Introduction to Fluid Mechanics and Fluid Engineering
Prof. Suman Chakraborty
Department of Mechanical Engineering
Indian Institute of Technology – Kharagpur

Lecture – 02
Introductory Concepts (Contd.)

What we left in the last time?

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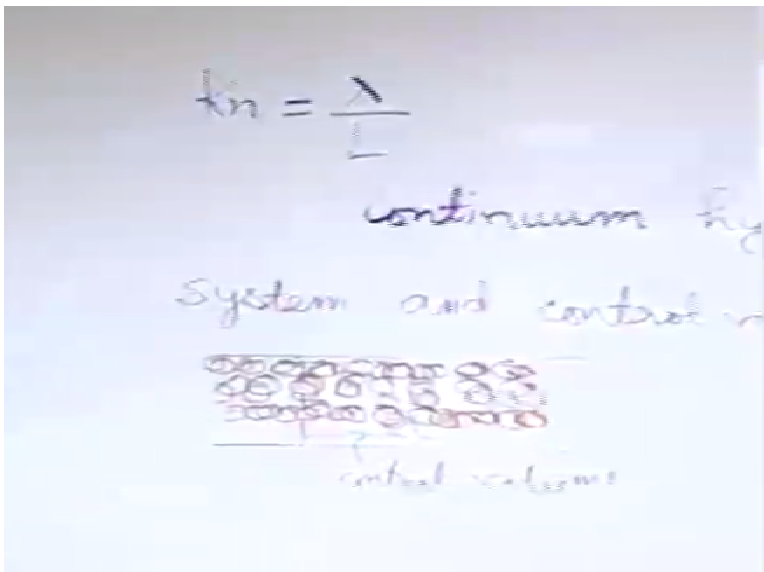

That is we define something called as Knudsen number, which is the ratio of the molecular mean free path and the characteristic length scale of the system. So this L we give it a symbolic note for a pipe it may be the diameter of the pipe for something else it may be some other dimension, but we call it as a characteristic length scale of the system. So if this ratio is small, what does it indicate? It indicates that the mean free path is much smaller than the characteristic length scale of the system which implicitly tells that it is sufficiently densely pack system.

On the other hand, if that is not the case, that if the Knudsen number is large, that means the means the free path it may be even larger than the characteristic length scale of the system if it is very, very rarified. In such cases what happens, in such cases you have very, very few number of molecules and then there are lots of uncertainties with respect to presence of molecules in individual elemental volumes.

So in those cases, the macroscopic point of view is not expected to work so efficiently or the macroscopic way of defining the characteristics of properties of the fluid might not work so efficiently. So whenever the macroscopic way works, we call it, we call the fluid as a continuum and the hypothesis concerned is known as continuum hypothesis. So what is a continuum hypothesis?

Continuum hypothesis tells that we treat the fluid medium as a continuous matter disregarding the discontinuities in the system. There are discontinues if you look into the molecular level. There are molecules there are gaps and so on, but if those are sufficiently compact, then you may treat it as a continuous matter. Once you can treat as a continuous matter, then you may use the well known rules of differential calculus to talk about the changes in properties from one point to the other, so you can talk about simple radiance, second order derivatives, and so on.

So continuum hypothesis works if there are sufficiently large number of molecules, so that the Knudsen number that we are talking about is very, very small. If the Knudsen number is > 0.1 so to say, then it comes to a state where your mean free path threatens to be 10% of the characteristics system length scale and when it goes on larger and larger there is a stronger and stronger deviation from the continuum hypothesis.

So there are situations when it cannot use continuum hypothesis and one needs to have a different treatment all together. Throughout this course we will be bothered mostly about situations and continuum hypothesis works. So that means there are sufficiently large number of molecules in the system so that uncertainties with regard to individual molecules do not influence the prediction regarding the fluid flow to a significant extent, because we are not looking from a molecular view point, but treating the fluid as a continuous matter.

So keeping that in view, what we will do is we will next see that no matter whether we are treating it as in a microscopic view point or in a macroscopic view point, how should we describe the fluid as a system. So for that, we will introduce to important concepts system and control volume. So we have identified an approach. Now we have to identify that what should be that fluid over which we apply that concept or approach.

