Fluid Mechanics
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Lecture - 7
Fluid Statics Part – IV

Good afternoon, I welcome you all to this session of Fluid Mechanics. Last class, we were discussing the force Hydrostatic force exerted on a plane surface submerged in a static expansion of fluid. And what we have finally concluded that if there is a free surface of fluid, and a plane surface; plane inclined surface is submerged in this expansion of fluid. Then, the pressure force is exerted perpendicular to the plane surface at each and every point. And the pressure intensity at any point as we know that it is equal to rho times, g times the height of that point or the depression of that point as the vertical depth of that point from the free surface. So, the pressure intensity varies at different points depending upon their depression from the free surface.

And finally, we recognized that the total hydrostatic force due to this hydrostatic pressures on any one side of the plane surface is equal to the pressure intensity at this center of area of the surface times the total area. That means, it is equivalently at the same area if it is placed horizontally at a depth equal to the depth of the center of area of the inclined surface. Then the pressure which could have been exerted that is the uniform pressure equal to rho times g times the height from the free surface that is the height of the centroid for the inclined surface. So, therefore, we are concluded that for an any inclined surface. The total pressure force is equal to the area of the surface times the pressure intensity at the centroid of the area or center of area.

And we also recognize the another fact that the pressure center. That is the point on action of these results and pressure forces passes through a point which is below the center of area. That means, it is more at a more depth than that of the center of area which is obvious because pressures are increasing with the depth. And the center of pressure can be found out by a simple expression that the movement of area about an axis which we take as the intersection of the plane surface with the free surface, we take an axis perpendicular to this. So, movement of area of this surface about this axes divided by the area, and the coordinates of the center of pressure. So, this we recognize in the last class.

Now, today we will be discussing the hydrostatic pressure forces or hydrostatic forces on submerged curve surface. If the surface is curve then how you will find out the Hydrostatic pressure force?

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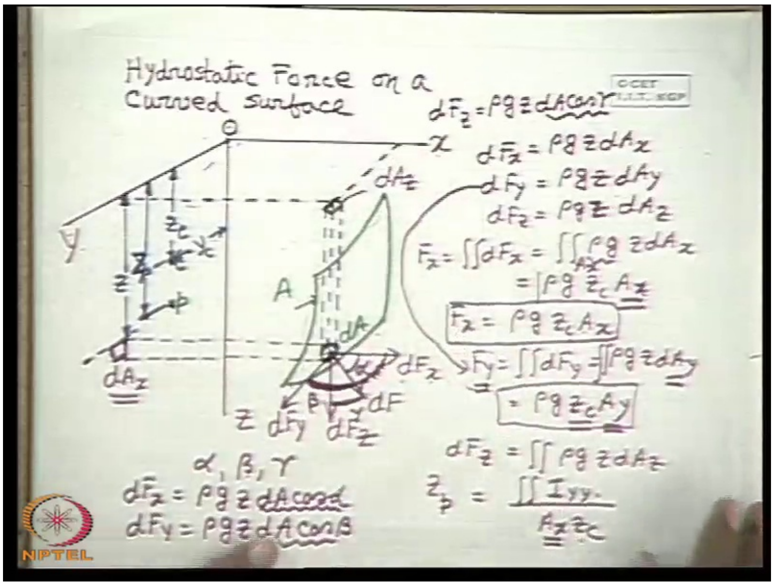

Now, let us see in general here. Let us first write hydrostatic force on a curved surface. Now, let us see here in general a curved surface. So, this is a curved surface you can see which in general has got curvature in both the directions, and which is emerged in a fluid whose free surface is this one in an expansion of fluid.

Now, in this case, we consider a three dimensional system, because of these curvature in two directions. We consider the axes o x, o y at the free surface. That means, this x y coordinate plane is at the free surface, and oz axis we consider vertically downward. Now it is true that at each and every point on the surface the pressure force intensity. Or the magnitude is equal to rho times the density of the fluid which is constant. In case of liquid times the g times its vertical depth from the free surface which we denote by the z coordinate. That means, rho g z, but the difference between this with that of a plane surface is that each and every point the force is normal to the surface which are not parallel.

