Heat Exchangers: Fundamentals and Design Analysis
Prof. Indranil Ghosh
Cryogenic Engineering Centre
Indian Institute of Technology, Kharagpur

Lecture – 06
Design and Simulation

This lecture is in continuation to our earlier discussion on Design and Simulation of Heat Exchangers. Today, we are going to talk about epsilon NTU method; in the last time we have talked about the LMTD technique.

Now here we are finding 2 new terms one is the epsilon or the effectiveness of the heat exchanger and the other term is the number of transfer units or in abbreviated form; it is the NTU. So, first of all we will look into the definition of each of them and then we will look how it is related to the design and simulation of the heat exchangers.

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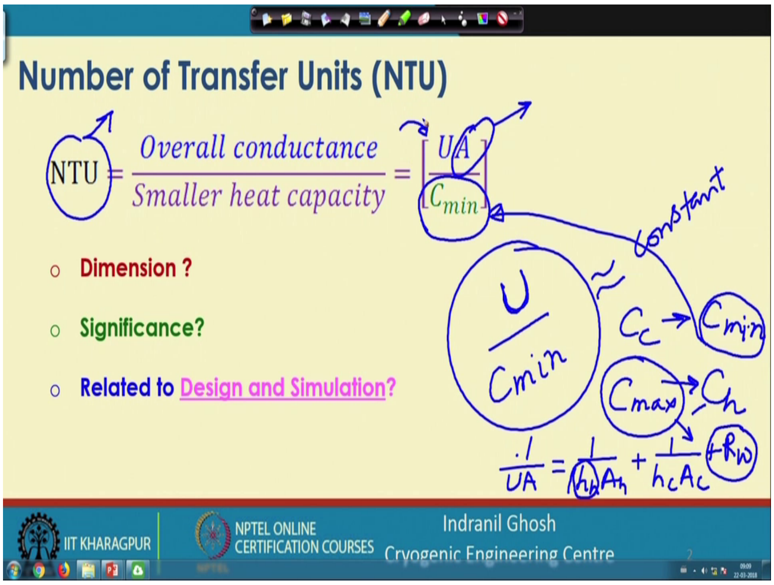

So, NTU is basically a ratio between the overall conductance and the smaller heat capacity.

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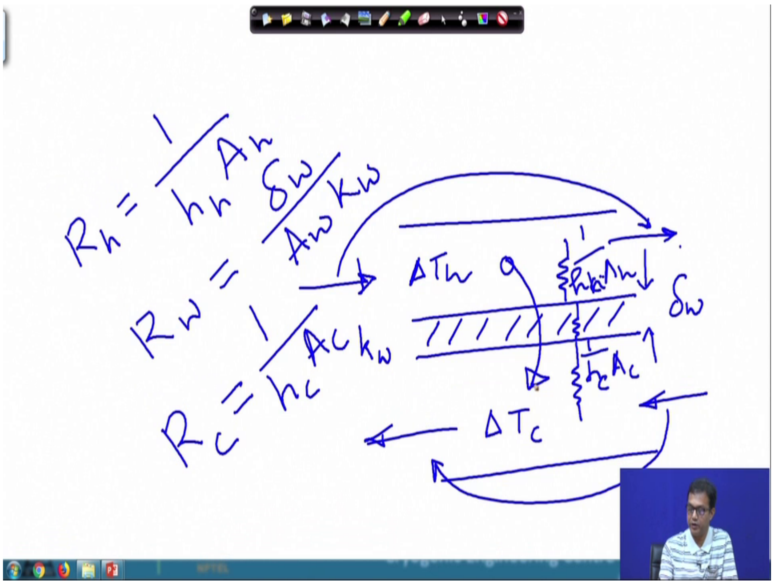

Now if we remember that we have earlier talked about a 2 fluid steam heat transfer where we had a hot fluid. And it is separated from the cold fluid by a separating wall; say this is the separating wall the thickness of the separating wall may be delta w. And its thermal conductivity may be k w and we had some hot fluid coming in and hot fluid moving out, cold fluid moving in; cold fluid moving out.

So, there is heat transfer from the hot to the cold fluid. Now this change in the temperature of this hot to cold; this is becoming colder, this cold fluid is becoming hotter. Now this change in temperature of the hot fluid that delta T h and the delta T c basically they are nothing, but the change in enthalpy of the cold fluid and the change in enthalpy of the hot fluid.

Now, this change cannot be unlimited or in a sense that this is restricted by some resistance what we have turned as the hot fluid resistance, cold fluid resistance and the wall resistance though we have neglected the wall resistance in some other cases. So, this is 1 by h; h A h this is h c; A c 1 by and so, in a nutshell we have the hot fluid resistance say R h is equals to 1 by h h; A h; R of the wall that is delta w by A w; k of the wall. And resistance offered the convective resistance offered in the cold fluid is h c A c; so, this heat transfer resistances will be faced by this hot fluid while transferring heat from the hot fluid to the cold fluid.