So when we talk about a system, by definition system is something of fixed mass and identity. So something which must have its mass fixed, it must have its identity fixed that means those are identified. For fluids, sometimes this is not a such a simple concept to implement, let us again take an example of flow through a pipe. So you have a pipe, there are many molecules or even particles whatever are entering the pipe and leaving.

So at a particular instance of time, these are the molecules which are present. So now at a different instant of time, you may have different entities, different molecules which are present. The reason is quite clear. Something is entering and something is leaving, so it is continuously being replaced. In such a situation, if you want to track the motion of these particles or individual molecules as something of fixed mass and identity it becomes difficult and tedious because then you have to put a tag on individual entities and follow it as it is moving.

So this kind of approach so called as particle tracking approach in mechanics is known as Lagrangian approach. In fluid mechanics it is not many times convenient to follow that approach. So what we do instead is like we focus our attention on a fixed region in space. So let us say that we have focused attention on this identified region as if we are sitting with a camera, focusing the camera on the zone. What we are observing?

We are observing whatever is coming into the zone and leaving that we are only keeping crack up to that much. We are ignorant about where from it has come and where it is going. So rather than focusing attention on individual particles we are focusing attention on a specified region in space across which matter can flow. So that region we call us control volume. So control volume approach is more convenient for fluid flow because you do not have to track individual particles and fluid is a continuously deforming medium.

So it is very difficult to track individual particles. It is much easier to focus your attention on a specified region and see what happens across that and this particular approach where you use a control volume and analyze what happens across that is also known as Eulerian approach. Just as the Lagrangian approach is following the name of the famous mathematician Lagran, Eulerian is according to the name of Euler.

So whatever we will be discussing in this context of fluid flow no matter whether it is a system approach, no matter whether it is a microscopic approach or a macroscopic approach, we will be mostly using the control volume concept for analyzing the flow behaviour. With this basic understanding, we are now going to the concept of a fluid. So till now we have loosely talked about fluid. We have not seriously defined what is a fluid?

Now common sense wise if you are asked that what is a fluid, obviously we will say something what flow is a fluid and loosely speaking this is not a bad definition, but there are many things which flow, but those are not so called classical fluids, those may be in a borderline between fluids and solids so to say. A very formal definition of fluids is like this, that fluids are substances which undergo continuous deformation even under the action of very small shear force.

So if you are applying a shear force, or there is some shear force acting on the fluid, then even if it is very small, it will continuously deform the fluid. For a solid obviously if you are applying a shear force, it will not spontaneously deform it till when it comes to a threshold limit when it will appear to be seriously deforming. Obviously, there are substances which are fluids, but which require a threshold shear to be deformed and therefore the borderline between the fluid.

And the solid is sometimes not so strict, but for both of the practical cases this definition is something which we will be keeping in mind and we will be classifying fluids or solids according to these behaviours. So one of the important consequences is that if there is a fluid, which is non-deforming. So non-deforming fluid may be fluid at rest. So if you have a container, within the container you have put some water and water is at rest.

What is the implication of that? The implication is very straight forward. There is no shear which is acting on it. So if there is some shear which is acting on a fluid, the fluid is deforming, converse is also true that means if the fluid is deforming, there must be some shear which is acting on it. So when there is a fluid which is there at rest that means there is no shear component of force that is acting on it.

That means there is only normal component of force acting on it and that normal component of force per unit area which is acting inwards is called as pressure. So obviously, whenever there is a fluid at rest, the entire situation of the forcing which is there on a fluid element, may be expressed in terms of pressure. When a fluid is moving, it does not mean that there is no pressure.

Obviously pressure is very much there, which would have been there if the fluid is at rest, but there are additional forcing components which come into the picture which are directly related to the deformation of the fluid and this situation all together may be tackled in continuum mechanics, that is mechanics of a continuous medium through the concepts of stress. So we will now introduce the concept of stress, which we will be introducing in the context of continuum mechanics so it is valid for both solids and fluids.

