Introduction to Fluid Mechanics and Fluid Engineering
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Lecture – 03
Introductory Concepts (Contd.)

Well, let us continue with what we left last time that is we were discussing about the index notations for the stress tensor components and how to relate the stress tensor components with the traction vector.

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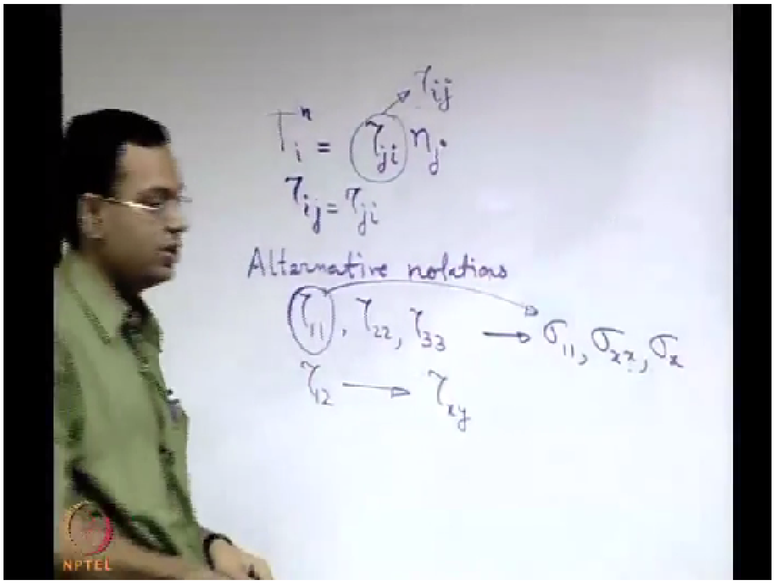

So this is what we came up with. We are not writing the summation notation because as I told that if you are having a repeated index, there is an invisible summation over that. So this i is called a free index because it may be whatever, 1, 2, 3, like that. So being in the right-hand side and in the left-hand side, in the right-hand side only once, this is like free but j is a repeated index and therefore it is a dummy index.

In place of j, you could write k, l, m, n, whatever. It makes no difference but whatever is this I, should also be the same. So this index and this index, they should correspond and we also showed that tau ij=tau ji; therefore, we can also write this as tau ij nj, which is the corresponding form of the Gauss' theorem by taking the moment balance into account. We will also discuss about some of the alternative notations because in text moves, different text books have different notations.

And of course the index notation is the cleanest one and I would say that most convenient one to use, but we will also look into the alternative ones. So when you have this tau11, tau22, or tau33, these are bizarre special components of the stress tensor. What do these represent? Normal components of the stress. So in many texts, you will see the corresponding symbol as sigma 11, say corresponding to tau11; sigma 11, sigma xx, or sigma x equivalently.

So there should ideally with 2 indexes but since both are repeated, sometimes takes only the repeated one and just use one x, so sigma x. Perhaps this is the original symbol that you learnt for the first time when you were learning the mechanics. So for tau or sigma with 2 subscripts would be a more convenient and more fundamental notation. So if you have for example tau12, then in the xy notation, you can also write it as tau xy.

So always remember 1 for x, 2 for y and 3 for z, that should correspond to any symbolic notation that you have with in the book, whatever book you are following and whatever notation that we are going to use in the class. Of course, we will try to use all sorts of notations in different contexts so that you feel comfortable with the general notation that is used in different cases. So what we sum up from the discussion on the stress tensor is that the stress tensor has different components.

We can clearly identify which are the normal components. The normal components are the components which are appearing in the diagonal of the corresponding matrix representation. There are off diagonal components which are like so called shear components. Now we have to see that how the characteristic of the fluid is related to the components of the stress tensor and we will subsequently see that how they are related to the components of the stress tensor is going to be strongly dependent on one important property of the fluid which is the viscosity of the fluid.

So we will come into the properties of the fluid subsequently but before going in to the viscosity, we will talk about one of the quantities or one of the properties which has a very important relationship with the stress tensor and that is nothing but the pressure of the fluid. So we will discuss something about the pressure of the fluid.

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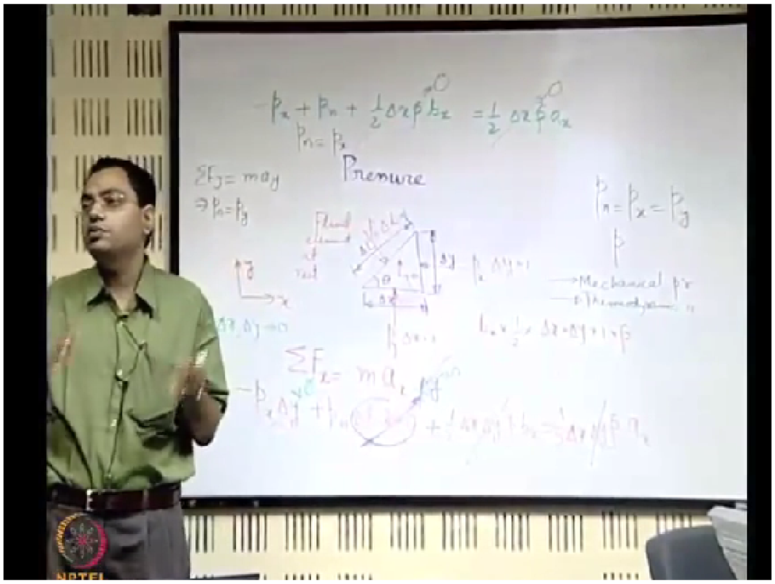

Whenever we start discussing about pressures of a fluid, we generally start discussing about fluids at rest. This somehow gives a misunderstanding to people that pressures are quantities very relevant only to fluids at rest. It is not so as we discussed earlier that when a fluid is at rest, there is no shear that is acting on it. So only the normal component of force is acting on it and therefore pressure becomes the only relevant surface force in that context.

