Introduction to Fluid Mechanics and Fluid Engineering
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Lecture – 06
Viscosity (Contd.) and Surface Tension

We continue with our discussions on viscosity. So let us consider second example.

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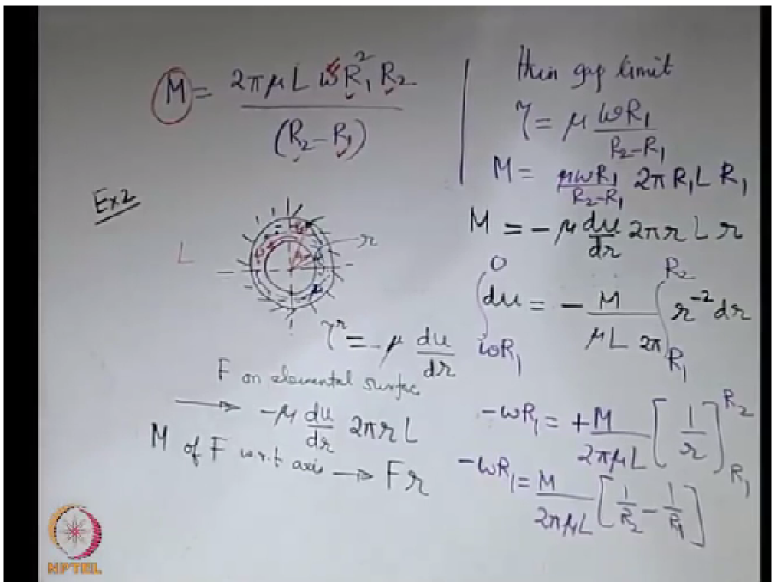

Let us say we have concentric cylinders and the speciality of this system is that maybe one of the cylinders is having a relative motion with respect to the other. So let us take an example where the outer cylinder is stationary, outer one is stationary and the inner wall is rotating with a particular angular speed omega. There is a fluid that is present in the gap between the 2 cylinders.

This kind of visual example that we have seen in our previous lecture. This type of example is important in many ways. I will give you 2 examples for application of this type of situation. One is if you consider an industrial application. In industry, there are shafts, cylindrical shafts which transmit power from say one point to the other. So think about the inner cylinder like a shaft. So that shaft is rotating and it is rotating but it has to be housed in a certain position.

At the same time, if you constrain it with a direct metal to metal contact that will give rise to a lot of wear and tear. So there is a lubricating material which is typically like oil which separates that from outer housing which sometimes is called as a bearing and the whole idea is to create a lubricating layer in between these 2 which prevents wear and tear because of the metal to metal contact.

So the shaft bearing type of arrangement in industry is very common and that is where you will find a lot of application of this kind. Even if you are looking from a more fundamental consideration of fluid mechanics, there is often a necessity to measure the viscosity of fluids and this kind of arrangement may be utilised to measure viscosity of fluids that is the case where it is known as rotating type viscometer.

Viscometer for measuring the viscosity and rotating type because of its particular nature of motion that you can easily appreciate. You will try to see that what is the physical situation that is going behind this type of example. When the inner cylinder starts rotating, it will try to move the fluid with it. Because of no slip boundary condition, the fluid immediately in contact with that will be rotating with the or will be moving with the same linear speed at different locations.

As you go radially outwards, let us say the inner cylinder has a radius of R1 and the outer has a radius of R2. So as you go from R=R1 to R=R2, what you will find? You will find that the velocity in the fluid goes down and the velocity is 0 at the outer radius, R=R2, that is also by no slip boundary condition. Now because of the presence of the fluid, the cylinder which is rotating, it is not rotating in an unhindered manner.

It is being subjected to some resistance. It has to overcome that resistance and maintain its motion. So it requires an external power to be imposed or so to say a torque to be there which is continuously rotating it overcoming the viscous resistance and let us say that we are interested to find out what is that torque or maybe power necessary to make it rotate with the uniform angular speed.

That is the objective of analysing this and therefore, if we apply that particular torque and if we see that it is rotating with the uniform angular speed that may be measured by something like a tachometer, then it is possible to relate these 2 in terms of the viscosity of the fluid. So everything else being measured from that expression we should be able to evaluate what is the viscosity of the fluid.

That is the basic principle by which one may measure or evaluate what is the viscosity of the fluid that is there in between. Typically, this gap is very narrow and we will see what is the consequence of that narrowness? Now let us say that we are interested about a section of the fluid. Let us say at a radius, at some intermediate radius, say R which is a local variable, smaller r.

Let us say that the length of the cylinder or both the cylinders is L which is perpendicular to the plane of the board and mu is the viscosity of the fluid which is occupying the annular space. So when we consider at a location R, we have an imaginary surface of fluid which is having a surface area of what? 2pi R*L. That surface of fluid is a surface on which there is relative resistance or there is a relative motion between the fluid layers, one is towards the inner and another is towards the outer.

Whatever is towards the inner, that tends to move faster. Whatever is towards the outer, tends to move slower. So that is a location where there is a shear stress that is present which is related to the rate of deformation. So if we want to write what is the shear stress at the radial location r or we should maybe use a superscript because it is not really tau with r as a subscript. Subscript meaning, we have preserved for something else.

So if we write this, then what would be its corresponding expression in terms of say Newton's law of viscosity? There is a mu, there is some sort of dudy type of term. So let us call it some du, du now we are using a coordinate of r and if we had used a coordinator of y, the only difference would have been that y is from the wall, solid from the 0 velocity wall, towards the inside, towards the inner one.

