Concepts of Thermodynamics
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Lecture - 38
Second Law: Illustrative Problems

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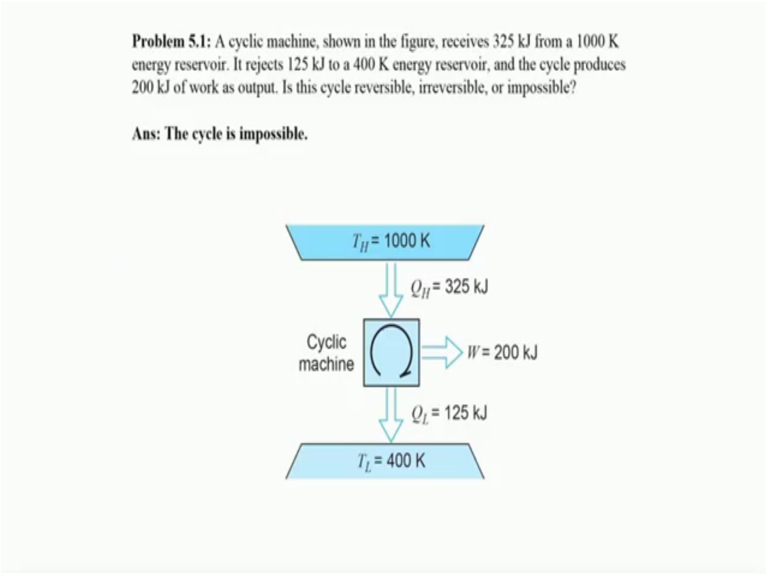

Today, we will work out some problems related to the Second Law of Thermodynamics. The first problem is projected here; a cyclic machine, shown in the figure, receives 325 kilo Joule from a 1000 Kelvin energy reservoir. It rejects 125 kilo Joule to a 400 Kelvin energy reservoir and the cycle produces 200 kilo Joule of work. Is the cycle reversible, irreversible or impossible?

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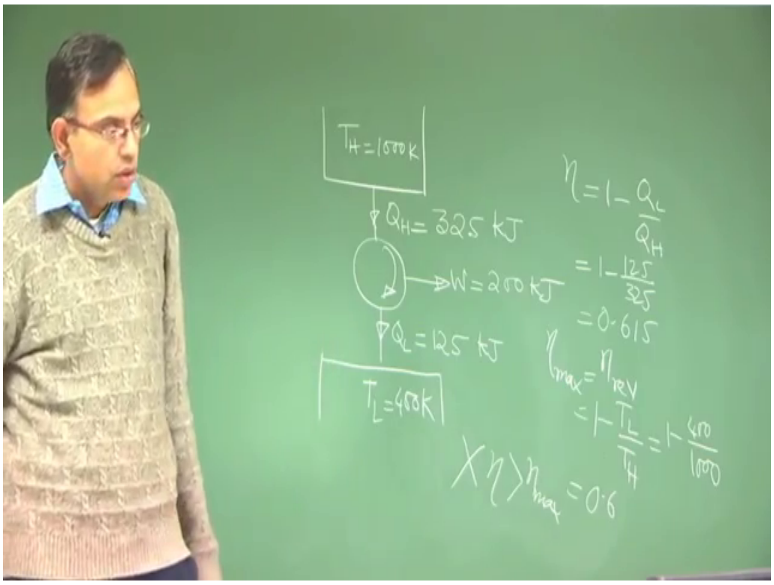

So, let us draw this. So, you have T H equal to 1000 Kelvin. So, I have put all the values. Given a situation like this, the first thing that you should check is does it satisfy the first law. So, how do you check whether it satisfies the first law; first law for a cyclic process is cyclic integral of heat equal to cyclic integral of work. So, here cyclic integral of heat is Q H minus Q L that is 325 minus 125 that is 200 and cyclic integral of work is 200, so it satisfies the first law.