In a case of a plane surface what happened? That force for example, there could have been a plane surface the force on each elemental area is normal to this surface and they are parallel to each other. So, therefore, a scalar summation of the elemental force was possible, which we did by simple integration of an elemental force throughout the area. But in case of a curved surface since, the normal at each end point of the surface which is the direction of the pressure forces changes. Because of the curvature is simple scalar summation. That means, a simple integration will not be possible.

So, therefore, the general procedure of simple basic mechanics is that. In that case, we will have to take an elemental force. And we will have to fix certain reference coordinate directions in which we will take the components and some of the components in a particular direction. And we will find out the components of the resultant forces in those directions. And then again vectorially some of the components to find out the resultant force, it is as simple as like this from elementary mechanics.

So, now we concentrate here, we will see to do that we recognize an elemental area d A which is at a vertical depth z. We just project this area here, and this is so near that is the z in the y z plane. So, z represents the vertical height or depression from the free surface of this area d A. Now, you see the hydrostatic force due to pressure is acting on this surface. If we consider the force acting by the liquid on this side I have told on one side we will consider. Other side we will consider as an open to atmosphere, then this pressure force is normal to this area d F. Now, this force can be resolved into 3 directions with respect to this frame of reference o x, o y and o z accordingly d F x is the x direction x component, d F y is the y component and d F z is the z component.

Now, if we assume that the normal to this surface makes an angle alpha with the x coordinates makes an angle beta. Let this is beta; this big angle normal to this with y axis is beta and normal to this with z axis. That means, this angle as gamma; that means, alpha beta and gamma are the angles made by the normal to this area with x axis, with y axis; that means; this angle and with z axis. Then we can tell simply that d F x is equal to what? d F x is d F. Now, d F is d A into rho g z. If we consider that this area is at a vertical that z from the free surface. So, simply rho g z times, what we can write? d A cos alpha. What is d F Y? rho g z d A cos beta. And what is d F z? That is rho g z d A cos gamma.

Now, you see this d A cos alpha physically represents the projected area of d A on y z plane; that we denote as d A x. That means it is the projected area of elemental area d A on a plane whose perpendicular is x; that means, on y z plane. So, therefore, we can replace d A cos alpha by d A x. And we can write d F x is equal to rho g z d A x. Similarly, we can write d F y as rho g z. Similarly, A cos beta will be the projected area of d A on a plane, whose perpendicular is the y direction; that means, it is on the plane x z.

So, that we represents as d A y. And similarly, d F z will represent rho g z times d A cos gamma which is the projected area of d A on a xy plane. That means, the plane perpendicular to z direction which is so near to this area is d A z, d A y area is not shown here it is difficult to show in this two dimensional paper. Now, we will see that if we now integrate. Now, the different direction component forces of these elementary forces, if it is made. Now, we can make a scalar sum. That means, now, we can tell F x as integration of d F x double integration, because this is equal to rho g z d A x.

Therefore, you see rho g z d A x. Now, you can recall this can be simply written considering a force on a plane surface that this is rho g z c into A x, rho g is common. If we take rho g out, then the double integral z d A x is made over the entire area Ax. Where A x represents the projectional area of this entire curved surface area A. That means, it is the projectional area on y z plane that is the plane perpendicular to x axis of the entire curved surface area A x. And z c represents the center of area of that projectional area. Or you can tell the projection of the center of area on the curve surface on y z plane, both are same.

Therefore, we see simply F x is equal to again I am writing it is very interesting result rho g z c A x. In a similar fashion I can write F y by integrating this d F y as rho g z d A y double integration. Which becomes is equal to rho g z c in to A y. Where z c is again the center of area the vertical distance or the coordinate of the center of area projected on x z plane. And A y is the projected area of the entire area or entire curved surface on the x z plane that is the plane perpendicular to y. And similarly, we can write d F z is equal to integration of rho g z d A z. Now, here we stop, we cannot write anything more fore here. Now, two very interesting result comes out from this expression for F x and this expression for F y. Now, we see, the x component of the hydrostatic force on the entire area is equivalent to the hydrostatic force on a vertical plane surface which is parallel to y z plane.

