Heat Exchangers: Fundamentals and Design Analysis

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Lecture - 09

Design and Simulation of Heat Exchangers – Numerical Problem (Contd.)

Welcome to this lecture on the Design and Simulation of Heat exchangers.

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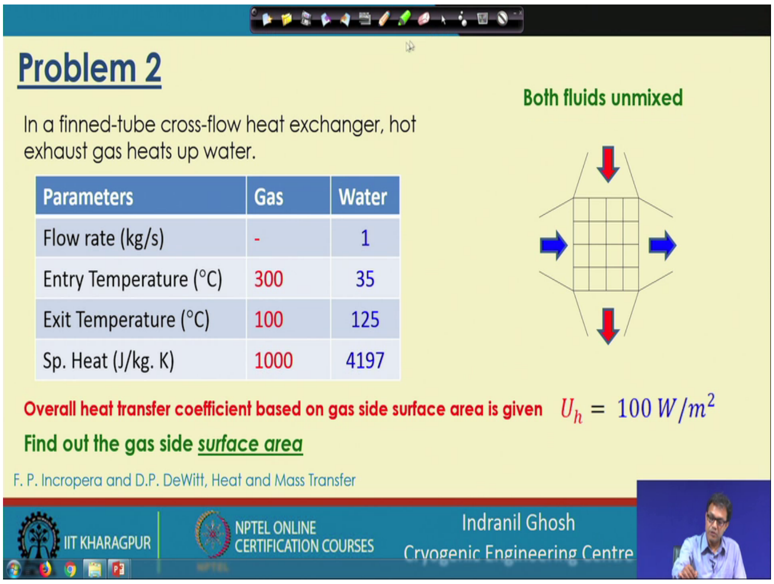

Problem 2

In a finned-tube cross-flow heat exchanger, hot exhaust gas heats up water.

Parameters	Gas	Water
Flow rate (kg/s)	-	1
Entry Temperature (°C)	300	35
Exit Temperature (°C)	100	125
Sp. Heat (J/kg. K)	1000	4197

Both fluids unmixed

Overall heat transfer coefficient based on gas side surface area is given $U_h = 100\ W/m^2$

Find out the gas side surface area

F. P. Incropera and D.P. DeWitt, Heat and Mass Transfer

In this lecture, we will try to solve some numerical problem. In the last class we have solved some numerical problem and we will try to solve the same numerical problem with a different approach. In this problem it is a cross flow heat exchanger where the hot exhaust gas is being cooled by the water.

So, the parameters are already known to us; we know the flow rate entry and exit temperature of the gas and the water stream. And since the inlet and exit temperatures are known we also know the fluid properties and that remain constant it is not changing with the length of the heat exchanger.

It is also given the overall heat transfer coefficient based on the heat transfer surface area and that is equals to U h 100 Watt per meter square. And what we need to find out is the hot side or the gas side surface area. Now before this is in a both the fluids are unmixed. So, before going into the details of this problem or trying to solve this problem. We just want to mention what is made by both fluids unmixed because this will time and again come that some of the fluids are makes some of the fluids are unmixed. So, we just want to clarify this one before we go to the next line I mean solution.

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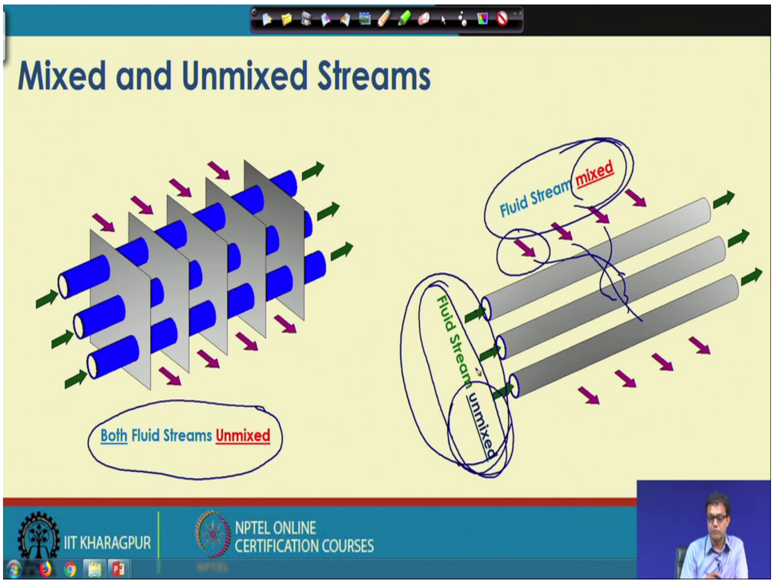

Mixed and Unmixed Streams

Both Fluid Streams Unmixed

Fluid Stream mixed

Fluid Stream unmixed

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So, in the mixed an unmixed stream fluid we find that in this configuration we are saying that both the fluid streams are unmixed. Here we have a finned tube of heat exchanger where the tube is having this flow. I mean the tubes are separated, I mean this fluid this is this fluid stream is coming from a particular pipe getting distributed to different sides. So, this is not mixing with this fluid or this fluid is not mixing with this fluid. So, they are separated by these fins.

