Concepts of Thermodynamics

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Lecture - 21
Supplementary Lecture: Problem Solving with the Aid of a Computer

Hello and welcome to this session of Problem Solving with Aid of a Computer and I hope by now you must be really familiar on how to use a computer to solve thermodynamics problems because in this problem we will tackle the problem in which there are two springs as shown in the figure.

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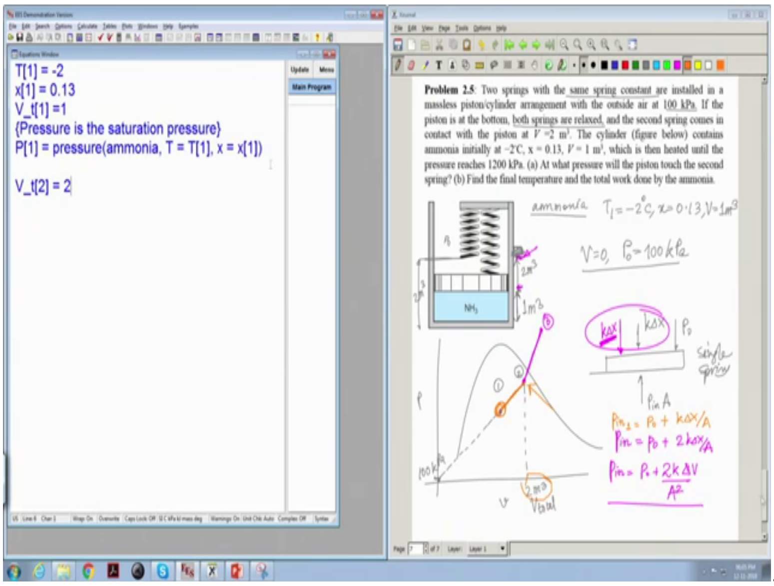

Two springs of the same spring constraint are installed in a massless piston cylinder arrangement with the outside air at 100 kilo Pascal.

If the piston is at the bottom, both the springs are relaxed and the 2nd spring comes into contact when the volume is equal to 2 meter cube. So, the 2nd piston comes into contact when the volume is 2 meter cube, ok no problem till then it is not engaged. A cylinder contains ammonia a very useful industrial gas invented as synthesized during the first world war by Fritz Haber, the famous Haber Bosch process.

So, ammonia is at T1 equal to minus 2 degree Celsius x equal to 0.13 and initial volume equal to 1 meter cube. So, this initial volume is 1 meter cube no problem. So, here we are showing these disks and all of thickness, but it is you can can take this actually this is 1 meter cube and then when this expands and when the piston is like this, you reach the 2 meter cube, ok. So, it is then when the 2nd spring also comes into contact with the piston no problem.

So, when the piston is at the bottom, when V is equal to 0, the spring is sort of relaxed. There is no, there is no V x by the spring, ok. When the piston is at the bottom the only pressure that acts is the 100 kilopascal. Why this information needed because we do not have a reference state to find out how the spring is, otherwise see the initial state is given somewhere at minus 2 degree Celsius.

And x equal to 0.13 is state 1. I do not know what the slope is, I dont know what the slope is because I do not have the spring constant, ok. It can be anything. So, what determines the slope is the back reference state because at P equal to at V equal to 0. This pressure becomes 100 kilopascal. So, this actually tells me the slope if I know the pressure at the first point. That is ok.

So, what is going on what is going on it is heated now and the volume will rise. How long will the volume rise up? Until it reaches the 2nd spring, then at the 2nd spring something will happen. What will happen? See if this is the p internal times area if there is only one spring is k delta x and that is p atmosphere for single spring, it is given that the same spring constant is there for the other spring as well.

So, the moment the volume expands to 2 meter cube, so this point should correspond to a total volume of 2 meter cube. The moment we reach 0.2 which is 2 meter cube, the 2nd spring comes into picture and thus we now have an additional k delta x takes which is already it was relaxed, spring was relaxed. So, now you have an additional compression in this direction and thus the P internal. So, P internal before was P naught plus k delta x by a, but after the two springs come into contact if there is a small change in volume, then both the springs will oppose that motion.

So, then you have extra force so, P naught plus twice k delta x by a. So, the slope of the line the slope of the line has become twice because I can write P internal is equal to P naught plus 2 k delta V by a square this delta V is obviously measured from the point where they hit the springs. So, from this point onwards it will be at a increased slope with something like this. This is nothing, but to say but the effective spring constant has become twice,. So, beyond this point this equation will be valid. I hope this is clear till this point this is the process.