So that y is just oppositely directed to r. So whatever is dudy, is just adjusted with a -du here. There is no other difference. Because y direction is preserved for the direction which is from this

0 velocity to the fluid and this is the R direction is just opposite to that, that is why this minus sign is there to adjust it and you may think also in a different way. As you are increasing with radius, you are having a reduced velocity.

So this is negative. If you want to make it positive, you want to adjust it with a negative sign. That is just a matter of sign convention but we have to be consistent with the sign convention. Whatever we have followed till now, we will preserve that. So that is a shear stress. If it is a Newtonian fluid, then what is a shear force which acts on this elemental surface. Let us say dF is the shear force on elemental surface.

We have already identified what is the elemental surface, that is the surface of the imaginary fluid with the dotted line as its radial envelop. So dF on that one is -mu dudr. We may just call it F, there is no necessity to call it dF because it is not like an elementary small volume that or area that we are talking about. So -mu dudr*2pi rL. So this force is a tangential force, right. So this force is like typically you will have this type of force which is tangential to these elements.

So this force will have a moment with respect to the axis of the cylinder. So what is the moment of F with respect to axis? That is F*r, we are just writing it in a scalar form not bothering about the vector nature because the moment vector is perpendicular to the plane of the board that we can understand very easily. So this is something, now you have to understand physically what is happening?

There is a particular power that is imposing, a motor is driving this. That means there is a torque that is being input to the system and the same torque is transmitted across different fluid layers; otherwise, it will not be able to rotate with the uniform velocity. So what it means is that if you call this as say m then m is something which is a sort of an input and it is balanced with the resistance moment that takes place at various sections so that you have a particular number, a particular value associated with that and that is dictated by the input, input power of the motor.

So you have m=-mu dudr*2pi rL*r. So now you can separate the variables in the 2 sides. So you can write du=-M/mu L2pi*r to the power -2 dr, right. I am very bad in algebra. So whenever I am mistaken, please correct it. Now when you integrate this, you can get a sort of variation in mu. Remember one very important thing. This u is the velocity in the fluid. So it has a variation from the inner to the outer.

For the inner cylinder, within the cylinder, there is no variation in velocity because it is a rigid body. Of course, linear velocity is varying but angular velocity is the same. Now the outer cylinder also is stationary but in between there is a difference in linear velocity because the fluid is deforming. It is not a rigid body. So at the inner radius that is at R1, what is the velocity? Sorry R1 on the right side we should write.

So R1=the velocity at, this is omega R1 and at R2, this is 0. So very quickly, we can write that -omega R1=-M/2pi mu L, this is that minus sign with get absorbed. You can simplify this and write as say M=2pi mu L omega R1, then another if you just simplify this, another R1R2/R2-R1. If there is any mistake, please let me know, okay. Now so you can see that there is a difference between R1 and R2 and here you are actually having R1 square R2 in this expression.

But if you neglect the variation of or if you neglect the velocity profile variation from the inner to the outer and assume that the gap is very thin, so that it is a linear profile. Then what would be the difference in expression that you get? So if it is a linear velocity profile that is taken, that is the velocity is varying from omega*R1 from inside to 0 outside in a linear manner. If the gap is this, then that is valid.

So for in a thin gap limit, in a thin gap limit, you have the tau=mu*omega*R1/R2-R1, omega R/h, dy if it is a linear velocity profile, it is just the ratio of the change in the 2. So if this is the tau, then what is the moment? So M=mu omega R1/R2-R1, that should be multiplied with 2pi. Now because the gap is thin, you can write 2pi R1L 2pi R2 L or if you want to be little bit more accurate, may be R1+R2/2L or whatever it will not make a lot of difference.

Let us write may be 2pi R1L*R1. So you can clearly see that as the difference between R1 and R2 tends to 0, these 2 expressions lead to almost the same thing. So if the gap is narrow, then the second approximation will give you a very quick estimation of what is the situation and from this or the more involved expression, you can clearly see that if you now know what is the power input to the shaft, the power input is this M*omega.

So if that is known, that means M is known. The dimensions will always be known, so R1 R2, that means R1, R2-R1, L that will be known. Omega can be measured with a tachometer. So that can give you what is mu from this expression. So if you are having a careful experiment where you are having the proper estimate of the power input.

As well as what is the angular velocity at which this inner cylinder is rotating, it will give you some good estimation of what is the viscosity of the fluid that is there inside provided it is Newtonian and that is how you may estimate the viscosity of an unknown fluid, that is a fluid for which you do not know the viscosity. Let us consider a third example.

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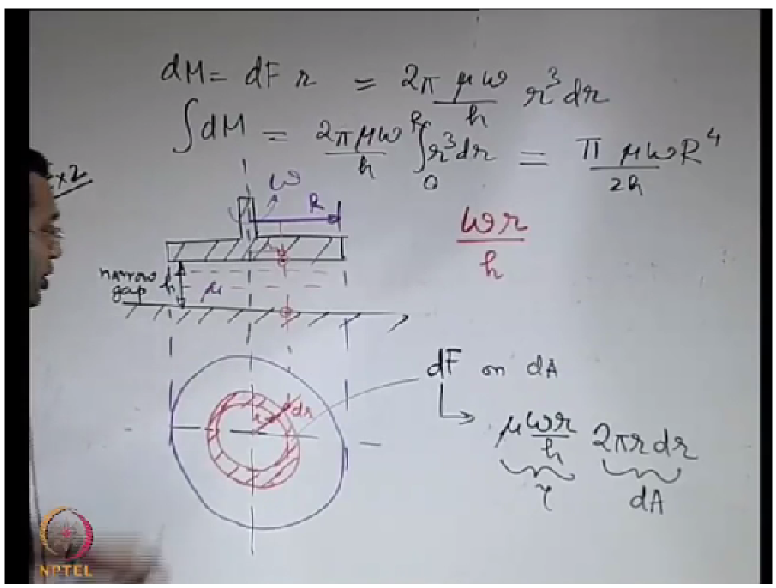

We consider that there are 2 plates, for example 2 circular plates. So these are the sectional views. If you draw the other view, this will be a circle. So if you draw the other view of say the droplet, it will be something of a circular nature. The problem is whenever I want to draw a circle, it becomes an ellipse. Whenever I want to draw an ellipse, it becomes a circle. So assume this is a circle, although looks more like an ellipse or maybe not even an ellipse.

