Heat Exchangers: Fundamentals and Design Analysis
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Lecture - 05
Design and Simulation of Heat Exchangers

This talk is about the Design and Simulation of Heat Exchangers. In particular we want to do the LMTD approach or the design of heat exchangers by LMTD method. So, before going into the details we want to look into the design and simulation.

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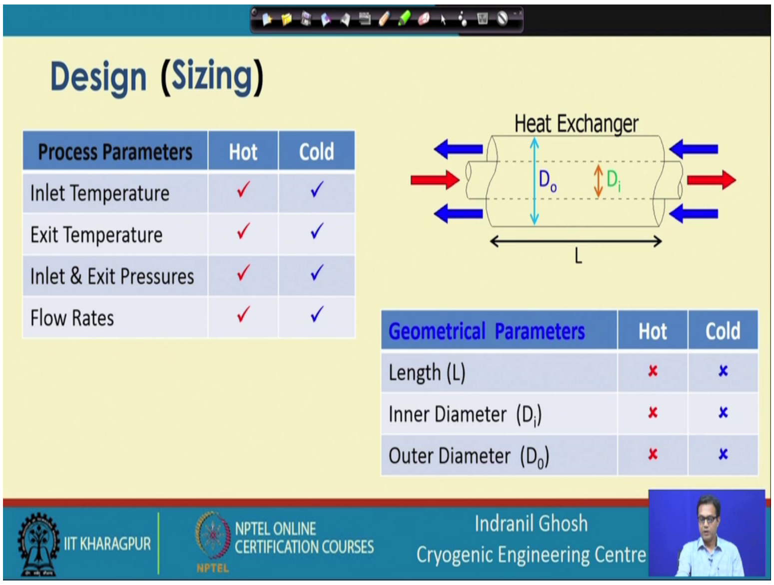

What do we mean by design? We want to see it with the example of a counter current heat exchanger. Here this heat exchanger we know what are the fluid streams the process fluid streams or the process parameters, the inlet temperature, the exit temperature, the exit and inlet pressure, the flow rates, all these parameter process parameters are known to us. And what we want to find out is the geometrical parameters like the length, the inner and outer diameter of the tubes.

So, in this counter current heat exchanger what we are basically trying to find out is the different dimensions or the geometrical parameters or the dimensions of the heat exchanger. And we often call it as sizing problem or the design problem, because it is related to the size of the heat exchanger, we call it sizing. Now, in contrast to this design we are also often we know we try to solve the simulation problem.

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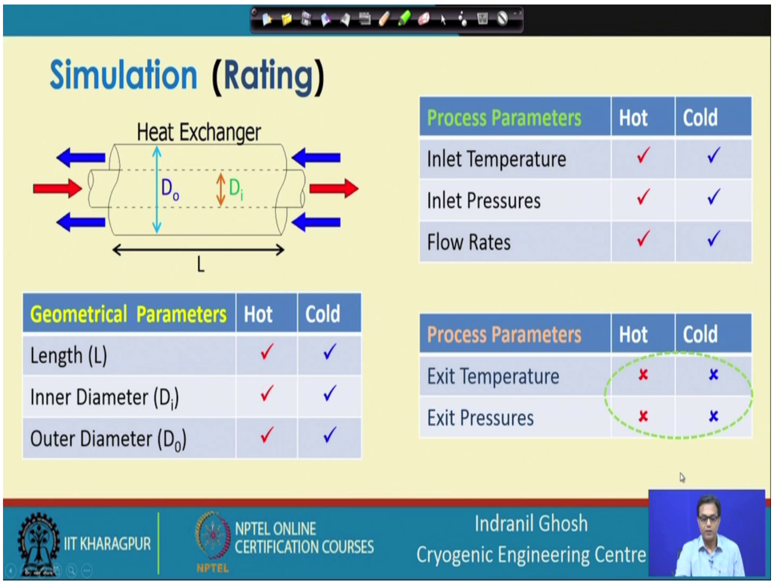

What is simulation? In Simulation if we take the same heat exchanger example that counter current heat exchanger. In this case, we know all the geometrical parameters like the length of the exchanger the diameter outer or inner, all these parameters are known. In addition to that, we also know some of the process parameters like the inlet fluid temperature, the flow rates are known that means we know this fluid flow temperature, we know this inlet temperature, we know this inlet temperature. And what we intend to find out is the exit temperature of the hot and cold fluid stream. So, this is something related to the rating or the simulation of the heat exchanger, we call it as simulation problem.

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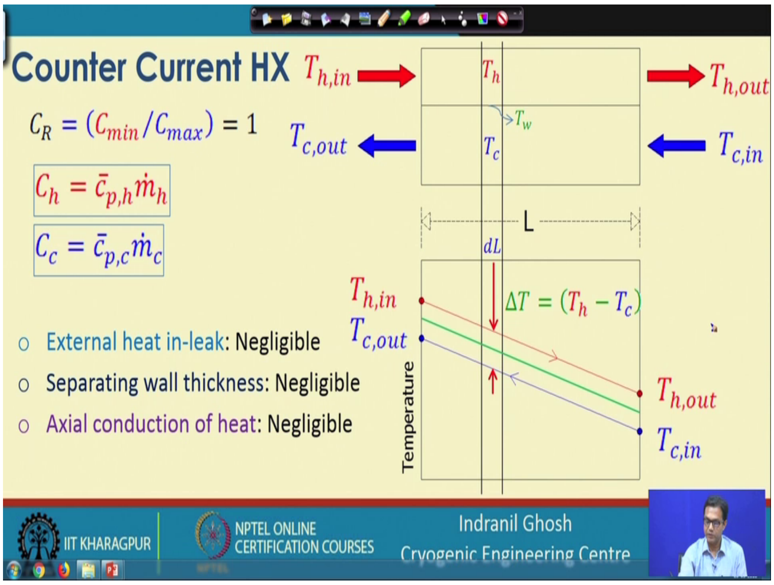