So, let us do some calculations and find out. So, we have been given T1 is equal to minus 2 x 1 is equal to 0.13 and we have V total 1 is equal to 1. So, with the value of the temperature and the quality we can find out the initial pressure. Can you tell me what this pressure should be? This pressure should be identical to the saturation pressure because it is inside the dome, alright.

So, P1 is known V1 is known. So, this point is known. What about 0.2 ? So, we know that at point to the total volume becomes 2 meter cube because 0.2 I have defined as the point where the volume reaches 2 meter cubes in total and this point V t 2 is equal to 2. So, now can I find out the P2? Yes I can because the slope of this line is known.

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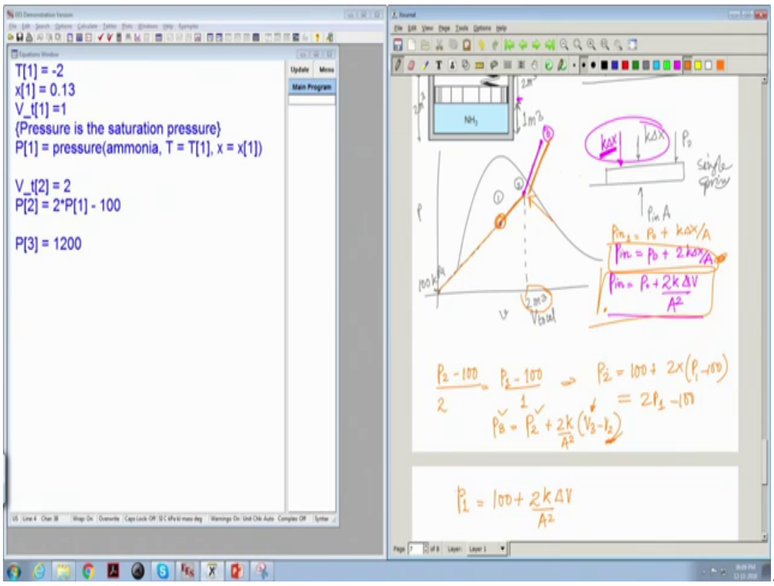

So, P 2 minus 100 by 2 is equal to P1 minus 100 by 1. So, thus we obtain P2 equal to 100 plus twice P1 minus 100. So, this is equal to 2 P1 minus 100. So, let us put it down P2 equal to 2 into P1 minus 100. So, thus we know what P 2 is, alright.

Now, what about state points 2 2 3, ok? What about this? What about this process? What what have what is the information that we have? So, the first question was at what pressure will the piston touch the 2nd spring. So, this is the pressure at which the piston will touch the 2nd spring and it is given that it is heated until the pressure reaches 1200 kilopascal. So, P3 is known. P3 is 1200 kilo Pascal, ok.

So, from points 2 to 3 this is the equation that we have. Essentially when we write this down in terms of the actual process parameters P3 should be equal to P2 plus 2 k by a square V3 minus V2 ok, but how do we know see I mean essentially this is the thing that we need to know. You need to know what V3 is, we know what V2 is, we know what P3 is. It is given. We know what P2 is. We do not we just need to find out what V2 is. For that we need to know the spring constraint. Can we find out the spring constant? Actually we can because when the spring is being compressed in the initial plot when in the initial process, so if you write this equation down over here essentially says P1 equal to 100 plus 2 k delta V by a square. Do we have the area of the piston? No we do not have the area of the piston, ok. No problem ok, but anyway we do not need to do all this. We can simply do the forming you know that the slope is twice and.

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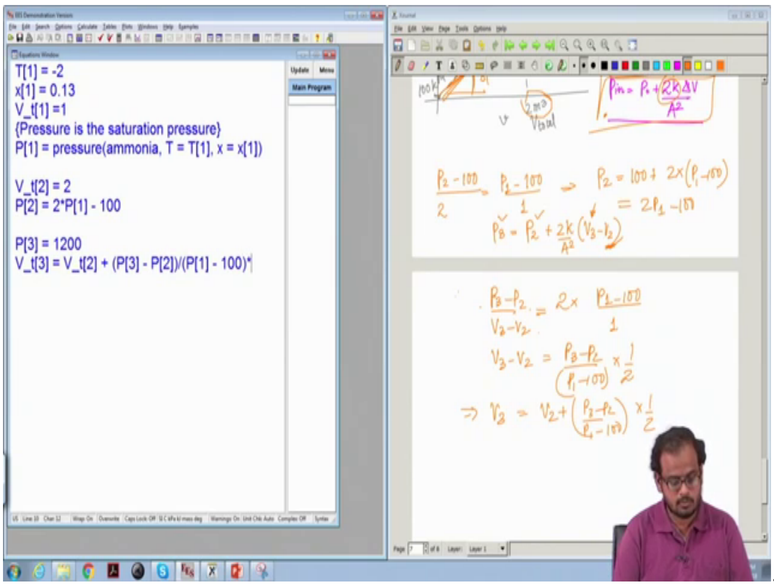

