

Fluid Mechanics
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Lecture - 8
Fluid Statics Part – V

Good morning, I welcome you all to the session of fluid mechanics. Now, if you recall, last time we were discussing about the buoyancy, phenomena of buoyancy, what is that? Again, just if you recall it, a body, if a solid body is either partially emerged or fully emerged in a fluid, the net horizontal force in any direction because of the pressure exerted by the fluid on the surface of the body is 0, but the body experiences net vertical force in the upward direction. That means, the resultant of the pressure forces on the surface of the body, that is, the mass body, either fully or partially, is in the vertical direction, upward vertical direction, and its magnitude is equal to the weight of the displaced volume of the fluid.

What is the displaced volume? That is the volume, emerged volume, that is the volume of this solid emerged in the fluid. In case of a fully emerged body, that is, submerged body, then the displaced volume is the volume of the whole body itself. So, weight of that volume of the fluid is the magnitude of the upward hydrostatic pressure force, which is known as buoyant force and this phenomenon is known as buoyancy.

Now, then afterwards we recognized that for an equilibrium condition of the body, first let us consider this submerged body, but it is very, it is valid for both, submerged and partially emerged body, that for equilibrium the primary condition is, that the weight of the body should balance the buoyancy force, upward buoyancy force by magnitude and also by the line of action. That means, weight of the body should be equal to that of the buoyant force and this two forces, which must be colonial for equilibrium of the body either in floating condition, that is, partially an emerged condition or fully emerged or submerged condition.

Now, the question comes, that probably I have already read in, earlier in mechanics, that equilibrium is of three types, one is stable equilibrium, another is unstable equilibrium, another is neutral equilibrium. What is meant by stable equilibrium in general is, that if a body is in equilibrium under several forces at a particular instant, now if you disturb the

body to just depart from its initial equilibrium position, whether the body is able to come back to its initial position or not? If the body is able to come back to its initial position that means force system acting on the body is such, that it makes it possible to restore its initial equilibrium position. Then, this type of equilibrium is known as stable equilibrium; that means equilibrium is stable. It is the question of reproduction; just the body is capable of reproducing its initial equilibrium position under any small perturbation to distort, to allow the body to depart from this initial position. It will again come back to its initial position.

Now, unstable equilibrium is such, that if the body is in equilibrium under such several forces at a particular position, if you disturb the body to depart from its initial equilibrium position the force system is such, that it does not allow the body to come back to its initial position. It goes on departing from its initial equilibrium position and in fact, the entire equilibrium condition of the body is destroyed. This is known as unstable equilibrium. So, equilibrium is there at a particular position and the particular instant, but this is unstable, unstable equilibrium.

Another type of equilibrium is neutral equilibrium, which means, that if you give a disturbance to the body, then the body will neither come back to its original position nor it will go on departing further and destroy its equilibrium condition, neither of these two, but body will remain again in the equilibrium condition at that point. That a sleeping person, that a sleeping person, if you take him from one position to another position, he will be keeping the (()), like that the neutral equilibrium.

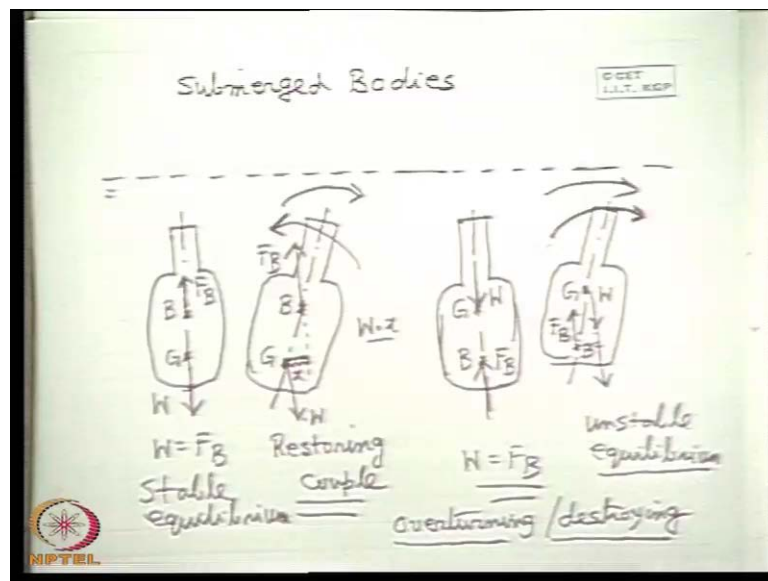
I mean, a very simple example, probably we have read it in school level, that if you place a marble on a hill or on a table of convex shape, if u place it, it may be in equilibrium, that the weight may be balanced by the reaction force if there is a contact surface. But if you slightly perturb it, that means, if you slightly push it, this will go down because of the convexity of the surface, that means, this is the unstable equilibrium.

Similarly, if you place a body on a surface, which is concave inward and you place a body, it is in equilibrium. The weight of the body is equal to the reaction force. If you give a displacement, then it will again come back to its original position because of the concavity of the surface. That means, this is a, and this is an example of stable equilibrium and neutral equilibrium. Example is very simple, on a flat table, flat table, if

you place a body for example, this is in neutral equilibrium. If you place it here it is in equilibrium; if you place it here, if you just do not consider the rolling of the, say it is in equilibrium. So, that means, this a concept neutral equilibrium.

In case of buoyancy, this perturbation to study whether the body is in neutral, stable or unstable equilibrium depends upon this perturbation in angular direction. That means, if you just displace the body or give a small rotation, then we see its stability. That is why sometimes this stable equilibrium, unstable equilibrium, neutral equilibrium is a couple or is referred to as angular stability of the body. So, body is linearly stable. When () and buoyancy force are equal in magnitude and are collinear, but weather in small angular displacement, allows the body to come back to its original position or not is referred to as angular stability or in general, to stable, unstable and neutral equilibrium.

