Introduction to Fluid Mechanics and Fluid Engineering

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Lecture – 05
Viscosity (Contd.)

We continue with our discussions on viscosity that we had in the previous class. So we were discussing about the nondimensional number, Reynold's number.

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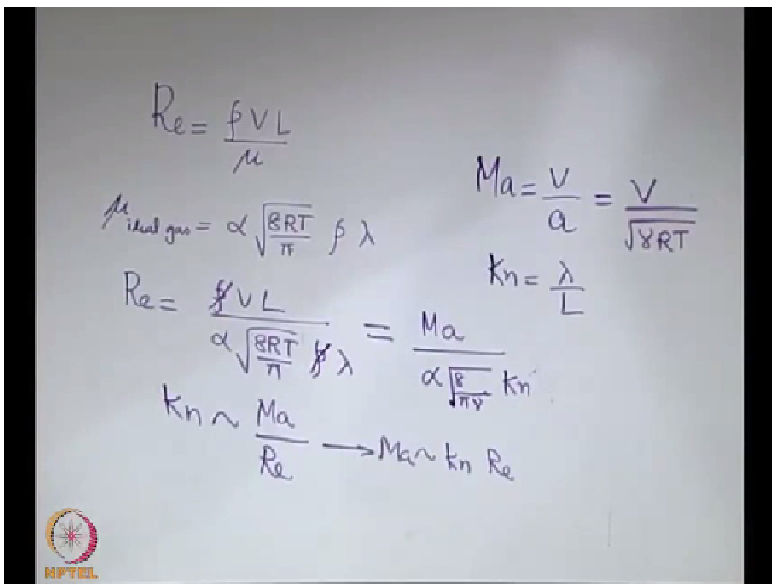

And this is what the expression that we got by sort of trying to relate the inertia force with the viscous force. In whatever problem, we are trying to see whether there is a relative dominance of inertia force over viscous force or not. This number is useful. Even in other contexts when the inertia force as such is not there that is the fluid is not accelerating, but it has some energy which would have been utilised to accelerate it to an extent.

Even in those contexts, that may be compared with the viscous effects which are present in the flow through this nondimensional number and this is a very important number. We will see later on that this will sort of dictate that what is the nature of the flow, is it laminar, is it turbulent and so on. These terminologies we will understand and appreciate later. There are many other nondimensional numbers.

Till now we have seen may be 3 non-dimensional numbers, Knudsen number, Mach number and now Reynold's number. Now let us try to see that can we develop a kind of interrelationship between these 3. Of course, it is not possible to do it for the most general case but perhaps for the most simple case, that is like the ideal gas. Let us see that whether we can develop a kind of relationship between these.

For that we recall that what was the viscosity for the ideal gas that we derived. Let us see. So it was some alpha*square root of 8RT/pi*rho*lambda. This was the expression. This alpha is typically fractional number like 1/6. So for this type of scaling estimation, its exact value is not so important. We are just trying to see sort of the nature of the functional relationship between these.

We substitute that mu in the expression for the Reynold's number. So what we get is rho VL/… So we can cancel out the rho from the numerator and the denominator. Then you can see that there is a group L/lambda which is 1/Knudsen number because here is the characteristic system length scale and lambda is the molecular mean free path. There is also a way to relate the other part of the expression that is V in the numerator and something varying square root of T in the denominator.

So if you recall, the definition of the Mach number, it is the ratio of the velocity of the fluid relative to the medium and the sonic velocity, that is velocity of disturbance propagating through the medium and for ideal gas, a can be expressed in terms of temperature. So how you can express it. May be in this form root over gamma RT where gamma is the ratio of the specific heat, CP and CV for gases.

Of course, this expression is not valid for all types of gases, only for ideal gases. Exactly whether it is square root of gamma R that may not be so important but at least the form is it is scaling with V/square root of T. So here if you try to substitute that form, so in place of V/square root of RT, we write the Mach number, then it should be adjusted with the coefficient alpha+adjustment because of the presence of this 8R pi, these types of terms.

So square root of 8R/pi gamma may be and only 8/pi, right, no R is there. R has already been adsorbed. So 8/pi gamma, then Knudsen number because lambda/L is the Knudsen number. So we can see that Knudsen number for an ideal gas will roughly scale with Mach number and then, so Mach number is roughly scaling with Knudsen number*Reynold's number. So if the Reynold's number is high that means the fluid is having a high inertia.

On the top of that, if the Knudsen number is high, then there is a high probability that it is having a great compressibility effect because it is trying to enhance the Mach number also and we have seen already that Mach number is a sort of indicator of how compressible the fluid is. So this is regarding gases. Of course again, this is not regarding all gases. This is just a special case of ideal gases.

If we go to real gases, the situation may be more complicated but at least what we can appreciate is that the viscosity of gases should try to increase with increase in temperature. The relationship is not as simple or straightforward as this one for real gases because for the difference between the real gas and the ideal gas in this context is straightforward. For real gases, you also have intermolecular forces of interaction and that needs to be considered for estimating the viscosity.

It is not just transfer of molecular momentum. So a combination of transfer of molecular momentum and the intermolecular forces of attraction for any real substance will determine that what should be the viscous nature of the fluid. If you come to liquids as we have discussed earlier that for liquids, the viscous behaviour is predominantly due to intermolecular forces of interaction.

At the same time, we need to appreciate that whenever we have qualifying something as viscosity for a liquid, that so-called viscosity even might not exist for a liquid because that is defined for a liquid only if it is a Newtonian fluid but there are many liquids which are not Newtonian fluids. Those are called as non-Newtonian fluids. The obvious connection with the name is that they are not obeying the Newton's law of viscosity.