And that is why it is easy to isolate all other effects and just focus on pressure if we are discussing about fluids at rest. If we are discussing about fluids in motion, obviously pressure does not get irrelevant. It may even get more and more relevant but to begin with we will consider that we are discussing about say an element let us say a wedge type element like this. Again I mean you may consider it is 3-D version by considering uniform width perpendicular to the plane of the board.

But let us just consider that it is a section of the wedge in the plane. So what we are interested to see is that what are the forces which may act on these wedge shaped fluid element when the fluid element is at rest, okay. So we are giving some names of these dimensions. Let us say this is delta L and may be this angle is theta. Just like what we did earlier for a general fluid element which may be subjected to normal and shear component, here we are going to discuss about a fluid element where there is only normal component of force on the surface.

The reason is quite clear; we are assuming a fluid element at rest. So there is no shear component. Keeping that in mind, so let us keep in mind this is a fluid element at rest. We will later on see that even if the fluid element is not at rest but internal deformation is negligible but the fluid element is moving like a rigid body, then also similar considerations may be valid but for the time being, we consider it at rest.

So we are identifying various forces which are acting on the surfaces and the volume, the surface force and the body force. So fundamentally we will treat any mechanics problem, continuum mechanics problem, in terms of the forces as a collection of body forces and surface forces, which we will try to keep it in either static or dynamic equilibrium. So for surface forces, we will first consider say the force on the surface.

So the normal force, and pressure by nature is acting normally inwards like the term pressure by that we qualitatively understand, intuitively understanding something which tries to compress the element. So it tries to act inward to the surface. So whatever is the outer fluid element that is trying to apply a normal reaction. It is like a normal reaction but the normal reaction is inward always.

So it is trying to pressing it, trying to press it so to say. So what will be this? This will be, let us say that we do not know whether pressure will be different along x, y, or z. So this is pressure along y. So we call it say py. When we call it py, this is the force per unit area, all of you know about that, and this multiplied by say delta x*width, say 1 is the total force. Similarly, and for this let us say that this surface has an orientation such that the direction normal is n, so we call it p with subscript n.

Why we are keeping these subscripts? We are still not sure that pressure should be varying as we change the orientation. Fundamentally we should be unsure to begin with because we have seen that any force acting on the surface, is likely to be strongly dependent on the orientation of the face that is chosen. So there is no reason to believe that pressures should not be where, whether it should be or should not be, that is what we are going to derive.

Now this will be this into delta L*1. There will be no shear component because it is a fluid at rest. There will be a body force which is acting on this. So let us write the body force component, say body force along x and body force along y. So what will be the body force along x? Let us again say that p1 is the body force or say bx is the body force per unit mass acting along x. So this multiplied by the mass of this.

So what is the mass of this. This is like a triangle. So 1/2 base*altitude that is 1/2*delta x*delta y*the width that is the volume that multiplied by the density, is the mass. Then bx is the body force per unit mass. So this is the total body force which acts along x. Similarly, you have total body force that acts along y, I am not repeatedly writing it just to save time and so we have written all the forces.

The resultant force if we write the force equilibrium, resultant force along x=the mass of the fluid element*acceleration along x. Well when we say acceleration along x, your general idea would be that we are considering a fluid element at rest. So fine, let us first consider fluid element at rest. So if we consider a fluid element at rest, the right-hand side is 0, obviously but we will see that whether the effect of right-hand side is there at all or not.

For that first let us write the left-hand side. So when you write the x component, you will have -bx*delta y, then the component of this pn in the direction of x, so you have x axis like this, y-axis like this. So what will the component of this pn along. So pn*delta L*sine theta, then +, there is a body force component along x. 1/2 delta x delta y rho bx, that is equal to mass of the fluid element.

So delta x delta y 1/2delta x delta y rho*acceleration along x. Of course you can write delta L sine theta as delta y, right. So we will make this simplification. In place of this, we will write delta y. Remember just like we in the last class tried to find out expression for the stress tensor components and traction vector in terms of stress tensor components at a point. So here also we are interested to do that so that we want a shrink the size of these as in the limit as delta x delta y all tending to 0.

So we will see what is the consequence of delta x delta y, all tending to 0. So if you take that limit, then what happens. Let us see that limit. So if you take that limit as delta y tending to 0, this will be 0. So our limit is delta x delta y tending to 0. This will be 0, tending to 0. These are tending to 0, so you can cancel delta y in all sides, that means this you cancel, this you cancel, this you cancel, this you cancel.

So what is left, you have -bx+pn+1/2delta x rho bx=1/2delta x rho ax, right. So when you take the limit as delta x tends to 0, then obviously these term goes away and these term goes away. Therefore, you get pn=px irrespective of whether this is accelerating or not. This is a very very important concept because in your high school physics, you perhaps have done the same thing but assuming that it is at rest and that might create a misconception that this will not work if it is moving like a rigid body with an acceleration.