Now, for this fluid stream this is also coming from some other pipe and getting distributed to different individual pipes. So, this fluid is not able to mix with this fluid inside this tube; so both the fluids remain unmixed. So, now in contrast to this if we consider this configuration it will find that this fluid stream; remain unmixed because they are not mixing with each other, they are not able to mix with each other whereas, this we have now removed the fins.

So, there is possibility that this fluid stream after passing through this one there is a possibility that they will be mixing up. So, this fluid stream is mixed and this fluid rain is unmixed. So, depending on the situation we may have both the fluid streams are mixed; some fluid stream mixed and some fluid stream unmixed and accordingly the epsilon

NTU relation or the NTU epsilon relation will change and that you have to keep it in mind.

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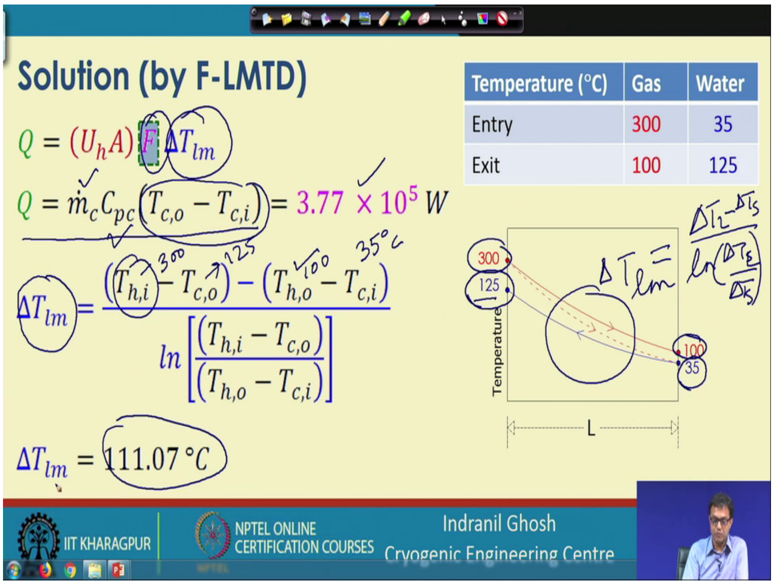

Solution (by F-LMTD)

$Q = (U_h A) F \Delta T_{lm}$

$Q = \dot{m}_c C_{pc} (T_{c,o} - T_{c,i}) = 3.77 \times 10^5 \, W$

$$\Delta T_{lm} = \frac{(\overset{300}{T_{h,i}} - \overset{125}{T_{c,o}}) - (\overset{100}{T_{h,o}} - \overset{35^\circ C}{T_{c,i}})}{ln\left[\frac{(T_{h,i} - T_{c,o})}{(T_{h,o} - T_{c,i})}\right]}$$

$\Delta T_{lm} = 111.07\,°C$

Temperature (°C)	Gas	Water
Entry	300	35
Exit	100	125

$\Delta T_{lm} = \frac{\Delta T_L - \Delta T_S}{ln\left(\frac{\Delta T_L}{\Delta T_S}\right)}$

So, this problem talks about the both fluids unmixed condition. Now, in this approach earlier we for the solution of that problem where both the cross flow with heat exchanger with both fluid stream unmixed, we tried to solve by some method. And now we are trying to solve it by F-LMTD relation because this since this is the cross flow heat exchanger, we know that we can correlate this heat exchanger, heat transfer by overall heat transfer coefficient and the area product multiplied by the fact F factor and the delta T log min temperature. So, in this case what is known to us both the entry and exit temperatures.

So, if we look at this temperature profile we will find it is a cross flow exchanger. So, the temperature profiles are not actually known to us, but we assume it to be as if this is a counter current exchanger, where the hot fluid stream is entering from this end and moving out at 100 degree centigrade; whereas the cold fluid is entering at 35 degree and moving out at 125 degree centigrade.

And this been a counter current exchanger, we would have easily determined the log min temperature difference by say delta T of the larger one minus delta T of the smaller divided by log min of delta T; large by delta T small. This we have already learnt and here in this case actually we know all this temperature exit temperature, but this is in a counter current configuration; our heat exchanger configuration is basically a cross flow exchanger. So, we cannot directly use delta T a delta T log min; we have to multiply this one with a correction factor A.