So this is the other view of the plate. There is a fluid which is there in between and our objective again is to see that what is the torque or power required if we want to rotate this one with a particular angular speed. The bottom one is stationary. Situation is quite similar to the previous one. So we should be able to work it out quite quickly. We have assumed that this gap is narrow. The viscosity of the fluid is mu and let us say radius of the plate is, the top plate is, R.

Because the gap is narrow, obviously it is expected that we may approximate it with a linear velocity profile from the bottom to the top but a key factor here is that, that linear velocity profile is now radially changing. So if you consider say a particular radial section like this, here you have 0 velocity here and what is the velocity that you will here? Omega*local R. So let us say small r is the local r.

So omega*local r will be the velocity; therefore, the velocity gradient at a section r will be omega r/h and this is because of linear velocity profile assumption from the bottom to the top. So if you take different radial sections, this will be different. So you cannot, if this was a constant, we could have easily calculated the shear stress by multiplying whatever constant it was with the total surface area of the plate that is being exposed to the fluid but now this being a variable, we must take it as summations of constants over small, small elements.

And that is how or that is why we have to choose small elements and integrate over that elements. So we take a small element at a radius r of thickness dr. So whenever we are solving any problem, these are very common situations. Many times because of systematically practising problems, we are habituated in taking elements in certain cases, doing integration and so on but many times we forget why we are doing it.

And it is very important to keep in mind that why we should do it. So here since it is continuously varying, we are interested to obtain estimation for the shear stress or the shear force which is our objective and that shear forces is locally varying because the shear stress is locally varying. We should take a small element at least over which it is a constant. So within this dr, it does not vary significantly.

Therefore, it may be treated like a constant over dr. So if we consider this area, we can multiply the local shear stress with that area to get the local shear force. So what is that local shear force. So let us say dF we call as local shear force on this dA, what is that? So fast you like the expression for shear stress, mu*omega r/h, that is tau, *the area dA, so what is that area? 2pi rdr. So with these as the shear, elemental shear force, this elemental shear force will have a moment with respect to the axis.

So what is that elemental moment? This *r. So that is dF*r. So that will be 2pi mu omega/h r cube dr. So the total resistive moment should be the integral of this... from 0 to r, that will become pi mu omega/2h R to the power 4. Know why, why it should be 2pi r? No, no, no. It is distributed over the entire surface. So it is like, when you consider a radial location, it is not a point.

It is like entirely distributed and that distributed force has a moment with respect to the axis. So it is like over the entire element. You can think it even more fundamentally. Do not just consider the full 2pi r but consider a small angular element with between theta and theta plus d theta and between r and r plus dr and then if you integrate that from theta=0 to 2pi, that 2pi term has automatically been taken care of.

So you should not take care of it doubly by considering 2pi here also, okay. So this is a very simple expression but it again tells that like there can be situations of variable velocity profiles and those may be taken care of in this way. So what we will do? We will post you some assignment or homework problems on viscosity, maybe very much related to these types of problems or maybe slightly different.

And you will find that in the course website, the homework problems and maybe we will give you a deadline in the next class that when to submit those problems.

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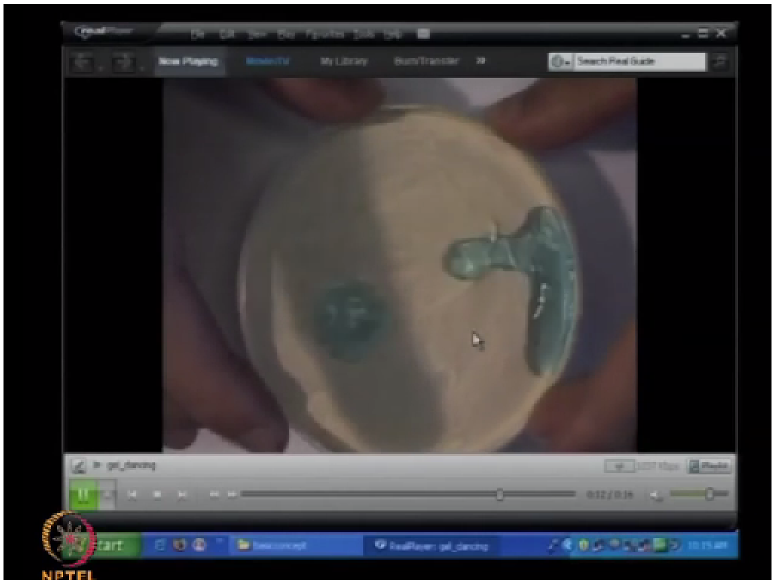

So we can sum up with our studies on viscosity or we will study more effectively viscosity later on in one of our related chapters that is equations of motion for viscous flows when we will learn viscosity effects more mathematically but just now we can sum it up to see that here there is some highly viscous gel and this highly viscous gel is being stirred and you can see that when it is being stirred, it tends to get broken and separated in parts.