And area of the surface equals to the projected area of these curve surface on the y z plane. Similarly, the y component of force on this curved surface is equal to the hydrostatic force of a plane surface area which is parallel to x z plane. And whose area is equal to the projection area of this surface on x z plane. Therefore, we can conclude from here that the hydrostatic force in any horizontal direction of a curve surface equals to the hydrostatic force exerted on a plane surface which is perpendicular to that direction. And whose area is equal to the projected area of the plane on that particular plane perpendicular to this direction.

So, again I am telling the conclusion is like that, if there is a curved surface, sometimes it helps us in solving problems. That if there is a curved surface submerged in a liquid, then the hydrostatic pressure force in any horizontal direction, it may be x, it may be y any way you can orient in any horizontal direction is equal to the force on a plane surface. Whose area is equal to the projected area of that surface in a plane perpendicular to the direction? That means, if we consider a plane perpendicular to the direction, horizontal direction in which we are finding out the net component of the total hydrostatic pressure force on the curved surface.

Then if we imagine a plane perpendicular to that direction then this is equal to the hydrostatic pressure force of a plane surface, which is the projection of this curved surface on that plane, which is perpendicular to that horizontal direction. So, this is a very important conclusion we get. Now; obviously, if we now define, want to define the center of pressure for these x component of force separately. We can find out p which is given by z p. If we consider this as the centroids so, this will be line on the same line vertical line. Because I have told that it may not be displayed. That means, y c and y p will be equal, because for a plane surface the centroid A l axis. Or the axis to the center of area is such that the areas are the movements of the areas on both sides are zero about this.

And, since the hydrostatic pressure force does not vary with hydrostatic pressure forces vary only with the area. So, that this is not displaced in along this line from the center of area. But this is displaced here, because the hydrostatic pressure force varies with the d A that we have already recognized earlier. So, if we that way find that p is the center of pressure with a vertical depression or coordinate z p. Then we know from our earlier discussion that z p in this case is given by this integral. We can write I about this; this is the y and z plane. So, this will be about the y axis, y y divided by a x into z c. That means, the movement of area of the projected area on y z plane about the o y axis, divided by this total area a x that is the projected area of the curved surface in the y z plane divided by the area projected area a x times the z c.

That of course, we can change that again by parallel axis theorem by transforming this second movement of area about o y axis to another axis parallel to o y through c that you can do as well in case similar as we did in case of plane surface. So, this is the similar way we can find out the pressure center for that is the point of application of F y by applying the same equation. That is the second movement of area of the projected area on x z plane. That means, that will be A y, Ay about x axis divided by this. That means I can write this well I can write this similarly, that z p point of application of F y as integration of I y y that is not the integration; this is I y y that takes the integration.

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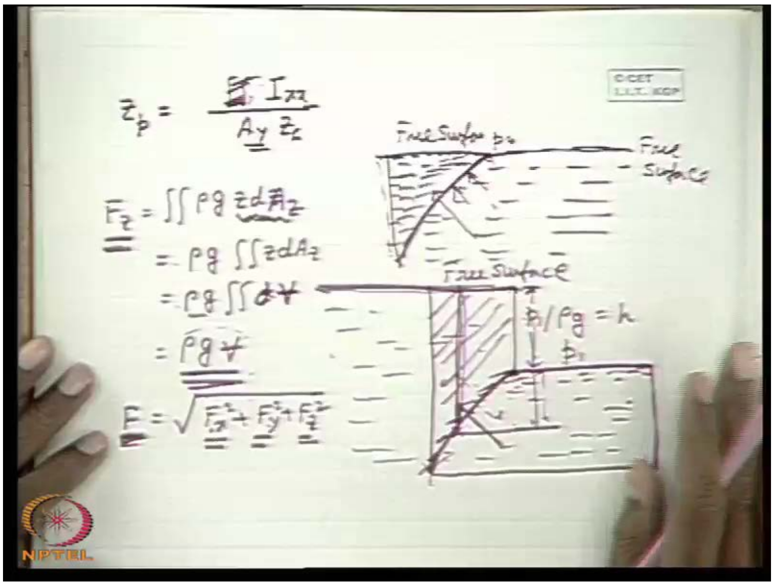

So, I can write this way that I; this is I x, because this is x, I x x of the area A y times the z c. That means, where A y is the area projected area of the surface on x z plane. And its second movement of area about this x axis divided by the area itself is z c. That means, we will treat as simple, simply as pressure forces on plane surface which are the projected surfaces on y z and x z plane. Now, what about the vertical force? Now, you see the vertical force component is this. This is not d F z; this is F z now F z. Now, let us write this F z is integration of d F z; that means, rho g z d z.