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So, let us now see, that let this be the free surface of the liquid. Now, let us consider first submerged body, first consider submerged body, well, first consider submerged body. Now, what is condition? Let us see a body like this, let us see a body like this, let us consider, now you see with the center of gravity and center of buoyancy should be in this line. Now, there may be three options, dividing point of application of center of gravity and center of buoyancy. One option is, that center of gravity may be, let this is center of buoyancy, center of gravity may be below the center of buoyancy. Another option is that center of gravity may be coinciding with center of buoyancy. Another option is, there

center of gravity will be center of buoyancy. This depends upon the relative distribution of mass over the (()).

When gravity is below the center of buoyancy, it is bottom heavy. When gravity is above the center of buoyancy, it is top heavy. And when the mass is distributed uniformly throughout this two coincide.

Now, let us consider when the centre of gravity is below the center of buoyancy what happens? Through center of buoyancy the buoyancy force B acts; through center of gravity weight W acts. They are collinear and equal always, W is equals to $F B$, so this is in equilibrium. Let us give a small tilt or angular of this body in this direction, towards the right. Let me draw this figure, now this tilt condition. Let us see the body, let us see the body. Now, this is the axis of symmetry of the original vertical axis, whatever you can tilt. Now, the centre of gravity with respect to the body is unchanged. This is, of course, not true for all cases, I will explain afterwards.

In certain cases if some solid part of the body moves, for example, when the ship moves, when a ship moves in a river some of the cargos within ship moves from one part to other part. So, therefore, the mass may change, but if we consider a tight solid body, the center of gravity remains unchanged and the weight acts vertically in center of gravity. Now, this center of buoyancy also remains unchanged in this case. This is because what the center of buoyancy is also the center of the volume and when in submerged bodies the entire volume is submerged, so here also entire volume is submerged. In both the conditions the bodies are submerged, so center of buoyancy also will not change through which the buoyancy forces acts.

Now, in this case you see these two parallel forces create a moment or a couple, which is now this is the direction of tilt, so a couple is generated in the opposite direction whose magnitude is W or $F B$, whatever you call, both the forces are equal, times this distance, times this distance. If you consider this distance as x , W into x , that means, simple, we can tell in this circumstances this $F B$ and W creates a couple, which is opposite to the direction of the tilt and this couple is known as restoring couple. So, this couple is restoring couple that means, which restores its position. That means this couple helps the body to come to its equilibrium position.

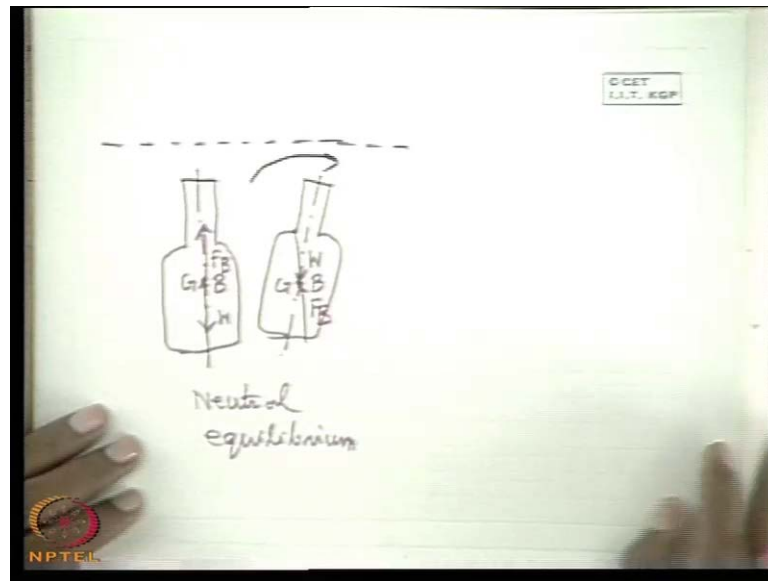
Now, if you give a tilt or in the left direction you will see even in the body buoyancy force and the gravity force makes a couple in the opposite direction to the angular of the body that means creates a restoring couple and helps the body to come into equilibrium. So, when G is below B we see the equilibrium is stable equilibrium. So, this is stable equilibrium, stable equilibrium, stable equilibrium.

Now, let us consider a case when G is above B that means, the distribution of the force is like this. Distribution of the force, sorry, distribution of the mass over the volume is such that means, it is top heavy that means G is, this is G and let B is below G. That means, through G is the weight of the body acts and through B the buoyancy force acts, but it is in equilibrium. That means, $W = F_B$, this is (()). Now, if you give a tilt, very simple, school level thing, so if you give a tilt in the same direction that means right-wise, towards the right if you give a tilt, this is the original vertical axis here, again in this condition the gravity remains the same, the center of the gravity and also the center of buoyancy.

Now, you see, this two forces, W and F_B , they create a moment, which is in the same direction to that of the angular tilt. That means, when you give a tilt, in this case the two forces F_B and W, whose colinearity is destroyed, they create a moment in the same direction or couple in the same direction and helps the body to tilt further there. Both the body departs from original position further and further and equilibrium is never attained. This case, it is known as unstable equilibrium, unstable equilibrium. And this couple under this condition the value of this moment that means, W or F_B times this perpendicular distance is known as overturning or destroying couple; sometimes we call overturning, this is the terminology in mechanics or destroying couple, the couple, which destroys the equilibrium.

Therefore, we see, now in this case if you give a (()) in the left direction you can again examine, that the couple created by the destruction of the colinearity between F_B and W, the couple created by this two force again will be in the same direction of the angular tilt and creates overturning and destroying couple and makes the equilibrium an unstable equilibrium.

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Now, the last possibility is the possibility, that G and B coincides, that means, this is the case when the weight, sorry, mass of the liquid is distributed, this is G , uniformly over the entire volume. That means, the weight acts to the center of gravity and the same point is the center of buoyancy, that is, $F B$. That means, the mass is uniformly distributed over the entire body. In this case, it is just like neutral equilibrium, as I have told earlier.