So when it is doing that, of course it is a highly viscous gel. There is an important additional force that is coming into the picture which is making it to behave in that type of way and that force is nothing but a surface tension force. So the next fluid property that we are going to learn is surface tension and what surface tension can do and what it cannot do, first let us look into some images before we go on to the more mathematical description of the surface tension.

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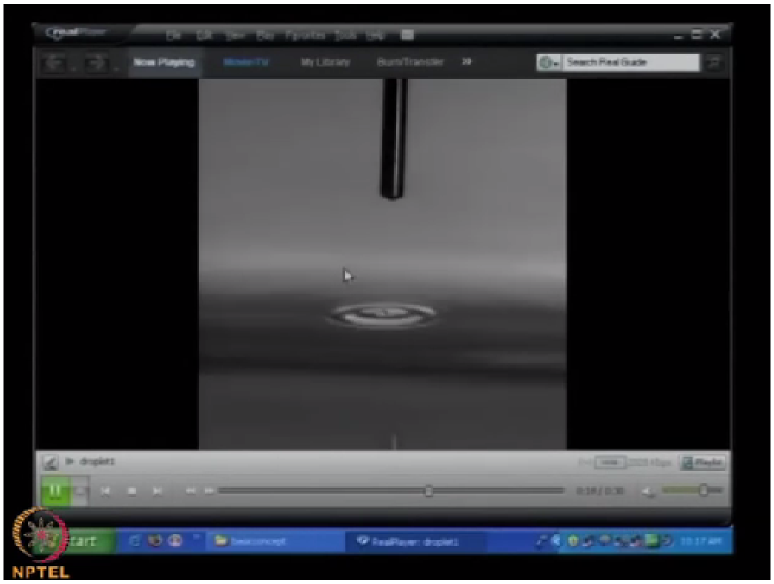

So this example is like you have fluid droplet which is interacting with another fluid and see that what type of behaviour is taking place and many times it looks like a magical behaviour and surface tension really can create magical behaviour. So it can create instabilities in jets and droplets and this type of instability is very very common. I will tell you that qualitatively why should you have...It seems you are liking this very much but I am not sure that you will be liking the mathematical details which go behind this very much.

So as a teacher my objective is to first addictive with this and then put a heavy dose of mathematics that goes behind. So that is what we are trying to do. So we will look into second example. Some of the examples later on may appear to be even better than what you saw earlier.

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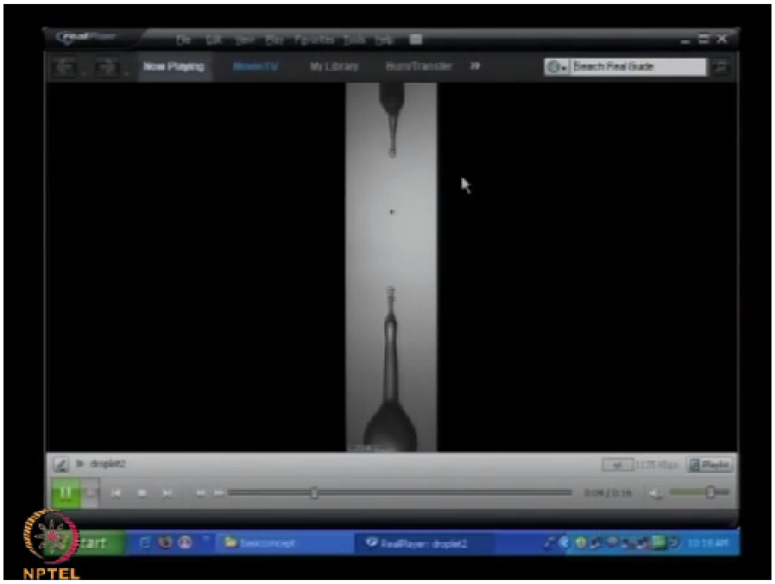

So this is breaking up or kind of making of a liquid column. So there is a liquid column. it Is getting broken up and the reason is that see what are the forces which are acting here. One is viscosity, viscous force, another is surface tension force, we will come across that and there are competitions between these 2. So if the viscous resistance is overcome, maybe surface tension driven instabilities can break it up or tear it up into nice pieces.

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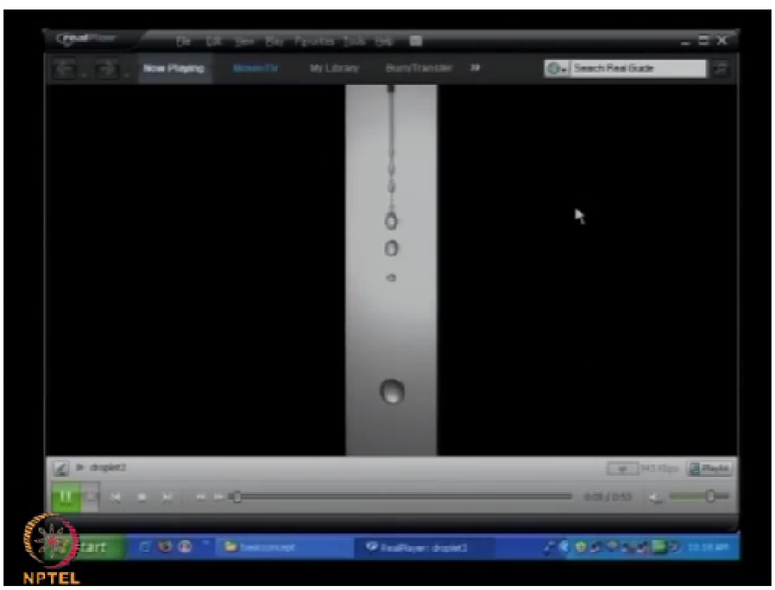

And the third example. This is like a train of droplets and you see the droplets are like, these are not like rigid spheres. So they are continuously deforming and you see the way in which these droplets are moving. So initially one big droplet and then you see that there are trains of droplets of different sizes and they are continuously interacting with each other. So at least you can appreciate that this is beautiful.