So, now we want to try the similar design of a counter current heat exchanger. And here this is an example we want to try it with a counter current heat exchanger, where we will assume a simple situation by assuming C R or the C min by C max equals to 1. When we assume that CR equals to 1, it will have a temperature profile shown in the figure. What we will find that the delta T, the temperature difference between the hot and cold fluid, it will remain same throughout the length of the heat exchanger.

So, since we are trying to solve a counter current heat exchanger design problem, here the fluid temperature, the inlet and outlet all for the hot and cold fluids are known. Here we have assumed C R equals to 1. Now, we want to find out what would be the length or diameter of this heat exchanger.

So, what is C R? It is basically a ratio between the heat capacity of the hot fluid and the cold fluid. This C h or the hot capacity ratio heat capacity is basically a product of the hot fluid specific heat multiplied by the mass flow rate of the hot fluid. And the cold C c or the cold capacity is equals to a product of the specific heat of the cold fluid multiplied by the mass flow rate of the cold fluid. So, these two ratio is basically ratio between the hot and cold fluid is equals to 1 we have assumed. So, in that situation only we can expect this kind of temperature profile.

Now, in this case this is the hot in, hot out, and Tc in and Tc out. Now, if we take a small elemental length dL across the length of the heat exchanger, we find that this is T h and this is T c, and we also assume that the internal temp I mean the wall temperature is T w. So, this T w is varying from this end to this end. Now, in addition to this assumption that C R equals to 1 that is the simplest case, we are making some more additional assumptions like there is no external heat in leak. That means, this fluid hot fluid is not connected with the environment neither this cold fluid is connected with the environment. The heat is being supplied only from the hot fluid to the cold fluid via this separating wall.

The separating wall thickness we are assuming it to be negligible. By saying that the wall thickness is negligible, we mean that the heat flow from the hot fluid to the cold fluid is not affected by the resistance of the wall. In addition to that, we also assume that there is no axial conduction. What is the actual conduction that it is that this wall is having a kind of temperature gradient, but we are not assuming any heat flowing from this end of the wall to the other end; only we are assuming the heat flow from the hot fluid to the cold fluid that is what is the assumption axial conduction heat is negligible.

So, now what will happen, the hot fluid will give heat to the separating wall, the separating wall in turn will give heat to the cold fluid. So, how do we estimate this heat transfer, we will take the difference between the hot fluid and the wall. So, T h minus T w, and the associated area with this fluid and the wall or the hot side heat transfer surface area. How much is it that is say, if it is A h then we have considered only dL length of this heat exchanger. So, if we assume that the heat transfer surface area is uniformly distributed over the entire length.

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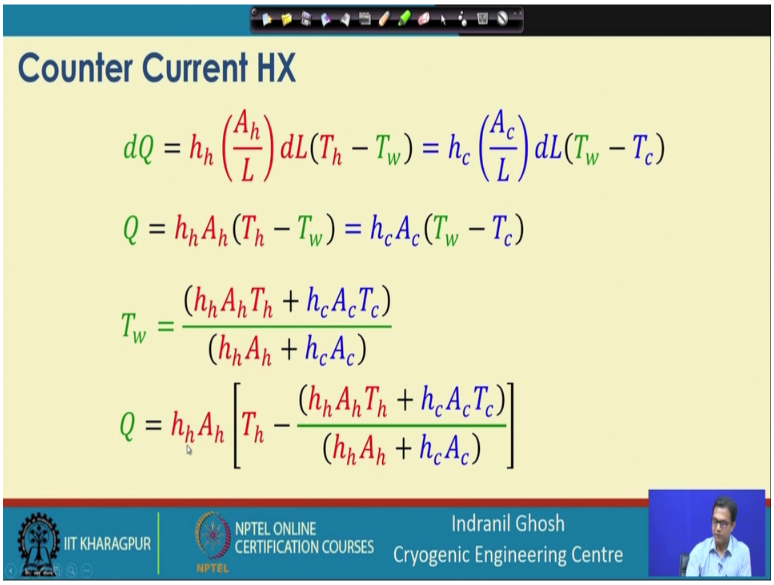

We will find that the heat transfer equation will look like, dQ is equals to as I told you that it is T h minus T w, and the area associated with this elemental length dL that is A h by L and dL is the small element and over and above h h the heat transfer coefficient of the hot fluid side. Similarly, we can equate it to the heat received from the wall to the cold fluid and the related heat transfer surface area and the heat transfer coefficient corresponding to the cold fluid side.