That means, if you displace the, sorry, if you displace this under the displaced condition, that means, if you give angular tilt in this direction, G , B , both will be same, so this will be the new vertical lines and they will be again collinear. This is the W and this is F , so it will be again collinear, that means, under any angular location and at any angular displacement and this position it will be neutral equilibrium. It will neither go further in this direction or will come back in this direction. This is known as neutral equilibrium, this is known as neutral equilibrium.

So, therefore, what we conclude is, that when a body is totally submerged the condition for stable equilibrium, that means, angular stability, that if the body is given a slight angular tilt in either direction, whether it will come back or not depends upon the fact, that will G , the center of gravity, is below the center of buoyancy. The body's angular; body has this angular stability means, stable equilibrium.

If the center of the gravity is above the center of buoyancy, if the body is in unstable equilibrium, that means, it does not have the angular stability. If you give a small angular

hill in either direction or part of the body, it will go on departing for the original equilibrium condition and the entering equilibrium will be destroyed. This is the case of unstable equilibrium when the center of gravity is above the center of buoyancy.

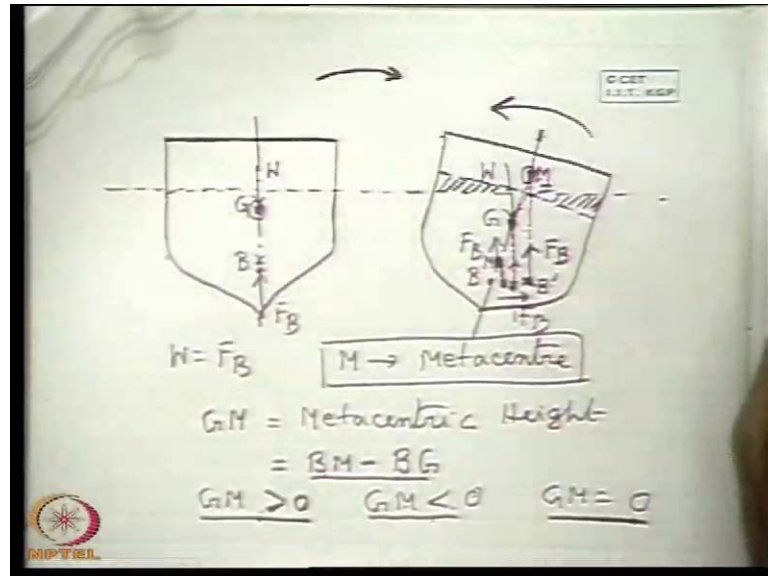
And the last case is the center of gravity and the center of buoyancy coincides, that is, for a special case when the distribution of mass is uniform throughout the volume of the body, in that case the body is in neutral equilibrium. That means, if you give any position with any angular position at any inclination, at any configuration, if you submerge the body, body will be at that particular configuration. That is the concept of neutral equilibrium.

Now, we will discuss the same thing under floating condition, but, there is a difference. First of all you must know, that why separately you are discussing this submerged and floating bodies? What is the key point of difference between a submerged and the floating body? First difference is, that as it is very simple, literally, floating body means a part of the body submerged, not the entire body submerged in the fluid. Therefore, the buoyancy force will be equal to the weight of the displaced volume of the fluid. That means weight of that volume of the fluid, which corresponds to the emerged volume of the solid; that is number one. Number two is that obviously, under equilibrium condition of a floating body, the weight of the body must balance the buoyancy force and next is that they should be collinear and buoyancy force is equal to the weight of the displaced volume.

Therefore, floating body in equilibrium is that the density of the body has to be lower than the density of the fluid so that the weight of the displaced volume of the fluid, which is the emerged volume, partially emerged, that means, not the total volume of the fluid must be equal to the weight of the fluid. This is very simple, we have rated school level. Now, what happens, when you give an angular hill for this floating body what is the most important is, that here though the center of gravity may remain unchanged, if there is no movement of mass within this solid body, but center of the buoyancy is obviously changing. This is because though the total displaced volume remains same to make it equilibrium and to be equal to weight, but some portion is getting more emerged and some portion is getting out of the water level depending upon the direction of the hill. So, they are both, the center of buoyancy to distribution of the emerged volume changes though the total emerged volume remains same, so that center of buoyancy

moves. It does not become same as earlier one, which we saw in case of submerged body, that is the key difference. So, they come separately for investigation of the angular stability in case of floating bodies.

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With that let us see, that what happens in case of a floating body. Good, now, let us consider this is the free surface and let us consider the floating body, typical cross-section of a ship, let us consider a floating body like this. This is the axis of symmetry, a symmetrical body like this. Now, let us consider a floating body, this is the part, which is emerged in the fluid and this is outside the fluid. Let us consider this is the initial position and here we will see one interesting thing.

Now, first of all, if gravity is below this center of buoyancy, center of gravity, it has always been in stable equilibrium, that you can see by yourself, I am not going to that. Let us consider the gravity is above its center of buoyancy. Now, we have seen in case of submerged bodies, we immediately, obviously, whenever the gravity is above the center of buoyancy, then obviously it is unstable equilibrium, but in this case it may not be. Then, angular stability may not always be destroyed if the center of gravity is always above B, you cannot tilt them. Now, center of buoyancy is the center of this volume and we consider this center of gravity is above B. One thing is true, that the weight and the buoyancy force, that is always true, they balance each other. Now, in this case what happens?

Now, what I have told just now, you see, that if you give a small tilt in this direction and let me draw the figure like this. Now, let in displaced position this is the old line of symmetry or the line, which contains the center of gravity and the center of old vertical line containing center of gravity and old center of buoyancy. Now, it is obvious, that if I retract or retrace this line, let this line be, this line, so therefore, we see, that this portion of the body portion has gone out of the free surface, which was earlier there underneath the free surface. And this portion of the body, which was above the free surface, has gone down below the free surface. That means, that if we give a tilt here more of the volume in this portion will go under the free surface and some volume in this portion will automatically come up. So, therefore, we see the distribution of the emerged volume change. It obviously, if this be the old center of buoyancy a new center of buoyancy will be shifted somewhere here in this direction. B dash is the new center of buoyancy, so what it does?