But this is very complex physical phenomena. It is not such a simple phenomenon. It is not like that there is a rigid ball falling from the top towards the bottom and fluid mechanics therefore is something which is fascinating but it is not as trivial as sometimes mechanics of simple particles. So we will see maybe one more example.

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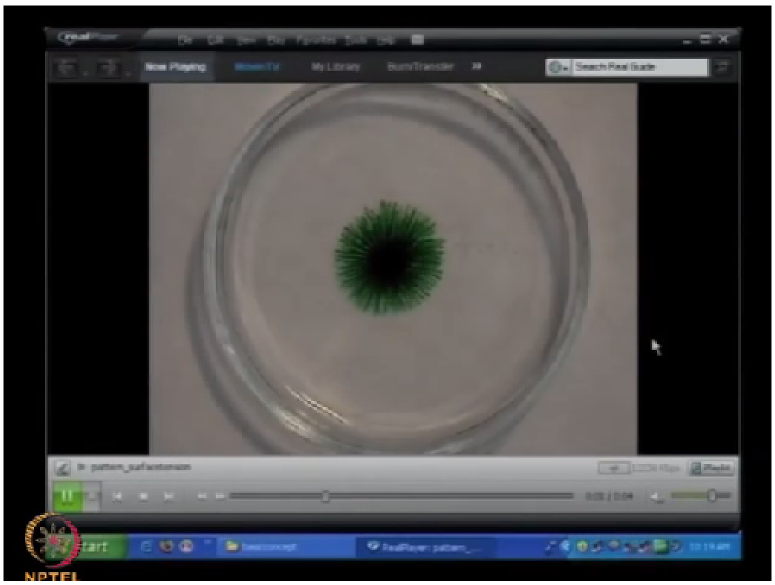

So you see the type of pattern that is being created by a die on a surface. So we will just play it once more to wrap this visual display up and go to see that what is the fundamental that goes behind, okay. So let us therefore with this motivation try to understand that what is the surface tension all about and what is its implication.

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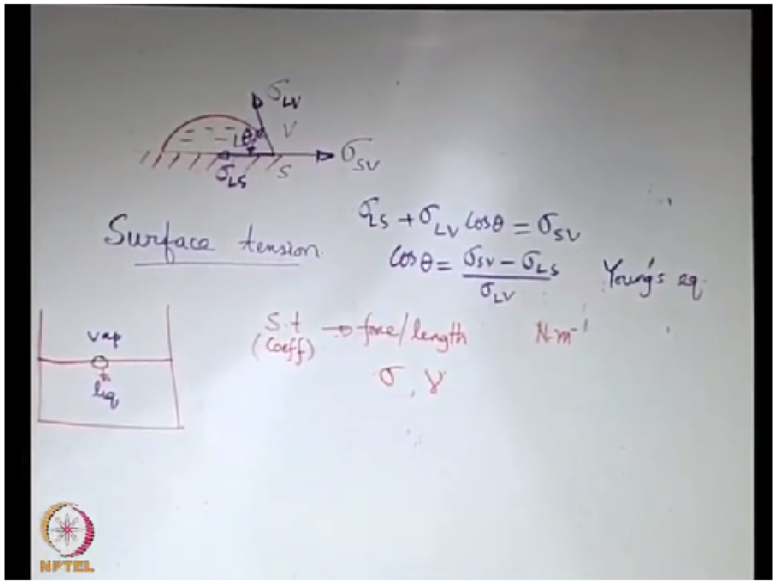

So we go to the description of surface tension. Again this is a very involved topic. We will try to develop very elementary qualitative feel of what it is about. Let us say that you have a container. In the container, there is an interface between so-called vapour and liquid, very common. You may have water and water vapour and liquid water and water vapour and there may be an interface.

Now what we are trying to do? We are trying to focus our attention on what happens to the molecules which are there at the interface. So let us identify some molecule which is there at the interface. Let us say this molecule is sitting on the interface and what are the forces to which this molecule is subjected. Let us try to investigate that. Clearly you can see that when there is a molecule at the interface, there are surrounding molecules and these surrounding molecules on one side, there is vapour, on another side, there is liquid.

Vapour is obviously much less dense than liquid and what you expect? You expect that which side will be pulling this molecule more, liquid side or vapour side? The liquid side will be pulling it more. So it is intuitively expected that this molecule will have a net resultant pull that is acting on it. When there is a net resultant pull that is acting on it, then there would have been a great chance that these molecules will get dissolved in the liquid but it does not happen like that because an interface is always form.

So there is something that makes sure that this interface is always form. What is that something that we will now understand. So one important concept that we can appreciate is that at the interface, there is a resultant force on the molecules which are there at the interface. Despite that resultant force, the molecule still holds its presence at the interface. That means it has some additional energy or effective energy by which it can, by virtue of which it can sustain its position at the interface by overcoming this net interaction and that energy is known as surface energy.