So, P3 minus P3 by V3 minus V2 has twice the slope as this particular line segment. So, this particular line segment has a slope which is twice this slope, ok. So, if I call this slope 2, this is slope 1, slope 2 is equal to twice slope 1 as seen from this pre-factor. So, if you do this we obtain this is equal to twice P1 minus 100 by V1 minus 0. So, V1 was 1 and with the help of this we know V3 minus V2 is equal to P3 minus P2 by P1 minus 100 times 1 by 2 which implies V3 that we actually want is equal to V2 plus V3 minus

P2 by P1 minus 100 times half and alternately the specific volume see the mass is only known. So, the specific volume cannot be used. So, this is the way you have to find V3. So, let us find out V3.

So, what is V total at 3? It is 2.844 litre meter cube. So, this was 2 meter cube, this was 2 meter cube and this final specific volume should correspond to something which is 2.844 meter cube should I mean it is not a specific volume, but we can divide by the mass.

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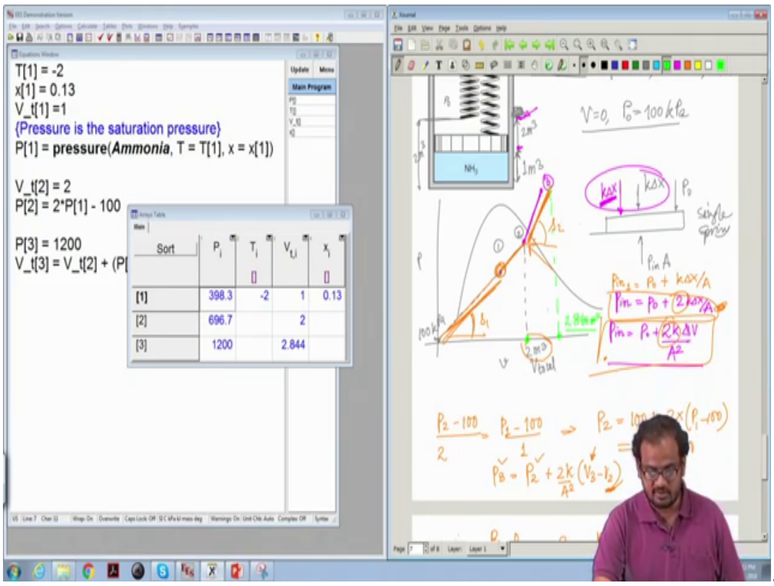

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So, what is asked let us see what is asked. We have you know the temperature ok, the temperature we can find out. So, for to find out the temperature we need to use two independent properties one of which is the pressure.

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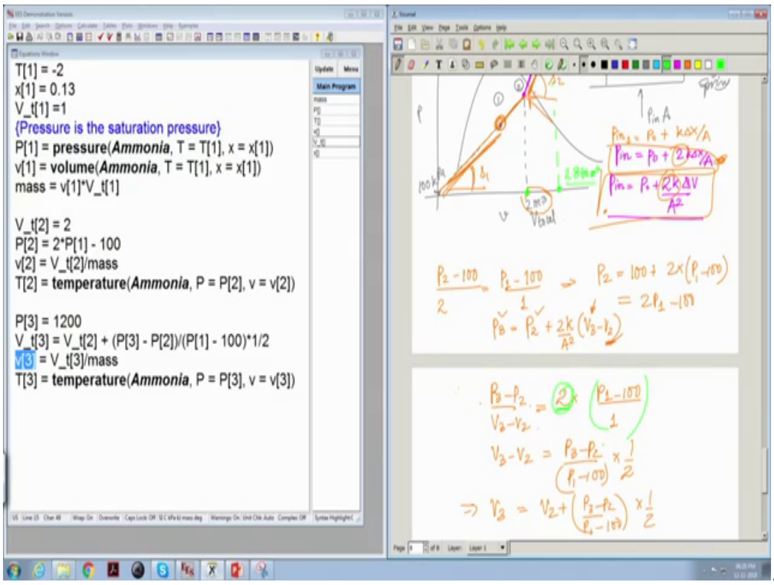

And the other we can find out by using the total volume and the mass. So, the mass of the system is the mass of ammonia taken is. So, let us find out first the mass of the total ammonia. So, the specific volume at 1 is equal to volume ammonia at T equal to T 1 and x equal to x 1. So, the mass is equal to the specific volume multiplied by the total volume.