Now, if we simplify this equation or rather, if we integrate this equation over the entire length, we will find that this dQ is becoming Q; and this heat transfer surface area is becoming A h. So, the overall equation becomes h h h into T h minus T w and on this side T w minus T c and the cold h c and A c. Now, if we simplify this equation to find out the wall temperature, we find this relation. Now, what we can do is that we can put this wall temperature; and replace it in either of these equations. When we replace this equation or replace the wall temperature to this equation, the earlier equation, we find that the heat transfer becomes this, like this.

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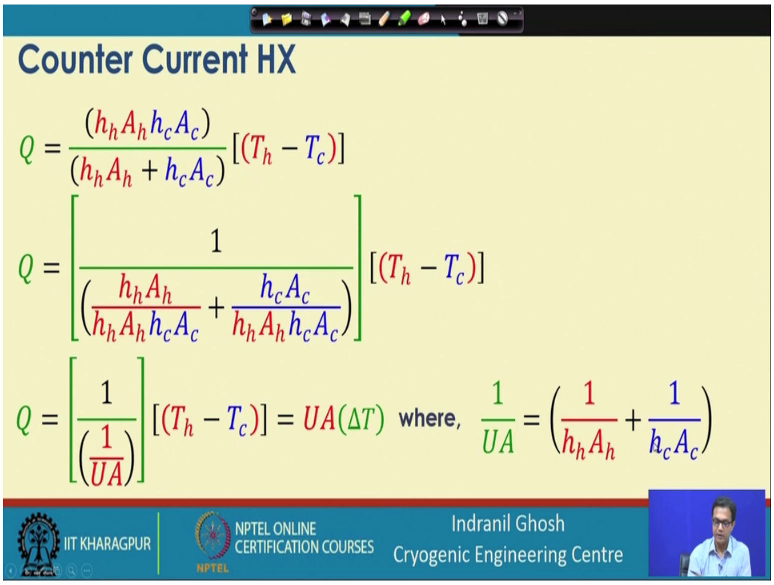

And if we simplify this equation, you will find that this will look like h h A h h c A c divided by h h A h h c A c and the difference between the wall temperature this is nothing but delta T. Now, when I take this numerator and divide this part, we find this will be simplified like 1 by UA equals to 1 by h h A h plus 1 by h c A c that is nothing but the this part is the hot fluid side heat transfer resistance, and this is the resistance offered by the cold fluid side convective heat transfer resistance. And this one can be represented by 1 by UA that is the overall resistance offered by this fluid.

So, now this can be finally, expressed in the form of UA into delta T, where this delta T is the difference between the two fluids, and UA is the overall heat transfer coefficient given by this expression.

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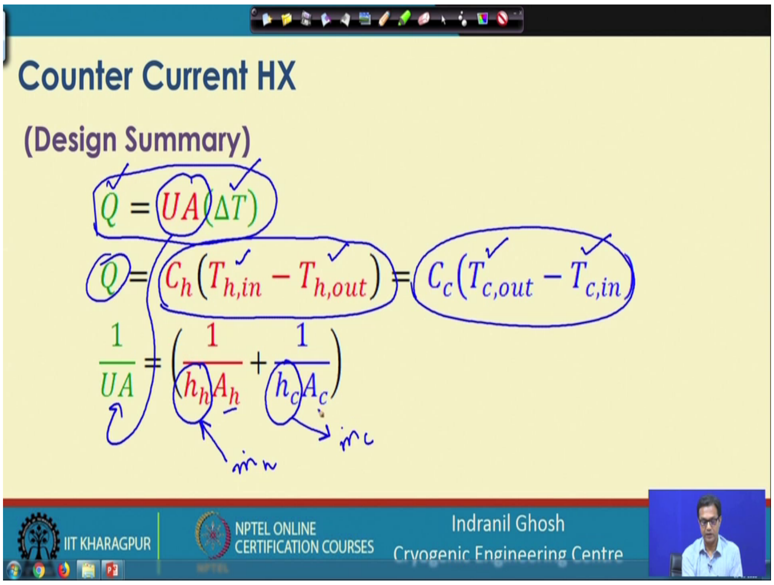

So, if we now go to the next slide so, in summary, we can say that the Q is given by UA delta T. And so, this is the final outcome we have seen, where UA is given by 1 by UA is related to the individual heat transfer I mean resistances offered by the cold and the hot fluid. And over and above, we have this individual mass balance for the hot and the cold fluid. The hot fluid mass balance is given by C h and T h in minus T h out. Similarly, for the cold fluid, we can have this equation.

Now, as we are trying to solve a design problem, we know the hot inlet; we know the hot outlet; we know the cold inlet; and the cold outlet also. So, we know essentially the amount of heat getting transferred. So, in this equation, we know Q; we know the delta T; we have assumed a simple situation, where C R equals to 1; delta T is throughout the length uniform. So, from this relation, we can calculate UA. So, this UA will now come to this equation, where it is the combination of 1 by h h A h and h c Ac.

Now, the hot side, heat transfer coefficient can also be obtained, because we know the mass flow rate m h dot. We can also find out h c, because the corresponding mass flow rate is also known m c dot is also known, we can also try to find out this h c. And then, we can try to find out the A h and A c. But, this is only one equation, where the number of unknowns are many like the length, internal diameter, outside diameter.

So, in order to find out those numbers, we have to get other equations. And by putting some other constants like the pressure drop, then the overall size, then we get other equations, and then we can try to find out all these A h and A c. So, this is the simplest situation, when we have assumed C R equals to 1. And we have tried to design a simple tube, in tube heat exchanger.

