

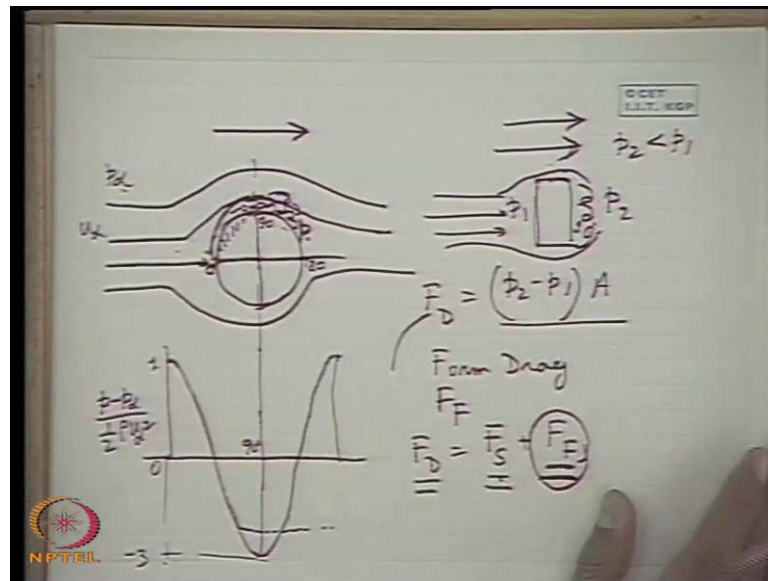
Fluid Mechanics
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Lecture - 48
Introduction to Turbulent Flow Part – I

Well, good afternoon and welcome you all to this session of fluid mechanics. So, today we will be beginning a new chapter, or a new section, introduction to turbulent flow. But before I start the turbulent flow, I would just like to conclude the earlier section, that is the boundary layer, introduction to laminar boundary layer, by continuing the earlier discussion that we were discussing about the boundary layer separation that we have recognized, when the real fluid flows against an adverse pressure gradient. Then what happens, the fluid particles very near to the boundary, that means, the fluid particles within the boundary layer, very near to the solid boundary, because of their low kinetic energy, flows in the opposite direction, cannot surmount the adverse pressure here. And because of this, what happens that eddies are formed; eddies are formed, because of this local flow reversal zone and which curtails the pressure in the fluid.

Therefore, the pressure in the fluid in that region becomes low, is reduced. So, therefore, when the fluid passed a body and boundary layer separation takes place, because of the typical nature of the flow, that means, the flow is decelerating against an adverse pressure gradient. Then the downstream part, where the boundary layer separation takes place, the pressure is reduced from that in the upstream part, for which a net force which comes from the difference of pressure in the direction of flow takes place, that is known as drag force and which is, because of the boundary layer separation, and this drag is known as the form drag.

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Just take an example here; if we see, we are discussing that flow past a circular cylinder. We know in case of an ideal flow, parallel ideal flow passes in circular cylinder, the flow is symmetric about both this and this axis. And the pressure distribution is like that; from this point, this is the stagnation point, where the velocity becomes 0; that is, the 0 degree measured here, the azimuthal degree, that the in the azimuthal direction in the 0 degree. So, as the flow approaches from 0 to 90 degree, this half of the cylinder, it is an accelerating flow and the pressure continuously is reduced. So, this part of the cylinder, the flow is decelerated; that means, the flow takes place against an adverse pressure gradient; pressure is again going to be high and ultimately, the entire pressure recovery takes place in case of an ideal fluid flow. But in case of a real fluid, the boundary layer forms near the surface. So, what happens? In this region, after 90 degree, when the fluid faces an adverse pressure gradient, the boundary layer separation takes place.

It takes place, some point near, around 90 degree, even some before 90 degree angle, because of this steep pressure gradient. The flow reversal pressure may take place even before 90 degree. So, at some point here, the flow reversal starts taking place; that is the boundary layer separation.

And ultimately, eddies are formed, turbulent eddies are formed and the pressure here, in this downstream part, is getting reduced from that in the upstream part. Upstream part pressure P_{∞} , which is more than P . So therefore, what happens? A net pressure