As we have discussed at the beginning of the second law, there could be several devices which satisfy the first law, but they are not practical, so that is what we are assessing that despite satisfying first law is it a practical device. So, what is the efficiency of this device 1 minus Q L by Q H right, so, 1 minus 125 by 325, so this is 0.615. But, working between two given temperature limits, the maximum efficiency is constant by the Carnot cycle efficiency.

So, eta max that is the reversible one that is 1 minus T L by T H, because for reversible cycle Q L by Q H is equal to T L by T H by the definition of the absolute temperature scale, so that is 1 minus 400 by 1000, so that is 0.6. So, the efficiency is becoming the maximum efficiency, so this is not possible right. Therefore, this device is an impossible device if some inventor claims that this device is working, then that claim is not valid ok. We will move on to the next problem. Let me erase the board before, we look into the problem.

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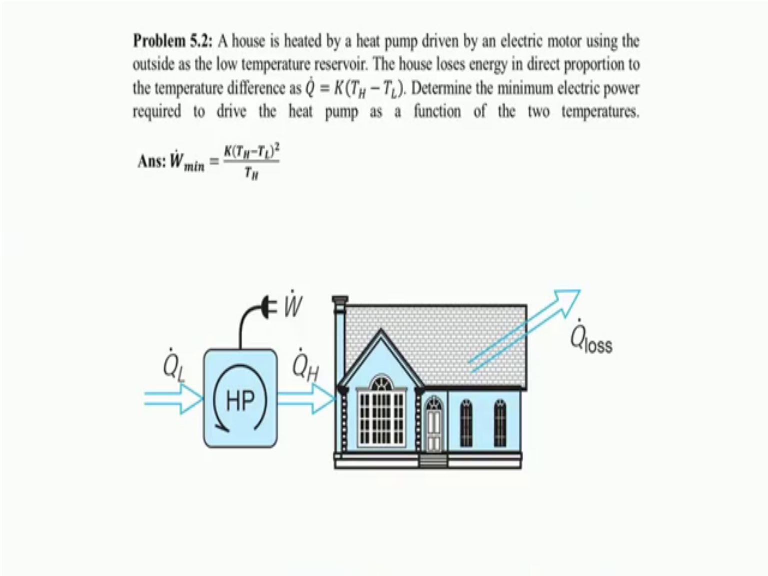

A house is heated by a heat pump driven by an electric motor outside as the low temperature using the outside a low temperature reservoir. So, this is typically a cold country based problem. So, you have a house, you can see the nice picture of the house in the diagram. So, this house is there in a very cold ambient. So, to keep this house hot, there is a heat pump which continuously drives heat into the house, this is normally what a do meter will do.

So, the house loses energy in direct proportional to the temperature difference between the house and the surroundings. So, Q dot is K into T H minus T L. What is the minimum electric power required to drive the heat pump as a function of the two temperatures. So, let us go to the board and solve this problem.

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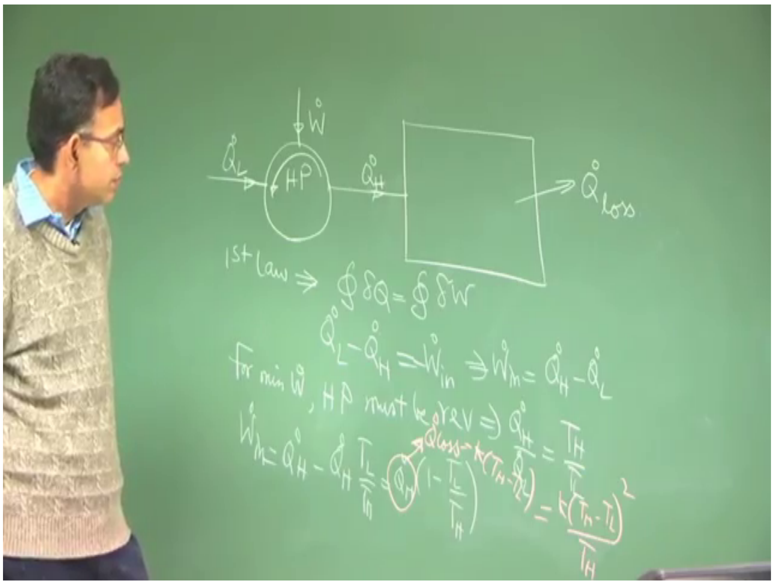