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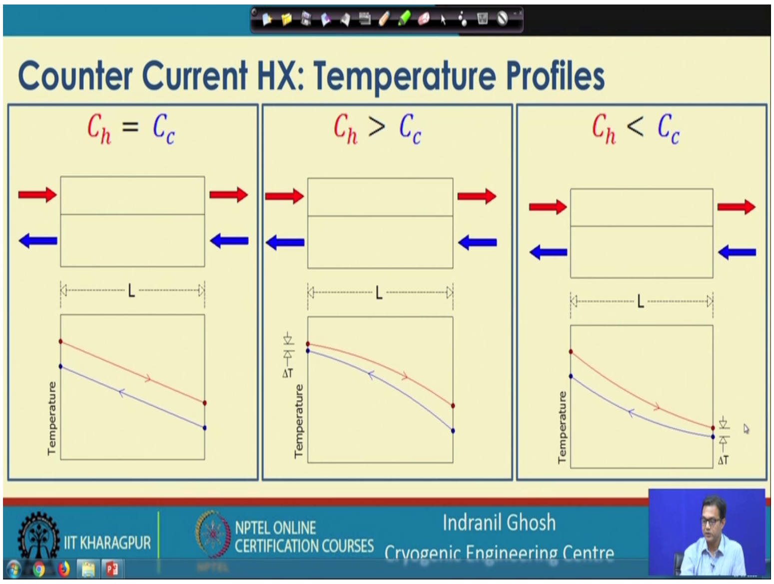

Now, we may not expect always that C h will be equal to the C c or C R will be equals to 1. There may be situation, when C h is greater than C c or it may also happen that C h is less than C c that means, C R either it will be greater than 1 or it will be less than 1. In case C h is greater than C c, we will find that the temperature profile is similar to the one like shown here, for the counter current heat exchanger.

When C h is greater than C c, we will find that the cold fluid outlet temperature is almost approaching the hot inlet temperature. On the contrary, if C h is less than C c, we will find that this hot outlet is approaching the cold inlet temperature. So, the temperature profile will look like this shown in this figure. So, now let us consider for time being that C h is greater than C c, and let us try to simulate the or design problem for this situation.

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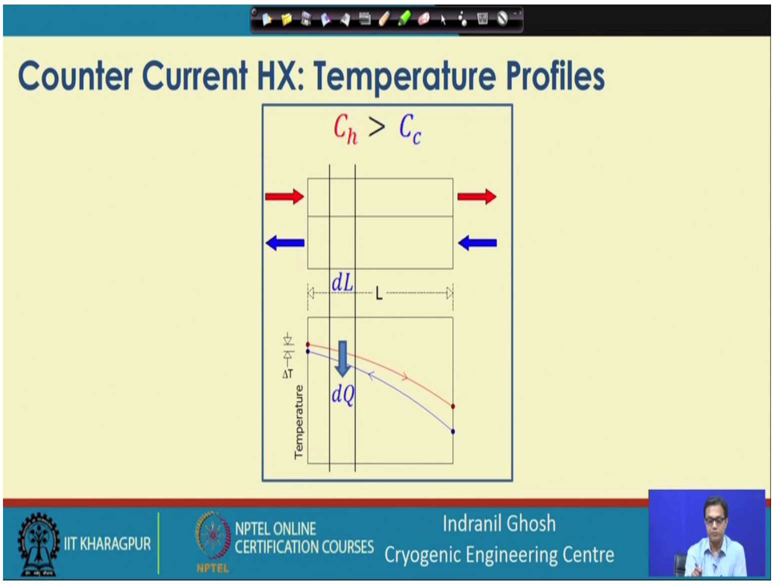

If we consider again a small elemental length dL, and then we can expect there would be a small amount of heat dQ getting transferred from the hot fluid to the cold fluid through the separating valve and all these assumptions which has been made earlier except C R equals to 1. All assumptions are still valid here, in this case also; and only exception is that we have assumed C h is greater than C c.

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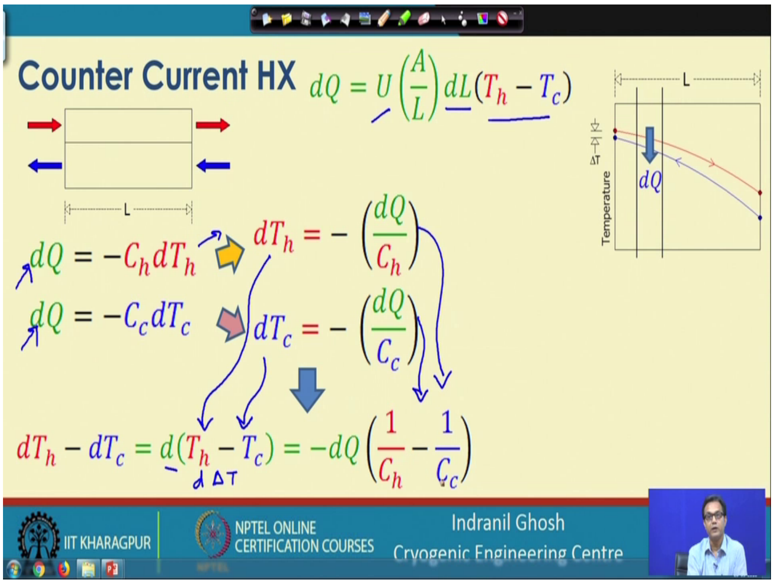

Now, in this case what we will find that the heat transfer coefficient or the overall heat transfer between the hot fluid to the cold fluid, can be related to the overall heat transfer coefficient U. And if we assume that the area is distributed uniformly over the entire length, then A by L multiplied by dL and T h minus T c; T h is the hot side temperature, T c is the cold side temperature at that point or in that small elemental L length dL.