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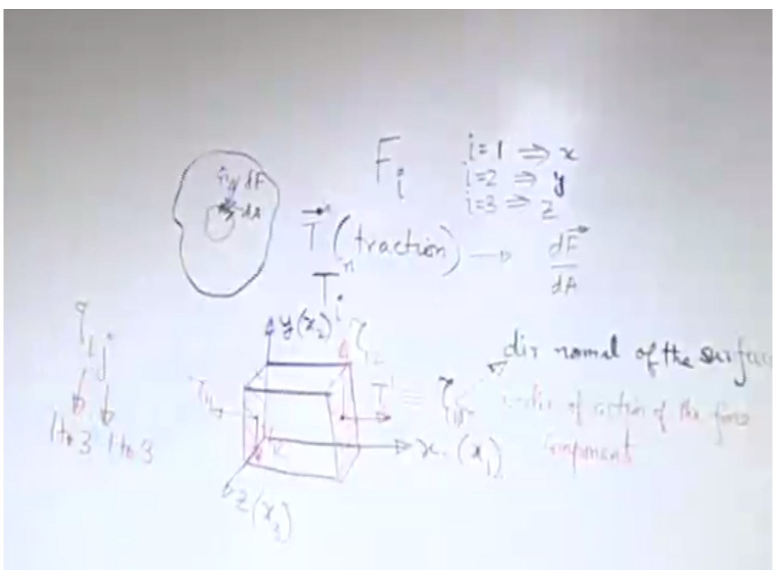

Now let us say that we have an element, when we say we have an element, it may be an element of a solid fluid whatever we are not very particular about it and we are identifying a small chunk from that. On that small chunk we are taking a small area say, dA. What we are interested is to see that whenever we are taking out this small chunk, the other part of the body will exert some force on this just by Newton's third law.

So we are interested to identify that force and let us say that force is directed like this, it is absolutely arbitrary. So it depends on many situations. So let us say that the force is dF. So if you want to define some force per unit area, then we are just giving it a name we are calling it T or traction. We will call it a vector because it is having the nature of a force. It will be implicitly determined by this one where F is a force, but we have to remember that it is not unique until and unless we specify the area. What it means?

Let us say that centered around the same point we take a different elemental area same dA, but differently oriented. So if we take that differentially oriented area now, if we find out the resultant force on that it is likely to be different that means given the point around which we take that differentially small area as fixed that is the location fixed, given the magnitude of the so called elemental area as fixed still this ratio is going to be different.

So this strongly depends on not just that dA, but how that dA is oriented. So it is important to give us kind of a superscript or subscript to this, so let us give us superscript n. So this n denotes that we are talking about an area which is having its outward normal in the direction of the n vector. So whenever we denote the orientation of area, it is customary to denote it by the unit vector in the outward normal direction. Let us say that n is such a vector.

Even if it is not a unit vector there is no problem because we can always normalize it in the form of a unit vector. The direction is what is important. If n is changed, that means the orientation of the area is changed. Obviously this traction vector will change. Now this traction vector therefore is not denoting something which is ordinarily like any other vector. So if you are talking about say a force, so whenever you are writing the component of a force, say F is a force.

You are using an index I to denote the component of the force. So this index I = 1 will mean that x component, I = 2 will mean the y component, and I = 3 will mean the z component. Therefore, by using 1 index I and varying it from 1 to 3 we may denote the components of a vector, but when we are trying to denote this traction vector s it has a component, because it is like a force per unit area, so based on the direction of the force it has its own direction.

It has its own components, but its specification depends on also another sort of index n which denotes the choice of the area orientation that has been employed to calculate this T. So it is something more general than an ordinary vector. How general it is? to understand that, we will take some special examples. What are the special examples? Let us say that we take an element of a fluid which is of a rectangular shape like this, we may orient axis like x, y, z in terms of the index we call this as x1, x2, x3.

Using the index is a very convenient way because just by varying the index you can vary the directions. Now let us try to see that, what are the special cases of traction vectors on the faces of this cuboids. So this has 6 faces and these faces are special surfaces. Why these are special surfaces? These have their normal directions either along x or y or z. This is absolutely an arbitrary area and what we are interested to do is to figure out what happens for an arbitrary area by referring to special areas which are either oriented along x, y, or z.