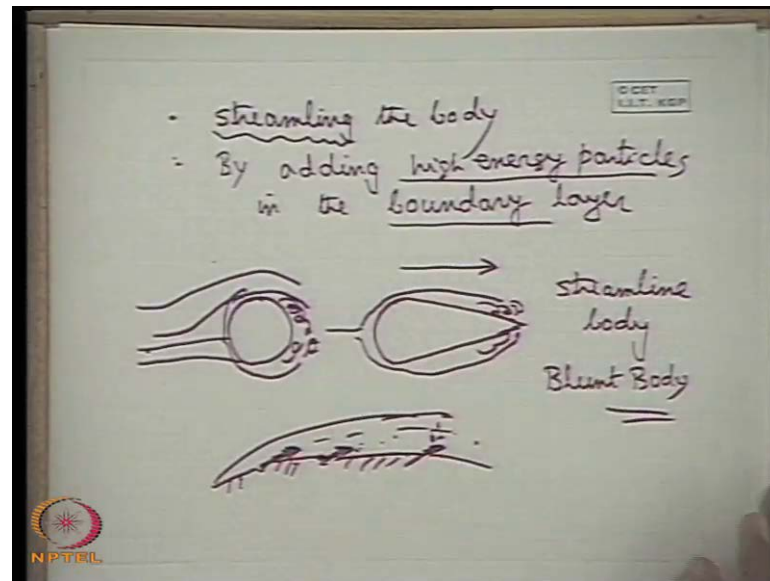
force act in the cylinder, in this direction; this is so for any body, if there is another body, any body, body of any shape in the flow takes place. So, you see, due to the boundary layer separation, so, the, what happens due to the boundary layer separation, the pressure here, in this part, this P_2 and this pressure is P_1 , P_2 is less than P_1 .

So, P_2 minus P_1 times the frontal cross sectional area A , gives a force, drag force in this direction, in the direction of flow. But there is another drag force, because of the ((skin)) friction, which is because of the (()) between the fluid and the solid surface. So, this is also acting in the direction of the flow of the fluid, with respect to the solid.

So, to distinguish this drag force from that one, it is known as form drag; form drag, or rather, it is expressed by F_P , form drag, F_F rather, form drag, form drag force. So, the total drag force F_D is composed of skin friction drag F_S and the form drag. Why, the boundary layer separation depends upon the formation of the body, shape of the body, because, it is the shape of the body which dictates the adverse pressure gradient, and therefore, the boundary layer separation takes place. So, that is why, it is named as form drag, which depends upon the form of the body, which is due to the boundary layer separation.

So, the drag force exerted by a body when it is placed in the stream of a real flow of a real fluid, then, it experiences two kinds of drag force; that is, the force in the direction of the flow of the fluid with the respect to the body; one is the ((skin)) friction drag, which is because of the shear force between the fluid particles and the ((skin)), that means, the surface of the solid; and, another is the form drag, which is the force that acted on the body in the direction of the flow of the fluid, because of boundary layer separation, which results in the reduction in pressure in the downstream part of the body, from that of the upstream part. Now, it is very essential to reduce this form drag. So, there are methods by which we can use reduce the form drag.

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There are two methods by which the form drag and boundary layer separation are reduced. One method is that, by making the body streamlined; that means, streamlining the body. Another method is by adding, by adding high energy particles, high energy particles, in the boundary layer; high energy particles in the boundary layer. Now, streamlining body means like that, that if a body is like that, for example, a sphere. So, there is a boundary layer separation. So, therefore, what happens, that boundary layer forms, then, there is a separation; that means, if the fluid flows like this, so, boundary layer starts forming like that, and there is a separation; so that, the boundary layer forms, and there is a separation. So, if we now make the body like this, a little elongated like this; that means, we make the body streamlined, so that, when the fluid flows over this, the boundary layer separation is delayed.

The boundary layer separations are delayed; that means, make the body such a shape, so that, the boundary layer separation is either prevented or delayed; that means, make the adverse pressure gradient more flat, so that, the boundary layer separation takes place at, almost at the trailing end, so that, the net pressure forces in the direction of flow is reduced.

So, a body where the boundary layer separation is delayed, much delayed, or prevented, is known as the streamlined body; whereas, a body for which the boundary layer

separation is advanced is known as a blunt body, which gives a more boundary layer separation, blunt body.

So, making the body streamlined. Now, another way of reducing the boundary layer separation is by making high, in adding high energy particles in the boundary line. We have recognized that, it is because of the loss in the energy of the particles within the boundary layer, because of the viscous interaction between the solid surface and the fluid, the fluid particles become unable to cross the adverse pressure in. so that, if we put more energy in the fluid particles within the boundary layer, so, boundary layer separations may be avoided.

This may be done, for example, by making a surface force. For example, let the fluid flowing over a surface like this, let the surface be...So, if there are pores through which the particles are injected at high velocity, fluid particles tangential to this surface, within the boundary layer, then, what happens, the boundary layer separations may be avoided.

By simply injecting fluid particles at high velocity, tangential to this surface, from surface pores, so, just near the solid surface, high particles, particles with high kinetic energy have to be injected, so that, these has, this gives the energy to the slow moving particles.