So, now if we make an energy balance for the hot fluid, we will find dQ is equals to minus C h into dT h, where dT h is the change in temperature within that small length dL. DQ is in terms of the cold fluid, we can write it that dQ is equals to minus C c dT c, where this dc dT c is the change in temperature that is happening in the small elemental length dL in the cold fluid. We can write this first equation in this form, where d h can be written as dQ by C h with a negative sign; and dT c can be written as minus dQ by C c.

Now, let us try to write or express d of T h minus dT c this is nothing but the delta T over the length d dL. So, d of delta T can now be expressed as minus dQ. If we take this one and this expression, I mean here in this equation if we put, we find that dT h minus dT c; this is we have taken from here; this we have taken from here. And, we are able to express d of delta T is equals to minus dQ into 1 by C h minus 1 by C c. Now, we can substitute this heat transfer from here to this expression. If we do that, we will find the equation is taking this form.

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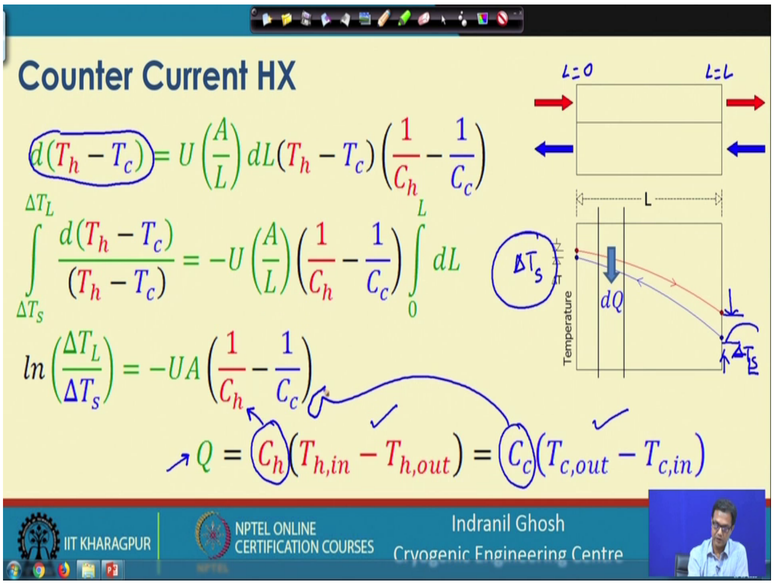

What is that form? D of d T h d T c is equals to U into A by L dL T h minus T c and 1 by C h minus C c. Now, what we are trying, we can now integrate it from this L equals to 0 to L equals to L. And when L equals to 0, the corresponding difference in temperature is delta T that is the cost of delta T smaller one, and this is delta T larger one, this is because we have assumed T c h greater than C c. So, we have the delta T s smaller on this side, and delta T large sorry this is delta T L large on this side.

So, if we integrate it now over this length, what we will find is that log of natural log of delta T large divided by delta T small is equals to minus UA into 1 by C h and minus 1 by C c. Now, we have this equation still remaining with us Q equals to C h into T h in minus T h out that is the mass balance of the cold fluid. And this is the mass balance of the sorry hot fluid and the cold fluid. This is for the hot fluid, and this is for the cold fluid this is known. So, if we know this one, we can take C h you know this expression and put it here; we can take out C c, and put this expression in this one.

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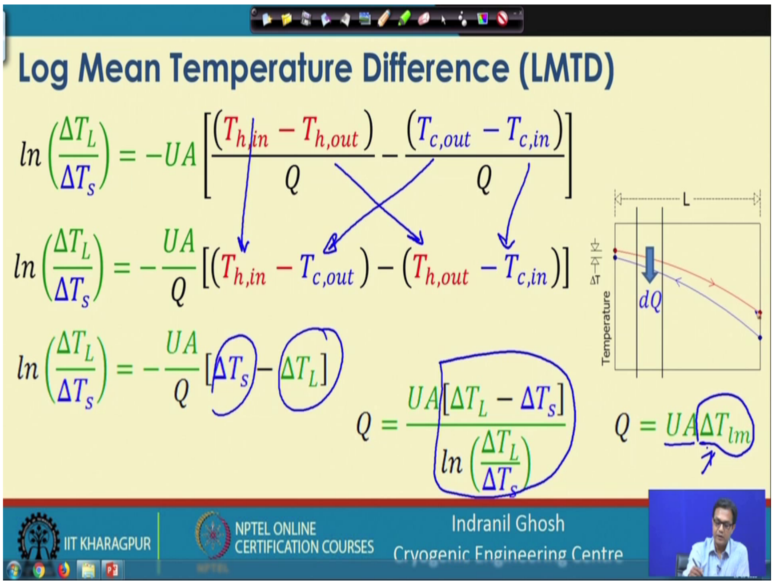

So, if we do that, what you will find is an expression like this. We have replaced that 1 by C h with this expression, and 1 by C c with this expression. Then with little algebra, if we do I mean we have rearranged with this term, we have taken T h same as this one. We have taken you know this T c out from there to here, and this one is somewhere here, and this is like this. This is just to you know, express this temperature in terms of delta T s and delta T L. So, the overall equation now becomes Q equals to UA delta T L minus delta T s divided by natural log delta T L by delta T s.