So, I am not a very good artist, so I will draw the house with a block not the nice picture that is given in the slide. So, this has Q dot H, this is the heat pump. So, now first of all, so it should satisfy two things first law and second law. So, for the heat pump the first law implies that cyclic integral of heat equal to cyclic integral of work ok. So, Q dot L minus Q dot H that is the net heat transfer to it and the net work done is minus W dot in ok. So, W dot in which you want to minimise to make your heat pump most efficient, so this is equal to Q dot H minus Q dot L.

Now, for minimum work input this must be a reversible heat pump that means, Q dot H by Q dot L is equal to T H by T L. So, W dot in is equal to Q dot H minus Q dot L into, so Q dot L is Q dot H into T L by T H. And for the house to remain in steady state, the heat that is supplied is same as the heat that is dissipated. So, Q dot H is same as Q dot loss, and it is given that Q dot loss is equal to K into T H minus T L. So, this becomes K into T H minus T L whole square by T H right. This is the minimum work input ok.

Student: (Refer Time: 09:03) cycle we have explained that the work output which is maximized if it is reversible (Refer Time: 09:11).

It is true that in the context of Carnot cycle, we have you know discussed that the work output is maximum. In the context of reversible heat pump, the same concept is true, but you know you have to use your common sense to understand that for a given Q H, what is that input that you require for a heat pump, the input is work. So, its coefficient of performance is the desired effect that is Q H divided by the work input.

So, the reversible heat pump will have the based performance that is its coefficient of performance will be the highest. So, to have the highest coefficient of performance to have the same Q dot H W dot in should be minimum. So, its a common sense extension of a reversible heat engine that we can apply to a reversible heat pump or a refrigerator. For a reversible heat engine, the efficiency is maximum for a reversible heat pump or a refrigerator the coefficient of performance is maximum the so in both cases in either cases, it is the performance parameter or performance index ok.

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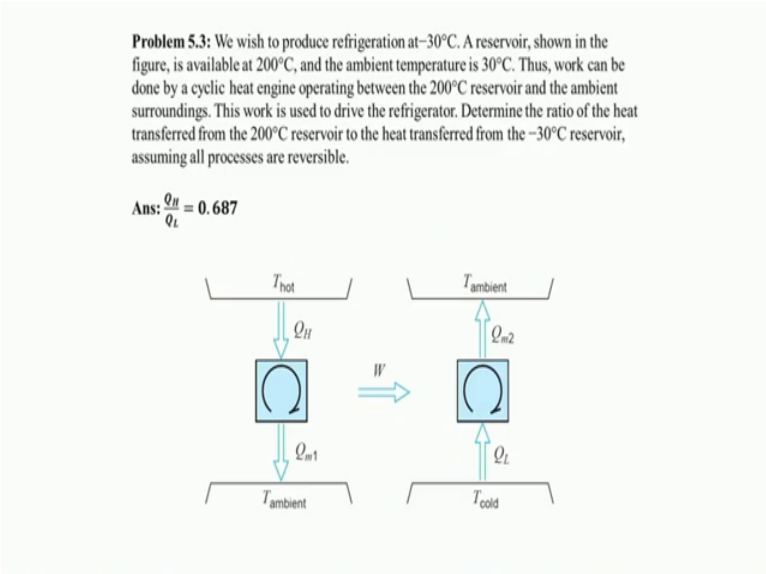

Let us go to problem number 5.3. We wish to produce refrigeration at minus 30 degree centigrade ok. A reservoir shown in the figure is available at 200 degree centigrade and the ambient temperature is 30 degree centigrade. Thus, work can be done by a cyclic heat engine operating between the 200 degree centigrade reservoir and the ambient surroundings. This work is used to drive the refrigerator. Determine the ratio of the heat transferred from the 200 degree centigrade reservoir to the heat transfer from the minus 30 degree centigrade reservoir, assuming all processes to be reversible.