And whenever you are having a fluid which is not following the Newton's law of viscosity, then its constitutive behaviour that is how the shear stress is related to the rate of deformation. It may be very complicated, it may be involve nonlinear function. We will not go into the details of how these nonlinear functions are derived or how these forms look like.

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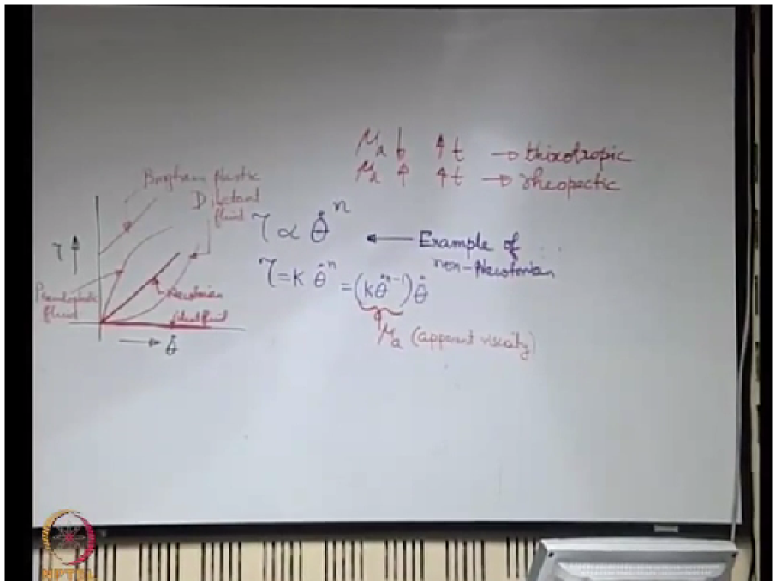

One of the examples is like you may have the shear stress is related to a power of the rate of deformation, that is theta dot, I think we used to denote the rate of deformation. So one may have these related to theta do to the power n. So this type of relationship is an example of a non-Newtonian behaviour and this type of specific example this is known as a power law of fluid, that means the shear stress is related to a power or index of the rate of deformation.

This index may be anything. I will give you an example if you think of blood typically. This index is close to 0.7. So this index is something which dictates the so-called viscous nature of this particular type of fluid again which obeys the power law. Not all Newtonian fluids, non-Newtonian fluids will obey the power law, neither blood is exactly a power law fluid. Blood is not exactly a power law fluid.

It has its constitutive behaviour much more complicated than that but if you just simplify as an approximation and try to express the stress and rate of deformation behaviour in that way, then that index will roughly be 0.7 for blood. Again it is not a constant, it is dependent on the composition of the blood and many other things. So this type of behaviour if it is existing. So you can write say tau=K theta dot to the power n.

You can also express this as some K*theta dot to the powers n-1*theta dot. The whole idea is a desperate effort to cast it in the form of Newton's law of viscosity. So if that is cast in this form, then if you know some local rate of deformation, from that this would be apparently like a viscosity. Of course, this is not really the viscosity because it is not a Newtonian fluid. So this sometimes is called as apparent viscosity, mu a.

Apparent viscosity is not really the viscosity in the proper definition sense but of course it has the same dimension of viscosity and it has a sort of similar physical sense as that of viscosity. The whole idea is that if you have such a definition, it is sometimes helps you to relate the behaviour of that complex fluid with that of an equivalent Newtonian fluid, that is the whole idea and that is why the name apparent viscosity.

It is not, never a true viscosity. Now if you try to make a sketch of how the shear stress relates with the rate of deformation for different types of fluids? Let us take some examples. Let me first draw 2 trivial examples which should be understandable to you very easily. Let us say we have this as one example and this as another example. So what special cases or special types of fluids these 2 represent? Yes?

This is Newtonian fluid and this is ideal fluid. So it is having 0 viscosity but in general, fluids may have behaviour which are different from these. Let us try to draw some examples here. One case may be something like this one, may be something like this. Even you may have something like this. So that is why if you take an example like this, this really puts a serious question into the definition of a fluid because it shows that there is something which is of fluid type but it requires a threshold shear to deform and there could be many such types of substances.

Think about a case that you have a toothpaste in a tube right. You need to press it to apply some shear before it starts moving out and with negligible shear, it will not flow. It will require a threshold amount of press before it starts moving. So that is also a fluid because it flows and many of its characteristics are explained nicely with the equations of motion of fluids but it is not that the classical definition that it will start deforming with infinitesimal shear.

So this type of example is a very interesting example but there are other examples which are more classical as fluids. For example if you consider this particular case. This is known as a dilatant fluid and this example is known as a pseudo-plastic fluid. By the way, this name is known as Bingham plastic fluid. These are names which have originated from the detailed studies of rheologies of substances and it is not so important that you have to remember these names.

But these are just to give you ideas that there are different categories of so-called fluids based on the shear stress and rate of deformation behaviour. So some of examples of these types of fluids, let us say dilatant fluid like water or printing ink. These types of fluids. These are known as dilatant fluid. So what are the characteristics of these. If you see that as you increase the rate of deformation, the shear increases with the rate of deformation.

And if you see that for a pseudo-plastic fluid, then also if you increase the rate of deformation, the shear increases, right but it comes to a sort of state like this. So the name pseudo-plastic comes from the fact that you may relate this type of behaviour with the plastic deformation of a solid so to say. So it is as if like there is some substance which is undergoing a plastic deformation as a solid goes.

So if you have say solid piece and you have heated it with a hammer where you good old forging processes. So then the material will be soft and that may be easily deformable. So a solid type of material will start flowing and in mechanics of solids, it is sometimes known as plastic flow. So it is not that a fluid is flowing but a solid is moving as if it is a fluid. So the critical demarcation between the fluid and the solid is often not there so to say.