And if you see, it does not matter even if it has a body force. So even if there is a body force that is acting, still this equality is valid. Similarly, by considering the force equilibrium along the y direction, it will follow that pn=py. So as a conclusion we can say that pn=px=py which means that we are talking about a quantity which does not sense the direction. So it is insensitive to the direction.

And it is acting always normal to the surface on which it is being evaluated. So this quantity we call as pressure. Since it is index insensitive or direction insensitive, we can just call it without any index and therefore unlike the general stress tensor, it is not a tensor because it a second-order tensor requires 2 indices for its specification, you should remember. Whereas this requires no index for its specifications.

So of course it is a tensor but tensor of order 0. So it is more easily termed as a scalar. So you can see that although stress and pressure, both are expressed in terms of force per unit area, mathematically and fundamentally their characteristics are somewhat different and that we have to clearly remember whenever we are discussing about these 2 related terms. There are more involved concepts on pressure which we will come across subsequently.

One important concept is that, there are 2 terminologies involved, once is called as mechanical pressure, another is called as thermodynamic pressure. So first we start with the mechanical pressure. We will briefly discuss about the concept; we do not elaborate but the subtle concept we will try to understand. So when we say mechanical pressure, what we mean? What we mean is that see if you are thinking about the normal stress, components of the stress tensor, you have tau11, tau22, tau33.

And if you feel that this p is a representative of the normal stress states of the element that is chosen because we are not considering the shear stress, effect of normal stress we are representing by p that means somehow there are is likely to be a relationship between these tau11, tau22, tau33 and p. The relationship may not be straightforward but since p is same in all directions, we can say that there may be a component of or a part of these tau11, tau22, tau33 which is like direction insensitive.

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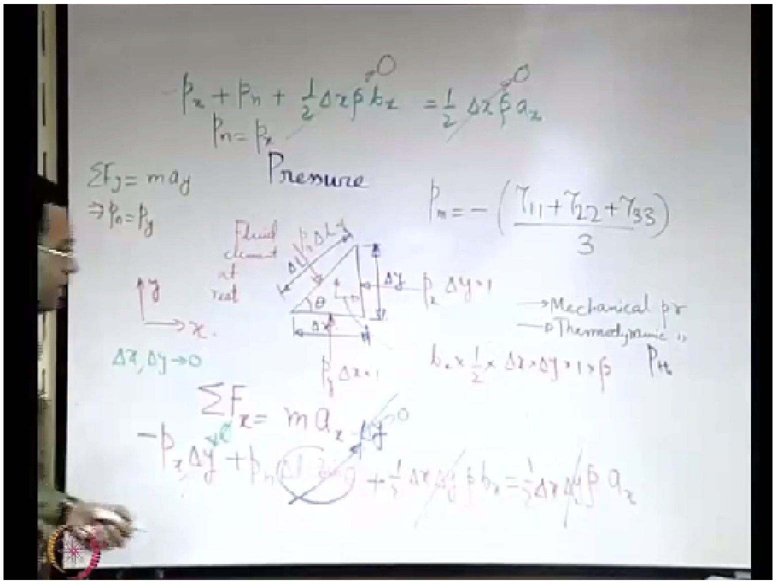

And that we may say just for the sake of definition of mechanical pressure, so it is a basic definition, that the mechanical pressure is defined as the arithmetic average of the 3 normal components of stresses with the minus sign. Minus sign is straightforward to understand because the positive sign convention of these were outwards from the surface whereas the pressure by nature is inwards to the surface.

So to adjust that, this minus sign is there. So this is also called as something like a hydrostatic component of stress that means you are assuming that it is like, it is representative of a state of stress where it is represented by a quantity pressure which acts equally from all directions. So it is as if like a fluid under rest that we are considering. So any state of stress which deviates from this hydrostatic part is known as deviatory part.

So that is what is something which deviates from a hydrostatic state of stress. We will come into the details of this concept later on whenever we are going to discuss about the equations of motion for viscous flows but this is just an elementary definition of mechanical pressure. Now when we talk about pressure actually, we are not really referring to this mechanical pressure always fundamentally because whenever we talk about pressure, think about say you are talking about pressure for gases.

You always relate pressure with density and temperature through a equation of state. So pressure from a thermodynamic point of view is something which satisfies the equation of state through the density and temperature. For an ideal gas, it is very simple. For non-ideal gases, it may be a more complicated equation of state but still equation of state is something which relates pressure, density and temperature in some mathematical form.

So thermodynamic pressure is that pressure p which will satisfy the equation of state. Now the question is, is the mechanical pressure going to be equal to thermodynamic pressure on not. So what is the fundamental mechanism that will dictate that whether they are equal or not. Say there is a bubble. Inside the bubble, there is a particular pressure, density and temperature. Now you are making a bubble to fluctuate its frequency of formation that is the bubble is changing its state very fast.

So what will happen? There will be a particular thermodynamic pressure, density temperature. Suddenly you are changing its state to a new state, new pressure, density, temperature. So in that way say the bubble is suddenly expanding and contracting, expanding and contracting like that and it is doing it very fast. Once it is doing very fast, the change is not so easily adjusted. So the system requires at least a threshold time to adapt itself to the change and make sure that it has its mechanical pressure which is like the kind of pressure that acts equally from all directions same as what is dictated by thermodynamic state.