So, how to determine this correction factor? Let us see we know already the amount of heat getting transferred because we have been told about the mass flow rate of the cold fluid, we know the C p of the cold fluid and the exit and inlet temperature of the cold fluid stream. So, the total heat is known in this equation. We also know that delta T log min and so that we can easily find out to be 111.07 degree Celsius, this is T h i that is 300 T c o is basically 125; T h o is hot outlet that is equals to 100 and T c i that is equals to 35 degree centigrade. And accordingly this hot inlet is 300 and cold in cold exit is 125; so accordingly if we put this value we will find delta T log min as 111.07 degree centigrade.

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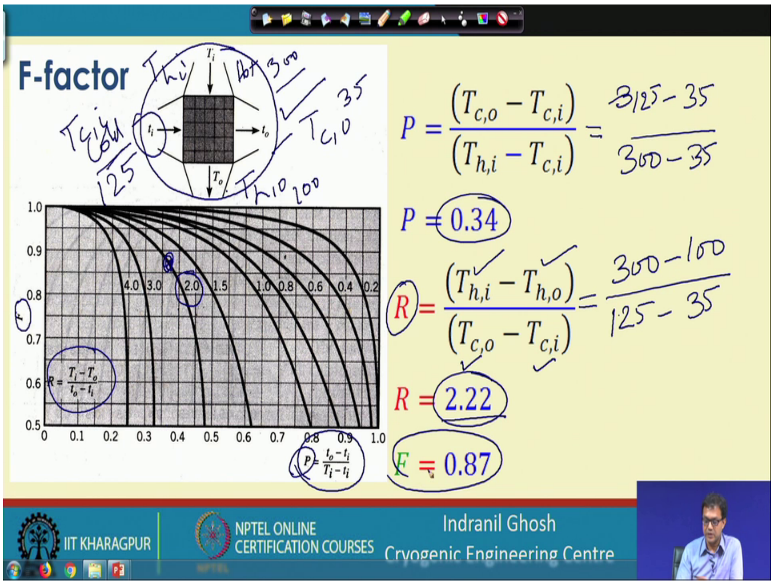

F-factor

$$P = \frac{(T_{c,o} - T_{c,i})}{(T_{h,i} - T_{c,i})} = \frac{125 - 35}{300 - 35}$$

$$P = 0.34$$

$$R = \frac{(T_{h,i} - T_{h,o})}{(T_{c,o} - T_{c,i})} = \frac{300 - 100}{125 - 35}$$

$$R = 2.22$$

$$F = 0.87$$

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So, now if we go look into this calculation of the F factor; we have to look for the F factor or the correction factor for this configuration, where both the fluid streams remain unmixed. So, in our configuration we have been told that both the fluid streams are not mixing with each other.

So, accordingly we have to identify and we find that when both the fluid streams are unmixed condition, this is the configuration of or this is the chart which will tell us the correction factor as a function of R and P; these are the 2 parameters. What is R? R is basically the hot inlet minus T hot exit and this cold exit minus cold inlet. So, basically this is nothing, but the capacity rate ratio that is R and how do we find it out? This is if you look at here this P is T 0 minus T i divided by capital T i minus small T I; this small T I is basically the cold fluid and this is the hot inlet hot exit cold inlet and cold exit.

So, this we have written T 0 as T c i here and cold inlet is written as sorry T c i this is nothing, but T c i and this is T c o and this is T h i minus T h o. So, accordingly we find that the there are 2 parameters; one is R and another parameter is P; P is T c o minus T c i divided by T h inlet minus T c i. So, T cold inlet is 35; sorry T if we look at this value this was 300 coming out to be 100 and this is 35 moving out at 125, if I am correct.

So, this is T c o; T c o is 125 minus, T c i is 35 divided by T hot inlet is 300 minus T c i is 35; so accordingly we will find that P equals to 0.34. Similarly the R value is T hot entry the inlet is 300 minus T hot exit is equals to 100. And then we have T c o that is equal to 125 minus T c i is 35 and we will find it to be R equals to 2.22.

So, now we have to find out for R equals to 2.22; we have for R equals; so these are the R values 0.2, 0.4, 0.6 like that. So, here some had this is for R equals to 2 and for P equals to 0.34; here we have 0.3, this is 0.4 in between we have somewhere. So, we can find that somewhere here our value will be. So, accordingly we find that this F factor is 0.85.