And there may be examples of these types of fluids like pseudo-plastic fluids as say polymer suspensions. So if you have suspensions of polymers. So if say there is a system in which there are aggregates or there are chains. Now it is possible that there are 2 things which are possible. One is if you are applying a shear, then the aggregates or the chains of particulates, they may be broken.

So you are having a system in which you have some liquid type of substrate in which you may have some particulate inclusions and they have form aggregates so to say. Now you are applying a strong shear. If you are applying a strong shear, those aggregates may be broken and it may give rise to the availability of new flow passages. So it may sort of help the fluidic motion. So viscosity is sort of opposite to fluidity, that means if it allows it to flow more easily, we intuitively understand it is less viscous.

On the other hand, sometimes it may be possible that because of formation of local aggregates and so on, the movement of the fluid ensures that fluid elements interact within the local aggregates and then they are in great entanglement and they cannot come out of those entrapment and flow. So different substances are therefore different. In some cases, it happens that because of this type of entanglement, the flow cannot take place so easily.

In certain cases, these aggregates or chains may be broken and it may help the fluid flow to take place in a much easier way. Obviously, we will not go into the details of the non-Newtonian fluids because that is not within the scope of what we are going to study as a part of this elementary course but at least we understand that there may be different types of the shear stress versus rate of deformation behaviour depending on the constitutive nature of the material of the fluid that we are looking for.

These expressions like the expression for the apparent viscosity will hide another important thing. Sometimes this many functions of time, that is you may have apparent viscosity increasing with time or you may have apparent viscosity decreasing with time and whether they are going to increase or decrease with time, that is going to be strongly dependent on again that how the rheological distribution or how the rheological property of the material is going to influence that.

So there are cases when the apparent viscosity will decrease with increase in time. So you may have the apparent viscosity decreases with increase in time, that type of fluid is known as thixotropic fluid and if the apparent viscosity increases with increase in time, that is known as a rheopectic fluid. At the end, it is a physical feel that how the fluid will look like and how the fluid behaves in presence of a strain.

So let us look into may be 1 or 2 examples with movies to understand that what type of flow behaviour we are going to expect for different fluids. So let us see may be one first very qualitative example.

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So we will see some examples which will tell us that like, I mean, these are not very classical fluids but these are like, it is deforming with a shear and sort of it as if it is a thin film that is being spread on a surface. There are many fluids which are also like thin films which are spread on a surface and there is no reason to disbelieve that this is also a fluid. Of course, we can clearly understand that this is an example of a non-Newtonian fluid. It does not obey the Newton's law of viscosity.

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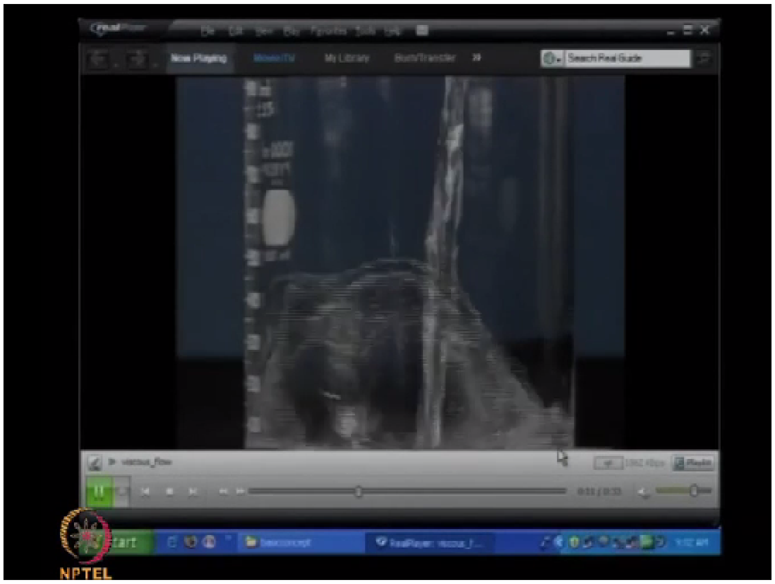

Now to get a qualitative idea of what is the difference between a highly viscous flow and not so much of a viscous flow. So let us look into these examples. So in one case, it is like water is being put or poured on in one of the beakers and see the other case.

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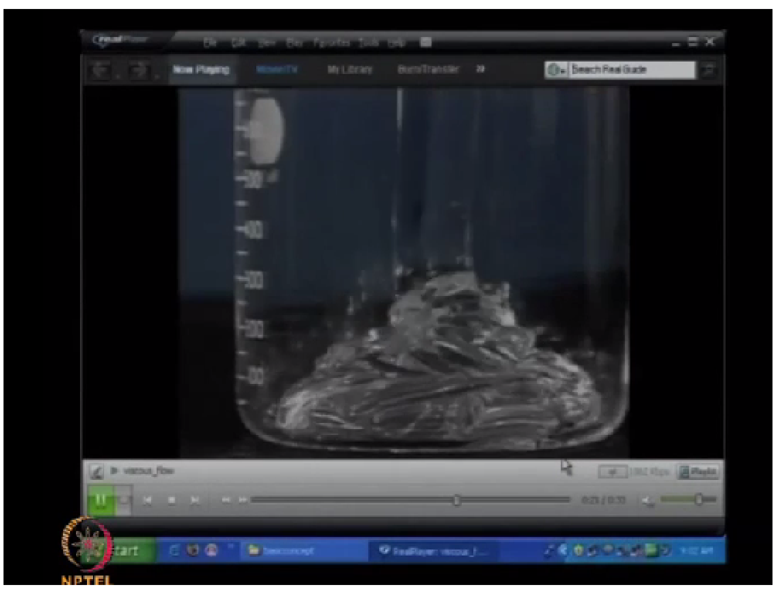

So this is also a fluid that is being poured and you can clearly appreciate that this is something which is of high apparent viscosity. So this type of qualitative feel is somewhat important and we can clearly recognise that the statement that it has something to do with like inverse of fluidity or flowability so to say. Now we will look into some bit more scientific way of looking into these effects of viscosity.