So that is the motivation of taking such an element. So once we have taken this element, what we are going to see is we are going to write such expressions for different faces. So when we come along this face, we are interested to write, the components of the traction vector. So let us write that. So components of the traction vector, there is a component along x. **“Professor - student conversation starts”** In terms of this notation, what should be the subscript? 1.

What should be the superscript? See what does this represents? It represents the unit vector outward normal of the surface chosen. So here the surface chosen has unit vector along x1. So this should be 1. **“Professor - student conversation ends”** Alternatively, we use a notation equivalent to this is tau11. So, 2 indices are there. What are these indices representing? The first index is representing the direction normal of the surface and the second index is representing the direction of action of the force component.

So in general, it is like tau ij where I represent the direction normal of the surface which we have chosen and j represents the direction of action of the force component itself. So let us take a second example. Let us say we write the y component on this surface. So this one. So tau what should be subscripts? First one is 1 and second one is 2 right. So this is very simple and you can do it for all the surfaces.

So what I will advise you to do is that you repeat the same thing for all the 6 surfaces to get a field that you understand the notation. This is like notation grammar. If you want to learn is a classical music you require to know the notation. So we are starting with the notation. The notation is important because that will help us in developing the basic equations in a very elegant manner. Interestingly let us look into the opposite face of this one.

So for this face, the outward normal is along negative x. So we will develop a sign convention that if the outward normal of the surface is along negative i, we will have the sign convention such that positive tau ij is along negative j that means here so this one we will call us positive tau11 for this surface y, because the first one is actually along negative x, the first index, therefore the positive sense of the j which is the second one is along negative x.

So on this surface if you want to draw positive sense of tau12. So that should be downwards. These are sign conventions. So if it actually is the other way, it will come as minus of this number just like in free body diagram you draw a force. The force might have come in the negative. That means it is actually in the opposite sense than what you have drawn in the figure. So these are also like that.

So as if you are drawing the free body diagram of a chunk or an element, so we are establishing sign conventions for that. So what we have learnt as the sign convention is if the direction I is along the negative of one of the coordinate axis, then the tau ij positive sign will be oriented along the negative j direction and if it is positive it is the other way. So this tau ij so when you write tau ij this I may vary from 1 to 3 and j may vary from 1 to 3.

So these are certain quantities in general you may have 3 * 3 9 tau ij components. We will later on see that actually out of this 9, you have 6 which are independent and utilizing those independent 6 components which are called as components of a stress tensor we can actually find out of the state of stress for any arbitrary plane which is neither oriented along x nor y nor z. So this particular quantity is which are like called as components of a stress tensor we may understand that these are not like vectors.

So what are the difference between these and the vectors? So very logically you can see a vector requires a single index for its specification i. This requires 2 indices for its specification. What are the special indices? One index is just like making it act like a vector, but the other index is specifying the direction normal chosen to calculate that quantity. So it is something more general than a vector. This actually is called as a second order tensor.

We will not be defining in general what is a tensor because it is an involved mathematical concept and there is not enough scope here that we discuss about that, but at least from common sense you can appreciate that the order of tensor in this Cartesian notation is like the number of indices that you are requiring to specify it. So vector is also a tensor. It is a tensor of order 1. Scalar does not require any index for its specification. So it is like a tensor of order 0.

So we have very easily come across 3 different orders of tensors, tensor of order 0 which is a scalar, tensor of order 1 which is a vector and tensor of order 2 example is a stress and we will see examples where we will be having tensor of order 4 as of course there may be nth order tensor in general, but we will see that there are certain important tensors in the context of mechanics in a continuum of fluid.

So fourth order tensor is one such example which we will come across later on in this course. So we will now go to our next objective that is given this components of the stress tensor, how we may utilize these concepts of these components to designate the state of stress on any arbitrary surface, which is neither oriented along x, y, or z.