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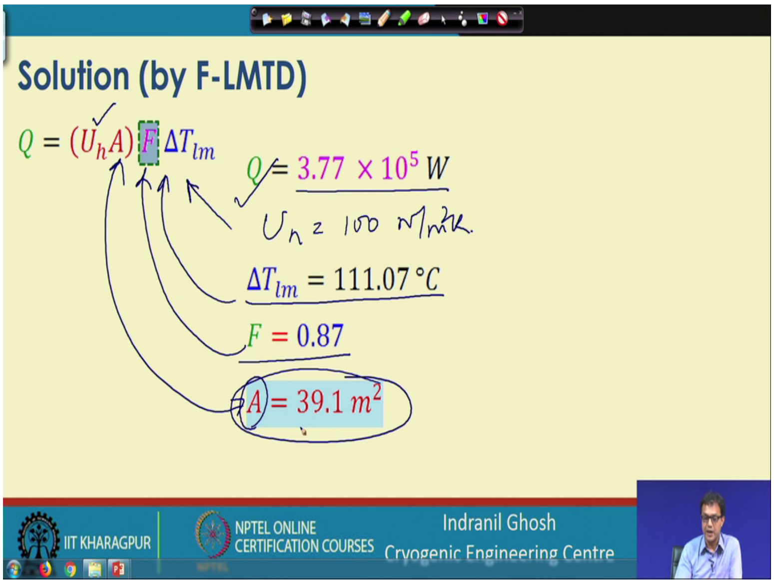

Solution (by F-LMTD)

$Q = (U_h A) F \Delta T_{lm}$

$Q = 3.77 \times 10^5\ W$

$U_h = 100\ W/m^2K$

$\Delta T_{lm} = 111.07\ °C$

$F = 0.87$

$A = 39.1\ m^2$

So, now if we go back to our discussion so, we have now almost all the parameters known; we know the Q the amount of heat getting transferred. We already have an idea about the delta T log min temperature; we have the correction factor A. So, accordingly if we put all these parameters in this equation, we can now calculate the area because already we have this overall heat transfer coefficient also U h equals to 100 Watt per meter square Kelvin that is also given.

So, accordingly we can find out this area to be 39.1 meter square; in our earlier calculation, we got similar almost similar value of 39.1 or 39.7 meter square. And that discrepancy is mainly because of I mean we have estimated the friction; the correction factor. And so there are inaccuracies involved with that one and that inconsistency in this area measurement or area calculation is because of that.

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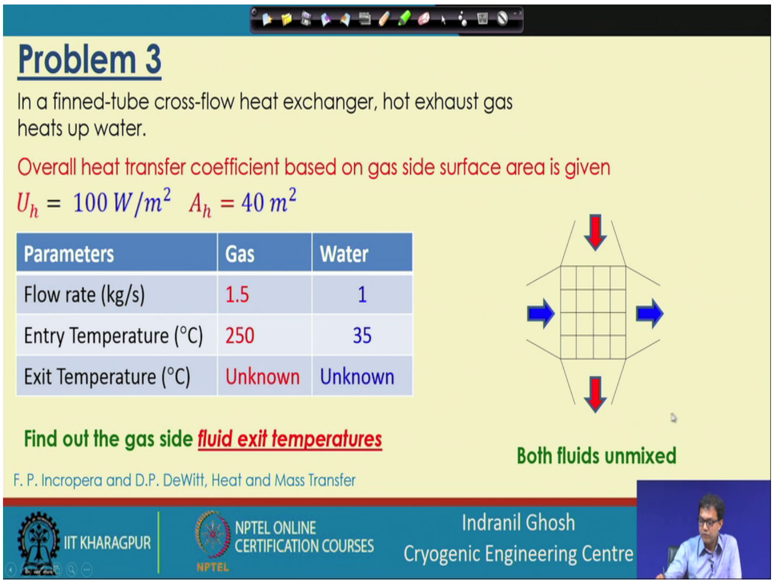

Problem 3

In a finned-tube cross-flow heat exchanger, hot exhaust gas heats up water.

Overall heat transfer coefficient based on gas side surface area is given

$U_h = 100\ W/m^2$ $A_h = 40\ m^2$

Parameters	Gas	Water
Flow rate (kg/s)	1.5	1
Entry Temperature (°C)	250	35
Exit Temperature (°C)	Unknown	Unknown

Find out the gas side *fluid exit temperatures*

Both fluids unmixed

F. P. Incropera and D.P. DeWitt, Heat and Mass Transfer

Now, we will try to solve the same problem, but we will change it to; I mean from a design problem to simulation problem. Like in the earlier calculation we were trying to evaluate the heat transfer surface area necessary, but in this problem as if this is heat exchanger which is already known to us it is in service. But somehow we have put according to that I mean design calculation, we have estimated the area to be 39.1 or 39.7.

So, accordingly we have put or manufactured the heat exchanger cross flow heat exchanger of area A h equals to 40 meter square. And the overall heat transfer coefficient is still around 100 Watt per meter square. And as if now we have changed the hot fluid flow rate, the gas flow rate from earlier value to 1.5 kg per second and the entry temperature has also changed from 300 to 250 degree centigrade. Now we want to find out how is the heat exchanger performance or how is the exit temperature of the gas side or the hot fluid and the cold fluid.