And whenever you have a surface energy, this surface energy is also manifested in form of a force because it appears that the molecules here are in a sort of a state of tension. So when they are in a sort of a state of tension because of this net pulling and pushing, that particular force which is responsible for keeping it in tension is also known as surface tension. So surface tension and surface energy are quite related.

And typically whenever we express surface tension which we express surface tension as force per unit length. What is that length? The length is the perimeter length on which this force is acting. So surface tension therefore is a force per unit length. So in SIs unit, it will be Newton per meter. So this is not the surface tension force but this is surface tension coefficient. So when we say surface tension, we loosely say surface tension but actually it is surface tension coefficient.

The surface tension force of course is this times the length on which it is acting, that is understandable. Typically, we use 2 symbols to denote this, either sigma or gamma, these are the common symbols which are used to designate surface tension. Now if we want to see that how the surface tension keeps a system in equilibrium, let us take an example of a droplet or a part of a droplet sitting on a solid surface.

So this is liquid and the right side, see this is vapour, so this is liquid vapour and this is solid. So you can see that at the interface between these, there is a triple junction that is created. You have a place where you have sort of contact between liquid, vapour and solid. Now if you want to see that what is the equilibrium that sort of keeps this in perspective or keeps this in equilibrium.

Then we can see that you have a force in this direction.

This force is because of the surface tension between or maybe let us just show in the opposite sense. Sense will automatically come out if we write the equilibrium but let us just show it in a opposite sense. Let us say that we will show it like this because just to appreciate that it is an element in tension. So this is because of the interaction between which 2 phases?

“Professor - student conversation starts” Yes? (()) (37:13) **“Professor - student conversation ends”** Solid and vapour. So let us give it a name. Let us give it a name, sigma SV, s for solid, v for vapour. So whenever we are talking about a surface tension coefficient, it basically talks about 2 different phases which are forming an interface and that is where the surface tension comes into the picture. If we are thinking of this one, this is between liquid and vapour. So surface tension is tangential to the interface and we can give it a name, sigma LV.

We could also write sigma VL, it is just the 2 phases which are important, not the order in which you write is important and regarding the liquid and solid, you have an interface here maybe sigma LS. There is an angle between these 2, say theta which is known as the contact angle. If you write the equilibrium along the horizontal direction, then you can clearly see that you can write sigma LS+sigma LV cos theta=sigma SV.

So when you are considering an elemental area or an elemental length, that elemental length is cancelled from all sides. So only the sigmas remain. It is basically a force balance but it looks like the surface tension coefficient balance because the corresponding length that is on which this acts, like it is maybe a unit length like that and that gets cancelled. So you can see that you get cos theta as sigma SV-sigma LS/sigma LV.

So if you know what are the surface tension coefficients between 2 phases taken at a time, then from that you can estimate the contact angle. This contact angle which is known as a static contact angle. Of course if it is dynamically evolving, then the contact angle will change and this is known as Young's equation. Just a simple equation that relates the contact angle with the surface tension coefficient at equilibrium. Now the question is, is it the only condition for equilibrium or is it something different.

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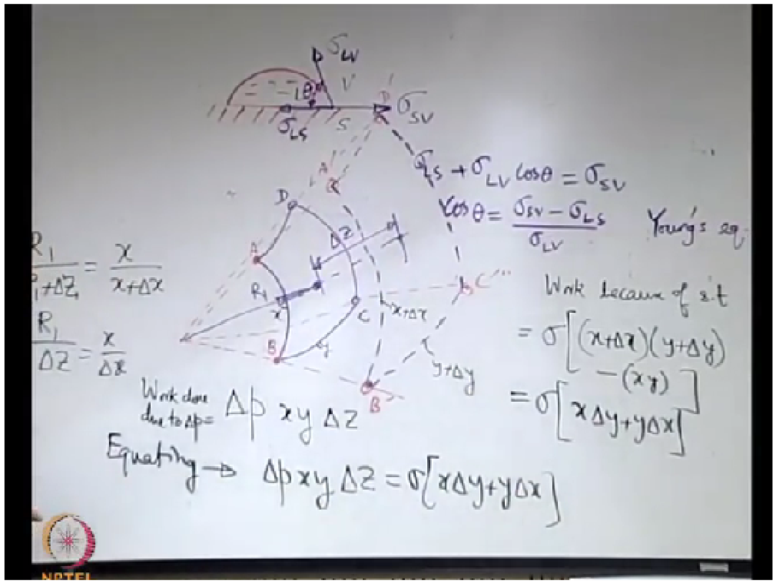

To understand that, we will try to consider a more invert situation when you have say a sort of element like this and say you are stretching this surface. This maybe a small element of the surfacing of a droplet and say we are trying to stretch it. So let us see that what are the kinematics or even the kinetics of this stretching. So kinematic aspect we will forget for the time being and we will just concentrate on the forces which are responsible for the stretching and the geometrical change which are responsible for that.

So to do that we will just draw some sketch. So this kind of thing we are trying to draw to see that as if there is a stretching which takes say the points AB to new locations A prime and say B prime. Similarly, if you draw the same type of radial lines, the other points which are forming the boundary of this, they will also go to different locations. So it is possible that you get a new location for the other points that is C and D because of the stretching.

So you can get a C prime and D prime. Let us tentatively try to locate some C prime and maybe some D prime. So we have a deformed element but maybe similar in terms of the geometrical characteristics because we have used the sort of stretching and it comes to a new configuration. You can clearly see that this element is made up of actually 2 different types of lines, one sort of parallel lines says AB and CD.