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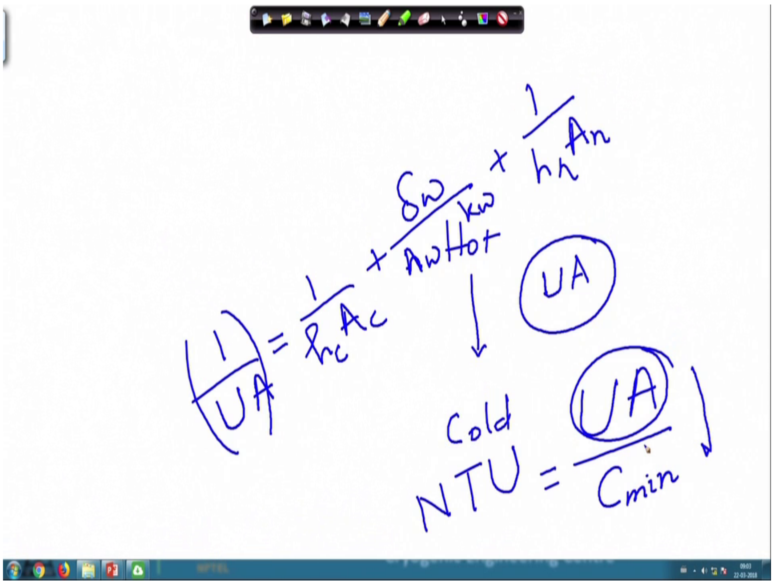

So, in a nutshell what we find that; this heat transfer from the hot to the cold fluid is limited by something which is known as the overall conductance UA or the overall heat transfer resistance UA; 1 by UA that is equals to 1 by as we have said in the earlier one this is delta w by A w, k w plus 1 by h h into A h; this is the overall heat transfer resistance or the conductance UA.

So, now we will find that the ratio between the overall conductance and the minimum capacity that is becoming important or that is what is giving say the amount of transfer. Or it is basically relating the amount of heat that could be transferred in from hot to the cold fluid. So, we define it as UA by C min; now, if we look into the dimension overall dimension of this number that is it is very easy to find out.

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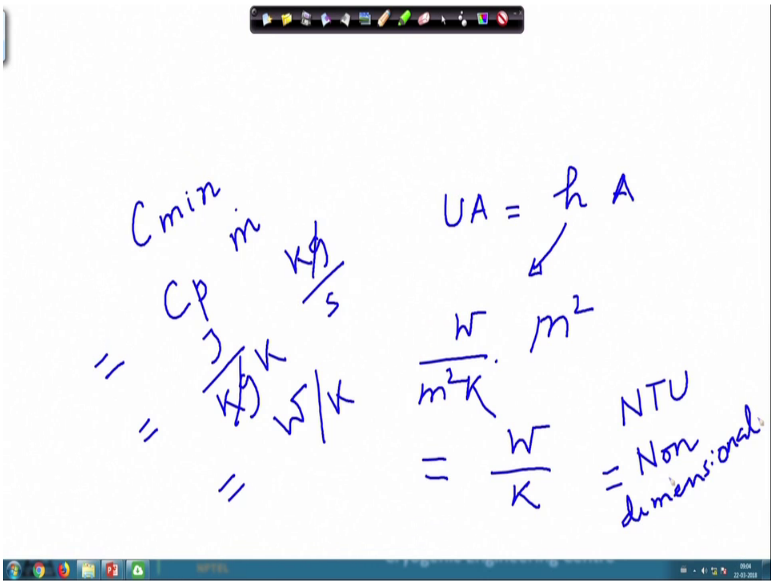

If we look at again it is individually, if we look at UA is basically nothing, but the dimension of h and A. So, this is the heat transfer coefficient that is equals to Watt per meter metre square Kelvin and this is area in metre square; so, this overall is Watt per Kelvin.

And on the other side what we have is C min that is nothing, but the C p of the cold or hot in the minimum fluid multiplied by the m dot. So, this is joule per kg Kelvin and this is kg per second; so, this is cancelling out. So, overall what we get is Watt per Kelvin; so, basically NTU is; a non dimensional number; non dimensional. So, this non dimensional number or often we call it as dimensionless heat transfer size or the thermal size of the heat exchanger. Now is it related to the physical dimension of the heat exchanger? Not necessarily; it is not necessarily related to the physical dimension of the heat exchanger.

This has been explained nicely with an example by (Refer Time: 08:09) they have given an example of a gas turbine regenerator where the NTU is 10.

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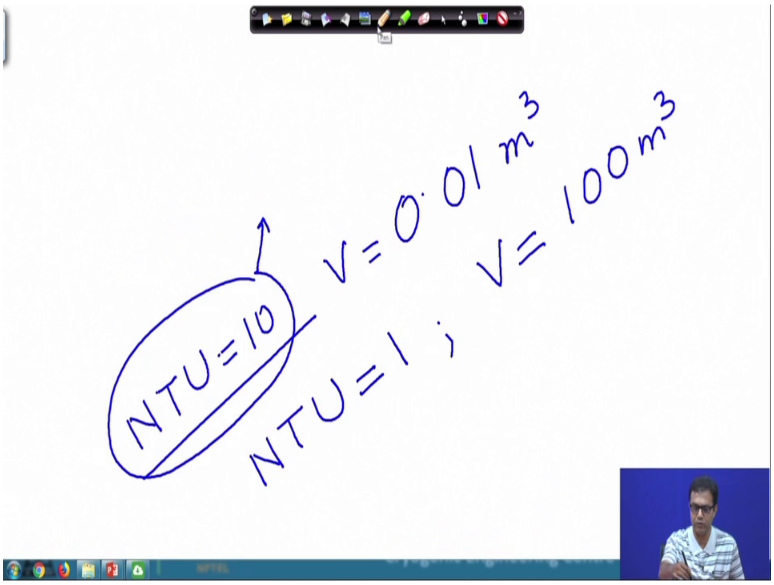