So, basically it is no longer design problem rather it is a simulation problem or the heated smear performance problem. So, here what is known is the heat exchanger configuration is known and what we intend to calculate is the inlet sorry the exit temperature of the fluid streams.

Now, if we look here as I told you that already we need to already some of the parameters it is known to us the entry temperature or both the fluid streams are known, the gas flow rate or the mass flow rate of water is also known. The exit temperatures are not known, but we know the heat exchanger configuration and the overall heat transfer coefficient. And it is also it has already been said that this is a cross flow exchanger with both the fluid unmixed.

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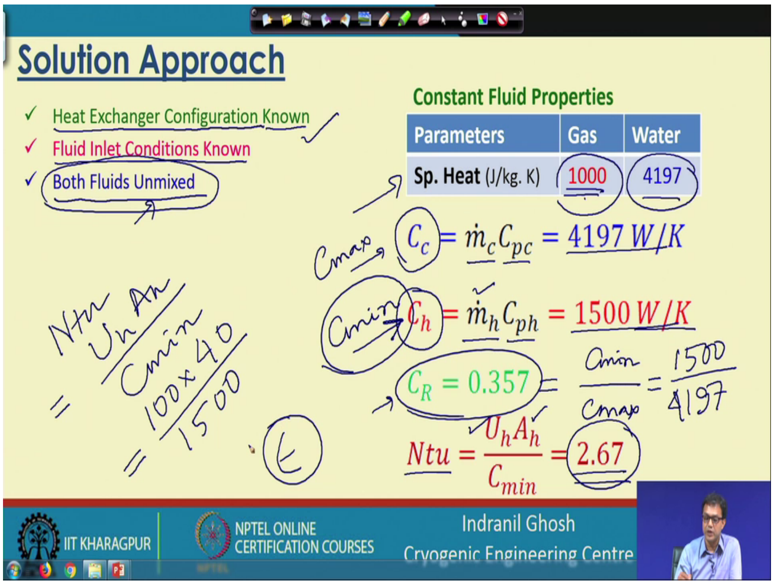

Solution Approach

- ✓ Heat Exchanger Configuration Known ✓
- ✓ Fluid Inlet Conditions Known
- ✓ Both Fluids Unmixed

Constant Fluid Properties

Parameters	Gas	Water
Sp. Heat (J/kg. K)	1000	4197

Cmax $C_c = \dot{m}_c C_{pc} = 4197\ W/K$

Cmin $C_h = \dot{m}_h C_{ph} = 1500\ W/K$

$C_R = 0.357 = \frac{C_{min}}{C_{max}} = \frac{1500}{4197}$

$Ntu = \frac{U_h A_h}{C_{min}} = 2.67$

$Ntu = \frac{U_h A_h}{C_{min}} = \frac{100 \times 40}{1500}$

ε

So, as I told you that the heat exchanger configuration is already known. That means, the heat transfer surface area and the heat transfer coefficient is already given, the fluid inlet conditions are known. The entry temperature and we are assuming that both the fluid streams are assuming or I mean it is already decided that they will be on remaining unmixed and we want to find out the fluid properties because this will also be necessary to calculate the heat transfer and we assume that the fluid properties are constant. So, this is this has been assumed to be specific it as equals it is 100 kilo Joule per kg Kelvin and for water it is 4197.

So, accordingly we can find out the cold fluid heat capacity. So, that is the mass flow rate of the cold fluid and the specific heat of the cold fluid. So, this is 4197 multiplied by 1 kg per second; so, that will come out to be 4197. Similarly for the hot fluid the capacity is m h dot and C p h. So, this is 1.5 kg per second and this is the specific heat; so it gives us 1500 Watt per meter; so, 1500 Watt per Kelvin. So, we can understand that this hot fluid stream is the C min and this is this cold fluid stream is the C max or maximum capacity. So, accordingly we can write that C R is equals to C min by C max; we can calculate this will become C min equals to 1500 Watt per Kelvin and this is C max is 4197. So, this will give us C R equals to 0.357.

Now, we have the C min value with us, we have the C R value with us we already said that the heat exchanger configuration is known. So, basically we will try to find out what is the NTU; NTU is equals to UA by C min and for the hot fluid stream it is already known U h A h by C min what is the U h? U h has been given as already is value is has been given A h value has also been considered as 40 and this is we have considered it to be 100 Watt per meter square Kelvin and C min is this is 1500. So, accordingly we will find that this is giving us 2.67 as the NTU.

So, now when we know the NTU we should be able to ideally calculate the epsilon and from there we will try to find out the other values. So, let us look I mean for this configuration what at the.