So, now, what is z d z you see that if this is z d A z. Now, what is z d a z? d a z is the projectional area on x y plane which means that if we consider a prismatic fluid element over the d A up to the free surface d a z is the cross sectional area. That means the projected area that means the area perpendicular to the z directions projected area on x y plane. Therefore, this quantity simply represents rho g, if we take out; this z d a z simply represents volume. That means, d v first I write this is the d v cut I will use always v cut as volume, because v will be using velocity afterwards. So, this is the elemental volume of this prismatic fluid element.

So, this is simply rho g v. That means, we can tell that the z component of the hydrostatic pressure force over an elemental area is equal to the weight of the fluid above that elemental area up to the free surface. And for the entire surface it is the weight of the fluid that is content in the region vertically above the surface up to the free surface. That means, if we make the vertical projections from the surfaces up to the free surface then the bulk of the liquid which is content within this vertical projection from the curved surface to the free surface, the weight of that liquid is equal to the hydrostatic z component of the hydrostatic force.

So therefore, next is that this force the point of action of this force is through the center of volume of this bulk of fluid which is content vertically above the surface up to the free surface. And it is little complicated to find out the pointer application. So, resultant force is very simply we just vectorially add F x square plus F y square plus F z square. But the pointer application of this force is difficult in a sense that we can find out the force F x direction. And we can find out the application of F y direction. And the meeting point cut with point we know, and if we draw a vertical line through it. Then we will see this is the actual force will pass through this point which will also go through the center of volume.

So, this I will explain while solving a problem. So; however, we can find out that this is the resultant force whose direction can be found out if I know the magnitudes of F x F y F z. And whose value the magnitude is found out in this way by vector summation of F x F y and F z. And whose pointer application is found out by finding out the pointer application of all the three forces. So next, I summarize the thing again by telling this. For a curved surface, the net hydrostatic pressure forces are found first by taking its components in refer directions in any horizontal direction.

The hydrostatic pressure force is equal to the force exerted on a plane surface which is the projected surface on a plane which is perpendicular to that direction. That means, if you want to find out the hydrostatic pressure force in x direction. We will take the projection of this curved surface on y z plane that is the plane perpendicular to x direction. And treat the plane surface that is the projected surface in a plane surface in y z plane as the vertical surface. And then we find out the hydrostatic pressure force in that plane. And that will represent the hydrostatic pressure force in the horizontal direction x direction or y direction for the curved surface.

And the vertical component if we take one of the coordinate axis in the vertical direction; obviously, we will have to take the vertical component of the pressure force is equal to the total weight of the fluid bulk of the fluid which is content within a region created by vertically projecting from the surface up to the free surface. That means, that is the weight of the liquid vertically above the surface up to the free surface. The bulk of the liquid that is content the weight of that is equal to the hydrostatic z component of the hydrostatic pressure forces. So, this way we can find out the pressure force in a curved surface.

Now, few things I must tell you. This is all about the pressure force and a curved surface. And there may be some situations where the free surface may not be defined properly. For example, this is a case that in an expense of water there is a card gate or surface like that. Now, this is the free surface which may not be extended in this direction. Now, what happens? This is not a submerged surface, but see that each and every point this pressure is atmospheric pressure p g. But each and every point there is a pressure force due to the height of this.

So, there is a net pressure force in this direction that if no. But to apply this formula we can do it analytically. But we want to find out the z component of the hydrostatic pressure force. And if we try to apply this concept that it is the weight of the liquid above this surface. If we are told to find out only this part, that what is the magnitude vertical component at the hydrostatic force? Then what we do is commonsense that we take an imaginary free surface. That means you extend the free surface, as if it is being submerged with this as free surface. Same thing, we can find out the submerged free surface. Hydrostatic force means if this is the surface I just take a projection. So, this is the weight of this liquid.