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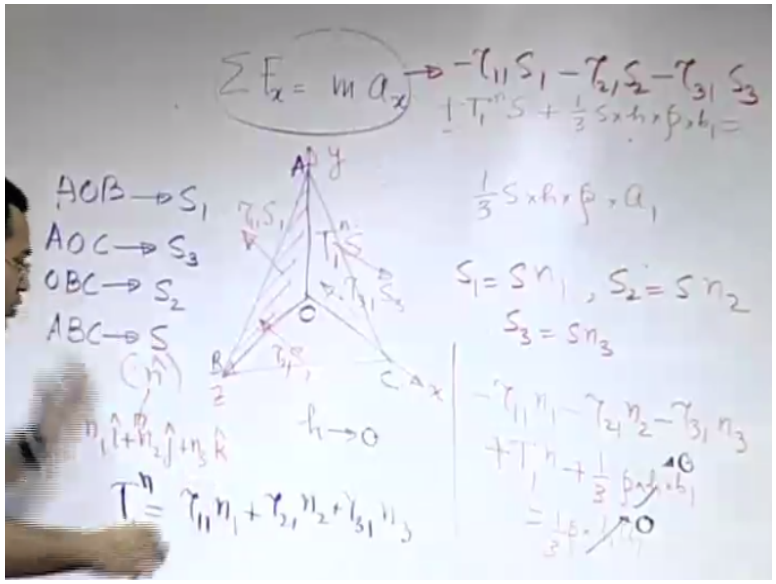

For that what we do is we consider an elemental volume like a tetrahedron. We give the points certain names for convenience. So there are surfaces like this. The surface AOB let us call it us S1, surface AOC we call it as S3 why such 1 and 3 because this index we are trying preserve for the direction normal of those surfaces so this one is for the fact that the direction normal of this AOB is along x. Similarly, this one is S2 and ABC let us give it a name S.

Can you tell, what is the motivation of taking such a volume? See whenever we are deriving something in the class it is like it will appear to you that it has to be done like that. Remember this is not a ritual. Do not accept anything whatever we are learning in the class as a ritual always try to ask yourself a question why have you taken such a volume? What is the motivation behind taking such an element? So if you see this element has 4 faces.

Out of this 4 faces, 3 are the special ones which I have their direction normal either along x, y, or z. The fourth one is not a special one. It is arbitrary oriented. So now by considering the equilibrium of this element by considering the forces which are acting on it, we will be able to express what is there on that odd surface in terms of what is there on the special surfaces. So, that is what is a motivation behind taking this one.

Now whenever we are coming to such an element, our objective will be, say to write the Newton's second law of motion for this. So just resultant force = mass * acceleration. Question is what forces are acting on this element. So when we say what forces are acting on the element, we will be classifying the forces in continuum mechanics in 2 categories. One is a surface force another is a body force. These names are almost self explanatory.

So when you say surface force it means that these are forces which are acting on the surfaces which are comprising the volume element chosen and body force is a fore which is acting over the volume of the body. Example is body force one of the examples of body force is the gravity force which acts throughout the volume of the body. Surface force, pressure is an example. Force due to pressure is a surface force. So whenever we are having forces we will categorize in terms of surface force and body force.

So wherever we have surface force, the surface force may be expressed in terms of the traction vector, because the traction vector we are defined in such a way that on a surface it represents the resultant force per unit area. So it represents a cumulative effect of all forces which are acting at that point on the surface. So this particular element has 4 surfaces. Let us write the forces which are acting on these 4 surfaces and write the Newton second law of motion along say x direction.

So what we are going to write is resultant force along x = the mass of the fluid element * the acceleration along x. So when we write the resultant force we will write it in terms of the surface force and body force. So first let us come to the surface say AOB. **"Professor - student conversation starts"** So, on this surface which component of the stress tensor will give a force along x? In terms of tau ij? Tau11.

So what would be the positive sign convention direction of Tau11 along this? So Tau11 this is the force permit area. So what is the area on which it is acting into S1? Similarly, for S2 what is the force which is acting along 1? Tau21. What will be its direction? Just like this because its outward normal is along negative 2 direction, therefore the positive sense of Tau21 on this surface is along negative 1. **"Professor - student conversation ends"** So this multiplied by S2 similarly this will be Tau31 S3.

There is a force surface which is really the back surface here which has its normal neither along x, y, or z. Let us say the normal to this is n cap. The outward normal vector of s. So this n cap say it has its components like this n1 I cap + n2 j cap + n3 k cap where n1, n2, n3 are the components of this along x, y, and z. We are now going to write the force on S. So the force on S let us say there it is T with superscript n so now we cannot use the tau notation for that because it is not a special surface.