So, once the mass is known we can find the specific volume at 0.3 which is equal to the total volume by the mass for completeness. Well also find out the specific volume of 0.2, this is equal to specific volume of 0.1 divided by 2 into 2 because the volume has doubled. If the volume is doubled, then we know the specific volume has also doubled because the mass is constant ok. Nothing sacrosanct about this step.

Temperatures, let us find out the temperatures T2 equal to temperature of ammonia at P equal to P2 and V equal to V2. Similarly t3 will be the temperature ammonia P equal to P3 v equal to v 3. The matter of fact let us not define v 2 like this let us do it the classical way which is the total volume at 2 divided by the mass and it does not make any difference. So, now we have we have the volume on the specific volume at 2 and the specific volume at 0.3 and v 3 we have estimated using this equation. So, v 3 equal to v 2 plus this because we we made use of the fact that slope is twice.

So, P2 minus P2 over here by P1 minus 100 the the P3 minus P2 by v 3 minus v 2 is twice that of this particular slope, ok. So, let us see what the temperature is and all we obtain, ok.

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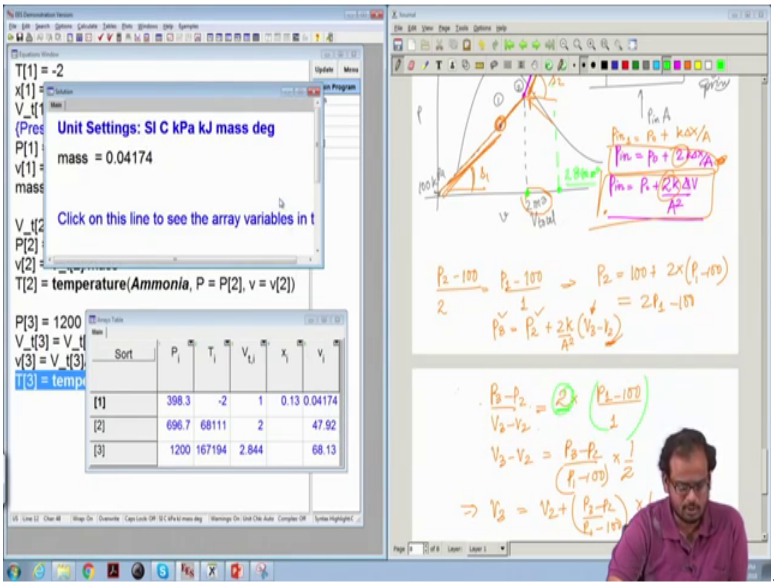

This seems to be a small error, but let us see how we can debug this. So, the pressure at 0.2 is 696.7 degree degree Celsius ah. The pressure is 696.7 kilopascal, but the temperature is showing something very absurd, ok. Let us let us open up the property plot and see what is going on for ammonia the P v plot let us draw which I minus 2 degree Celsius, ok.

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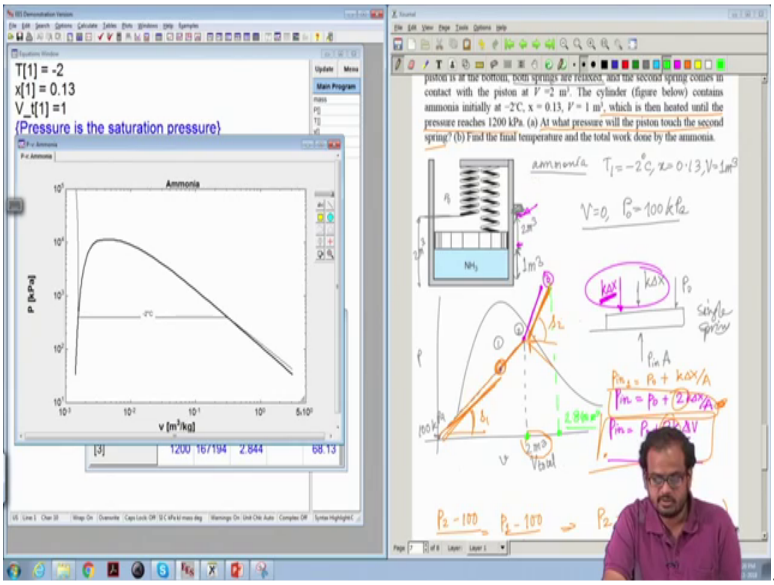