So, if we put this expression as the natural log sorry the log min temperature difference, so then it becomes Q equals to UA delta T lm or log min. So, this is the log min temperature of this entire temperature difference.

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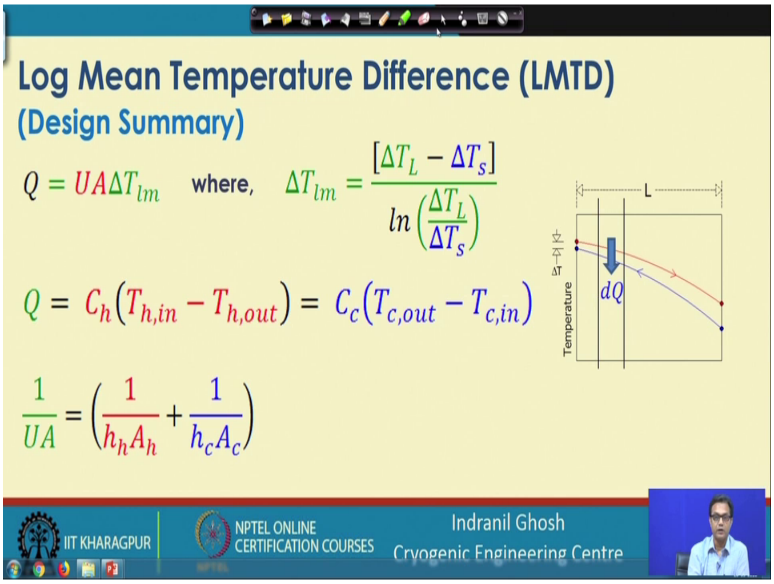

And then we can write the heat transfer as this is the design summary. Now, what is known to us, the Q this is known, because we know all the exit temperatures. Now, because this exist temperatures are known, we can also find out this log min temperature difference. This Q is known in this equation, the log min temperature is known. So, we can find out the overall heat transfer coefficient, which is nothing but 1 by h h h plus 1 by h c A c. So, again as we have discussed earlier, once we know this expression, we can try to find out the hot side and the cold side parameters, but this may be it may be necessary that there will be certain number of iterations.

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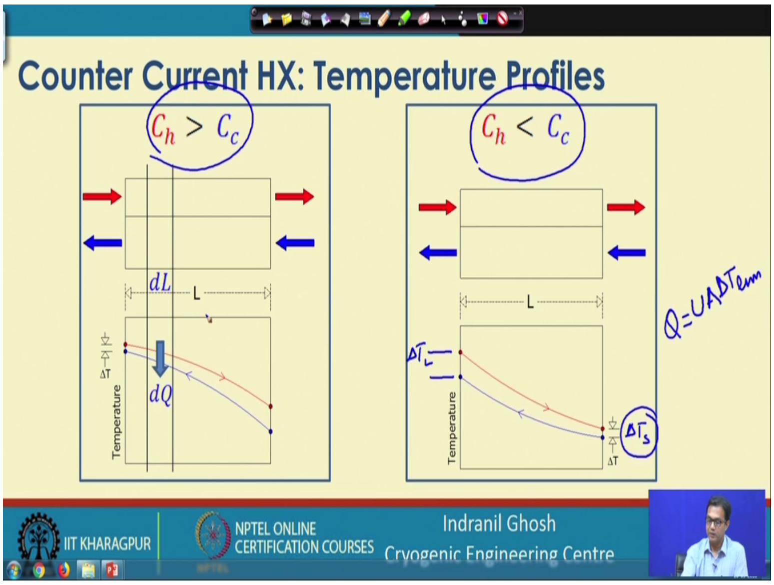

Now, we have discussed about this particular situation, C h greater than C c. And we have seen that Q equals to UA into delta T of lm. Now, is it valid for C h less than C c, it can be shown that this is also valid for this situation also. Only thing is that this time that delta T large is occurring on this side; and delta T small is occurring on this side in contrast to this situation. Other things will remain same, the sign etcetera will also be taken care accordingly, and the same expression also remains valid for this situation. Now, we want to look for the counter current, already situation we have already look we have already discussed.