So, here you can see that in the diagram there is a heat engine and some work of the heat engine is used to run a refrigerator. The whole understanding of the problem is that there are many numerical values of temperatures given, which ones corresponds to what. So, here we have T hot, T ambient, T cold. So, these are the things that we will sort out in the board.

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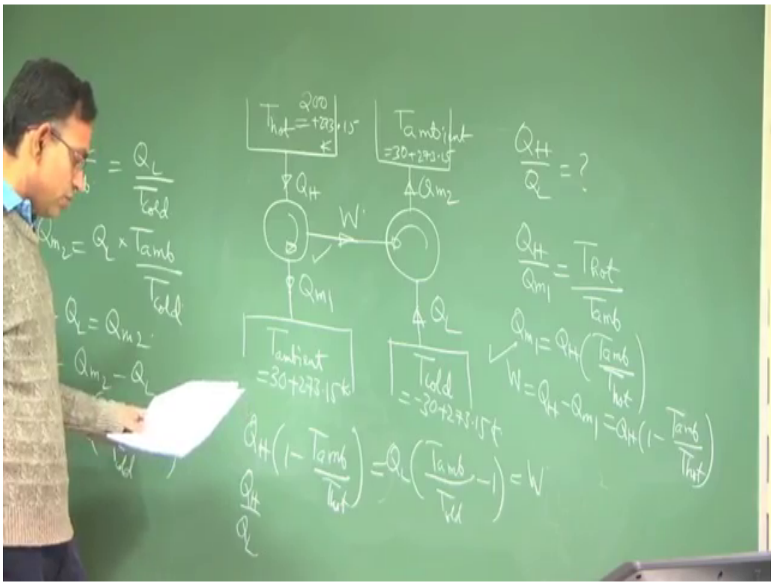

So, let us draw the schematic, so T hot ok. So, let us look into the given conditions. So, let us complete this is W ok. So, the question is what is Q H by Q L that is the question, so we have to first understand, what is the question? Then T hot is 200 degree centigrade. So, 200 plus 273.15 Kelvin ok. T ambient is 30 degree centigrade, so it is 30 plus 273.15 Kelvin. Then T cold is minus 30, so minus 30 plus 273.15 Kelvin. And this is T ambient which is same as 30 plus 273.15.

So, you have both are reversible right. So, you have Q H by Q m 1 is equal to T hot by T ambient. So, you can write Q m 1 is equal to Q H into T ambient by T hot right. So, W is equal to Q H minus Q m 1 that is cyclic integral of heat equal to cyclic integral of work ok. So, for this is becomes Q H into 1 minus T ambient by T hot.

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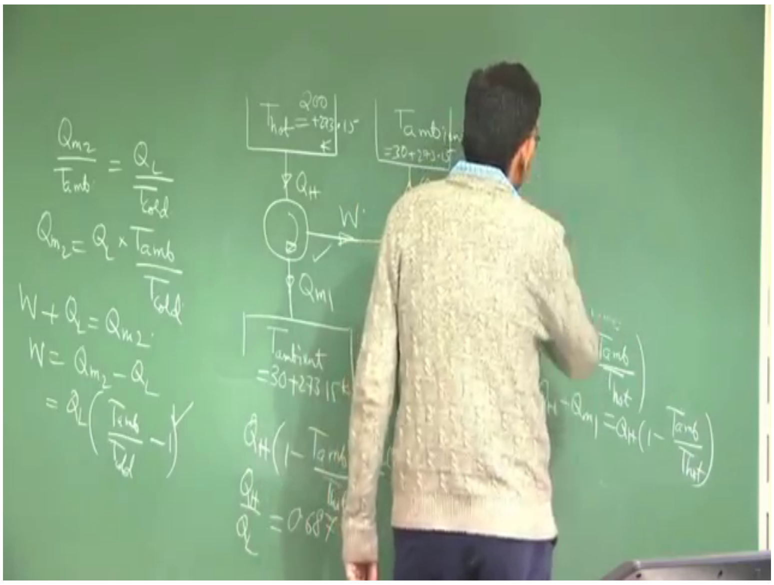