So, as a whole, the bulk energy of the particles within the boundary layer, that is, the energy of the slow moving particles within the boundary layer, gets enhanced, so that, the boundary layer separation is delayed.

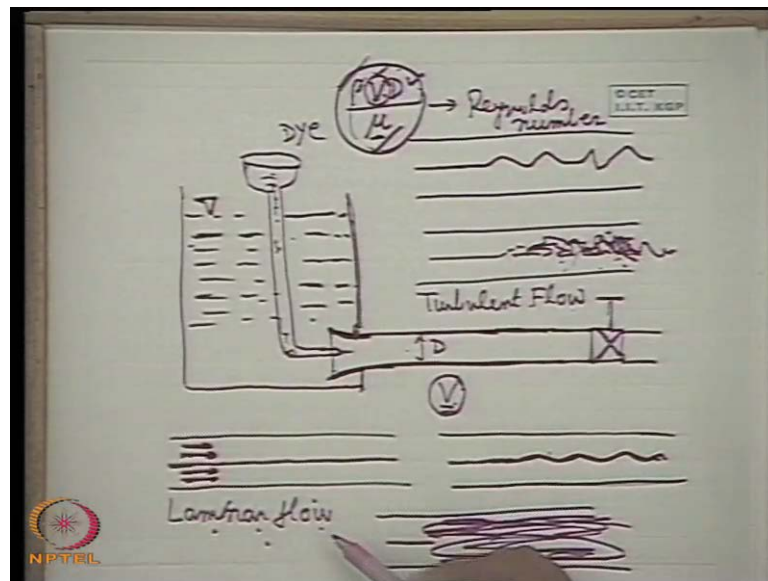
So, there are two methods by which the boundary layer separation is delayed so that, ultimately, the form drag on the body is reduced; the body experiences a less amount of form drag. So, well, now, I will start the turbulent flow. We will start the today's lecture, turbulent flow. Now, we have probably mentioned in many places earlier, while discussing different principles of fluid flow, that, laminar flow and turbulent flow.

There are two kinds of flow for a real fluid, known as laminar flow and turbulent flow. So, the concept of turbulent flow, first came in the mind of Reynold, at the end of 19th, by the end of 19th century, from a very classical experiment, which he performed, from where he first came up with the concept of Reynold's number, as a similarity parameter;

and, firstly, (()), two distinct kinds of flow qualitatively; one is a laminar flow, and another is a turbulent flow, which was named afterwards.

Now, let us therefore, start from that classical experiment by Reynolds, at the end of 19th century, when he first demonstrated the two distinct kinds of flow; one is a laminar flow and another is a turbulent flow. Well, let us see the experiment.

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So, experiment is like this. He first started with a tank, with a liquid; for example, water. He started with water, water in a tank with a constant height. One tube was attached at the side of the tank, at this position, with a flared entry and a valve was kept at the downstream part of this tube, which was initially closed.

Now, what he first started in the experiment, what was his basic intention? First, the water in the tank was allowed to settle fully, so that, no disturbance is there in the water. And then, the entry of the water, to make a smooth entry without any disturbance of this water in the tube, the entrance of the water was made flared, like this, which is known as the ((whale/well)) mouth entry.

Then, a valve was kept at the downstream to change the flow through this tube, so that, the entry of the liquid takes place without any disturbance. Then, what happened, he used, he injected a color dye, just at the centre of this tube, to see the visually, the nature

of the flow, by observing the color dye which is injected, the position of the color dye in the flow.

So, what was observed that, as the valve was opened gradually, in the region of very low flow, he observed that, the flow was like this; that, if this is the tube, the flow was like this, that the color dye which was injected at the middle, it goes throughout, as an unbroken color streak. Then, what happens, after some, after increasing the flow, that means, if the valve is gradually opened, so, if the flow is increased, it was found that, when the flow increases, with the increase in the flow rate, it was found that, the color dye is, which was unbroken throughout earlier, gets a little wavy, after some length from the inlet, it gets little wavy in nature. And still, with further increase in the flow, he found that, the waviness in this color dye, which was injected here, that is, this thread becomes more, and then, ((reacted)) like this. And, at still higher flow, it was found that, it is like this; it is more or less, waviness is much irregularly, like this; it moves like this. And, still at a higher flow, it was found that, the entire liquid becomes evenly colored; that means, the dye mixes everywhere, due to this movement and the entire liquid becomes colored liquid; though it is maintained little here; ultimately, the entire liquid becomes colored liquid.