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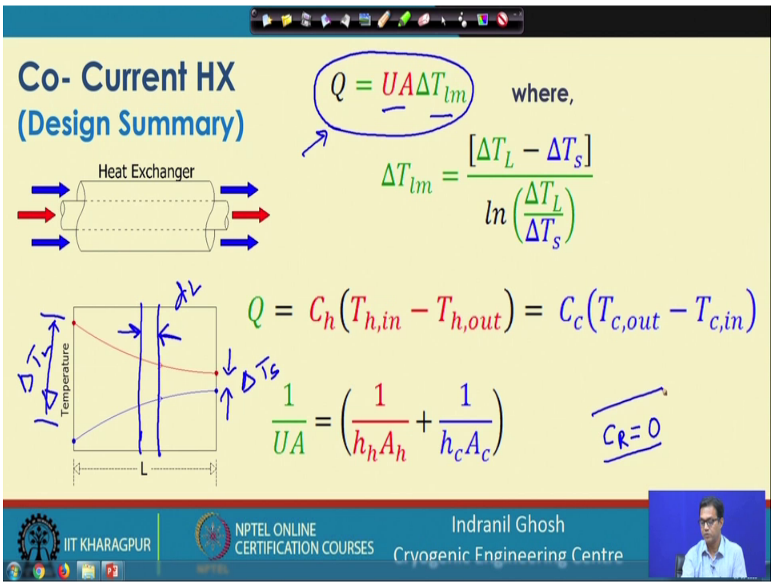

Now, if we go for a parallel flow heat exchanger or co-current heat exchanger, again by the same way if we take a elemental length over this heat exchangers length, if we take and small delta L, if we consider. And if we consider the temperature difference, if we write the energy balance, we will find that the heat transfer overall heat transfer is given by UA into delta T m. This delta T m in this case, we will have this as delta T large and the difference on this side is delta T small on this side; other things are remaining same.

So, in a nutshell, we can see that all the counter current and parallel flow or co-current heat exchanger can be expressed by Q equals to the heat transfer in this cases can be expressed as Q equals to UA delta T lm. In fact, for all counter current, parallel flow, as well as for situations where C R equals to 0 that is because to condenser or I mean boiler situations also. We will find that this relation remains valid.

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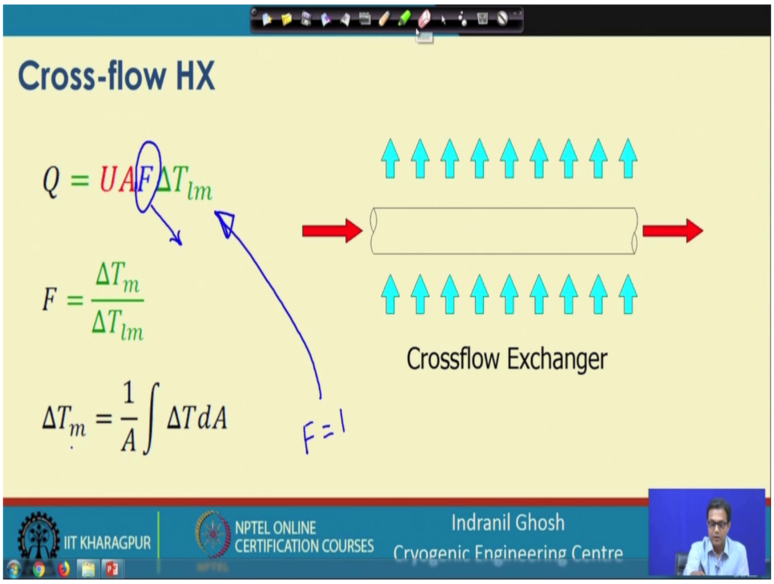

Now, if we go to the next slide, I mean if we find that there is a cross-flow heat exchanger. Do we think that still we can use this correlation, we can use this I mean relation, Q equals to UA delta T lm, but with small correction factor, because in this situation this delta T m or the min temperature or the log min temperature that is not the same.

In case of cross counter flow and parallel flow heat exchanger, F was taken as 1. And if we put that relation here, we will find that this relation is valid for the counter current and the co-current heat exchanger. But, as we are going to talk about a cross flow heat exchanger, we see that this will become less than 1.

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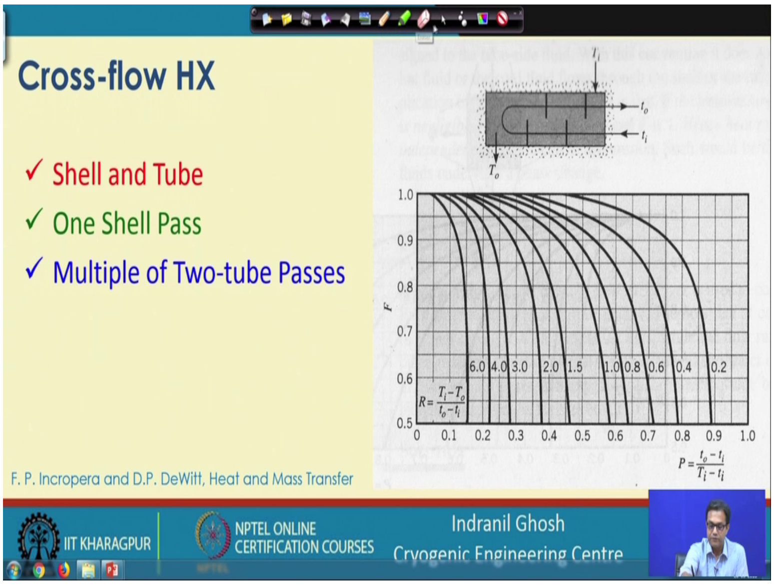

So, how to find out that relation? We will see that for different heat exchanger configuration, this has already been calculated for our ease. And this is related to the it is expressed in terms of R and P, two parameters which are related to the two temperature differences; here this is T inlet minus T exit, and this is the tube side that is T 0 minus T i, this is one ratio. The other ratio is T 0 minus T i divided by the maximum temperature difference that is T i capital T i minus small t i. This expression has been taken from this graph has been taken from F. P. Incropera and Dewitt Heat and Mass Transfer book.