Look into the heat pump. So, for the heat pump you can write Q m 2 by T ambient is equal to Q L by T cold. So, you can write Q m 2 is equal to Q L into T ambient by T cold ok. So, and energy balance says that for the heat pump W plus Q L is equal to Q m 2.

So, W is equal to Q m 2 minus Q L. So, this is Q L into T ambient by T cold minus 1 right. So, this expression for W and this expression for W, they are the same W right, so that means, you can write Q H into 1 minus T ambient by T hot is same as Q L minus 1 each is equal to W. So, from this you can find out what is Q H by Q L, all the temperatures are known in Kelvin. So, Q H by Q L is 0.687 ok. So, we will work out a couple of more problems, let me erase the board.

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Problem 5.4; consider a Carnot-cycle heat engine operating in outer space. Heat can be rejected from this engine only by thermal radiation, which is proportional to the radiated area and fourth power of the absolute temperature. So, this is related to the Stefan Boltzmann's law of radiation. So, Q dot radiation is some constant K into A into T to the power 4, where A is the surface area at over which the heat transfer is taking place, so that for a given engine work output an a given T H, the radiator area will be minimum when T L by T H is 3 by 4.

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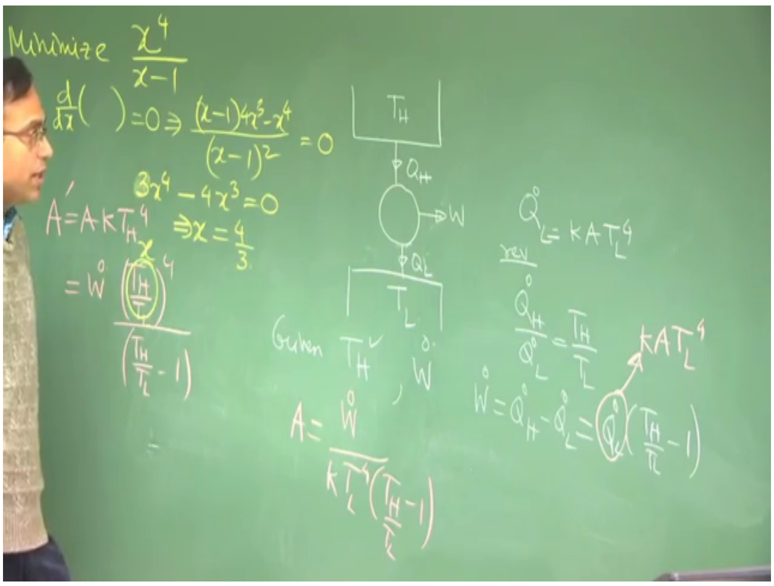

So, let us try to see what are the given data and I will try to draw a schematic of this problem. So, what is given is Q dot L is K A T L to the power 4, this is what is given. Also given you have a given T H and given W dot. So, what you have is Q, because it is reversible, you have Q dot H by Q dot L is equal to T H by T L right.

And w dots is Q dot H minus Q dot L, here everything is expressed in terms of rate of heat transfer, because you know the radiation is also expressed as the rate equation. So, Q dot H minus Q dot L, so you can express it in terms of Q dot L. So, Q dot H is Q dot L into T H by T L minus 1. And Q dot L is given as K A T L to the power K A T L to the power 4 ok.