So thermodynamic state is a change that imposes a kind of disturbance to the system. System requires a time to attain equilibrium so that it will eventually have mechanical pressure could thermodynamic pressure and therefore, we generally do not distinguish between mechanical and thermodynamic pressure, we say that it is just a pressure but if the change is so fast that the system at intermediate states does not get enough opportunity to attain equilibrium so that whatever change is in the thermodynamic state, the system does not get enough opportunity to adjust to those subsequent changes.

And then in such cases you will not have mechanical pressure equal to thermodynamic pressure but those are very rare cases. So for most of the practical engineering applications, the changes are such that those changes will be adjusted or adopted to by the system in a way that you will have mechanical pressure equal to thermodynamic pressure and therefore whenever we will be talking about pressure, we will not be distinguishing the mechanical and the thermodynamic pressure, we will be just calling it as a pressure.

So that is how we will be going about it. Now we have discussed about one fluid property which is pressure. Whenever we are talking about effects of compressibility, there are other related fluid properties which come into the picture in a very related manner and those properties we will look into one by one briefly.

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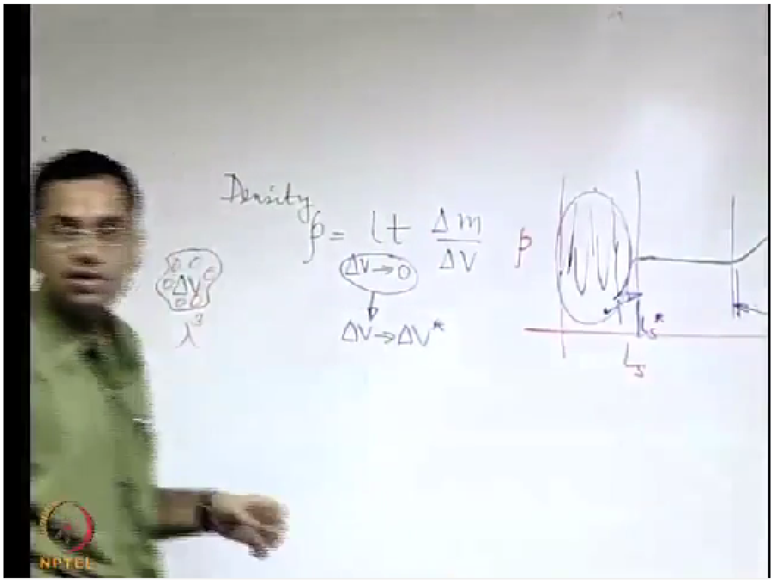

So one of the important properties will be density. So loosely what we say that if we have a volume, elemental volume, say delta v, we have the mass of the molecules which are there in this delta v. So we take mass per unit volume. All of us like to write limits. So we will write limit this as delta v tends to 0. We will think that nicely it should give a limiting definition of what is the so-called density.

Mathematically very nice, we will see whether it works or not. So what it says? It says in the limit as delta v tends to 0 that means limiting is small volume, you find out what is the mass of molecules inside. So you find that mass per unit volume, get local density at a point, that is what this definition is saying. Whether it works or not, we have to comeback to the continuum hypothesis to adjudge.

So if we remember that in the continuum hypothesis, we disregard the molecular nature and we just consider that it is a continuous medium, it does not mean that there are no molecules but obviously were abstracted off the molecules and we adjust representing that they are gross effect but whenever there are molecules, we have to see that what is the number of molecules within this elemental volume.

Again if the number of molecules within this elemental volume is very small then because of the statistical fluctuations, even uncertainty one molecule will give a lot of error and will give a lot of fluctuation. So it is critical that what is that elemental volume that you should choose. It cannot be too small. What is the smallness. The smallness will come with a length scale. The smallness of the length scale here is the mean free path, lambda.

So a very small volume when we say, then that will scale with say lambda cube. Lambda is like a length scale which will correspond to a very small volume. So when the volume is of the order of lambda cube, elemental volume, then it will have lots of uncertainties in the statistical fluctuations of the molecules because within that length scale, you really have uncertainties related to collision.

On the other hand, if you take this volume delta v very large, then also you can calculate the density but it will not be able to capture the local variations. It will give a global average; therefore, one has to choose a threshold length scale for calculating this density and how it should be sensitive to the length scale if you make a plot of say the length scale that you choose, say we call it Ls and the density that you predict.

So you will see that if you close a very small length scale, you will get a variation, these type of fluctuation, then it will come to a steady one and then if you choose the larger length scale, it will be changing like this. So what is the significance of such a plot? These length scales are small enough so that you have merely random fluctuations because of the uncertainties. This length scale is fine.

Beyond this length scale, you have variation. This variation is because of over the system length scale, the density is varying from one point to the other. So a correct choice of length scale may be something which can be say in between these 2. So in between these 2 limits. Therefore, if we say delta v tends to 0, that is not fundamentally correct because delta v tends to 0, will make you fall on these wedging because delta v tends to 0 means mathematically delta v is as small as possible.

So as small as possible will obviously be something smaller than as big as possible. So obviously in that context, one has to remember that this delta v tends to 0 should be corrected and how it should be corrected? We should change it as not delta v tends to 0 but delta v tends to some delta v star which is like, if we call this as Ls star, then maybe it is of the order of Ls star cube. So it is a threshold length scale beyond which you are not having such uncertainties and fluctuations affecting your density calculations.

