is 0.015 by 6 meter cube per kg may be it is all right, maybe it is in the liquid state.

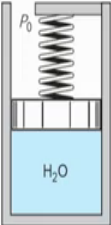
So, what is this? So, if you calculate this; this will be less than v critical, right. So, it is this state a , if you heat this the liquid level will rise ok. On the other hand if m is equal to 1 kg then v_2 then your v_1 dash the new v for case b it is 0.015 meter cube per kg, right. So, this is greater than v critical this means on heating the liquid level will fall, ok.

So, to summarize if you heat a substance in a closed space on heating you cannot trivially say whether the liquid level will rise or fall it depends on which side of the critical point the volume belongs to, ok. So, we will work out a last problem actually we will not work it out in full, but we will give you a hint of how to solve this problem we have solved enough number of problems.

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Problem 1.5: A piston/cylinder arrangement is loaded with a linear spring and the outside atmosphere. It contains water at 5 MPa, 400°C with the volume being 0.1 m³. If the piston is at the bottom, the spring exerts a force such that $P_{\text{int}} = 200$ kPa. The system now cools until the pressure reaches 1200 kPa. Find the mass of water and v_1 , the final state (T_2 , v_2) and plot the P - v diagram for the process.

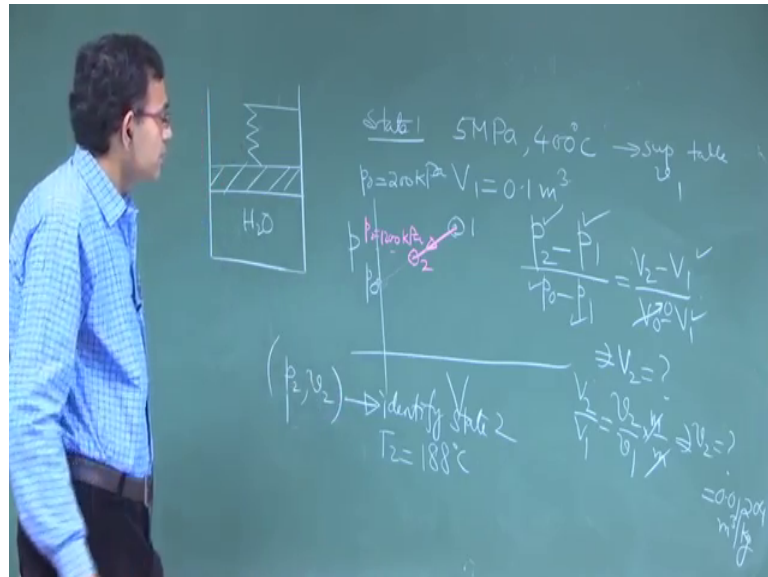
Ans: $m = 1.73$ kg, $T_2 = 188$ °C, $v_2 = 0.01204$ m³/kg



So, this is problem 1.5. You have a piston cylinder arrangement it is loaded with a linear spring and an outside atmosphere. It contains water at 5 MPa 400 degree centigrade with a volume of 0.1 meter cube. If the piston is at the bottom the spring exerts a force such that p

lift equal to 200 kilo Pascal. It is a hypothetical state; that means, had the piston being at the bottom that is volume equal to 0 to 100 kilo Pascal equilibrium 4 pressure will be required to lift the piston. Now, from state one the system cools until the pressure reaches 1200 kilo Pascal. Find the mass of water and v 1 final state and plot the P-v diagram of the process. So, let us try to do this.

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We draw the schematic of this problem. You have water; state 1 5 MPa, 400 degree centigrade. See from now onwards you try to develop an intuition since pressure and temperature as given as a independent property it cannot be here two-phase system and at such a high temperature it will definitely be superheated vapor for water. So, that common sense intuition you can develop without you know having to look deeply into the table to figure out what where is this state point located.

So, and the volume is 0.1 meter cube. So, what it is says is very interesting that had the piston being at the bottom that is volume is equal to 0 your p 0 p will be equal to p 0 p 0 is 200 kilo Pascal this is not an actual state point, but a hypothetical state point. State 1 is an actual state point where the pressure, volume, temperature everything is given.

Now, the system cools until the presser reaches 1200 kilo Pascal. So, when it cools because it is encountering a linear spring it should cool along this path only, the straight line joining this two. But, you know the full part of this is not real it is just a hypothetical

set of data points, so that you can generate the direction along which the process will occur. So, the process will occur after cooling from 1 to this point 2 and this p_2 is 1200 kilo Pascal.

So, the question is what is V_2 ? So, how do you calculate this? You use the equation of this straight line; $p_2 - p_1$ by $p_0 - p_1$ is equal to $V_2 - V_1$ by $V_0 - V_1$ where V_0 is 0. p_2 is 1200 kilo Pascal, p_1 is 5 mega Pascal, p_0 is 200 kilo Pascal. So, all these are known and V_1 is 0.1. So, from here you can calculate what is V_2 . V_1 is known you can only find out V_2 from here.

So, now V_2 by V_1 is specific volume at state 2 by specific volume at state one because mass gets cancelled from both numerator and denominator. So, from here from superheated table you can calculate what is v_1 . So, once you have calculated what is v_1 , you substitute it here you will get what is v_2 , that is one of the questions. So, once you know what is p_2 and v_2 this p_2 , v_2 combination from table will give you what is T_2 . So, this will enable you to identify state 2.

So, you identify state 2 by looking into the combination of p_2 which is 1200 kilo Pascal and V_2 is the answer to this is 0.01204 meter cube per kg ok. So, by looking into 1200 kilo Pascal and 0.01204 meter cube per kg, you have to identify the state whether it is two-phase or superheated or whatever. How do you identify see at 1200 kilo Pascal you see what is v_f and v_g if this v is between v_f and v_g it is in the two-phase region; if this v is greater than v_g then it is in the superheated region. Then from the superheated table with this combination you have to find out what is the temperature that is one of the questions. So, T_2 is 188 degree centigrade, ok.

So, it is therefore, a problem where you will develop a practice of how to identify a state point given not just pressure temperature may be a combination of pressure and volume. With this we conclude this lecture.

Thank you very much.