So, A is W dot by K T L to the power 4 into T H by T L minus 1 sorry yes A is equal to W dot by K A T L to the power 4 T H by yes this is all right yes. Now, because T H is given you can you know have a different parameter which you want to minimise which is A into K T H to the power 4, the reason is that in that case we will have W dot by W dot into T H by T L to the power 4 by T H by T L minus 1.

So, if you consider T H by T L is equal to x, then the problem is as good as minimize x to the power 4 by x minus 1 right x to the power 4 by x minus 1 minimize area means minimize this, so that means d dx of this is equal to 0, so x minus 1 whole square into x minus 1 into this 4 x cube minus x 4 is equal to 0 right.

So, you have 4 x to the power 4 minus x to the power 4. So, 3 x to the power 4 minus 4 x cube is equal to 0 that means, because x is not equal to 0. So, x is equal to 4 by 3 ok. But, you also have to check that the second derivative of this is less than 0 by greater than 0, because it is a minima.

So, second derivative of this has to be greater than 0, so that please check I am not doing that trivial algebra here, but just check that if you make a second derivative of this with x equal to 4 by 3, it indeed becomes positive that means, it is indeed a minimum of the area are not, maximum of the area. Why minimum of the area is important, because for making a compact engineering design to achieve a particular heat transfer, you want the minimum area to achieve that heat transfer. Otherwise, if you increase the area anyway, heat transfer we will increase. So, the good engineering design is that you achieve the same heat transfer, but using less area that is why minimising the area is a matter of concern for this problem.

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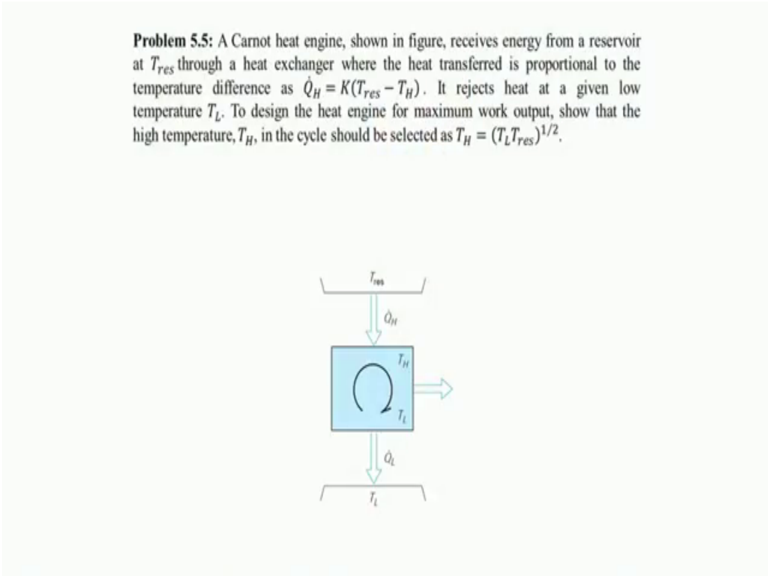

So, we will work out a final problem, before we conclude this lecture, so that is problem number 5.5. A Carnot heat engine, receives energy from a reservoir at T reservoir through a heat exchanger, where heat is transferred proportional to K in to T reservoir minus T H. It rejects heat at a given low temperature T L. To design the heat engine maximum work for maximum work output, show that the high temperature T H is square root of T L into T reservoir ok.