So this delta v star therefore we can say is the smallest elemental volume over which continuum hypothesis is valid. So it is not tending to 0 but tending to a limitingly small volume, delta v star, over which still continuum hypothesis works. Below this limit, continuum hypothesis might not work and therefore this definition will not work because this definition is on the basis of a continuum description of fluid properties like density. So we have talked about density. We have talked about pressure. Next let us talk about bulk modulus.

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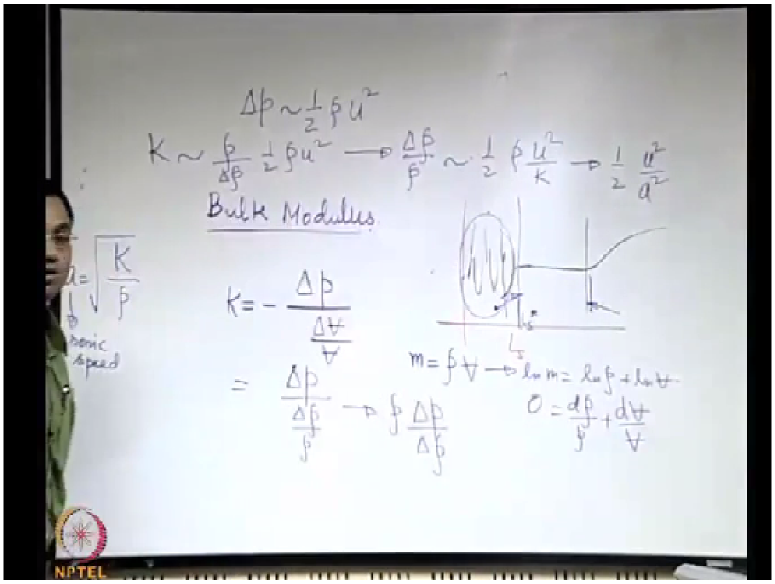

So all of you are aware of the basic concept of bulk modulus but let us just see that how you have defined it. Let us try to define it first in a very loose manner. It is always important to get a qualitative feel and then of course you can have more sophisticated definition. We will not go into very detailed sophisticated definition of bulk modulus because it requires a detailed understanding of thermodynamic processes.

And therefore, we are not going into that type of definition. So in a lose sense, if you are applying say a pressure differential, delta p, that is expected to give rise to a change in volume of a fluid element. Let us say that change in volume is delta v. So original volume was V. So this is the rate of, or this is the total changing volume per unit volume. So this is a kind of a volumetric strain and this is the pressure differential which is responsible for the volumetric strain.

And you expect that if delta p is positive, delta v is negative because if you press a fluid element, it should compress, its volume should decrease. If you want to give the corresponding fluid property a positive number definition, then you should adjust it with a negative sign. Now you can relate the change in volume with the change in density. How is it possible to relate the change in volume with the change in density?

So you have, consider the mass of a fluid element. So that is the density*volume. From now onwards whenever we will be discussing about the volume, we will be using a symbol knot v but v with a strikethrough because we will be using v for velocity also. Just to avoid that confusion between the symbol of velocity and symbol for volume, we will be just distinguishing those in this way.

So I will not be repeating the symbol many times but once I will be using this type of symbol, you just take it that we are talking about the volume, not the velocity. So if you want to, see that what is the relationship between the elemental change of density with element change of volume, what you can do simply, just you can take log of both sides and differentiate. So if you differentiate keeping in mind that the mass of the fluid element is conserved.

So its derivative should be 0; therefore, loosely like if you were following this definition, we can relate delta v/V with delta rho/rho. So that we just absorb the minus sign and it will be like this. So it is like rho delta p/delta rho, like this. Now we can relate delta p with the velocity of flow in a order of magnitude since the delta p and the velocity of flow.

This is just like if you consider that there is an equivalent pressure change which is brought about by the change in kinetic energy of fluid which is moving with a velocity u, then this is not that they are exactly equal, it is just to say that one scales with the other in this way. So you can therefore write a scale of K as, so rho... Let us try to see that what is delta rho/rho scale, that means what is the change in density relative to its original density.

So if we do that, it will be 1/2rho u square/K. Remember one thing that this K/rho, it is something which is a very fundamental quantity which you have studied in physics. What is this or square root of K/rho, if it reminds you more? This is fundamentally sonic velocity, sonic speed so to say. What is sonic speed? Sonic speed is not just speed of sound. Sonic speed is the speed by which a disturbance propagates through a medium.

And here we are talking about these type of disturbance to the elastic property of the medium. So it happens to be the speed of sound, okay. So this is the sonic speed a, where a we will call as sonic speed. This is a very basic high school physics base definition. So keeping these in view, we can write this as 1/2u square/a square. So you can see that the relative change in density is related to a quantity u square/a square. What is this u square/q square? This is a non-dimensional quantity that you can see because it is a ratio of 2 velocities.