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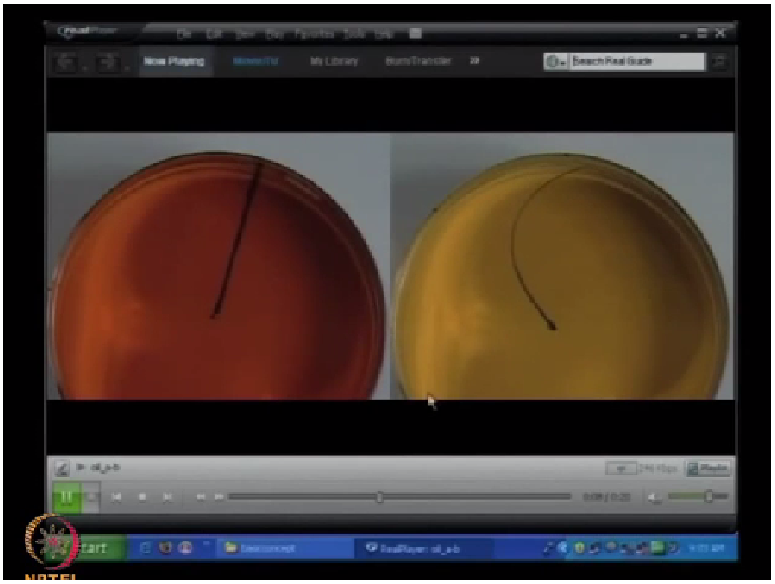

So let us look into one example or rather 2 examples shown side-by-side. These are two fluids with 2 different viscosities. There is a line, coloured line which is moving with the shear and that is a line which sort of represents the deformation and we just see it again. Hopefully if it works, yes. Now can you tell from these 2 examples just qualitatively, whether the red one has more viscosity or the yellow one has more viscosity.

“Professor - student conversation starts” The red one. The red one has more viscosity. How, do you understand that. (()) (23:33). **“Professor - student conversation ends”** Say you are still applying the similar type of influence or similar type of motion actuator for these 2. But if you see, the red one, let us play it again and then understand. See in the red one it responds to the change of momentum almost throughout, right.

And it is so particular about responding to the change of momentum that it follows the change of momentum exactly, how, it is almost like very rigidly. On the other hand, in the second case, it is not doing like that and there is a region where it actually shows a kind of lead or lag depending on whether you are describing the fluid or describing the bounding solid which is making the fluid move. So it may be either a lead or a lag, it depends on how fast or how slow these movements are taking place. The first case, there is no such lead or lag.

And it is like the entire fluid is feeling the effect of the disturbance and getting adjusted to that.

So with the high viscosity only, that is possible and when you have high viscosity or low viscosity, we have the characterizations to the Reynold's number and let us see a particular case, say a low Reynold's number case.

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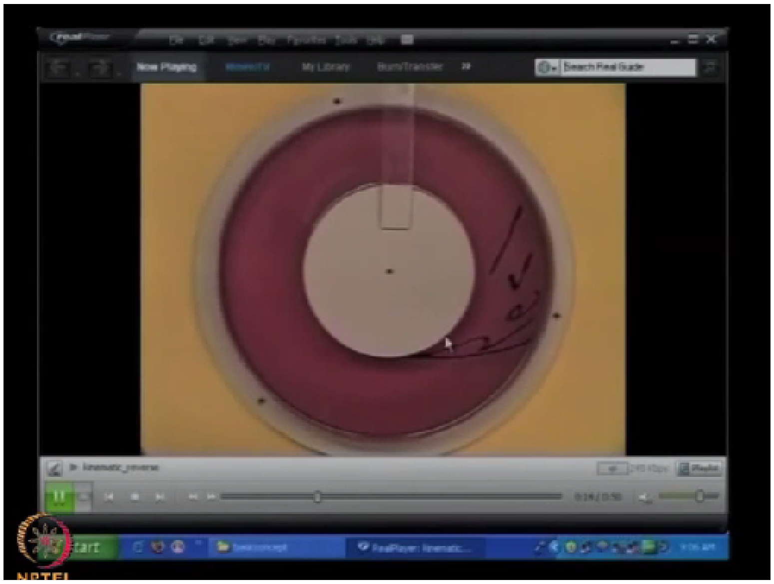

So we are just trying to now give a bit of quantification to what we have observed till now. Say it is a very low Reynold's number. See what happens? These type of arrangement we will work out may be one example to illustrate that what it is. Sometimes this type of arrangement is used to measure viscosity of fluids. This is called as rotating type viscometer. Just like any meter is for measuring things.

So this type of device may be utilised for measuring the viscosity of a fluid kept between 2 concentric cylinders in the annular space. Of course this example is not for illustrating that measurement but if you see now the direction of rotation has got reversed and because of a low Reynold's number, what it ensures is that viscous effects are very strong, so it adjust to the change so nicely, again if you see that the marker Reynold's number < 1 has come back to its original shape.