Another sort of parallel line like BC and AD. Therefore, we say that it has 2 different curvatures in 2 different planes. So this like AB and CD. Let say they have a particular curvature and AD and BC, they have a different type of curvature. So let us say that we are calling this radius of curvature as R1 one let us say that with respect to this R1, now there is a displacement. So when we say R1, we basically mean up to the centre.

So you have to imagine that this is like a surface which is a curved one as if this goes to the centre of this one and then from this, so maybe you can just stretch it like this and then from here to here, let us say that this is delta Z is the displacement. Let us say that originally the dimensions of these curved surfaces were like x and y. Now x becomes x+delta x and y becomes v+delta y.

So we can write from the similarity of the entire geometry that R1/R1+delta Z=y/, it should be x or y, you tell? It should be x/x+delta x because x is that dimension that is in the same sense of the radius of curvature R1. The other one will have a different radius of curvature R2. So we just concentrate on R1 and we can therefore write R1/delta Z=x/delta x. Now we are interested to see what are the forces which are acting on it.

Say what stretches it? Let us say there is a pressure differential between the outer and the inner of this. So this is a membrane. So the membrane has a difference in pressure from the outer and the inner and let us say that difference in pressure is delta p. When you have a difference in pressure of delta p, that gives rise to a force. What is the force that acts on an area xy? This is almost like a rectangle.

These are small elements because if you take big elements, the local radius of curvature change has to be taken into account. So this is just magnified for clarification but these are small. So delta p*xy is the resultant force because of pressure differential between the inner and the outer surface of the membrane and that *delta Z is the work done because of the pressure difference. So this is work done due to delta p and that contributes to the surface energy.

So this should be the corresponding work because of the surface tension. So what is the corresponding work because of the surface tension? That is the surface tension coefficient*the change in area. So the work done because of surface tension, let us say sigma is the surface tension coefficient between the 2 phases interacting here, that *the delta A. So what is the delta A? x+delta x*y+delta y is the new area, -xy, is the old area.

So this is basically work associated with stretching of a surface and this pressure differential is creating this displacement. It has undergone a stretching. The surface has some energy now to sustain that and retain its form. That is by virtue of the surface tension. So this will be x delta y+y delta x, delta x delta y, that product being small, we will neglect in comparison to the other terms. Now if we equate these 2, that is what we can write. So on simplification what will follow?

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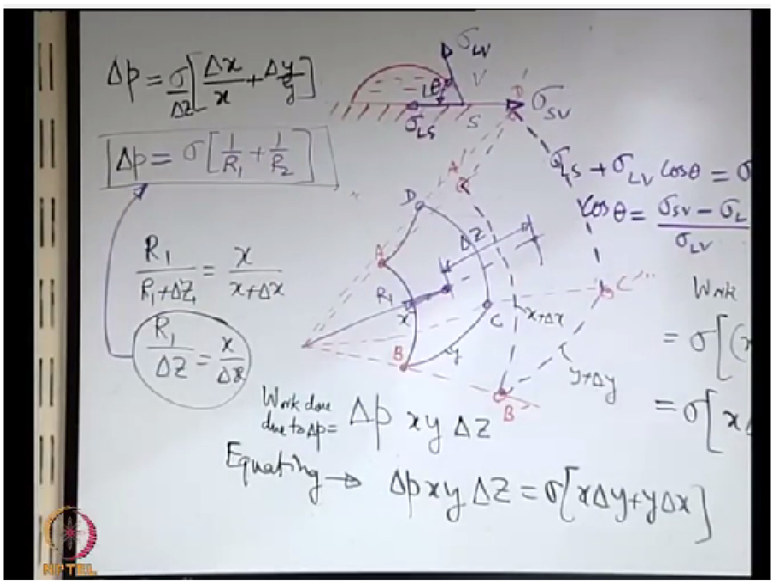

Delta p=sigma, you can write delta x/x, right, +delta y/y, this entire thing, there should also be a delta Z at the denominator, right. So now if you use this relationship, delta x/x is delta Z/R1. So this relationship can be utilised. Similarly, if you write in terms of y, it will be R2, that is the only difference, where R2 is the other, the radius of curvature for the other elements. So this can be simplified by taking help of this and similar expression as sigma*1/R1+1/R2.

So if you have an element of an interface in equilibrium, then this is how you can relate the pressure differential across that with the surface tension coefficient and the radius of curvature or the radii of curvature of the elements that are constituting the surface. We can take examples as special cases which is convenient examples to take. We consider first spherical droplets or maybe a spherical bubble. So if you consider a spherical droplet, then what is the situation?

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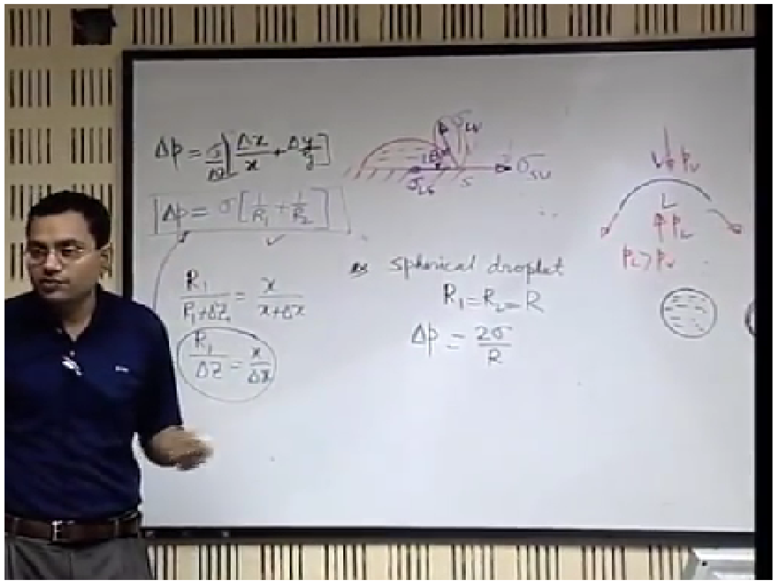

If you have a spherical droplet as an example. So when you have the interface as a sphere, it has same radius of curvature at all points. So for a spherical droplet of radius R, you have R1=R2=R and therefore, you have delta p=2 sigma/R. In case of this droplet, if you have a sort of a bubble say, what will be the difference between a droplet and the bubble? See droplet is like you have this full thing liquid and outside ambient maybe vapour.