Tau notation you can write for a special surface where the normals are along x, y, or z. So we use the T notation. This is for the ABC. This component wise it is along 1 and the area on which it is acting is S and obviously by default we are taking it along a positive x. These are the surface forces. What is the body force? Say, we call that b is the body force per unit mass. So if you find out what is the mass of the fluid element, so what is the mass of the fluid element.

So let us say that we find out first what is the volume of the fluid element, so for this type of element, we can say that it is one third into the area of this ABC * the perpendicular distance from O2 ABC. Let us say that perpendicular distance is H. So we first find what is B1? So what is B1? First, the volume one third * s*h where h is the perpendicular distance from 0 to ABC is the volume.

Mass of this you multiply it by the density so rho is the density so this the mass and this multiplied by the body force per unit mass along x will give the total body force along the direction 1 or x. So this is the mass, this is the body force for unit mass along x. So the product is the total body force along x. So we write the Newton second law of motion. This particular expression now we write the forces - Tau11 S1 - Tau21 S2 - Tau31 S3 + T1 with superscript n*s that is the total surface force + the total body force that is the net force which is acting on it and that net force = its mass * acceleration.

So that is = what is this mass one third is H rho that is the mass. Let us say A is the acceleration so A1 is the acceleration along x1. **"Professor - student conversation starts"** Sir one thing I want to ask yes; we will interrupt you. Can you tell what is surface force and body force? The physical definition and the mathematical definition is identical here. Surface force is a force which is distributed over the surface which is the envelope of the volume that is being considered and body force is something which is acting within the volume of the body within the body.

Like surface force is a shear force. What is the sheer force of (()) (37:35) also normal force just like pressure is a normal component, it is not a shear component. Actually one term is trying tau11 S1 is also acting normally to the body, but it is a force which is acting on the surface I mean whether normally to the body or not it is a matter of direction, but the force may act on the surface or the force may be acting throughout the volume of the body that is how it is classified.

So obviously when we are considering this, this is something which is acting on a surface obviously it will have a direction there is no contradiction with that with the surface force and body force. Surface force will have a direction. Body force will also have a direction. **"Professor - student conversation ends"** So when you have this expression. Next is, you can write S1, S2, S3 in terms of S. How can you write it? See S1 is like the projection of S on the yz plane.

So how do you find out the projection? You find out basically the component of this so called vectorial representation of ABC on the yz plane. So that means when you want to find out the component you basically find the dot product of the corresponding unit vectors. So this has a unit vector in the direction of ABC has unit vector in the direction of n, AOB has unit vector in the direction of – I, but obviously here the + dot –

You are already taking care of through this sign convention so you are not duplicating it once more. So the dot product of those 2 directions will be in one is this vector another is i. So the dot product will be n1. So in terms of these are all magnitudes. Their senses have already been taken care of with + and -. So S1 is nothing but Sn1 by taking the component of so called S in the direction of the so called S1. Similarly, S2 is Sn2 and S3 is Sn3.

So in that way if you substitute in this equation you will see that S gets cancelled out so what you will get - tau11 n1 - tau21 n2 - tau31 n3 + T1 + 1/3rd rho * h * B1 = 1/3rd rho * h * A1. When you have these equations the next consideration that we have to make is something which is subtle, but important to understand. We will shrink this volume to a point such that this entire volume as if converges to the point 0 because our end objective is to find out the state of stress at a point in terms of any R chosen around that point.

So we will be considering a vanishing area a vanishing volume so to say not a vanishing area so that everything converges to 0 that means we are taking the limit as h tends to 0. So when you take h tends to 0 the entire volume will converge to the point 0. Then whatever we describe basically is the description of state of stress at a point 0. So when you take that limit as h tends to 0 we will see very beautifully these terms will tend to 0.

So in that case you are left with a very simple expression for T1 that is tau11 n1 + tau21 n2 + tau31 n3. You can see that this is a very excellent expression because it relates the traction vector on an arbitrary surface with the components of stress tensor. What are the inputs? The inputs are that you must know the state of stress on those planes with specified orientations.