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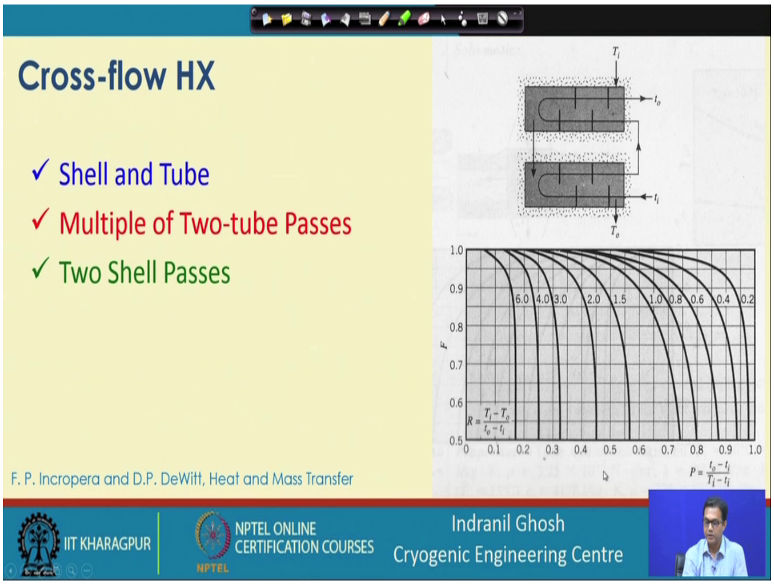

So, similarly if the geometry is changing from shell and tube, this is multiple to tube passes I mean it will be either two-tube or two-tube multiples, and there is two shell side; this is one cell side, another cell side. So, this is the expression for now this is the correction factor corresponding to that configuration.

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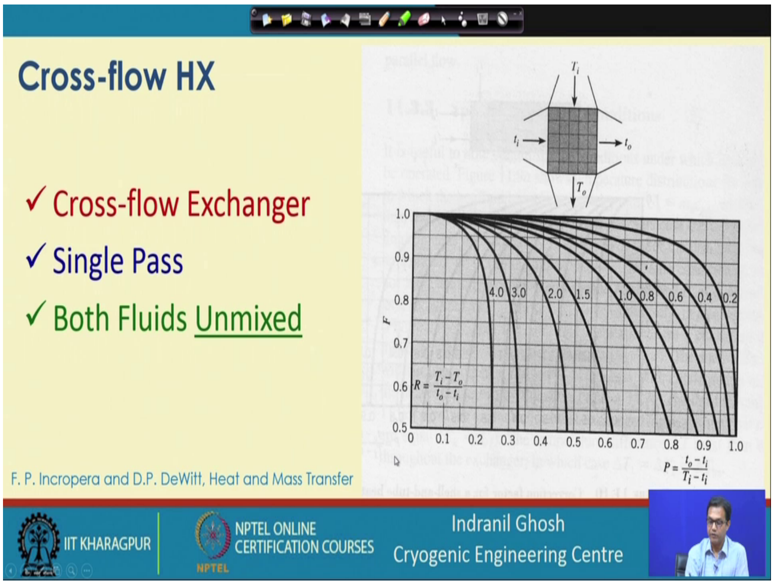

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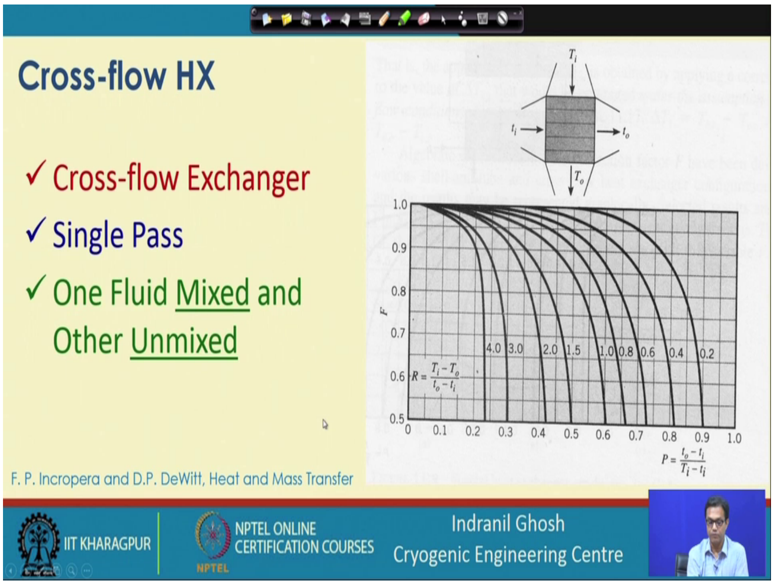

Similarly, the other configurations will be like when we have a cross-flow heat exchanger with single pass, where both fluids are unmixed or both fluids are mixed; one fluid mixed another fluid unmixed. This will correspond to a different correction factor. So, based on this values of the F factor, we have to add some correction to that LMTD method to solve the Q equals to UAF delta T lm, and then we can design the cross-flow heat exchanger.

Thank you.