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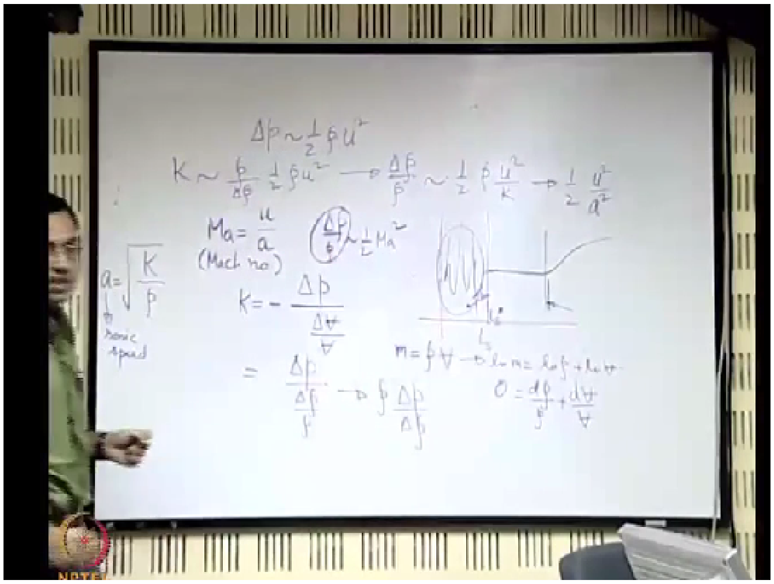

$$\Delta p \sim \frac{1}{2}\rho u^2$$

$$K \sim \frac{\rho}{\Delta\rho}\frac{1}{2}\rho u^2 \rightarrow \frac{\Delta\rho}{\rho} \sim \frac{1}{2}\rho\frac{u^2}{K} \rightarrow \frac{1}{2}\frac{u^2}{a^2}$$

$$Ma = \frac{u}{a} \quad \boxed{\frac{\Delta\rho}{\rho}} \sim \frac{1}{2}Ma^2$$

(Mach no)

$$a = \sqrt{\frac{K}{\rho}}$$

sonic speed

$$K = -\frac{\Delta p}{\frac{\Delta V}{V}} = \frac{\Delta p}{\frac{\Delta\rho}{\rho}} \rightarrow \rho\frac{\Delta p}{\Delta\rho}$$

$$m = \rho V \rightarrow \ln m = \ln\rho + \ln V$$

$$0 = \frac{d\rho}{\rho} + \frac{dV}{V}$$

So in the numerator you have u, in the denominator you have a. So u is the velocity of flow and a is the velocity of a disturbance which is moving in the medium in which the flow is occurring and these 2 ratios is known as Mach number, I mean, ratios of these 2 numbers is known as Mach number. So you have heard about the Mach number like a jet moving with a Mach of this. So to have the Mach number, here is the velocity of flow relative to the velocity of the disturbance with which the disturbance propagates within the medium.

And therefore, we say that it is having a more and more compressible effect. The reason is if you just write this delta rho/rho, you see that it will scale with 1/2 of square of the Mach number. Therefore, higher the Mach number, higher is the effect of the change of density relative to its original density. So Mach number therefore is a very important indicator of something which is called as compressibility of a fluid.

So what is the signature of compressibility of a fluid? We will say that our fluid is compressible when it has a change in density because of a change in pressure. So in that way, all fluids are compressible, right because all fluids will have some change in density because of change in pressure but when we say that a fluid is incompressible, what we mean is that, that effect is negligibly small. So a compressible fluid and an incompressible fluid, these are just conceptual paradigms.

There is no fluid as such which is incompressible but when we say that a fluid is incompressible, we mean that its compressibility effect is very very small. Again how small or how large, that is something which may be debated. So let us say that we are talking about a change, this relative change say 5%. So let us say that if we say that this change is < 5%, we say that it is almost incompressible. So if we want to see that what will lead to that 5%?

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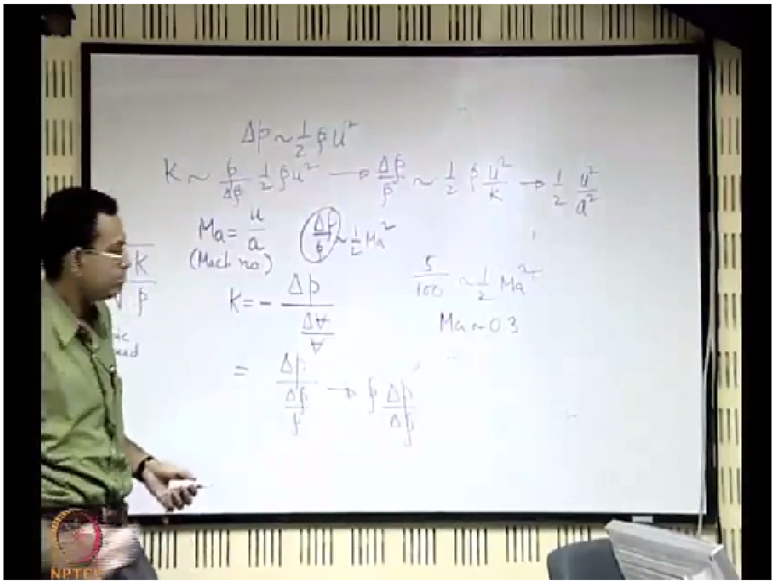

So one way to work it out with say 5% means 5/100, so what would be the threshold Mach number for this. Roughly 0.3, right. So 0.33 or whatever, but roughly 0.3. That means if we say that a relative density changes < 5% is something which we do not consider as a compressibility effect, that implicitly means that a Mach number < 0.3 is something which is not going to give us any serious compressibility effect.

So this is important because whenever you are analysing an engineering flow, nobody will tell you that whether the flow is compressible or incompressible. As an analyser, it is your responsibility to make a judgment of whether you are going to use the concept of compressible flow or incompressible flow for the analysis of your problem and then you have to be confident that whether a particular analysis methodology is going to work or not.