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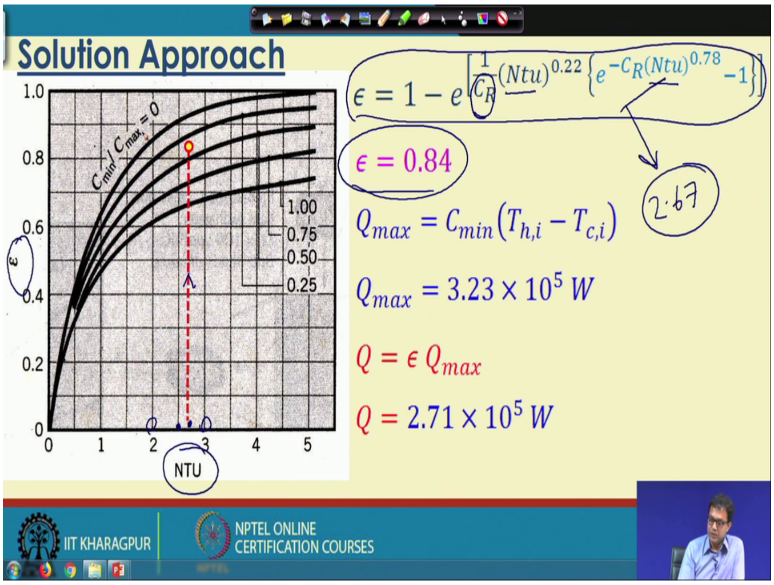

Solution Approach

$$\epsilon = 1 - e^{\left[\frac{1}{C_R}(Ntu)^{0.22}\left\{e^{-C_R(Ntu)^{0.78}} - 1\right\}\right]}$$

2.67

$$\epsilon = 0.84$$

$$Q_{max} = C_{min}\left(T_{h,i} - T_{c,i}\right)$$

$$Q_{max} = 3.23 \times 10^5\ W$$

$$Q = \epsilon\, Q_{max}$$

$$Q = 2.71 \times 10^5\ W$$

There are 2 approaches again we can take help off the chart for the epsilon NTU as it has been shown in the earlier case. I mean when both the fluid streams are remaining unmixed the epsilon into relation is can be plotted from this relation, but now we see we have this NTU value known to us this NTU is 2.67.

Now just now we have calculated it to be 2.67 and from that NTU value ideally we should be able to put this value from this relation in this relation and we should be able to find out the value of epsilon. Here if we look at the CR value it is already known, NTU value is already known and we can ideally calculate and in reality also we can calculate epsilon and we will find that this epsilon is coming to be 0.84.

We can also take help of this chart and we can fix this NTU; it is somewhere this is 2 and this is NTU equals to 3 in between this is 2.5. So, 2.67 it will be on the somewhere here; so, if we go up and also we have the C R value known to us. Thus, here just in the previous slides we have calculated in the previous slide in the previous slide we have calculated C R equals to 0.357.

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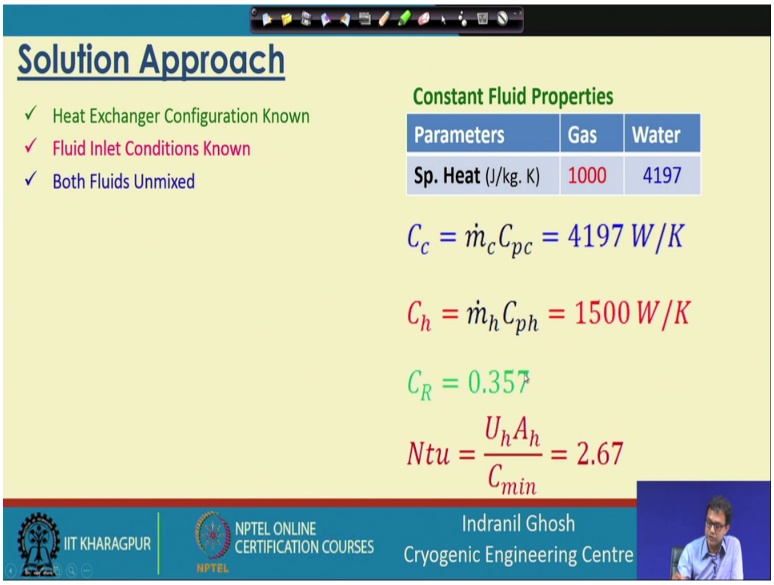

Solution Approach

- ✓ Heat Exchanger Configuration Known
- ✓ Fluid Inlet Conditions Known
- ✓ Both Fluids Unmixed

Constant Fluid Properties

Parameters	Gas	Water
Sp. Heat (J/kg. K)	1000	4197

$$C_c = \dot{m}_c C_{pc} = 4197\ W/K$$

$$C_h = \dot{m}_h C_{ph} = 1500\ W/K$$

$$C_R = 0.357$$

$$Ntu = \frac{U_h A_h}{C_{min}} = 2.67$$

So, now if we go to this 0.357 if you look at this graph this is for 0.25 and this is equals to for epsilon C R equals to 0, this is equals to 0.25 and this is equals to 0.5. So, it will be in between somewhere and we can find out; say it will be somewhere like this and here this corresponding value we can see that it will be about this. So, it will be about 0.8283 or 84 I mean this is obviously an approximation where approximate value will often from the graph whereas, if we put directly this relation in this relation; the value of NTU and C R will directly obtain the epsilon.