So, from geometry if I can find out the weight of this liquid easily I can tell this is the vertical component of the hydrostatic forces. Sometimes the situation is like that there is a pressurized container there is no free surface a pressurized container. This is the pressurized container where the fluid is at some pressure. So, the pressure intensity there is with its depth. And here at the top surface the pressure is p 1. So, pressure is here p 1 but at this point the pressure is p 1 plus the hydrostatic pressure due to height. This is a container.

What is the force acting on this? So, you can go with the analytical expression by integration. But if we have to use this concept if we have told to find out the vertical component of the hydrostatic forces. Then what we do? This we cannot treat as an submerged surface below the free surface of the water. But we can transfer that by imagining a free surface whose here we will go up to a vertical height is very simple p 1 h is equal to p 1 by rho g. That means, we create some height which is equal h p 1 by rho g and make it that is as if this is the free surface.

And we consider that this is emerged in the free surface. Then what is the hydrostatic z component of the hydrostatic pressure forces? This volume of the liquid, weight of this volume of the liquid; that means, we can create a free surface and consider it as if it is submerged on this free surface. So, only thing is that the direction will be like this if this is a free surface. And it is submerged then if we consider the force on this surface this side. It will be in this direction here, because of this pressurize chamber it will be in this direction.

So, this will be the free surface, because pressure here will be this pressure plus this height, which will be same as this vertical line. That means to extrapolate from the pressure a free surface this is known as imaginary free surface concept. Sometimes we have to make to find out the vertical component of forces easily. If we feel that geometrically it is easier to find out the weight of the liquid vertically above the curved surface up to the free surface. Then an imaginary free surface depending upon the situation we can think off.

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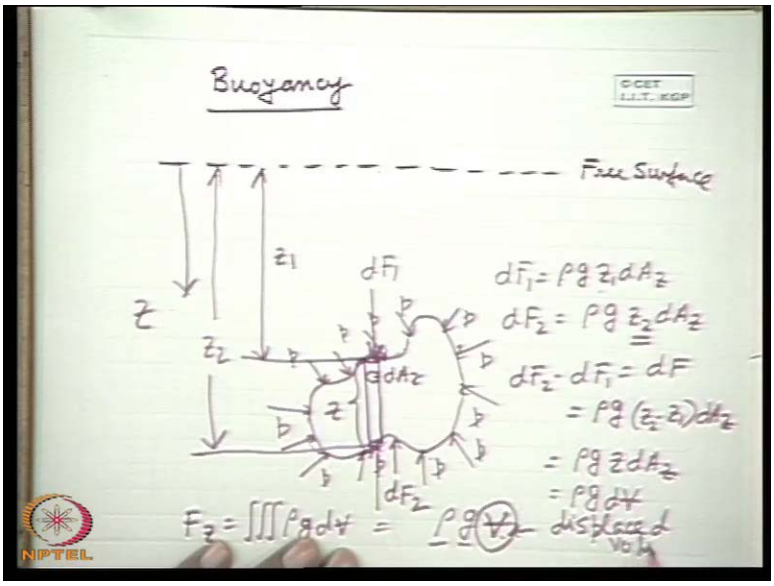

Now, we come to the next part Buoyancy, what is Buoyancy? Now I come to buoyancy. Probably, this you have heard at your school level, there will be a little recapitulation of your things which you have done at your school level buoyancy. Now, you know that if a body is totally or partially submerged. If totally submerged or partially emerged, then the body experience will be in a liquid or even in a fluid body experience is upward trust, where from this upward trust comes.

Now, this phenomenon is known as buoyancy. And this upward force that a body is a experienced we are also experienced, because we are totally submerged that emerged in fluid air. So, we are always experiencing a upward trust. So, it is a fact that a body if it is emerged in a fluid then it experiences totally emerged or partially emerged it experiences an upward force. This phenomena is buoyancy probably at a very little let; that means, in the class 7 or 8 level we have read. And this phenomenon is known as buoyancy and the force is known as Buoyancy force.

So, where from this upward force comes? Please tell; where from this net upward force comes? Where from it comes? Where from this force come? Tell me the force come from a force concept, not because on another physics, then I will as write that that physics creates a force. Force comes from some force concept. Why these force come?