Now, if this be the G, now the force acting on this body is W, vertically downward through G and then we see, that this F B. Now, it is not very easy to say, that always it will be overturning moment or a destroying moment. In this case what do you see, that if I make projection of the vertical through G like this under this particular condition this is which moment? Restoring moment, very good, so under this case it is the restoring moment, that means, it is angularly stable, but can we tell always if B shifts in this direction, just commonsense, you tell always there will be restoring moment?

No, if this B dash comes here, that means, if I draw the vertical projection or a vertical line from the centre of gravity and if B cannot cross this vertical line towards this direction, direction of hill, then it cannot create a restoring moment so long it is here. So, this force will always create a overturning moment, that whether there will be a restoring moment or an overturning moment is not conclusively decided by the position of G above B, but this shifting of B with respect to G. If the shifting of B takes place like this, that it crosses the vertical line in that direction, then it creates a restoring moment, otherwise it creates an overturning moment. So, this is the condition.

This is better stated analytically by this fact, that if we make a vertical projection from the new center of buoyancy, make a vertical line, extended vertical and let it intersect with the old vertical line containing the center of gravity at the old center of buoyancy at the point A and this point A means metacentre. This is very important concept,

metacentre and usually, for a small angular hill, if we do not consider a very big angular hill, that means, small perturbation with small angular hill in either direction. So, you can see, if you give a tilt in that direction the same condition will (()), that means, B either create an overturning or restoring couple depending upon its relative displacement with respect to G. Now, this M metacentre is usually for a small angular hill, is a geometrical criterion depending upon the dimension of the body and its geometrical shape, so it becomes a geometrical criterion.

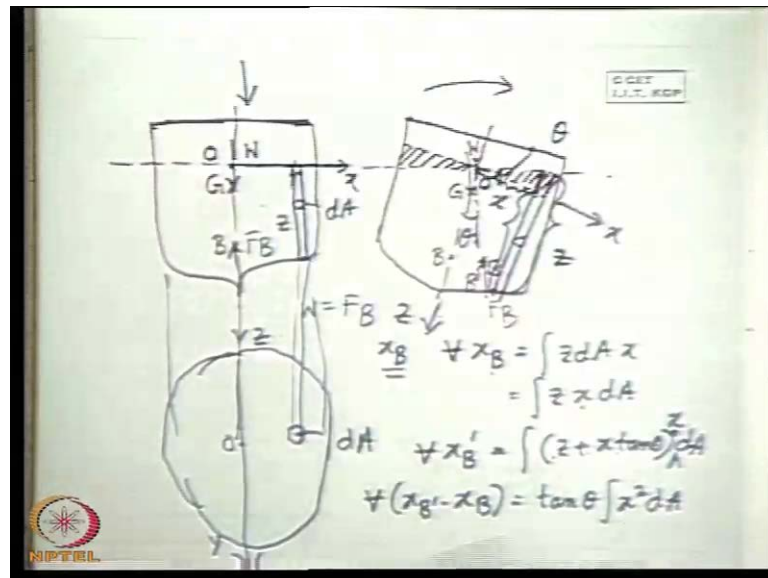
Now, we see that the way we appreciate, that if B crosses this vertical line in the direction of the hill, then it will give a restoring moment otherwise overturning moment can be told with respect to this. Very important point, metacentre, that if metacentre is above the centre of gravity, in this case, it is that then the body is under stable equilibrium and if the metacentre is below, that means, in this case if this is the shape, then metacentre, then the vertical line will always intersect at a point, that the same point known as M, which is below G. That means, when metacentre is below G, then we can tell, that the body is not in stable equilibrium and the height GM, that means, the height from the centre of gravity to the metacentre, if it is below, this is the height, this is known as the, along this old vertical line or the line of symmetry you can tell, is known as the metacentric. That means, GM, this distance is known as the metacentric height, which can be also written as $GM = BM - BG$.

If you write like, that when metacentre is below the centre of gravity, then GM is negative. That means, we can tell GM is greater than 0, GM less than 0 and GM equal to 0, three cases. When GM is greater than 0, metacentre is above the centre of gravity. By this formula we define as analytically, that it is greater than 0, M is above G and the body is in stable equilibrium. When GM less than 0, means, M is below G, so we define it as BM, so that GM is less than 0, then the body is in unstable equilibrium, that the couple created because of the tilt by the weight and the buoyancy force is an overturning one, that is, the unstable equilibrium. And when metacentric height coincides with G, that means, B is displaced in such way, that the vertical line through B just intersects G that means, again in that displaced position F B and W become collinear, if it satisfies that, that means, GM is 0. That means GM is equal to BG, which means, the vertical projection from new center of buoyancy intersects this point, then this is the neutral equilibrium.

So, therefore, it is not conclusive only from G above B , which was therefore, submerged body, it will be under unstable equilibrium for a floating body. The key point is, that in this case, since the center of buoyancy moves towards the direction of the hill it gives an advantageous condition, that means, the center of buoyancy moves towards the direction of the hill, it gives an advantageous condition. If even if it is top heavy, that means, center of gravity is above the center of buoyancy till it is in equilibrium, stable equilibrium, provided the metacentre is above G , that means, the buoyancy shifted in such a way, that it crosses the vertical line from the center of gravity, so that it creates the restoring couple to allow the body to come back to original position. Well, this concept is now clear.