So, once we know the epsilon we can try to find out the actual heat getting transferred because the Q max or the maximum amount of heat that can be transferred in this heat exchanger is also known. We know the minimum capacity fluid we know the entry temperature of the hot and the cold fluid.

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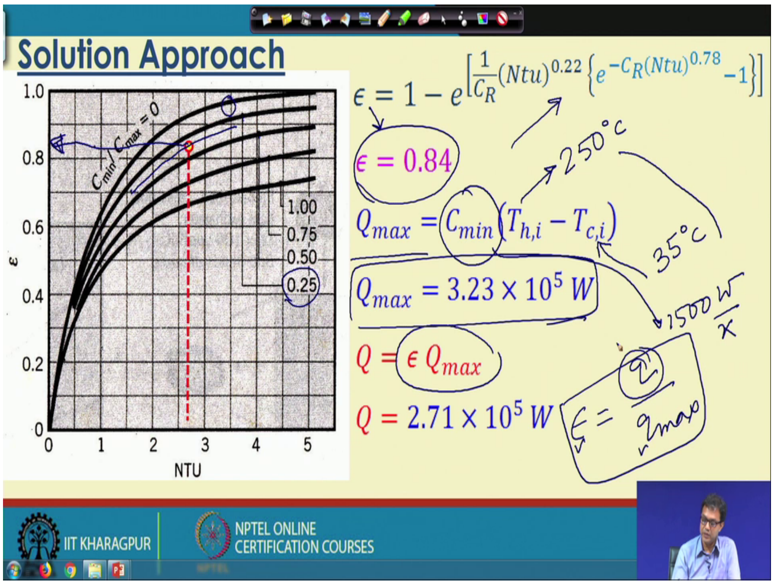

Solution Approach

$$\epsilon = 1 - e^{\left[\frac{1}{C_R}(Ntu)^{0.22}\left\{e^{-C_R(Ntu)^{0.78}} - 1\right\}\right]}$$

$\epsilon = 0.84$

$Q_{max} = C_{min}(T_{h,i} - T_{c,i})$

$Q_{max} = 3.23 \times 10^5\ W$

$Q = \epsilon Q_{max}$

$Q = 2.71 \times 10^5\ W$

250°C

35°C

$1500\ \frac{W}{K}$

$\epsilon = \frac{q}{q_{max}}$

So, what is the hot entry? This is 250 degree centigrade and the cold fluid is remaining same that is equals to 35 degree centigrade. So, we can and this C min also we have seen this to be 1500 Watt per Kelvin.

So, accordingly we can try to find out this Q max and that will come out to be 3.23 into 10 to the power 5 watt. So, when we know the Q max we can find out the actual heat getting transferred from this relation because we know the definition of q is equals to q by q max or this is what is known to us. So, we know already epsilon we have calculated we know the q max we can find out the actual heat transfer. So, that we can find out to be 2.71 equals to into 10 to the power 5 assuming a value of epsilon equals 0.84.

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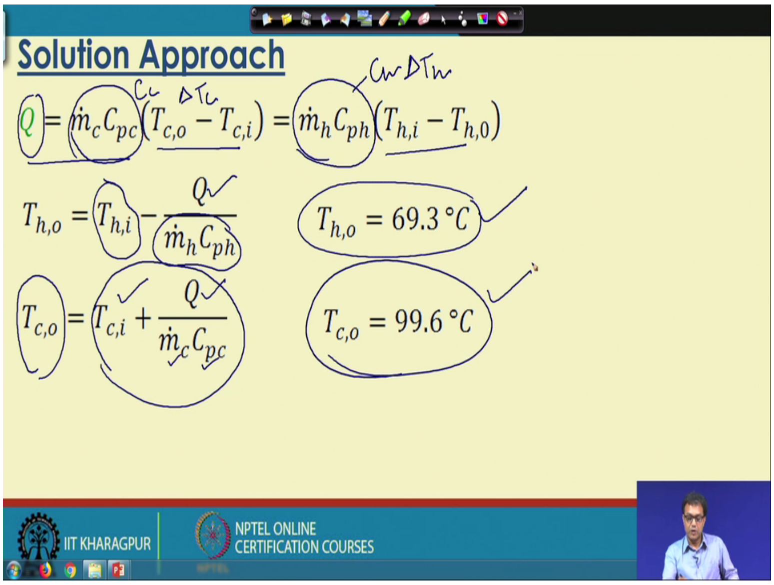