But the volume the code volume of the heat exchanger is only point 0.01 metre cube, this is corresponding to an NTU of 10. So, in another case in a chemical heat exchange chemical plant shell NTU of heat exchanger may have an NTU of 1, but it may have an overall code volume of say 100 metre cube.

So, this is in contrast that; this is not necessarily that a larger NTU will correspond to a larger dimension of the heat exchanger. But for a particular type of heat exchanger if we find that if UA by C min; UA by C min, if this parameter is remaining constant or this is nearly constant then we can say that this A is directly related to the NTU; that means, for a particular type of heat exchanger if we find that the u by C min is nearly constant. So, if we increase the NTU the physical dimension of the heat exchanger A will also increase.

Now, what is the significance of this one? We can have an idea about the overall size for a particular heat exchanger from this parameter. And if it is so happen that this C min, it will directly control the NTU, but we will also find that C max that can also have effect on the overall NTU. Because if the C max is changing; so or this C h if it is say the hot fluid is the minimum capacity fluid; sorry the cold fluid is the minimum capacity fluid. And C h is the hot capacity fluid is the maximum capacity fluid, in that case what we will find?

This C c will be in the denominator, but the other ones C max that is also controlling the overall heat transfer coefficient because that 1 by UA is related to 1 by h h; A h plus 1 by h c A c. And since; here in this one we have another term R w that is the wall resistance, we may consider it or sometimes we neglect this term. Now what we find that C min is directly controlling the NTU in this relation, but the other ones C h will also have an influence on this overall heat transfer coefficient by controlling this h h term.

When there is a change in C h that will change the h h and that will in turn change the UA and that will in turn have an effect on this overall I mean NTU. Now this NTU can be an effective means of in the design I mean when it is related to the effectiveness of the heat exchanger; we will find that this is giving certain advantage in the design and simulation of the heat exchanger. So, first of all we need to look into the definition of the epsilon or the effectiveness of the heat exchanger.

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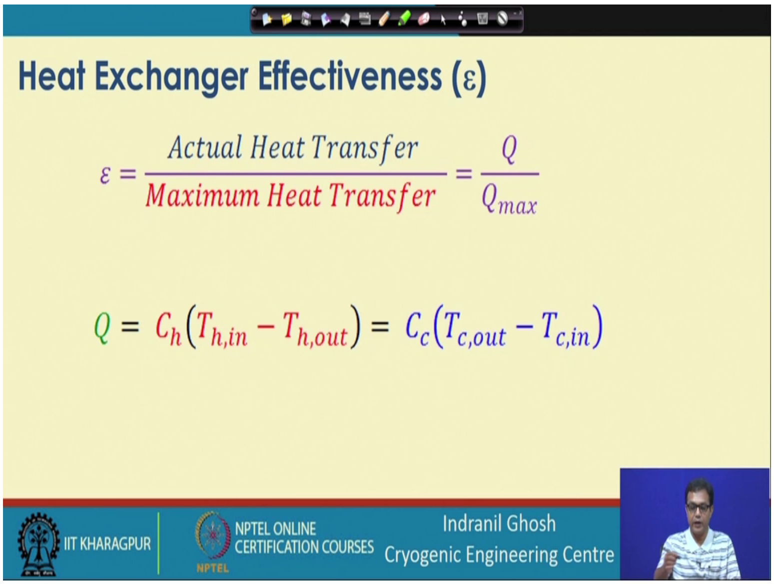

So, the effectiveness of the heat exchanger is defined as the actual heat transfer to the maximum heat that can be transferred in the heat exchanger. Now what is the actual heat that is getting transferred?

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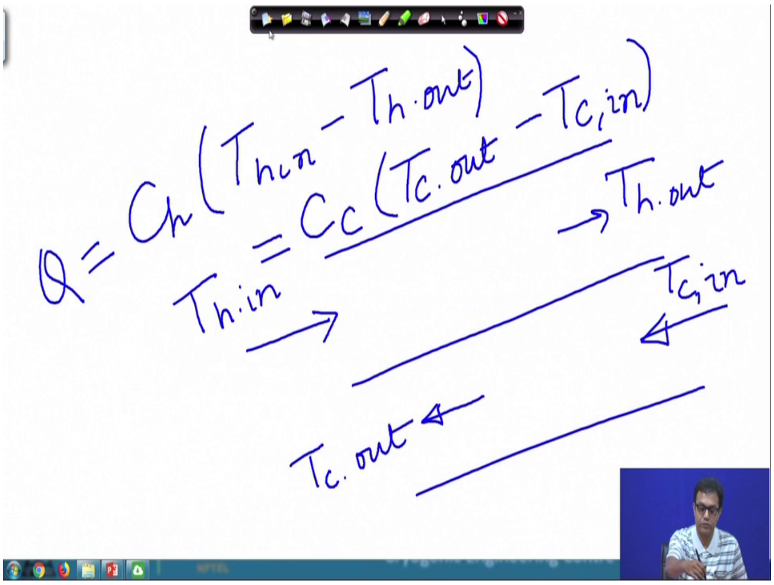