Now, with this concept in metacentric height I will derive now a very important (()), what exactly is required for designing a floating body impact. This for example, a ship, that the metacentric height should remain positive, so that for any angular floatation or angular disturbance of the ship about any horizontal axis, ship is stable. Now, this metacentric height, I have told of the metacentre position of this, metacentric is the function of the geometrical shape of the ship or the geometrical shape of the floating bodies. So, if we know beforehand by, calculate this metacentric height from the, perform a functional relationship of the metacentric height with the geometrical dimension of the body. Accordingly, we can design the ship, that make the geometrical dimension, show that the metacentric height is always positive and it should be as big as, that means, MG should be high, high positive value, so that stability is ensured even under a greater perturbation. So, for that we tried to seek or we seek a functional relationship between the metacentric height or the position of the metacentric in terms of the geometry of the body.

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Let us do that. Well, again we, again we make the same thing, that this is the body, we have floating bodies body like this, the floating body is like this, that is the cross section of the ship, rather taken a cross section of ship, this is the plane area. Now, after the hull let us, it could have been better, different, so this is after hull is in the displaced position. Now, let us see, first we consider a simple case always, that G is above B. That is not the simple case; simple case is G remains unchanged.

Let us consider this is G, but the old B, obviously, when you give a tilt in this direction what will happen? Old B will be shifted here, B dash. So, the weight act in through this and this is acting F B, F B and this is W. Now, if we trace this old line of, line for the plane floatation we can write or we can, sorry, draw line like this, let this is the point O, point of intersection of the original plane of floatation, well this is the original plane of floatation and the new plane of floatation and let them intersect along their centroidal axis.

Now, one thing is true, so long W is equal to F B, well, W is equal to F B, that means, this is the W, this always we consider equilibrium, only we are considering the stability, that is angular stability, then the buoyancy force W is unique; so, buoyancy force is unique. So, therefore, by the principle of Archimedes the emerged volume should remain same whether it is given a tilt or not, which means, the volume, which has come up, that means, this hatched portion mass be equal to the volume at this hatched portion, which

has gone inside the liquid. This is the key point, nothing else, so this volume is equal to this. So this is alright.

Now, if we simply define the center of buoyancy with respect to any frame of reference, now we will have to do. Let us consider this as O in this original figure, we consider the axis like X and this is as Z, vertically downward Z and the horizontal axis is on the plane of floatation and take OY, the perpendicular to the plane. That means, if we see this cross-section like this, a cross-section like this, cross-section, just an arbitrary cross-section like this, so Y will be along this direction. That means, if we define O somewhere here, this is the OY that means, this is in this perpendicular XY and vertically downward Z, I think it is alright.

Now, if we define this center of buoyancy X coordinate as x_B , as x_B , which is 0 under the condition because this is the Z-axis. That means, this is from Z-axis, so x_B . Then, we can write, that V, emerged volume, into x_B is nothing but if we take here a prismatic body of the fluid or the volume emerged whose cross-sectional area is dA and Z is the coordinate, that means, this is the height of this prismatic element, that means, a Z coordinate of this point, whatever you can tell, that this becomes equal to z into dA into x , sorry, Z into dA into x , rather we can write $Z \times dA$, alright.

Now, you see, that here what happens, that is the dA is the volume and moment of the volume, that is, the center of buoyancy. Now, this case, it is 0 if we define center of buoyancy on this line, this case it is 0. Now, this case what we can do? We can, now the axis are defined like this, therefore Z and x will be like that, but here if we take, now this is emerged here, so here if we take a prismatic volume like this what is its height? So, this dA we can show it here, this is the dA , this is the typically elemental area in this plane that means, this view. Now, where here if we take the same prismatic element dA here, the height is equal to Z, that means, A is this one with respect to this coordinate axis x and this is Z plus this extra amount, which has become emerged in the fluid.

So, if this is the x , that means, the x coordinate of this prismatic element from the coordinate axis origin, then this will be $x \tan \theta$ where θ is the angle of hill. That means, this is θ , if we make this θ or the vertical through this O with the old vertical θ because angle between two lines is equal to angle between their perpendicular, so θ is the angular hill, that means, θ is the angle between the old

plane of floatation and the new plane of floatation or between the old vertical line and the new vertical line, so this is the theta.

So, therefore, if this is x and this is theta, so this extra part is what? x tan theta, that means, now we can write V x B dash. If x b dash is this, the B dash, that is, this x coordinate of the new center of buoyancy, what we can write? We can write that is equal to Z plus x tan theta into dA. Simply, we subtract this upper one from this one, we get V x b dash minus x b is equal to tan theta is constant because angular tilt is constant throughout the body. So, therefore, we can take tan theta into what? x square dA, sorry, x, one x is missing for the moment, so Z x plus tan theta, dA is the volume, elementary volume and moment for, moment (()), simply x square tan theta, alright. So, this is equal to, now you see we can write this. Is it ok?

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The image shows a whiteboard with handwritten mathematical derivations. At the top, there is a circled expression $(x_B - x_B) = \tan \theta \int x^2 dA$. Below it, the equation $\forall BM \tan \theta = \tan \theta \int x^2 dA$ is written. This is followed by $BM = \frac{\int x^2 dA}{\forall}$. The final result is $BM = \frac{I}{\forall}$, where BM is underlined and I is circled. An arrow points from the underlined BM to the text "Metacentric radius". In the bottom left corner, there is a logo for "NIPTIL" with a star-like symbol.

Now, we can write, well, now what we can write? We can write that V into x B dash minus x B is equal to what? Tan theta, tan theta into x square. Now, x B dash minus x B, for small angle of theta if you make a projection from this to this axis, this is the axis z for small angle of theta, this can be written as B, this is B and if this vertical line intersects at a point, let this is M, so this BB dash can be written as BM tan theta, BM tan theta, for small angle we can write this.