So it is a perfect adjustment to the change. Let us look into another example of…**"Professor - student conversation starts**" Yes. (()) (26:31) see inertia force as I told you that it is not exactly always the inertia force as such. If it is not accelerating, it is not inertia force but it is having some kinetic energy which if could have been utilized to accelerate it, it could have given rise to some inertia force. So when we are having low Reynold's number, keeping that effect unaltered, the low Reynold's number, viscous effects are stronger and stronger.

So when we are comparing 2 cases, the other effect which is present in the numerator is almost something which we are not disturbing but we are trying to see is that what is the relative change in the viscous effect in the 2 cases. So if you see this example now it is being reversed and it does not come back to the same situation. If we play it again, we can see that it does not come back to the same state with which it started.

So we are having a particular deformation, the fluid is trying its best to adjust to the deformation by propagating the momentum disturbance. That propagation of momentum disturbance is something which is not as efficient as in the highly viscous fluid. So when it comes back, there is always a lag in that propagation of the disturbance and imposition of the disturbance and therefore, it is not possible that it entirely reverses back its state.

So this is a qualitative feel of 2 different cases. In one case, you have a highly viscous flow and in another case, not such a highly viscous flow. Yes? (()) (28:23). Always whenever we are talking about forces, we are comparing. So when we say that viscous effects are important, we have to see that important in the context or in comparison to what? So as we have mentioned in our initial discussion, then when we talk about the smallness of a dimension, we always ask a question.

It is small with respect to what. Like if you have a 1 millimeter dimension, to me it may be small, to somebody, it may be very large because it is very large in comparing to the atomic length scale but may be to me, I am more happy in thinking about kilometers and then I will say that yes, 1 millimeter is very small in comparison to 1 kilometer. So whenever we are talking about smallness or largeness, we are really making a comparative assessment.

And therefore, it is not just like a viscous force that is there, maybe there is some other force also which is competing with it and always whenever there is a system in a sort of dynamic equilibrium type, then there are competing forces. Otherwise, the effect will perpetually grow. So there are competitions and the understanding of mechanics is just to understand how this competitions are working in a system. So there are always competitive forces and we will see that how these competitive forces are important. **“Professor - student conversation ends”**

So we will not go too far looking into these examples and now let us move on to some of the typical examples in terms of problems, not just qualitative examples but where we try to illustrate the concept that we have learned through example problems.

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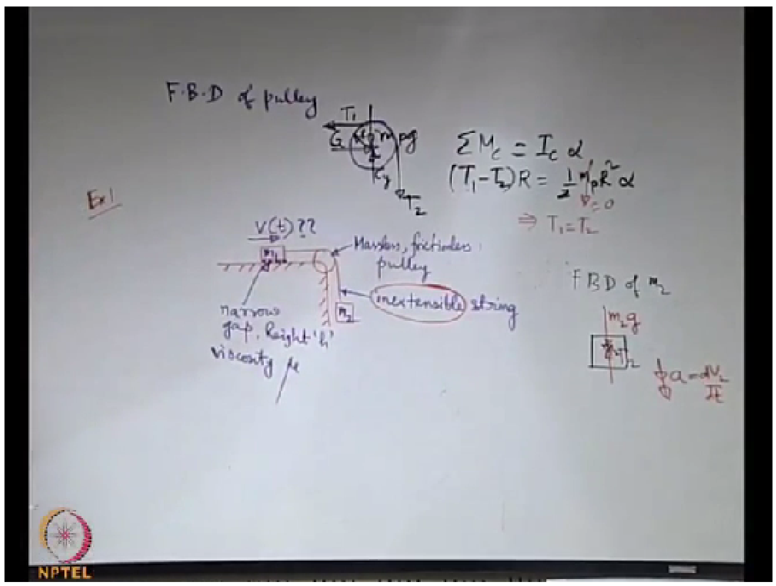

So let us consider an example 1. Let us say that you have system with mass and pulley and swing, the kind of system that you always love to deal with in mechanics problems. So we start with such an example because it will have a good transition from your earlier studies in mechanics to the mechanics of fluids. So where we make a change is like we put some fluid, say oil in the narrow gap between the block which is there on the plane and the plane.

So this qualitatively is a narrow gap of say height small h. Let us say that mass of this is m1 and the one that is hanging is m2. We make father statements that this pulley is mass less and frictionless and also this string is inextensible. Assume that the fluid which is there in between the block and the plane is Newtonian with the viscosity mu. Our objective say is to find out what is the velocity of this block of mass m1 as a function of time.

So this is the question that we would like to answer, okay. This is a very simple problem but we will try to go to as much fundamental depth as possible for such a simple problem. We may start with the expression and you will see the expressions are very intuitive. We may write those easily but we will start with a free body diagram of different elements which are present in the system to have a more thorough insight on what happens.

Let us draw first the free body diagram of the pulley, okay. Most of the times, you will never draw it but if required, we should. The pulley is hinged to this surface. So the centre of the pulley at the centre, there is a hinge. Now what should be the forces which are acting on this. Normal reaction if you say, normal reaction by whom to whom. **"Professor - student conversation starts**" (()) (33:59) Okay, one by one. **"Professor - student conversation ends".**

So first we are talking about the hinge. So what type of reaction you expect for the hinge. So there are contact forces. It is something which is occurring in a plane. So in general, we will be having general force in a plane which may be resolved into 2 components. So let us say that we are talking about 2 components, Cx and Cy which are like arbitrary orthogonal components of the forces which are there at the hinge.

Then tension in the string. So here you have some tension, T1, here you have some tension, T2. For the time being, let us forget about these assumptions which have been given, mass less pulley, frictionless pulley inextensible string and then we are not exactly using the consequences of these in drawing the free body diagram. So in the free body diagram, we are keeping everything as general.