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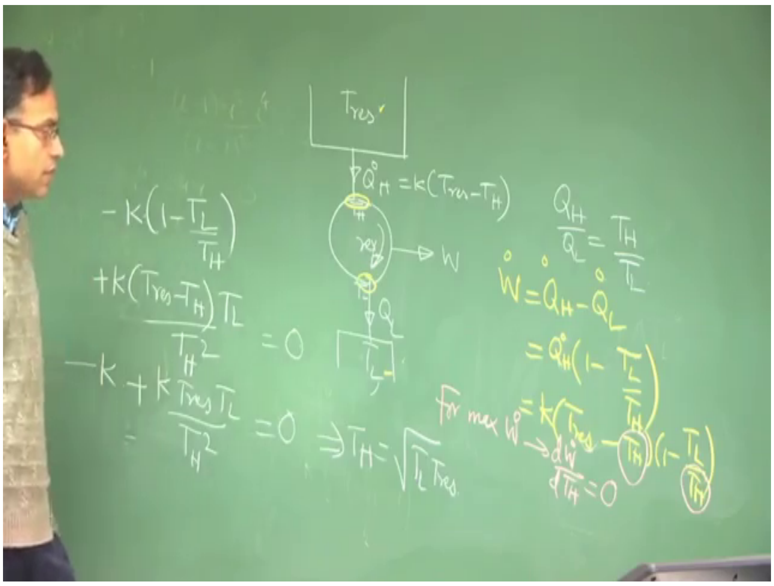

So, let us work out this problem. And this is a very interesting problem and I will explain why it is so interesting from a conceptual point of view. So, this is a this is claim to be a reversible heat engine, now you may argue that here heat transfer is taking place across a finite temperature difference right. T reservoir is not same as T H, this is the difference between this and the Carnot cycle which we discussed in the theory portion of the lecture that Carnot cycle was considered to be both internally and externally reversible Carnot cycle.

Now, this is the typical Carnot cycle, where the external universe external reversibility is sacrificed, because you know that is the more practical approach where you have a temperature difference across, which the heat transfer is taking place, but internally it is a reversible cycle. So, this is called as an internally reversible Carnot cycle or endo reversible Carnot cycle.

So, still for this cycle you can write Q H by Q L is equal to T H by T L. Remember when you write Q H by Q L is equal to T H by T L that T H and T L are not this T H and T L anymore for endoreversible cycle, it is the T H and T L across which the heat transfer is taking place in the system boundary, this is the adaptation that you have to make, if the cycle is not externally reversible.

So, you can use because you know this is Q H by Q L equal to T H by T L, so long as everything within this domain of T H and T L is reversible. And here the domain of T H and T L is brought down from reservoir to this much. So, within this two, it is reversible. And therefore, you can write Q H by Q L equal to T H by T L. But, if you write Q H by Q L equal to T reservoir by T L that will be wrong, because it is externally reversible. So, this is a very very important concept that you have to keep in mind.

So, once you write this the net work done W is Q H minus Q L, so W dot is Q dot H minus Q dot L and Q Q dot L is Q dot H into T L by T H. So, Q dot H into 1 minus T L by T H. And Q dot H is equal to K into T reservoir minus T H into 1 minus T L by T H ok. So, then the only variable here is T H right. And what you want for maximum work, so you want maximum work output.

So, for maximum work output d W dot d T H should be equal to 0 ok. So, d W dot d T H should be equal to 0 that means, let us just differentiate this. So, K into 1 minus T L, so minus K into 1 minus T L by T H plus K into T reservoir minus T H into T T L by T H square right. 1 by x will become minus 1 by x square, so this is equal to 0. So, minus K plus K T reservoir T L by T H square, so that becomes equal to so the remaining terms get cancelled out right, so, this is equal to 0.

So, your final answer is T H is square root of T L into T reservoir. Again you can check with the second derivative, if it is less than 0, then only this represents the maximum. So, we have discussed about various problems, where the processors are either reversible, both externally and internally or we have discussed about processes which are internally reversible, but externally irreversible.

The next question that comes to us is that well we can understand that irreversible is irreversible that means, it is deviated from ideality or reversibility. But, if you have two irreversible processes, both are deviated from reversible, but how much they are deviated from reversible process, so that means the deviation from reversibility needs to be quantified. So, in the next lecture, we will discuss about a quantitative parameter that will give us the strength of irreversibility that means, deviation from the extent of deviation from an ideal or reversible process.

Thank you very much.