If we look again into that heat exchanger configuration; we will find that the hot fluid is changing its temperature from T h in to T h out whereas, this is T c in the counter current configuration this is T c out. So the actual heat that is getting transferred or the enthalpy change that we have said already that Q can be related as C h; T h in minus T h out.

Similarly this can also be written as C c; T c out minus T c in, so this is the actual heat that is getting transferred. So, what we need to now look at is the maximum heat that can be transferred in this; processes. So, how to estimate this maximum heat transfer? This maximum heat transfer we can estimate and this is an ideal situation like it will be of a heat exchanger of infinite length and we have to also keep it in mind that so, already we have looked in to this heat transfer that that is actually taking place.

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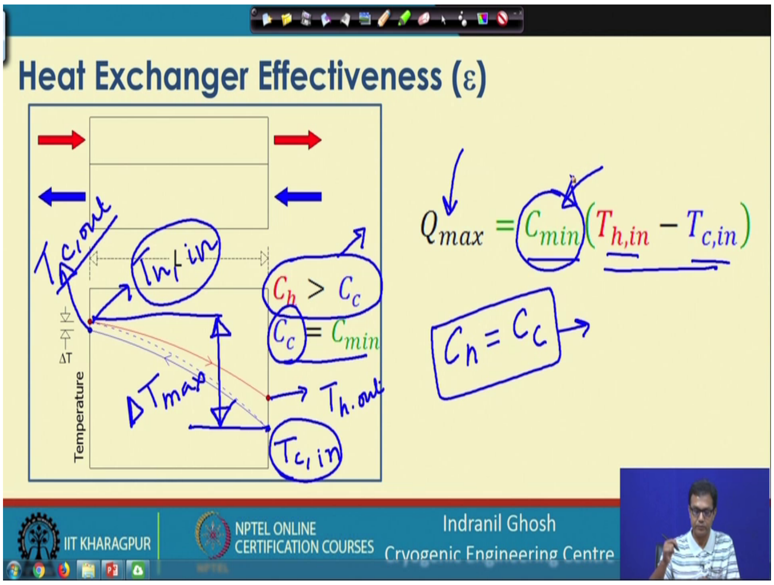

And now we will look into the overall I mean the maximum that heat can be transferred. So, here we will consider a situation where the hot fluid heat capacity is greater than the cold fluid heat capacity. Why we are choosing this one? There is an obvious condition where you know we have the simple condition rather that C h equals to C c; we will look into that one that is a simple situation, but the more complex situation is with the when the hot or the cold fluid capacity are unequal. So, in case we have this C h greater than C c we find that the hot fluid is coming from T h, this is T h in and this is T h out and this; this is T c in and this is T c out, this is T h in.

Now, we will find that this cold fluid being the minimum capacity fluid or the lean fluid; I mean that will experience the maximum temperature difference like when someone is weak he has to suffer most. So, this fluid T c will have the maximum difference in temperature. So, when we have assumed that C h is greater than C c; that means, the cold fluid is the minimum capacity fluid and the minimum capacity fluid is experiencing the maximum difference in temperature.

So, we have to keep this one in mind that while finding out the maximum difference in temperature; we have to identify the minimum capacity fluid. And the minimum capacity fluid is supposed to have the maximum difference in temperature. But please remember that this condition is only valid when; under certain conditions it is valid. And the conditions are like this that there is no external heat in leak, there is no axial conduction of heat, there is infinite amount of heat transfer; surface area involved in the process.

So, basically this is an ideal heat exchanger and we are trying to find out then ideal heat transfer that is taking place. And it is also inherently taking care of the second law of thermodynamics by not exceeding the temperature at any time; the coldest fluid is not crossing, the maximum temperature of this is the hottest temperature in this process T h in; that will not be able to cross that T; this T c in will never be able to cross the T h in. Or in other words T c out can never be more than T h in; this is possible only in a 2 stream heat exchanger, but in case of multiple stream heat exchanger this is they are not really this case.

So, we will discuss about that part later; while discussing about the multi stream plate fine heat exchanger or plate fine heat exchangers. Now if we look at this one, so the maximum heat that has been transferred is with the minimum capacity of fluid and what is the maximum difference in temperature? This is the maximum difference in temperature or T max; delta T max is with the cold in and the hot in.

So, T h in minus T c in multiplied by the C min will give you the maximum amount of heat transfer. It may occur to our mind that Q max it should be C max; so, we find from here that which is the minimum capacity fluid? The minimum capacity fluid is the C c, the cold fluid that is experiencing the maximum difference in temperature; so, that is why we have multiplied it with the C min.