So let us consider that the pulley might have a mass also. So just to see that if it has. Of course, it has a mass. When we say mass less, we do not mean that it is literally mass less. We mean that effect of that is not significant, that is all. So whether the effect of mass is significant or not, how will we understand? These types of statements are very important, like mass less, frictionless, inextensible and so on.

Most of the times, you think that these are for beautification of the problem statements and at the end, you come up with a conclusion which sort of abstracts you from the path by which you have come to the conclusion. For examples if we say T1=T2, right. We will of course see T1=T2, all of you have learnt from your mechanics that it should be like that. Now can you tell that whether T1=T2 is because of mass less pulley, frictionless pulley, inextensible string, or what?

"Professor - student conversation starts" (()) (36:25) see now I am getting 3 different answers from… So like if it is multiple choice question, mass less, frictionless, inextensible, all the above, none of the above, or many choices are given to you. **"Professor - student conversation ends"** Let us see which one is the fundamental and which one is dominating. So if you think that the mass less is the thing that should be put in question.

So let us consider that it has a mass and see that what is the consequence. It will help us in understanding that what would be if it is mass less. Let us say that mp is the mass of the pulley. So it has its weight also, okay. Now what we do is, we want to get an expression between T1 and T2. So if we take moment of all forces with respect to C, the 3 Cx Cy and mg, these get cancelled. It helps us in obtaining a relationship between T1 and T2.

So what is the basic equation that we are looking for? Resultant moment of all forces with respect to an axis passing through C perpendicular to the plane of the board is the moment of inertia of this pulley with respect to an x axis which is the same as axis with respect to which you have got the moment * the angular acceleration.

So if say capital R is the radius of the pulley, then you can write this straightaway as T1-T2*R=, if it is like the pulley is like a disk as an example, say we call it 1/2MR square… So if the mass of the pulley is neglected, then automatically it will give rise to T1=T2, right. So it does not matter whether it is frictionless or not, so far as these goals. Is it a totally correct statement, now another question I am asking you?

You should try to understand it through a contradiction, say you have encountered cases where you have a pulley with a string or a belt around that and there is a difference between these tensions and the difference is given by say if it is an impending slip, T/T2 is E to the power coefficient of friction * the theta, angle of ramp. This you have learnt in the statics. So now here we are seeing something different, right.

So what is the anomaly? You think about it, I will ask you next time. Let us continue with this problem because for this particular problem, that is not going to be important. So much I can tell you. Now we come to say the free body diagram of m2 may be. So you have the weight, then you have the tension which is T2. Of course T2=T1 and then if the system is released, what would be the direction of motion that you expect?

So this is coming downwards and this is expected to go towards the right. So understand the physics of this problem. When it is moving downwards, it is tending to go towards right. There is a resistance at this interface which does not want to make it go towards the rights so easily. So it is somewhat like a friction and that friction here is sort of lubricated. It is not the direct contact between 2 solids but there is a thin film of liquid in between.

So that means if that is the case, then if it comes down, let us say it comes down with an acceleration a. So if this comes down with an acceleration a, which is you may say this a is like dV2dt. So when it comes down with an acceleration a, you expect that m1 also moves right towards right with an acceleration a. Again by which assumption, it is there. **“Professor - student conversation starts**” (()) (41:43) Inextensible string. **“Professor - student conversation ends”**

So inextensible string had a role to play here. Mass less pulley we have seen had a role to play here and yes frictionless pulley also has a role to play here because the belt is being wrapped around the pulley and because of friction, there may be a difference in the contact forces between the belt and the pulley. So that will be more apparent if you draw the free body diagram of the belt and see its interaction with the pulley.

So when you are thinking of interaction between the belt and the pulley, that is not considered here and that is not considered implicitly because it is frictionless, so important interaction is through friction. Normal reaction will always get nullified even if it is considered because you are taking moment with respect to the centre. So all these have been used in some way or the other. Let us draw the free body diagram of m1 which is the important matter so far as the understanding of viscosity goes. So if we draw now the free body diagram of m1.

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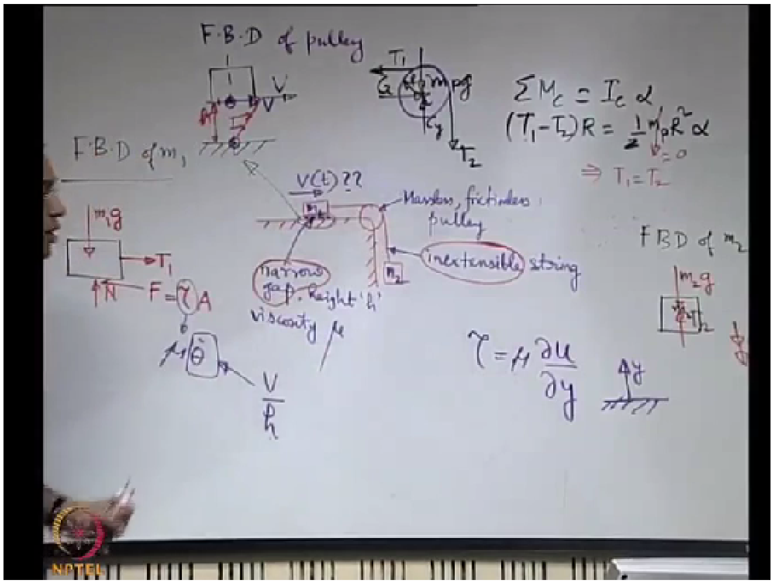

Now you tell what are the forces which are acting on m1? Yes, one is T1. There is a normal reaction to the… What is the origin of these normal reaction? This is not a direct contact between the solid boundary and the block. Yes? There is some fluid is there and there is a pressure distribution at the interface between the fluid and the solid. So the resultant of that pressure distribution gives rise to this normal reaction.