This is nothing, but the distribution net resultant of the hydrostatic pressure forces. Nothing else the body is submerged or partially emerged. So, there is hydrostatic pressure forces distribution around the body. And then net defect of the hydrostatic pressure forces is a vertical upward force which is the buoyant force it comes from the distribution of hydrostatic pressure force.

Now, let us see what happened? That let us consider a free surface of water free surface. And let us consider a body like this, let us consider a body totally emerged or submerged body. So, what will happen? The pressure force is will act on the body like this p. Though I am writing it as p, but do not misunderstand that this p is same everywhere, because this p depends upon it height from the free surface. So, on the surface of the body pressure forces and experience because of the fluid that is the surface force we have discussed.

Now, you tell me the common sense tells that the resultant force due to this surface force pressure forces in any horizontal direction becomes 0. This is because; this is; obviously, found out this balances each other; that means, horizontal force is 0. Now, if we apply our earlier knowledge that the resultant force component in any horizontal direction on a curved surface is equal to the projected area on a plane perpendicular to that direction. If we apply that concept now this curved surface is a close curve surface for a body and whose projected area in any plane is 0.

So, therefore, the concept tells that the net hydrostatic force in any horizontal direction is 0, but it is not so, in the vertical direction. Why you see; obviously, vertical direction the pressure force at these point and pressure force at these points can never balance because they differ in the height. Because this is z pressure increases with the increasing z. Now, if we take a prismatic liquid, a prismatic liquid whose cross sectional area is d A z. And in such away that these point the upper surface is at a height from the or a depth from the free surface z 1.

While this lower surface, this lower surface of the prismatic fluid is at a depth z 2. Then we can write that the pressure vertical component of the pressure forces on this prismatic element of the surface of the body is on this surface is rho g z 1 in to d a z. Similarly, this is acting, let this is the d F 1. So, I can write d F 1. Similarly the upward force at the bottom base of this prism in the vertical direction is d F 2 which is equal to d F 2 rho g is very simple we have done earlier a d a j. So, therefore, this is more than this. So, therefore, a net force acting d F 2 minus d F 1, let d F vertically upward in the prismatic sys fluid is rho g z 2 minus z 1 into d a z. Now, this becomes is equal to rho g z d a z where z is the height that means the height of the prismatic. That means, this is simply rho g z d v.

So, therefore, if we integrate then we get F z is equal to triple integration. That means, over the volume rho g d v; that means, we get rho g v. So, therefore, we get the idea that the net hydrostatic force pressure force in any horizontal direction is 0. But in the vertical direction there is a force which x upward in the body. Whose magnitude equal to rho g times the volume of the solid body the entire volume of the solid body? Now, you see that when solid body exists here under submerged condition; that means, it has displaced the liquid in this portion.

So, therefore, we can tell that the same volume of the liquid has to be displaced by the solid. So, that solid makes its space. So, that is why this v is sometimes refers to as displaced volume displaced; that means, the volume displace and displace volume always will be volume of the body immerged in the fluid. In case of a submerged body which is totally immerged in the fluid. This displace volume of is its total volume because it has to occupy by its total volume. But when the fluid is partially submerged that partially immerged not fully immerged. Then this displace volume is the volume of the solid that is within the fluid; that means, that part of the volume. So, this is referred technically as displaced volume.

So, therefore, we come to the conclusion that the buoyant force that is the buoyancy force which is the net upward force. And it is as a consequence of the hydrostatic pressure force surrounding at on the surfaces of the body is equal to equal in magnitude to the weight of the displaced volume of the fluid, weight of the displaced volume of the fluid by the solid. Or you can tell it is equal to the weight of the fluid whose volume corresponds to the volume immerged in the fluid volume of the solid immerged in the fluid.

That means, in case of a submerged volume total volume of the solid. In case of a partial immerge solid it is the volume that is variant different. That means, it is equal to the volume displaced weight of the fluid whose volume equal to the displaced volume of the fluid by the solid. And this phenomenon was discovered by Archimedes which is the starting point of the fluid mechanics. And that is why this phenomenon is known as Archimedes principal. This phenomena is known as let us write with pride the name of this great man Archimedes principal. So, this is the buoyancy.