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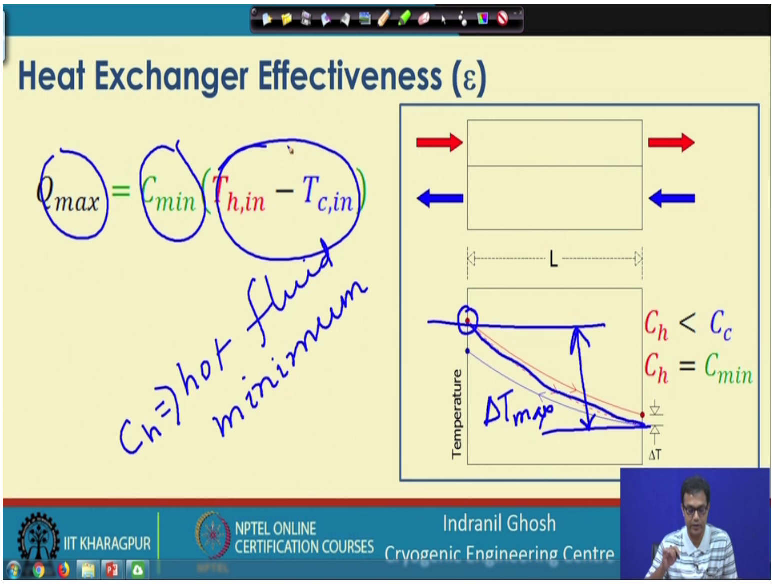

Now, if we look into here is the other situation where we have considered C h less than C c or the cold fluid is the minimum capacity sorry the hot fluid is the this is the hot fluid; the hot fluid is the minimum capacity fluid. Hot fluid by assuming C h less than C c; the hot fluid C h is the minimum capacity fluid. And that will experience the maximum difference in temperature, the temperature profile will look like this and this is the T h in.

So, it will come to the lowest temperature; that means, it will have the maximum difference in temperature T max. So, here also the maximum difference in temperature is occurring with the minimum capacity fluid. So, we can write that the Q max is basically the C min multiplied by the T h in minus T c in. So, we have an idea about the Q max we have an idea about the heat transfer Q the actual heat transfer that is taking place. So, we can now try to find out the heat exchanger effectiveness.

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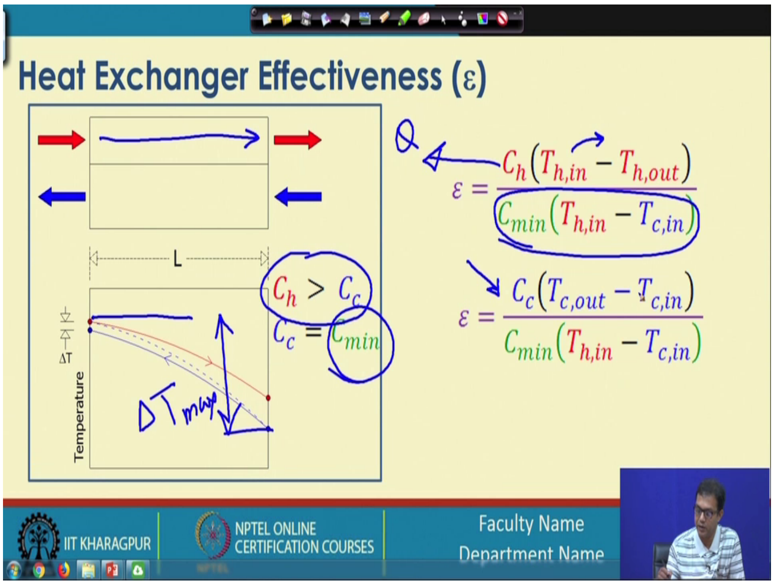

So, in case of the first situation that we have considered is C h greater than C c that is the cold fluid is the minimum capacity fluid. And in that case we can write this is nothing, but Q; Q can be written as a change in the hot fluid enthalpy change, divided by this is the maximum enthalpy that could have been changed; so C min which is C min?.

The cold fluid is the C min and T h in minus T c in that is the maximum difference in temperature this is T max delta T; max the maximum difference in temperature that is multiplied by the C min and this is relating to the hot fluid enthalpy change. We can also write this Q actual heat transfer in terms of the cold fluid enthalpy change. So, what we will find here is that we will have this C c and C min term cancelling out in this expression, but we cannot cancel this C h and C min ok.

So, this is in this case this becomes a simple expression epsilon equals to T c out minus T c in divided by T h in minus T c in. So, this is T c in, this is T h in and this ratio is telling the overall; I mean the heat exchanger effectiveness.