Of course the same normal reaction we have drawn but we have to understand that physically where it originates. Then there is some resistance also. So what is the resistance? There is let us call it F just with analogy with the problems involving standard mechanics of solids. So this F is sort of friction force but here again, the origin is not the direct contact between the plane and the block.

So what is origin of this friction force? Viscosity, right. So it is viscosity or viscous effects at the interface between the block and the fluid. So that should be expressible in terms of the shear stress and the shear stress is expressible in terms of the rate of deformation through the Newton's law of viscosity because we are assuming it is a Newtonian fluid. So what we expect here is that this should be some equivalent shear stress at the interfere*the area.

So let us say that a is the bottom surface area of the block. So that is another input to the problem. Now only what that is left is to relate the tau with the rate of deformation. Now the rate of deformation is something which takes place where? Which takes place if you look at the magnified view of what happens in this thin film. So in this thin film, you see that there is a solid boundary at the bottom which is stationary.

At the top, there is a block that is moving. So this is the block. So this block is moving with a particular velocity and this is like this block is idealised as a particle. So when you idealise sort of rigid body as a particle? When you do not have rotational effects. The fundamental difference between a particle and a rigid body is, a rigid body has rotation or it is capable of having different rotational components but particle cannot rotate.

So this of course we are not considering that this could rotate in this type of situation. So it is just like a particle or point mass. So everything is moving towards the right with a particular velocity which is changing with time. So let us say that this has the velocity V. So when this has a velocity V, of course that is the function of time that you have to understand. Now if you draw the velocity profile at any section which includes the fluid between the block and the plane.

At the wall, because of no slip boundary condition, it is 0. Here what should be the velocity of the fluid? It should be V. That is also because of no slip boundary condition. So no slip boundary condition is not 0 velocity of the fluid but 0 relative velocity. So if the solid moves with that velocity, fluid will also move with that same velocity. Now see the catchword. Again a very nice catchword is there.

There is a narrow gap. So narrow gap means this thickness h is so small, it is so small that this variation of velocity between 0 to V, may be assumed as linear, okay. Now if you see that this shear stress, let us now complete the expression for the shear stress. It is mu*the rate of deformation. Rate of deformation could be different at different y locations but if it is a linear profile, it is same everywhere because rate of deformation is related to dudy, that we have seen.

So rate of deformation in such a case is dudy, where u is the velocity. So if u versus y is the straight-line, dudy is the constant. So this for our present case will become V/h. So remember what are the approximation or simplifications which have led toward this. If this is not a narrow gap, it has to be what is the rate of change of mu with respect to y at the interface between the block and the fluid, not at anywhere but since now it is a straight-line, it is as good as it is anywhere.

So the very important understanding is we are using that tau=some mu, it is better to say that it is a partial derivative of u with respect to y because u could be function of many other things. Here of course u is a function of time but no space coordinate. So still it is like a partial derivative. If we write dudy which we wrote for the first time when we wrote such an expression, just for simplicity in understanding but it is in general a partial derivative.

It assumes that there is only one component of velocity and the partial derivative comes from the fact that that one component of velocity itself could be a function of many things, x, y, z, time like that and it could itself be a variable and what is y? The important thing to keep in mind is that if you have a solid boundary, y is the coordinate which is normally directing outwards from the solid towards the fluid.

So y is a genetic thing. It is not that any y-axis you define, it is that dudy. So this y has a special meaning. This y is the axis which is normal to the surface into the fluid that we are considering. So if you are having a different type of coordinate axis, then you have to adjust this with a + or - sign. So that you have to keep in mind that this is the orientation that we had considered for describing the Newton's law of viscosity. That sanctity should be preserved.

With that understanding, now we write the equations of motion by Newton's second law which should be straightforward. So for the block, m1, of course along y direction, it is equilibrium. So n=mg that is not a part of the requirement of the problems.

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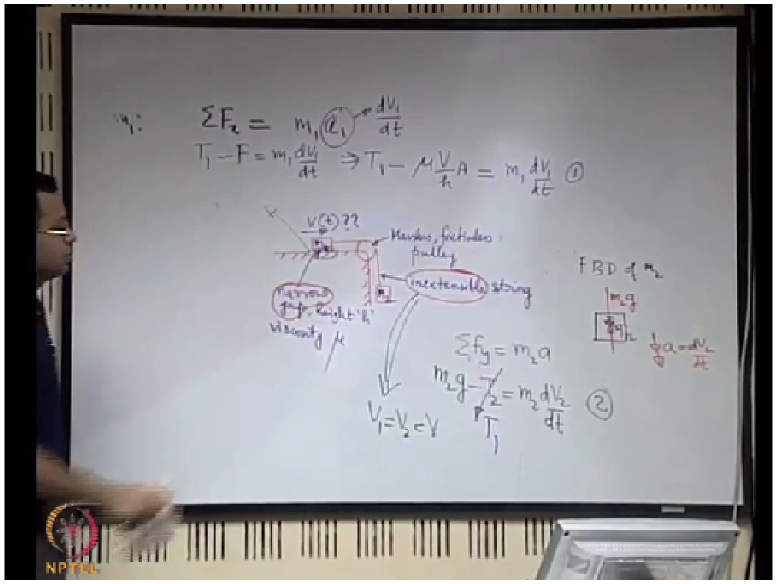

So we are just writing the resultant force along x for m1=m1 * the acceleration of a that is a1 that is nothing but dV1dt. So resultant force along x is T1-F. Since it is an inextensible string, we will have V1=V2=V. So very soon we will write in place of V1 as V or V2 as V where V is the common velocity with which the system is moving. So you can write T1-mu V/h*A=m1dV1dt. Let us say this is the equation number 1.