So, this is minus 2 degrees celsius and let us draw the initial point. So, what is the initial point on the x axis? We have specific volume on on the y axis, we have the pressure. So, this is this is the initial point. The initial point is running way far off. So, there must be something wrong with this slope. So, let us see if there is something wrong with the slope. This is how this is how you have to debug the problem. So, let us see what is wrong.

So, we have obtained the mass let us see what is wrong, what is the specific. So, let us see state 1 makes sense. State 1 makes sense. Yes 0.13 and the mass is also equal to 0.04174 because that is the specific volume equal to the total volume divided by the specific volume. So, the mass is equal to this,.

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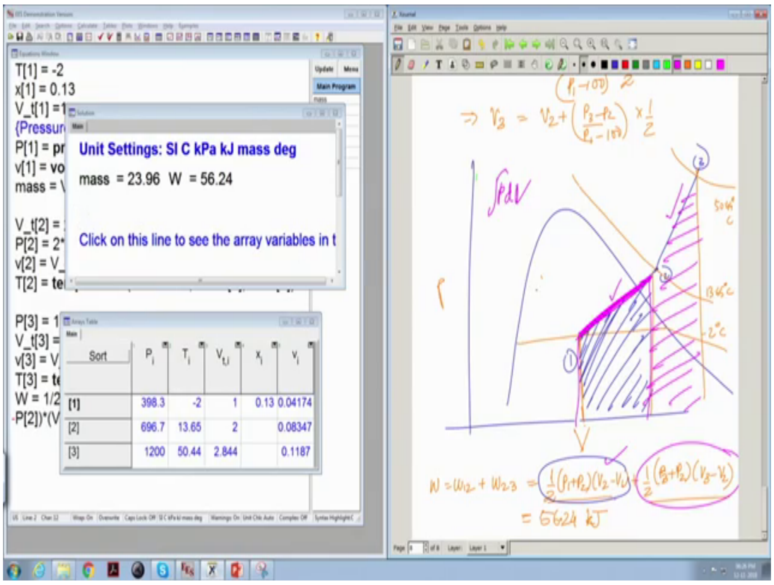

So, let us see alright there we go. So, mass is equal to 23.96 kg. So, we have we had missed up a small that is a very silly mistake, were a very innocuous mistake, but anyway it is good to see that even while we do this, we can make a small mistake, but you should take you should not perform this kind of mistakes and you should take motivation from this, ok.

So, what is the final temperature that we obtain? The final temperature is 50.44 degree Celsius ok. So, the final temperature is 50.44. Let us draw the schematic.

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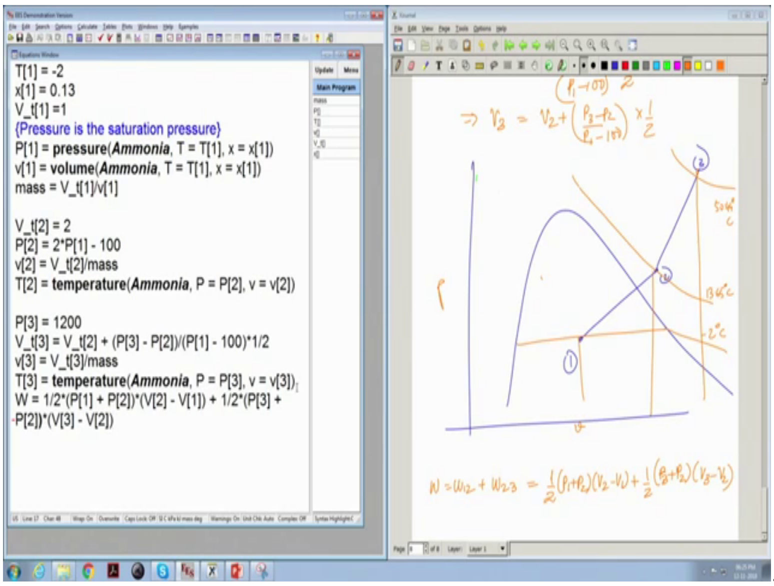
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So, we have something like this going to this and and with a higher slope 123 and this is minus 2 degree Celsius. Isotherms for ammonia 0.2 was maybe like this. This is 13.65 degree Celsius and 0.3 is 50.44 degree Celsius. So, these are the three points and let us do the plot let us do the actual plot let us do property plot ammonia on the P V and let us have two more isotherms 13.65 and 50.44, ok.