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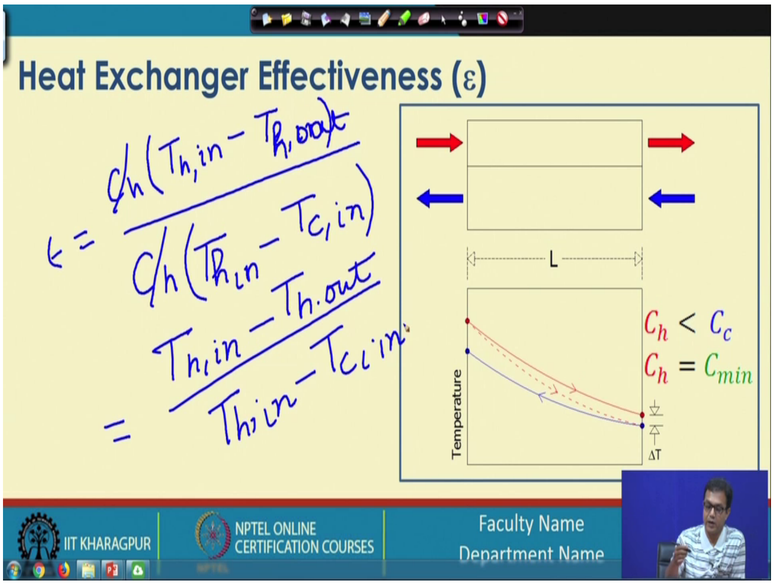

So, now if we similarly try to find out the effectiveness in other situation say in this case what is the minimum capacity fluid? C h is the minimum capacity fluid. So, if we have to now find out the heat transfer sorry the effectiveness of this heat exchanger; how can we find? We can write it as epsilon is equals to the actual heat that is getting transferred.

What is that actual heat that is getting transferred? Say in terms of C h, it will be C h multiplied by T h in minus; T c in I am sorry this is T h in minus T h out divided by what is the minimum capacity fluid? C h. So, this is nothing, but T h in minus T c in; so, this is cancelling out. So, this essentially becomes T h in minus T h out divided by T h in minus T c in.

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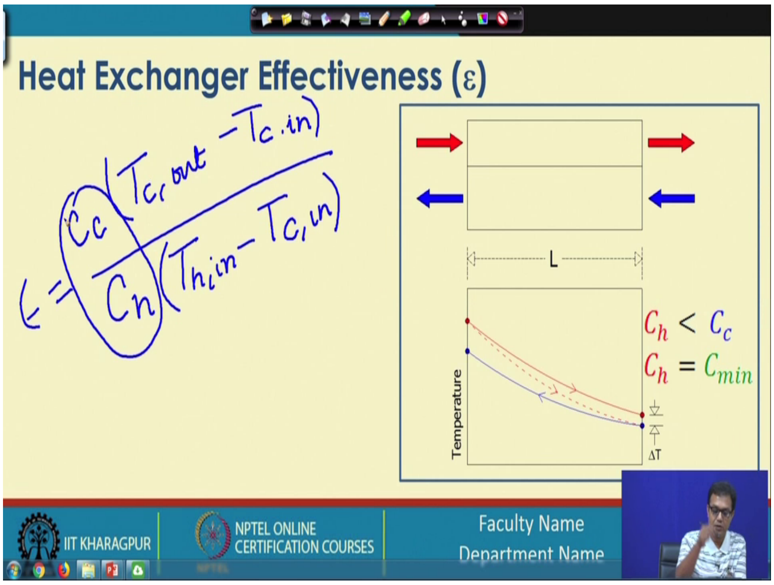

The same thing can also be written in terms of ; we can also write it in terms of the cold fluid enthalpy change. So, that becomes C c multiplied by T c out minus T c in divided by C max is Q max is equals to C h and that is equals to T h in minus T c in. We can also write this way, but this will not cancel out; so, it will remain there.

So, now let us look into an expression where we can try to relate this epsilon and the NTU.

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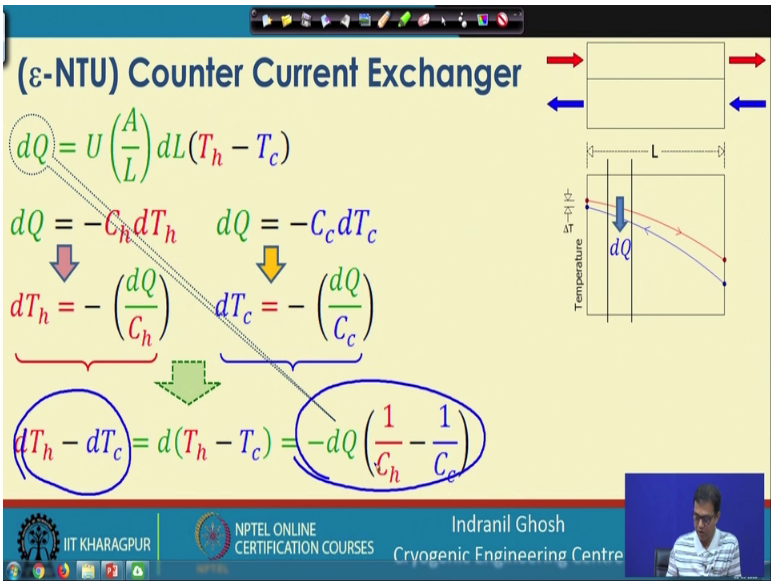

So, in an earlier discussion we have seen that in this heat exchanger; we have taken an small elemental length d L where we can write d Q; I mean in terms of the overall heat transfer coefficient or conductance multiplied by the difference in temperature.