Solution Approach

$$Q = \overset{C_c}{\dot{m}_c C_{pc}} \overset{\Delta T_c}{(T_{c,o} - T_{c,i})} = \overset{C_h \Delta T_h}{\dot{m}_h C_{ph} (T_{h,i} - T_{h,0})}$$

$$T_{h,o} = T_{h,i} - \frac{Q}{\dot{m}_h C_{ph}} \qquad T_{h,o} = 69.3\ °C$$

$$T_{c,o} = T_{c,i} + \frac{Q}{\dot{m}_c C_{pc}} \qquad T_{c,o} = 99.6\ °C$$

So, when we know the actual heat getting transferred; so, it is now easy for us to calculate the exit temperature. The hot fluid stream is I mean for both the heat transfer the C C multiplied by the difference in temperature and C h multiplied by this is nothing, but C h multiplied by the difference in temperature and this is C c multiplied by the delta T c this is C h multiplied by delta T h.

So, in this equation what we know is this Q already we have calculated. So, this Q is known; we know the hot inlet temperature entry temperature, the mass flow rate multiplied by C p h is already known, and accordingly we will find that the exit temperature is 69.3 degree centigrade. And the cold exit temperature will be related again you know from this relation; if we just manipulate you will find that the exit temperature is related by this equation where on the right hand side parameters are known Q, T c i, m c, C p c. So, we would be able to find out the exit temperature of the hot fluid stream of the cold fluid stream.

So, now we can see that depending on the configuration or the operational parameters of the heat exchanger; we may have to adopt different techniques for solving different kind of problem. It may be design problem to find out the heat exchanger surface area or the details of the tube or the finned configuration though we have not, I mean encountered the fin.

So, but we also I mean the same problem when we have designed the heat exchangers; we may have situation that some of the parameters or some of the operating conditions will change or with the time odd you know deliberately, we are forced to change the operating condition and how the heat exchanger performance is varying with the I mean with that change in the operating condition is it is an interesting parameter.

So, that design problem now will become simulation problem and we as we have seen that we have designed the heat exchanger, where the operating parameter has been changed and the flow rate of the hot fluid has been changed to 1.5 kg per second.

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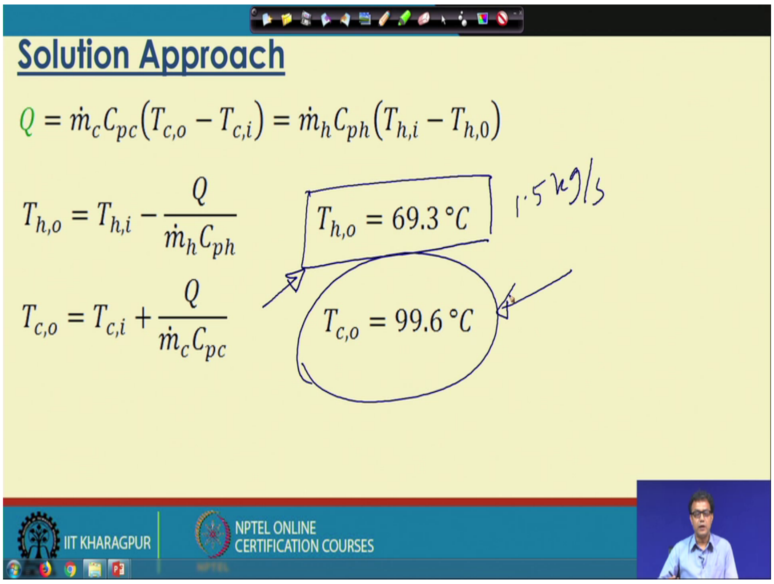

Solution Approach

$$Q = \dot{m}_c C_{pc}(T_{c,o} - T_{c,i}) = \dot{m}_h C_{ph}(T_{h,i} - T_{h,0})$$

$$T_{h,o} = T_{h,i} - \frac{Q}{\dot{m}_h C_{ph}}$$

$$\boxed{T_{h,o} = 69.3\,°C}$$

1.5 kg/s

$$T_{c,o} = T_{c,i} + \frac{Q}{\dot{m}_c C_{pc}}$$

$$T_{c,o} = 99.6\,°C$$

And now that has changed the exit temperature to 69.3 degree centigrade whereas, the cold inlet condition; I mean exit temperature has now come to 99.6 degree centigrade.

So, depending on the heat exchanger configuration whether they are mixed on unmixed, whether it is a simulation or it is a design problem; we have to take help of this epsilon NTU or we have to take help of that LMTD or it may be an F-LMTD. And we have to look for the appropriate equation to solve that correction factor F or we have to find out the appropriate or we have to take help of the appropriate chart and for calculating the correction factor or I mean that R or the P parameters. And accordingly find out the heat transfer or we have to design or design parameters.

Thank you.