Of course for all flows, compressible flow, analysis will work because all flows are compressible but it is like if you have a mosquito, you will not like to kill it with a canon. So if you are ready or if you are having a possibility of doing a relatively simple analysis, one should not go for a complex analysis, that is what all of us have learnt in engineering that do not go for unnecessary complication until and unless it is absolutely required.

So whenever compressible flow analysis is not required, we should not go for it and this Mach number of flow will give us a guideline of whether we should go for a compressible analysis or not. A couple of other important points or remarks are there regarding these definition of bulk modulus. One is see in this definition, we have talked about a change in volume because of a change in pressure or equivalently a change in density because of a change in pressure, but pressure effect of change of density, it depends on the type of process.

All of you have heard of certain thermodynamic processes like adiabatic process, isothermal process and so on. So given a particular system, how the density will change with pressure, will depend on the nature of the thermodynamic process. So this definition as such fundamental is not incorrect but incomplete because it does not talk about the thermodynamic process by which you are trying to have this change of state.

So there are more fundamental or correct definitions of these in terms of specifying it as, say either a reversible isothermal process, reversible adiabatic process and so on but we are not going into those details here because thermodynamics is not the scope of this particular course but we should keep that in mind because whenever you will be studying thermodynamics, again these type of definition will come into the picture and there more detailing will be done in terms of whether it is a reversible adiabatic process, reversible isothermal process and so on.

So that is one of the important concepts. The second important concept is as follows. Say you are interested to identify whether a flow is incompressible or not and in that respect, there is a subtle difference between the concept of incompressible fluid and incompressible fluid. These are very very subtle concepts. So when you talk about an incompressible flow, what you mean is that if you have a volume element of a fluid, that volume does not change.

So incompressible flow means that there is no volumetric strain of the fluid element. There is no change in volume but you cannot directly always relate it with this definition because the change in volume may not always be due to change in pressure directly. It maybe because of something else also. There are reasons for which you might have change in volume of a fluid element not because of the change in density due to change in pressure.

But maybe because of change in density due to change in temperature, not directed due to pressure. So whenever we are talking about incompressible fluid, we are talking about that we are asking ourselves a question that is there a change in density because of a change in pressure. If that answer is that yes, it is significant, we call it a compressible fluid but compressible flow definition is something more general, compressible flow means a fluid element which if it is going to have a volumetric strain or change in volume per unit volume by whatever reasons.

It need not be just due to pressure or it maybe because of anything, then we say that it is a compressible flow. So compressible fluid and compressible flow are related because of course one of the reasons of being a fluid compressible or being a flow compressible is because the fluid itself is compressible. So the density changes due to change in pressure is significant but there could be other effects that are creating the change in volume. So this is a concept that we should remember. Next what we will do is we will try to learn about very important property of fluid which is called as viscosity.

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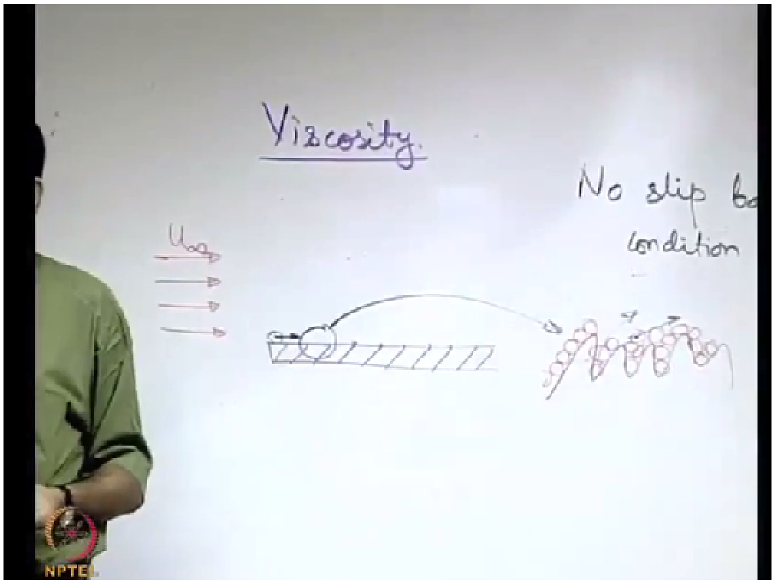

So when a talk about viscosity, we will not try to just learn it in abstraction but we will start with an example. Let us say that you have a flat plate just like the top of a table, a flat plate and fluid is coming from far stream just like say fluid is being blown from that side, it is coming on the top of the table and going away. So the top of the table will be like a flat plane. So let us say that the fluid is coming with the uniform velocity from a freestream.

In fluid mechanics, usually we give such a symbol infinity with a subscript to indicate that it is a freestream condition. So infinitely subscript is like a freestream velocity. So it is coming with a uniform freestream velocity. Now that freeways will be disturbed because of the presence of the plane and let us see that what is going to happen? So when these fluid first comes in contact with the plate, what happens?

Let us first try to understand that say there is a fluid molecule which comes in contact with the plate. So what will the plate like to do with the fluid molecules? Let us consider 2 different examples. So one is for a gas and another is for a liquid. Usually whenever we discuss about fluids, we are either talking about gases or liquids but sometimes there physical behaviour, it is better to discuss distinctly or differently.