In the previous class, we have discussed about this there we have obtained an expression where d T h minus d T c is related to d Q and 1 minus C h minus 1 by C c.

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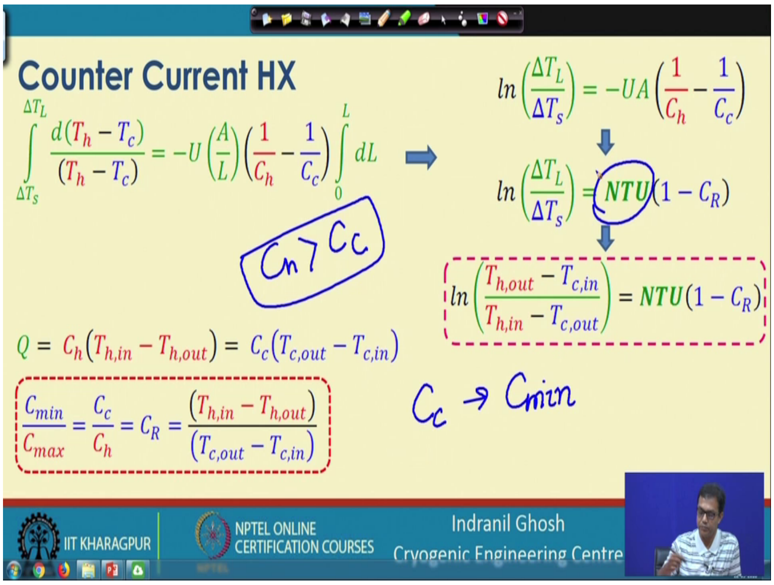

And from there we got an expression like this where we had an expression like this and from there we obtained this relation. Now you see this was the relation we have obtained; in the earlier discussion. Now here we have made a small change; now in this case we have assumed that C h is the minimum capacity fluid sorry, this is a we have taken as C r.

So, this is the case C h is greater than C c we had discussed in that earlier case. And now what is the minimum capacity fluid? C c is the minimum capacity fluid if C; is C c is the minimum capacity fluid we have taken that one out C c, we have taken out and that becomes with the change in sign it becomes NTU. Let us look; how? If we take this C c out that becomes UA by C min and this becomes C c by C h minus 1 and C c by C h that is C min by C max by C min by C max; we know that C min by C max is equals to C R.

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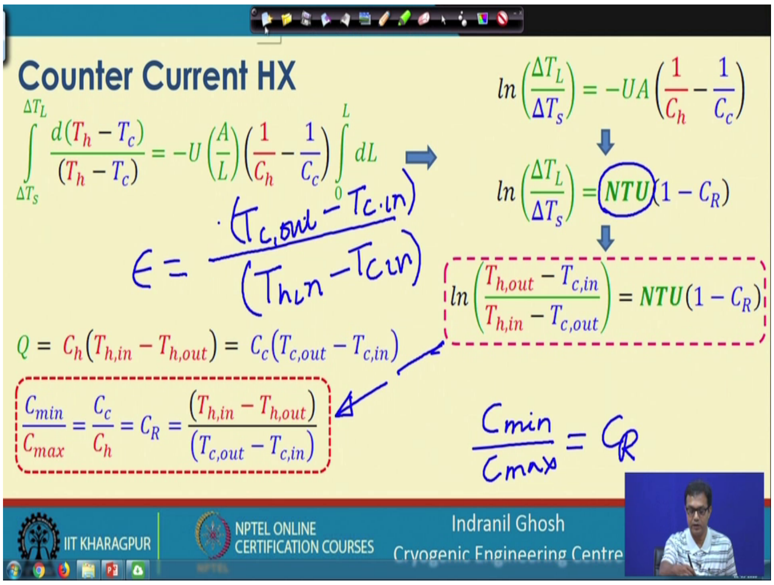

So, we can write it as 1 minus C R into NTU and this negative sign has been taken inside ; so it is 1 minus C R.

So now if we put this value of delta T L; what is delta T L? It is T h out minus T c in divided by T h in minus T c out. So, we have this relation, we also have this relation about the actual heat transfer and the if we slightly manipulate this 2 and if we take the heat exchanger effectiveness expression epsilon is equals to; here what is the minimum capacity fluid? C c is the minimum capacity fluid; the cold fluid is the minimum capacity fluid.

So, we will try to relate it in terms of T h in minus T c in and this will be T c out minus T c in. So, this C c and C c is cancelling from both denominator and the numerator.