We write next the question number 2 which should be for the mass m2. So let us draw the free body diagram of the mass m2, that is already there, so we just write its equation. So it is now along y we are considering. So this is coming down. So m2g-T2=m2dV2dt. From the inextensible string, we can write V1=V2=V and we have already seen T1=T2. So if we call this as equation number 2.

In equation number 2 we have this T2 same as T1. So we can get what is the expression for T1 and substituting equation number 1, let us do that.

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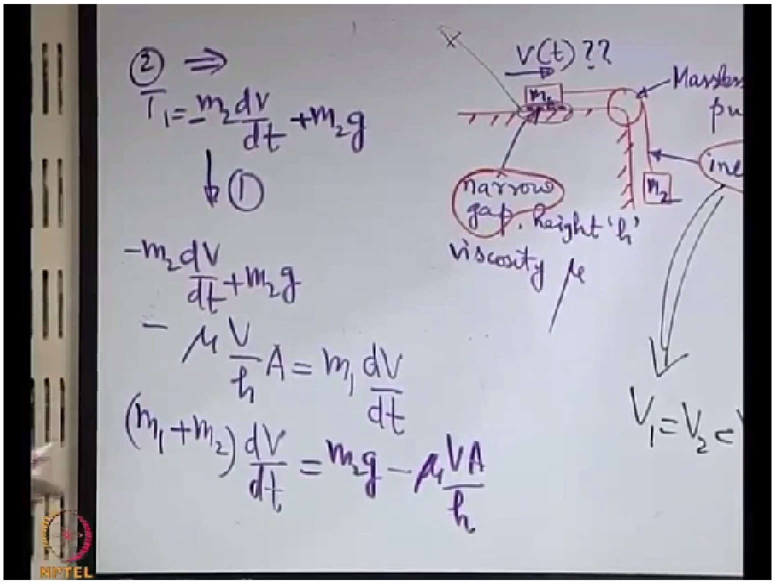

So from 2 we get, T1=m2dvdt, sorry T1 is -m2dV/dt+m2g and that we substitute in 1. So -m2dV/dt+m2g=, that is T1-muV/hA=m1dVdt. So you have m1+m2dVdt=m2g-mu VA/h. It is very easy to integrate it. You can just separate the variables. So you bring this whatever function of V is there on one side and whatever function of T on the other side.

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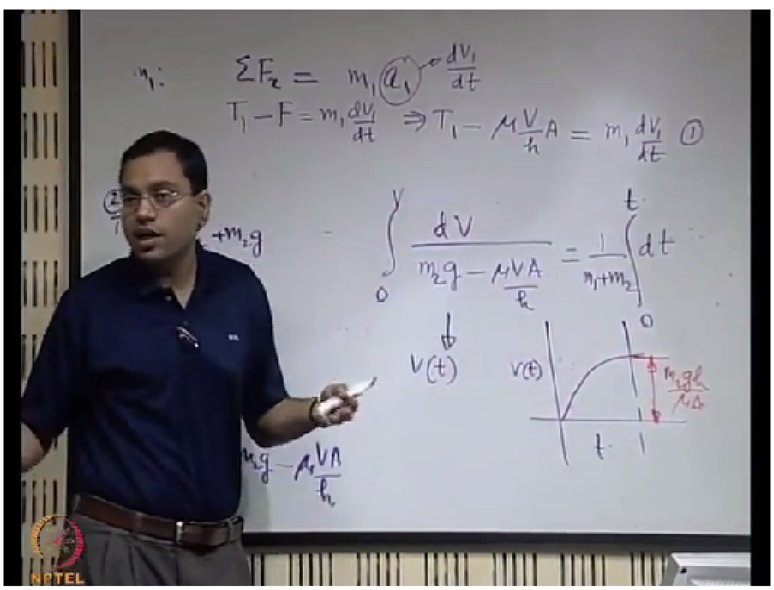

So you have dV/m2g-mu VA/h=… So you can integrate it with respect to time, say at time=0, the velocity was 0 or V0 in general may be if it was having an initial velocity and at time=T, say the velocity is V. So from this if you integrate, you will find an expression for V as a function of time. Of course in an exponential form it will come because it will be in a logarithmic form the integral that will appear.

So it will be some function of time and if it is released from rest, its velocity should increase with time or decrease with time? Increase, right because the m2g that is falling down and making it to move towards the right. At the same time, there is a viscous resistance, so it will come to a sort of asymptotic state when it has a balance between these 2 forces and then there is no further change in velocity.

So without working it out, may be the velocity versus time characteristic could be something like, maybe like this and let us say that it comes to a sort of steady state at sometime which is large time, theoretically tending to infinity, practically large time. So what should be this velocity? So can you tell that what should be this velocity? **"Professor - student conversation starts**" (()) (56:18) **"Professor - student conversation ends"**

Yes, m2gh/mu A. So this is… and the reason is straightforward that when you have a steady state, there is no more change of velocity with respect to time and therefore it should be an equilibrium between these 2 forces. So that would give rise to this type of expression. So whenever we work out a problem, it is important that we also try to get a physical feel of what is happening, try to formulate it in terms of the physical feel, try to get a sketch may be of what is the variation.

And then it will be so easy to interpret whatever results you are getting whether they are correct or not or whether they are having some physical sense or not, okay. With this we will have a short break and then we will continue with the discussions on viscosity and surface tension in the next part of the lecture. Thank you.