And the components of the unit vector direction of the arbitrary plane that you are considering everything at a particular location. The other thing is that there is a way of writing this symbolically in a more compact manner. You can see that here you are having 2 indices. So the first index is what is vary? The second index is something which corresponds to this one. So you can just generalize it and say.

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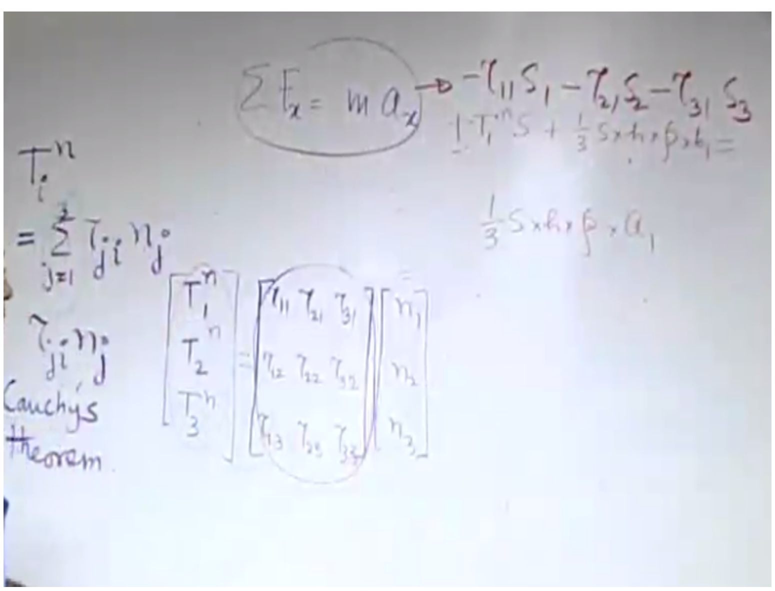

So you have 1 index I which is fixed, the other index j which is a variable which varies from 1 to 3. Now because this type of notation is very common the general rule is again the general notation is that this summation is only omitted. So this becomes invisible. So this is also written as just tau ji nj. How you will know that there is a summation. Whenever there is a depicting index you have to keep in mind that there is an invisible summation in it so from now onwards many times.

We will be using this notation without using the summation symbol and I have to keep in mind that whenever there is a repeating index there should be an invisible summation over that. This is known as Einstein index notation. This was first introduced by Einstein. So this type of notation gives a very compact way of writing this traction vector in terms of the components of the stress tensor so this is known as Cauchy's theorem.

So what it does is? It expresses the traction vector on any arbitrary plane in terms of the corresponding stress tensor components. It is also possible to write it in a matrix form. So what we can do we are having components of the traction vector on a given plane so with orientation 1, 3 components you have. So this if you just follow this expression you will see that. So you can see that this is nothing but a matrix way of writing the 3 components of the traction vector.

So whatever equations that we have, this is not something new, just put I = 1 it will correspond to the first row of this I = 2 and I = 3. So you can see is that here you get those so called 9 components of the stress tensor and all these are not independent we will see, but this is something what it is mathematically doing you can see here look into these quantities. What is this? This is a vector. It has a 3 components n1, n2, n3. This is also a vector. So this is acting like a transformation with maps of vector on to a vector.

So it is a very important characteristic of a second order tensor that a second order tensor maps a vector on to a vector. Similarly, like a fourth order tensor, maps of second order tensor on to a second order tensor like that as an example. So tensor is also like a transformation tool or a transformation which tries to transform one vector into another vector if it is a second order tensor. Next thing that follows from this is that are all these independent or all these tau ij is independent or we should be in a position to express this tau ij some of this in terms of the other. So for that we will quickly do one exercise.