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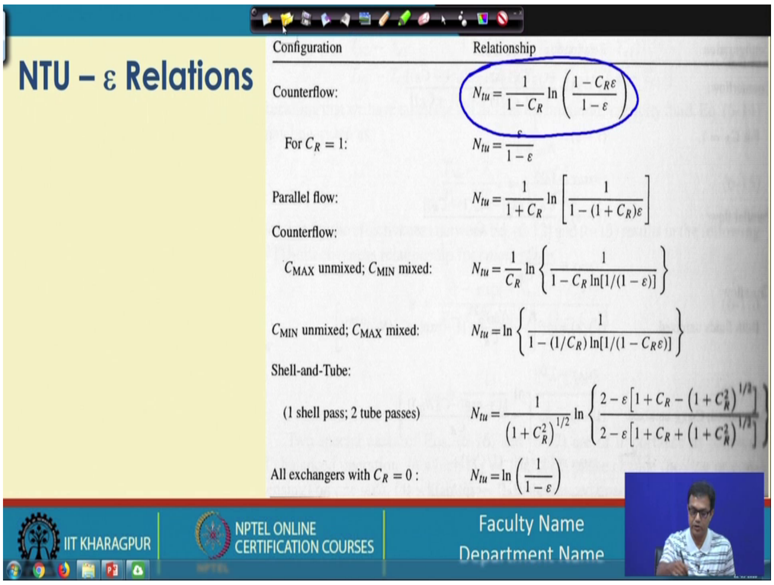

So, with this 3 equations if we now manipulate; we will find the final expression is coming like this; we will have a relation like this. So, if we have to derive this equation just we need to take help of that expressions; this 3 expressions that we have discussed in the earlier page. So, with the help of this equation, this equation and the definition of epsilon; we can with the little bit of arithmetic, we can get this equation. So, we will try to see what we have.

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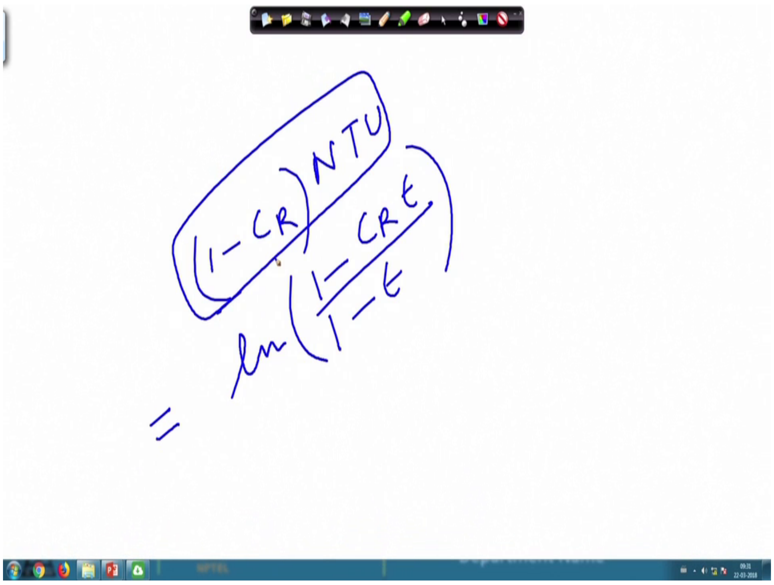

Now, just now we have seen that 1 minus C R into NTU is equals to l n of 1 minus C R epsilon by 1 minus epsilon.

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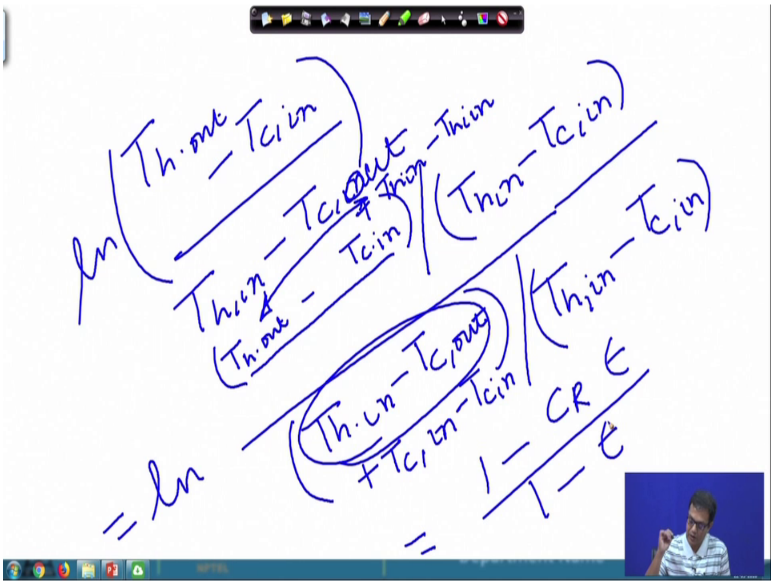

So, how did we arrive at this configuration? So, on the one the right hand side we had this expression. And on the left hand side what we had is l n of T h out minus T c in divided by T h in minus T c in.

So, we will add and subtract the first of all we will divide this expression both denominator and the numerator by T h in; oh sorry this is not T h out ok. So, what we have to do is that T h out minus T c; this numerator will be divided by the maximum difference in temperature T h in minus T c in. And the denominator also will be divided T h in minus T c out divided by T h in minus T c in. So, this will not change the equation as such.

Now, with this part we will add and subtract T h in and subtract T h in from that expression. And for this one we will add and subtract T c in and T c in; so, this will eventually not change this overall expression, but now we can combine this T h out and T h in and this will finally, come as 1 minus C R into epsilon by 1 minus epsilon.