So, with this let us draw the overlay on the x axis. We have the volume and on the y axis we have the pressure. There you go. It is a very nice looking plot. Just as we expected it has a slope like this and then has a double slope. So, what is the work done? What is the work done? During the process now the last question is find the final temperature which is 50.44 and we have to find the total work done. What is the total work done? So, total work done will be work done from 1 to 2 plus work done from 2 to 3 which is equal to half P1 plus P2 times V2 minus V1 plus half P3 plus P2 times V3 minus V2, ok. So, let us write it down in the computer.

So, the work done is equal to 56.24 kilo joule. So, I hope it is clear how it is some of these two trapezoids because work. So, this is specific volume, but anyway it is the volume enclosed by the curve in the P n total volume plot, ok. I might as well have drawn the schematic in the total volume because the mass is 1. So, it is this area. So, this is the area of the trapezoid that I have just hatched and this particular area and this is that particular area. So, these are two trapezoids because when you have a linear function in P, the area that it sweeps is a trapezoid. This is one trapezoid. So, area under the curve is the P dv, ok. Integral P dv is the area under the P V plot.

So, PV is going like this essentially the area below this which is a trapezoid and hence, it is this formula. It is very easy and this has been covered in the theory class as well. So, this area of trapezoid plus this area trapezoid gives you the total work done. This is 56.24. So, in this particular problem we have dealt with a problem in which there are two springs. So, in the initial times there is one delta VP and delta V relationship and beyond a certain point when two springs attached, there is another P delta V relationship. So, we have made use of one delta P relationship, but not this one, but this one initial time and the final time relationship.

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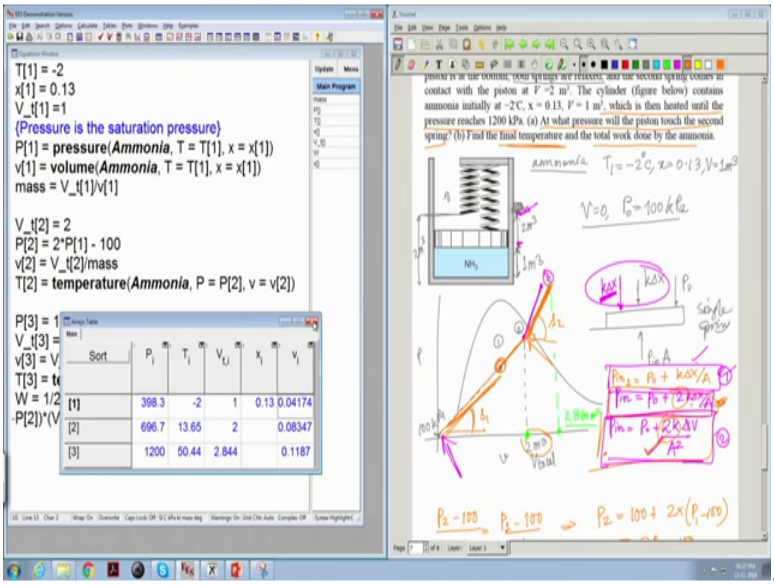

So, from process 1 to 2 the pressure varies as this and the reference state was this particular 0 point. Once that slope is known, the slope from 0.2 to 3 is known to be twice this because you are using the fact that now you have two springs. The fact that you have two springs increases the spring constant. So, you have twice the slope and because of that you you can now refer directly that the slope from 0.2 to 3 is twice the slope from 0.2 to 1. So, with this and we have seen how the actual plot looks like. You should revisit this problem. You should do this problem again.

Because it it involves slightly more concept than the previous problems, but nothing out of the extraordinary. You should be able to justify what you are doing by clearly understanding what the process is. It goes from 0.1 to 2 and then, 2 to 3 where there are two springs 1 to 2 only one spring for the single spring. You had a reference reference point at 0 where the spring was having no displacement with that slope. You could do all the calculations in this problem ok. So, with this note and with this slightly tricky problem, we finish here and I will see you next time.

Thank you.