If we take, if we make a perpendicular from B dash on this old vertical line, which is the z axis rather, then we can tell for small angle this perpendicular does not make any

change from B to this point of projection. So, we can take simply from this right angled triangle, this vertical line meets at M, so this is BM tan theta. So, we can write BM tan theta for small angle; tan theta, because this projection may not coincide with B, that is why small angle concept. We can write, this is, this is almost equal to BM tan theta, so tan theta tan theta cancels, well, so we get what? BM, sorry, this is tan, BM tan theta. So, V is there, so BM is equal to x square dA divided by V.

Now, what is this x square dA? What is this x square dA? If we just see here x square dA, now we see to this figure, that means, x is the distance of any elemental area here from this y x, that means, this x square dA, which is there in the numerator of this expression of BM represents what? The moment of, second moment of area of the plane of floatation, this is the plane of floatation. That means, I see here as a section from, here if I take a section from here and look from top, that is sectional plane we see, that is plane of the floatation. So, x square dA is the second moment of area of the plane of floatation about the centroidal axis about the centroidal axis, which is perpendicular to the plane of floatation. This is the plane of floatation, that means, this is the second moment of area, be very careful, this one, second moment of area of the plane of the floatation about this OY, which means physically about the centroidal axis. That means, this axis here, which is perpendicular to the plane of floatation, plane of floatation is the exit plane, so y axis perpendicular to that or you can tell the centroidal axis of rotation about which there rotation is taking.

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The image shows a whiteboard with handwritten mathematical derivations. At the top, there is a small diagram of a right-angled triangle with vertices B and M, and an angle theta. The vertical side is labeled BM tan theta. Below this, the following equations are written:

$$\int (x_B - x_G) \cdot \rho g x^2 dA$$

$$\cancel{V} BM \tan \theta = \tan \theta \int x^2 dA$$

$$BM = \frac{\int x^2 dA}{\cancel{V}}$$

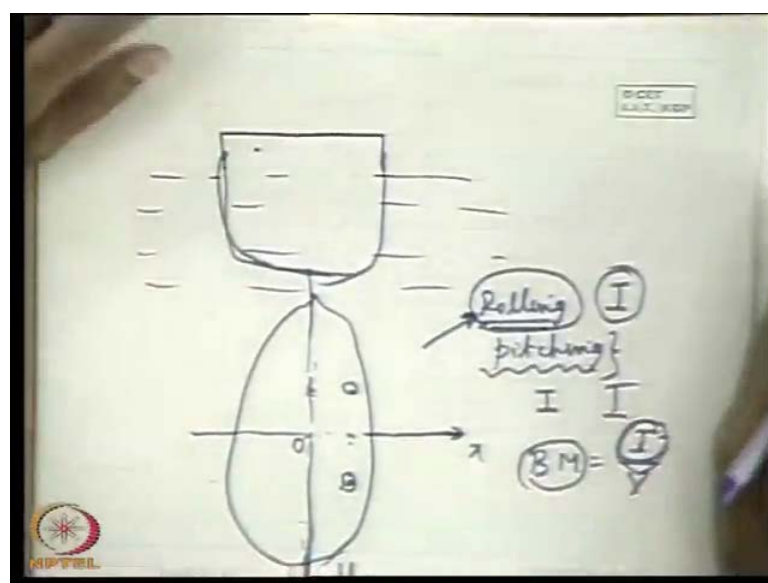
$$\underline{BM} = \frac{\underline{I}}{\underline{V}}$$

An arrow points from the underlined BM to the text "Metacentric radius".

So, if you can understand this nomenclature, $x^2 dA$, as we defined, as I simply, I , we make I y , again y makes a confusion, that you will have to take this as the y -axis. So, simply I tell I , so that I can write BM is equal to, this is a very important formula, I by V , so nomenclature is like this. BM means, what is BM ? It is the distance from the center of buoyancy to the metacentre along the old line vertical line where center of buoyancy and center of gravity was acting; so, along this B end, where the GM is the metacentric height BM . Therefore, this is the definition. And usually it is told as metacentric radius, not very common, but we will see in my book I have written, that not all the books they tell because it is not a very common terminology as GM metacentric height, but BM you can (()), metacentric radius distance between the buoyancy and the metacenteric along the old vertical line. So, this is BM , which is equal to I by V .

So, we will have to recognize the nomenclature like this, V is the emerged volume, that is, the volume of the solid, which is emerged in the fluid displaced volume. And I is the moment of inertia of the plane of floatation, moment of the inertia of the plane of the floatation about a, about the centroidal axis, which is perpendicular to the plane of rotation or the axis of the rotation. That means, if you simply (()), plane of floatation is the horizontal projection of the horizontal, that they, this plane and this perpendicular axis is the centroidal axis perpendicular to the plane of floatation. So, the nomenclature of I should be very important.

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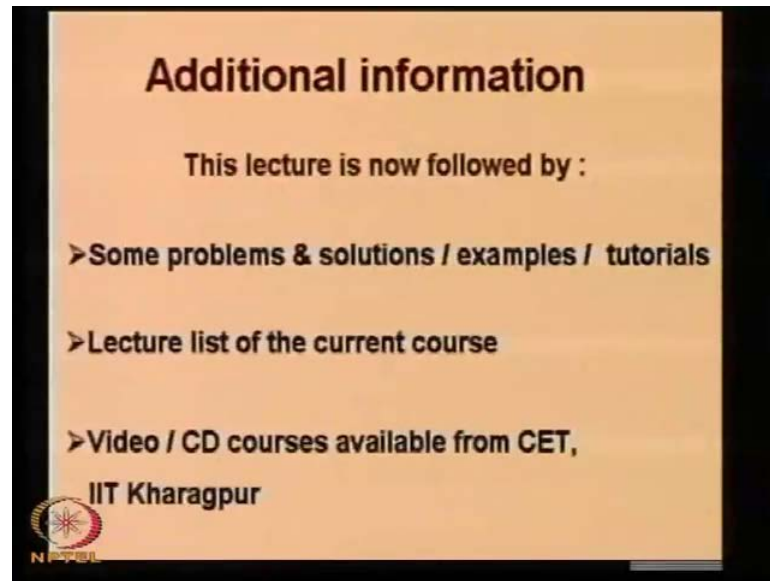
Now, you see, in case of a ship if you see the structure of a ship, now the ship, if you see a ship, now this is the, now if you see the ship, this is the elevation if you see its plane. Now, the structure of the ship is such, that like this there may be two axis, vertical axis and horizontal axis, let x and y . Now, movement of the ship about, now you see one thing, that the movement of the ship about this y axis, that means, in this direction, is known as rolling, rolling, transverse axis. And movement of the ship about this axis, that means, about the longitudinal axis, that means, in this direction is known as pitch.