So let us say that now there is a gas molecule as a first example which is coming, falling on this plate. So what will happen? There will be first a tendency that the gas molecule is adsorbed on the surface. So once it is adsorbed on the surface, then what will happen, that it will exchange some of his momentum with the surface. So it will try to have it slowed down and then again it will try to be getting ejected from the surface.

So it is like a molecule falling on the surface, adsorbed on the surface, getting ejected from the service like this. So in this process, many molecules are colliding with this and they are exchanging their momentum with the wall. So if there are very large number of collisions, so to say theoretically infinitely large number of collisions, then these kind of momentum exchange will bring on an average the molecules in equilibrium with the surface.

So if the surface is at rest, the molecules will also be at rest. So that will imply that there is 0 relative velocity between the fluid and the solid at the point of contact and this is something which is known as no slip boundary condition. So fundamentally what is a no slip boundary condition? It is 0 relative tangential component of velocity to be more accurate, 0 relative tangential component of velocity between the fluid and the solid at their points of contact.

We are not talking about the normal component because still the molecule will be colliding like it may have a sort of elastic collision. So it will bounce back. So it may have a normal component. Obviously regarding the normal component, there are issues like if the molecules are sufficiently large in number and they are at the wall, they cannot penetrate and go through the wall.

Wall is not having holes. So that is called as a no penetration boundary condition, then there even the normal component of velocity will become 0 but no slip boundary condition does not talk about that. That is a separate consideration. No slip boundary condition talks only about the tangential component of velocity. So 0 relative tangential component of velocity between the fluid and the solid at the point of contact.

Now as I am telling these to you, you are tending to believe that this is always the correct picture and this has happened really for a long time. So for a long time, this no slip boundary condition was taken as something which is like a ritual which should not change and the reason was that, for many or for most engineering flows, it is still valid or it has been experimentally found to be very very accurate.

But whenever we are understanding this concept, we should ask ourself a question. Are there conditions in which the no slip boundary condition maybe valid? It is important because in many of the modern day applications of fluid mechanics, especially fluid mechanics in small length scales, these condition is something which is put under serious question. So obviously we need to see that or we need to appreciate that this is just a conceptual paradigm.

It is not something which is a ritual and which is expected to work always. Let us see, let us try to look into an example in the context of gas flow that no slip boundary condition does not work. So let us say that you have gas molecules but not very large number of gas molecules. Then what will happen? The molecules will be exchanging momentum with the wall but there will not be very large number of collisions.

Because there will not be very large number of collisions, the momentum exchange will not be complete. So there will be some velocity of the fluid relative to the solid boundary, even if otherwise we tend to believe that there should not be any slip. So that is just because of the rarefied nature of the medium that there is not sufficiently large number of molecules to have a theoretically large number of or infinite number of collisions.

On the top of that, there may be local strong gradients in density and temperature and that might itself induce motion of molecules of gases over the solid surface. So these are called as phorectic motions. If these are induced by temperature, these are called as thermophoresis and these may be induced by any other effect but temperature is one of the common effects by which by introducing a very high gradient of temperature, you can introduce local flow of molecules of gases over the solid boundary.

So we can see that there may be situations and there are likely to be such situations when the no slip boundary condition is not valid but well in most of the engineering systems that we are talking about, the no slip boundary condition will work for gases except for rarefied gases or maybe gases which are not having sufficiently large number of molecules or gas is being subjected to very high local gradients of density or temperature.

For liquids, it is difficult to believe that the no slip boundary condition will not work because liquids are very compact systems. So liquid molecules will not be in sufficiently number to have inadequate collisions with the wall but for liquid molecules, there may be slip because of certain reasons. So to understand the picture of what happens in the liquid molecules, let us consider a small element of surface like this.

The surface may look very very smooth on the top but if you look it into a very powerful microscope, it will be much much worse than what I have drawn here. So it will have lots of peaks and valleys and what will happen is that the molecules will nicely sit on these peaks and valleys. So some will be entrapped like this and because of the compact nature, what will happen?

Whatever is entrapped is not easily being escaped and that will make us believe that yes it will be a no slip boundary condition. At the same time, if you are having a very high shear rate which is being introduced on the liquid. Say a very high rate of shear strain, what will that try to do? That will try to forcefully dig this out from or take this out from these locations. So then in that kind of a context, the liquid molecules may also slip on the surfaces.

Otherwise, if you have very smooth surfaces, say you must have heard about carbon nanotubes. These days those are very sophisticated and fascinated technologies to produce carbon nanotubes. So those are very smooth tubes and if you are having liquids in contact with them, now obviously there will be the Van der Waals forces of interaction between the surface and the liquids but in such case, water flowing through those nanotubes will have very ordered hydrogen bonding.

And then the motion of that water will be such that it can overcome the van der Waals forces of interactions those are relatively weak in comparison to these strong bonding in the water to overcome the wall attraction and flow on the top of such surface. So it may actually slip and these are called as highly slipping surfaces. Therefore, we have to keep in mind that no slip boundary condition is a paradigm which will work for most of the engineering problems that we are going to consider.

But at the same time, we should not take it as a ritual. We should keep in mind that there are situations in which it might be violated but for practical purposes for almost all the problems that we are going to solve in this particular course, no slip boundary condition will work, okay. So let us stop here today and in the next class, we will take this up and introduce the concept of viscosity through these no slip boundary condition that we are discussing. Thank you.