If you have a bubble, so you have a thin layer of liquid here and you have something outside and something inside. So you have here say a vapour, then you have here also a vapour. So you have basically 2 vapour liquid interfaces to consider. One is as you jump from the inner vapour to this liquid line and then you jump from the outer liquid to the outside location that is the vapour. So you have 2 interfaces which are formed because of this one.

So maybe this should be multiplied twice to get the net pressure difference. So you can see that depending on the physical situation, this needs to be adjusted but this basically talks about that if there is one interface, across that interface if there is a pressure difference, then how can that pressure difference be related to the surface tension coefficient. Qualitatively if we try to understand that if we have say such an interference, let us say that you have vapour on one side, you have liquid on another side.

Now can you physically tell that on which side the pressure should be more if the interface looks like this, vapour or liquid? So if you think about the surface tension, it is a acting something like this. So when you have a pressure from this side, you have a pressure from this side say pL, you have a pressure from this side pV. Since the surface tension is already acting, is having its component downwards, so the upper component should be more strong to overcome that, that means pL should be > pV in the center.

So when we are talking about delta p, we are talking about the magnitude, the difference between pL and pV but out of these which one is more, that should come from your physical understanding of the problem. That is quite clear. The other point that we will mention here is that in a very nice way I introduced this equilibrium to you but you must have seen or if you have noticed it carefully, we have not really shown the equilibrium in a y direction.

It should come to your mind that yes very nicely we have seen the equilibrium along x but if you think about equilibrium along y, there is only a poor chap which has a vertical component. There is nothing else to balance it, then the droplet should go up, takeoff from the surface. You have never seen it just like taking it off like that.

“Professor - student conversation starts” (()) (54:20) **“Professor - student conversation ends”** We are talking about a surface, an interface, so it is an equilibrium of the interface when we are talking about it is not the weight but interaction, some interaction between the surface and the fluids, fluid molecules, which are manifested in forms of different intermolecular forces. All intermolecular forces are not together brought in the category of surface tension.

So on and affect, the net effect is there is a normal reaction, just like what you have as a normal reaction on a block on a plane. In a very similar way, what is normal reaction? it is a manifestation of some molecular scale interactions on a larger scale. So that type of normal reaction is also there, we have not drawn it here explicitly but that type of normal reaction is something that takes care of these type of interactions.

So we should keep in mind that this is one of the situations where you are having a statement of equilibrium, the normal component, we are not always keeping in view but that also has its role to play. So the 2 conditions of equilibrium, one is this one and of course, the other is the expression for cos theta that we have derived but we have to remember that these are necessary conditions for equilibrium but not sufficient.

That means that these conditions may be more fundamentally derived by minimising the surface energy of the system. So a system, any system in equilibrium minimises its energy. So if that is the stable sort of configuration. So if we express the surface energy and set out its partial derivative with respect to say R and theta to 0, then we will get the corresponding expression for equilibrium but these expressions do not automatically ensure that the second derivative is positive. That means it ensures 0 gradient but it does not ensure that it is the minimum.

Therefore, these are necessary conditions for equilibrium but not sufficient. So even if these conditions are satisfied, still you may get interesting instabilities in the droplets and bubbles and so on and some of those pictures that we have already seen. So maybe we wind up here for the day and we will just see one very small movie to wind up the study for the day.

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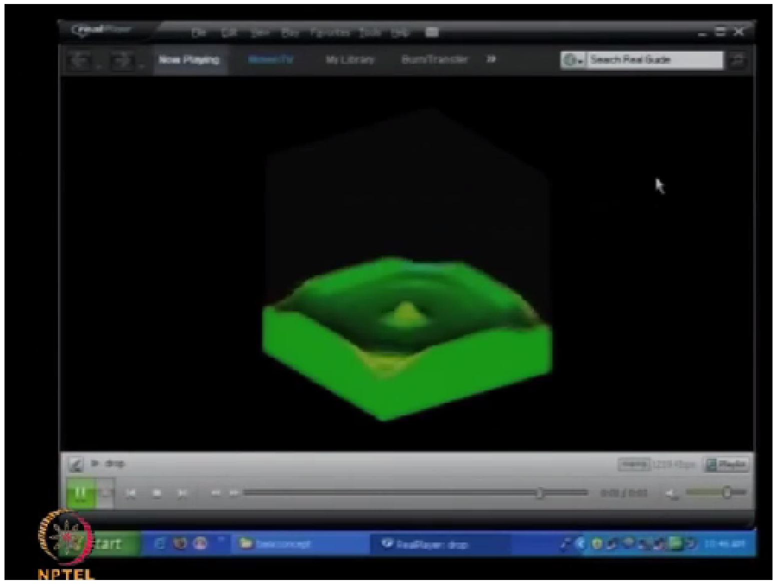

So we will just play it again and see that what effect surface tension is creating here and maybe that is enough for the day and we will continue with that in the next class. Thank you.