Now, when the angular stability for rolling, that means, about this OY axis, pitching is about the transverse axis, that means, moves like this and rolling is about is longitudinal. When rolling, angular stability with rolling is considered, then the I corresponds to the second moment of area about OY . Similarly, when the angular stability with respect to pitching is considered, that means, it is, its angular movement about x axis, then the moment of inertia about the axis x , OX is considered. Let this is (I_x) . Now, you see from the typical geometry of the shape, the angular, the second moment of area about y axis is much lower than the second moment of area about axis because the longitudinal dimension is more. So, therefore, I is lower in case of rolling as compared to that in case of pitching, clear.

Therefore, so BM is I by V . So, therefore, in case of rolling BM is small because I is small. Rolling is about OY axis, about OY , the inertia moment of inertia is small. Therefore, stability with respect to rolling is more important than this stability with respect to pitching. See, if it is almost stable with respect to the rotational motion or with respect to pitching about x axis, the moment of inertia of any element about the x axis is much higher. So, rolling, for rolling the stability is more important. That means, we will have to see the angular stability point of view, that it is the moment of the inertia of the plane of floatation about an axis makes this more vulnerable for its angular stability in a particular direction.

Well, thank you.


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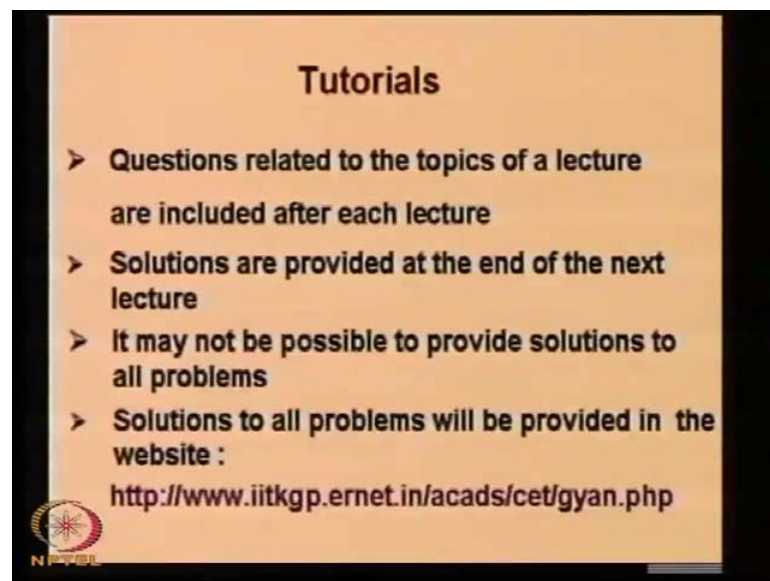
Additional information

This lecture is now followed by :

- Some problems & solutions / examples / tutorials
- Lecture list of the current course
- Video / CD courses available from CET,
IIT Kharagpur




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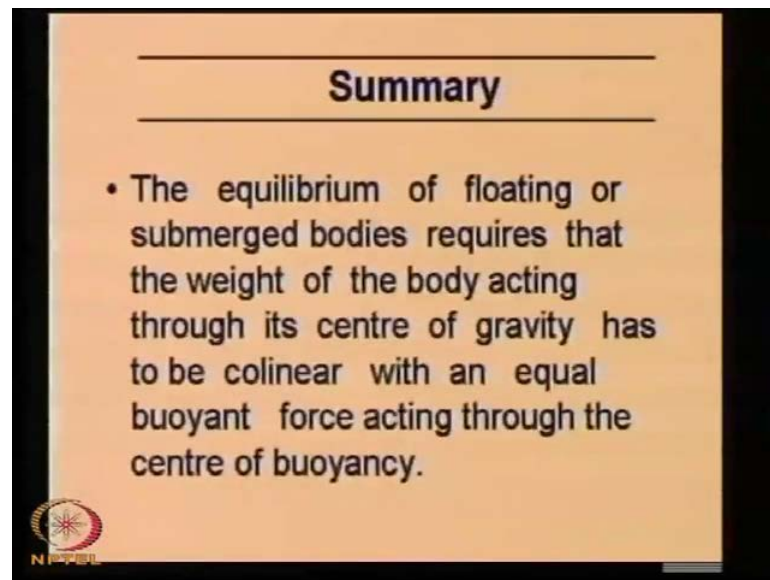


Tutorials

- Questions related to the topics of a lecture are included after each lecture
- Solutions are provided at the end of the next lecture
- It may not be possible to provide solutions to all problems
- Solutions to all problems will be provided in the website :
<http://www.iitkgp.ernet.in/acads/cet/gyan.php>