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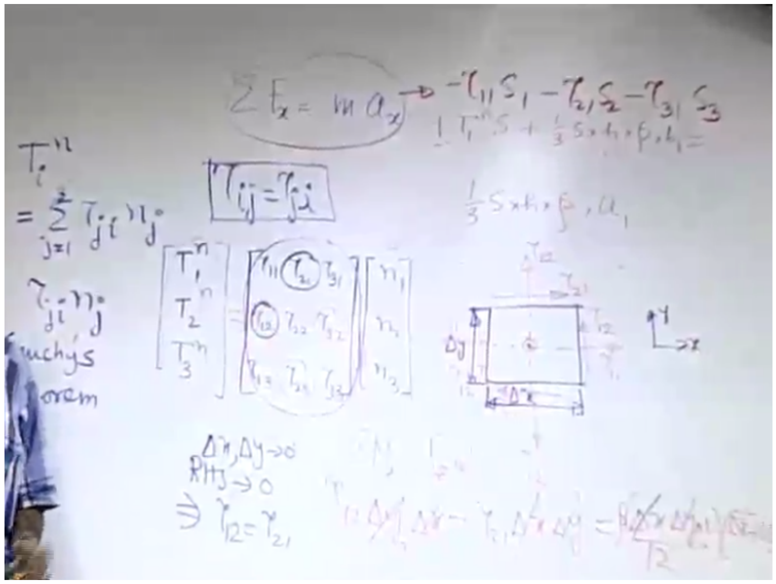

We will consider now a 2 dimensional element. Two dimensional element is something where everything is occurring in a plane just for simplicity so we are assuming that the third direction is like unity or whatever. So it has its length like say this is delta x, this is delta y, just imagine that it has faces perpendicular to whatever has been drawn in the figure having all width as 1. So let us write the components of the stress tensor on these surfaces.

So, very quickly we will write it, because we have learned by this time how to write it, so this is Tau11, this is, what is this? Tau21, here this is Tau11, this is Tau21, here this is what will be here? Tau 12 it is along the positive one because the outward normal is along positive 2. So these 2 will be reversed. So I would expect that you always correct it. So the first index is what direction normal. So direction normal is what? 1.

Second index is the direction on which it is acting. So this is tau 12 and this is Tau21. Here same. Now we are interested about the equilibrium of this element. So when we are interested about the equilibrium of this element we will consider the rotational equilibrium as an example. So rotational equilibrium let us consider that as if it is an element where we will be writing an equivalent form of Newton's second law for rotation.

As if like we are writing rotation of a rigid body with respect to a fixed axis something like that when the axis is this center. As if we are writing a rotational equilibrium equation with respect to an axis which passes through this center o and is perpendicular to this plane of the board. So that is why it is a 2 dimensional thing. We are considering a rotation in this plane basically. So what we can write, resultant moment of all forces which are acting on this is what say we are writing the moment with respect to which axis?

Z axis here is equal to what tells that it is 0? It might be having an angular acceleration. So it is I with respect to the same axis which is passing through this one and perpendicular to the plane of the board say z times the angular acceleration say alpha. So the resultant moment of all these forces what will be that? So we will see that tau11 and - tau11, tau22 and - tau22 they cancel. So moment contributors will be tau12 and tau21.

So tau12 and this tau12 they found like a couple. So it is tau12. What is the area on which it is acting * delta y * 1 which is the width * the arm of the couple moment delta x. For the other one it is clockwise so - tau21 delta x * delta y. We are assuming there is no body couple which is acting on it just like body force there could be body couple. Fluids usually do not sustain body couple.

So there is no body couple which is acting on it = the moment of inertia is like it is having a dimension of aim * length square. So aim is like delta x * delta y * 1 * rho that is the m. If you write it properly it is delta x square + delta y square/12 with respect to this axis times the alpha. Keep in mind the delta x * delta y are all small and tending to 0 so if you cancel by considering this as tending to 0 but != 0 so you are left with what?

In the right hand side, you have terms because delta x and delta y are tending to 0 you have the right hand side tending to 0 and that will give you a very interesting result tau12 = tau21. So in general tau ij = tau ji. That means tau21 and tau12 are same. Tau31 and tau13 are same and tau32 and tau23 are same. So you are left with 6 independent components in this stress tensor and you see that tau ij = tau ji.

What are the assumptions under which it is valid? there is no body couple that is the only assumption. It does not depend on whether it is at rest or in motion. This is a very common misconception that people have that is valid only for static systems and it might be accelerating but it does not matter because in the limit as delta x delta y tends to 0 the angular acceleration term becomes insignificant and that is how you get tau ij = tau ji. So we will stop our discussion today. We will continue in the next lecture.