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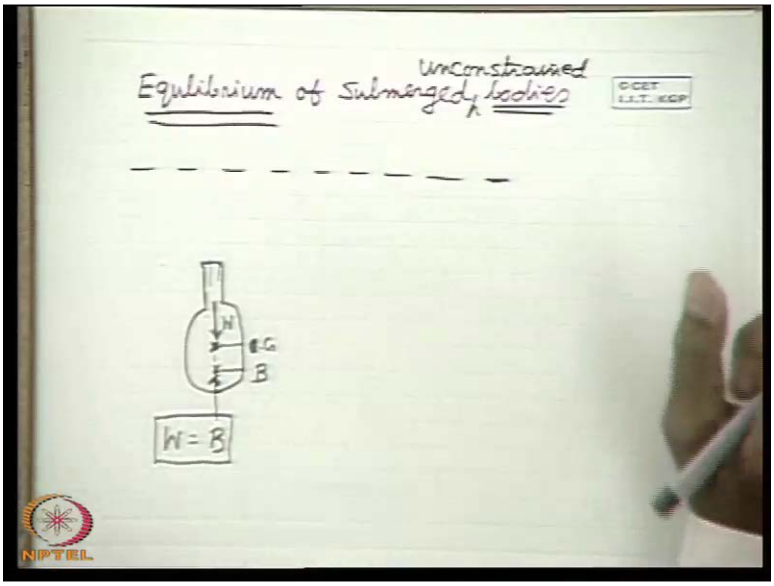

Now, we come to the equilibrium. Now, we will discuss the fourteen equilibrium of submerged bodies. First, we consider equilibrium of submerged bodies. Now, if a body is submerged in a liquid. So, you see a vertical upward force that is the buoyancy force. Whose magnitude is the weight of the total weight of the fluid whose volume is equal to the volume of the solid, there is acting upward. What is another force acting? That is the weight of the body.

So, one thing you will have to write equilibrium of submerged unconstraint bodies. That means, the body should not be constant otherwise, there is no other force acting the body is kept free. Another vertical another force which is another force acting is the weight vertically down words. There is no other force because in a static free there is no friction force. So, only the friction forces which gives rise to buoyancy forces another is the weight. So, therefore, if the weight is more than the buoyancy force the body will go down sink and sink. So, there is no equilibrium it will only rest if there is a surface solid surface.

Where the body can be kept at this, because of the reaction force exerted by the solid surface we will balance the net value of the weight minus the buoyancy. But if it so happen? The density of the fluid; the density of the solid, and the volume of the body is such that buoyant force equals to the weight of the body.

That means weight of the body equals to the buoyancy. That means, the density is same can only happen if the densities are same. In that case, what happens? The weight and the buoyancy are equal in magnitude. And if they become collinear and if they become also collinear; that means, vertical with the same pointer application on the same vertical line. Then the body will be in perfect equilibrium condition under submerged state. So, therefore, the condition equilibrium of submerged bodies here we can write unconstrained. That means free that means, only weight and buoyancy is acting.

So, now you see what happens? Let us consider a body which is like this, let us consider a body like this. Now, the condition of equilibrium is that the weight acting from its C G, let this is a C G. And buoyancy force acting through the center of buoyancy B, let us call B or let us call as G they have to be equal W is equal to B. And they should be collinear. So, they should be in the same vertical line. But there may be a displacement between G and B along the vertical line G may be upward than the B or may be down to b that depends upon the distribution of mass, relative distribution of mass and weight. If weight is uniformly distributed, mass is uniformly distributed throughout the volume. Then of course, G and B coincides, but not necessarily in general they have to be coincide, but even if they are not coincident.

But they should the collinear. So, this is the condition of equilibrium of a submerged body. Now, as you know in mechanics there are 3 types of equilibrium; one is stable equilibrium, another is neutral equilibrium, another is unstable equilibrium. So, we will discuss this stable, unstable and neutral equilibrium in the next class. So, this is the condition that the weight and the buoyancy force have to be collinear for equilibrium. But whether it will be stable equilibrium or unstable and neutral equilibrium will discuss in the next class. That there are 3 types of equilibrium will define these 3 classes of equilibrium as you already know in your mechanics. Then we will find conditions under which the body will be in stable equilibrium under submerged condition or neutral equilibrium or in unstable equilibrium. The next class well

Thank you.