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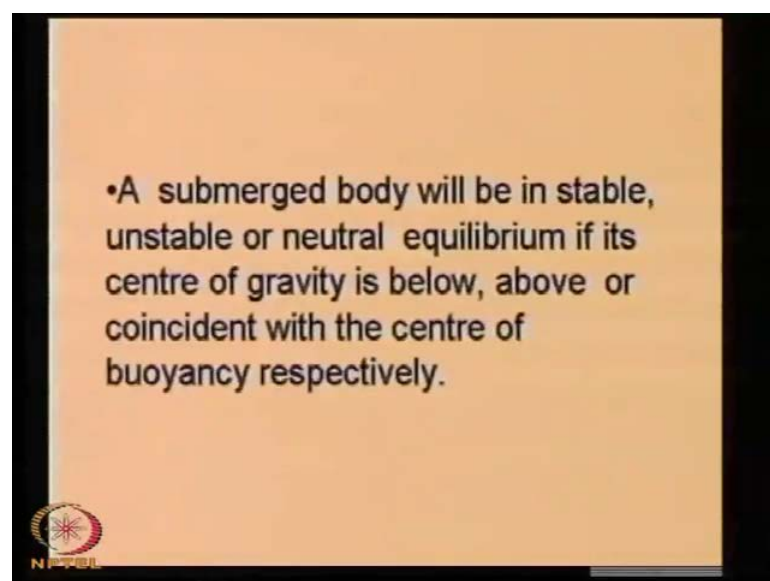


Summary


- The equilibrium of floating or submerged bodies requires that the weight of the body acting through its centre of gravity has to be colinear with an equal buoyant force acting through the centre of buoyancy.



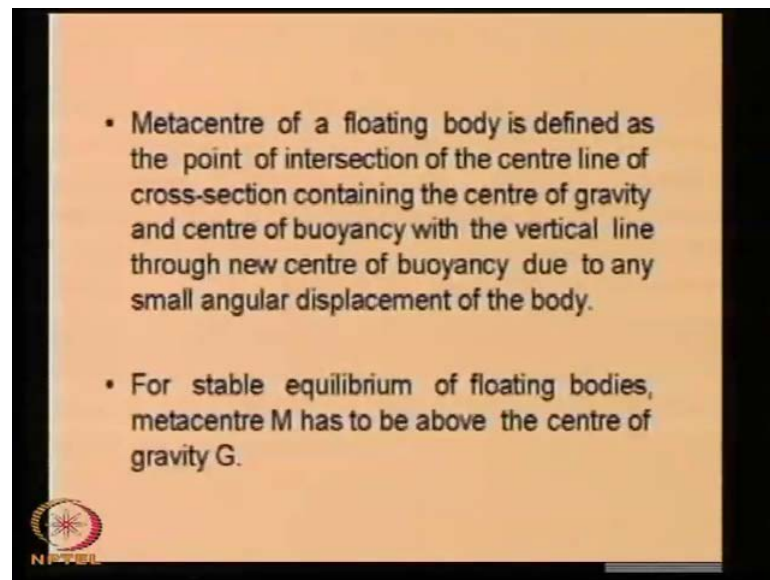
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- A submerged body will be in stable, unstable or neutral equilibrium if its centre of gravity is below, above or coincident with the centre of buoyancy respectively.



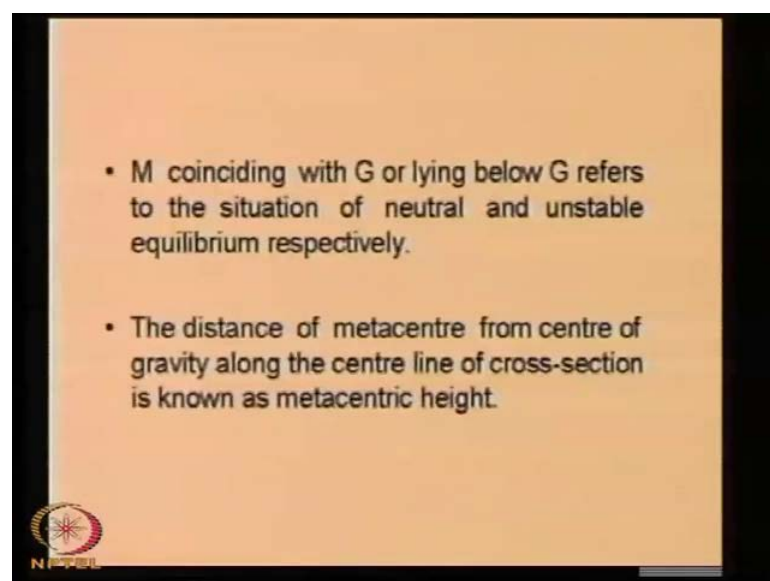
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A slide with a light orange background and a black border. It contains two bullet points. In the bottom-left corner, there is a circular logo with a star and the word 'NIPER' below it.

- Metacentre of a floating body is defined as the point of intersection of the centre line of cross-section containing the centre of gravity and centre of buoyancy with the vertical line through new centre of buoyancy due to any small angular displacement of the body.
- For stable equilibrium of floating bodies, metacentre M has to be above the centre of gravity G.

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A slide with a light orange background and a black border. It contains two bullet points. In the bottom-left corner, there is a circular logo with a star and the word 'NIPER' below it.

- M coinciding with G or lying below G refers to the situation of neutral and unstable equilibrium respectively.
- The distance of metacentre from centre of gravity along the centre line of cross-section is known as metacentric height.

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Problems

(Objective types with multiple choice)

1. For stable equilibrium of floating bodies, the centre of gravity has to

- (a) be always below the centre of buoyancy.
- (b) be always above the centre of buoyancy.
- (c) be always above the metacentre.
- (d) be always below the metacentre.
- (e) coincide with metacentre.

[Ans: (d)]




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2. For typical geometries of boats, the stability in pitching movement is more than that in rolling movement. This is because

- (a) the moment of inertia about the longitudinal axis is very low.
- (b) the moment of inertia about the transverse axis is very low.
- (c) the moment of inertia about the longitudinal axis is very high.
- (d) the moment of inertia about the transverse axis is very high.

[Ans: (d)]



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3. A floating body will be in neutral equilibrium when its

- (a) metacentre coincides with the centre of gravity.
- (b) centre of buoyancy coincides with the centre of gravity.
- (c) metacentre coincides with the centre of buoyancy.
- (d) metacentre is above the centre of gravity.

[Ans: (a)]