So, therefore, we observe, after certain flow, certain velocity of flow, the orderly motions of the fluid particles, like this, gliding one over another, at different lamina, from the, this is the, what from which it has come, it flows at different lamina; that means, one plane gliding over another plane is distorted; that means, there occurs a transverse velocity, which is fluctuating with time and space, so that, the thread becomes wavy.

And, it becomes still more wavy, as the flow is increased and at still higher flow, the flow becomes totally irregular in nature. The fluctuating flow becomes totally irregular in nature, so that, the entire flow becomes colored; that means, the fluid particles intermingling with each other in the transverse direction and so that, the color thread gets an opportunity to mix with the entire fluid and the fluid becomes colored.

So, with this, he concluded that, there are two kinds of flow; in one kind of flow is this, where the fluid flow is gliding over each other, in orderly manner; that is known as laminar flow, laminar flow. And, another flow which takes place at a higher flow rate, it is the turbulent flow, where the fluid has erratic fluctuations; fluid flow has erratic

fluctuations. And, in this case, the flow has got different dimensions; that means, though the bulk flow is in this direction, the fluid particles has got a fluctuation in the transverse direction, which is absent in the case of laminar flow.

Now, he had the idea that, is it so, that, it is, there is a unique velocity, or ((the critical)) velocity above which it happens. Then, he did the experiment with different fluids. Then, he tried to find out, what is the velocity range, or what is the value of the velocity, critical velocity, over which the laminar flow changes to turbulent flow, and, he found that, it varies from fluid to fluid.

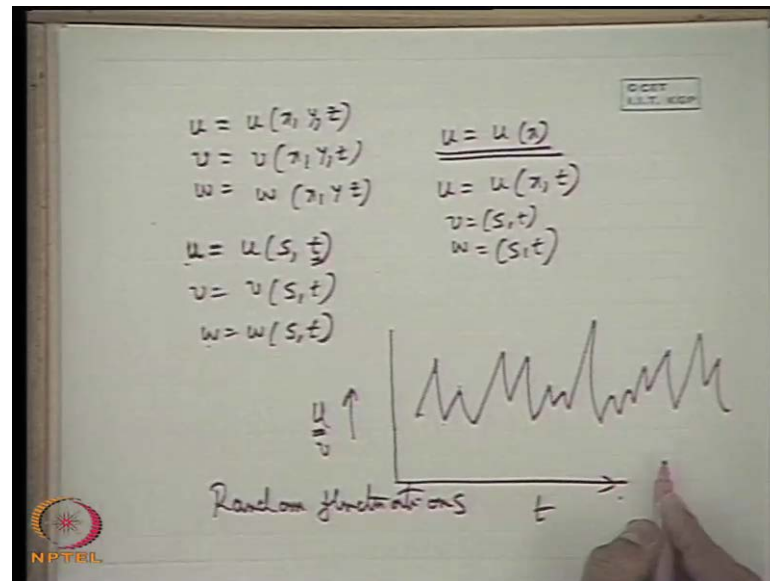
And also, he found that, it depends upon the diameter of the pipe also. And ultimately, he found out an unique value of this parameter, the combination of this $\rho V D$ by μ , ρ is the density of the fluid, μ is the viscosity of the fluid, V is the average flow velocity of the fluid and D is the diameter of the fluid, D is the diameter of the pipe; sorry. This combinely decides the criteria, after which the flow changes from one kind, that is, the laminar flow to another kind, irregularly fluctuating flow; it is turbulent flow. And, this number is known as Reynolds number, according to his name and this Reynolds number becomes the criteria for a fluid to change from this laminar flow region to turbulent flow region. So, when the Reynolds number is small, then, the flow is laminar. When the Reynolds number is high, the flow is turbulent; that means, when...Therefore, the concept of velocity comes in these.

If the fluid has got more viscosity, fluid becomes laminar, even at a very high velocity; but if the fluid has got less viscosity, fluid becomes turbulent even at a lower velocity. So, as a whole, the combination of these parameters, known as the Reynolds number, is the criteria for the fluid flow to change from laminar to turbulent flow. Now, with this classical experiment of Reynolds by the end of 19th century, I now come to the concept of turbulent flow. Today, turbulent flow has developed like anything, and there is a vast course for turbulent flow itself. So, we will not discuss as a whole, the turbulent, the concept of turbulent flow; for this course, we will give only a brief introduction of the turbulent flow today. In almost, all the flows in practice are turbulent in nature. So, therefore, we must require the analysis of turbulent flow, or the concept of turbulent flow analysis.

Now, therefore, we first define turbulent flow now, in this session. Turbulent flow is a flow, where there is an irregular fluctuations of the velocity components; that means, if the velocity components exceeds some value, that means, when the flow Reynolds number exceeds some value, this particular critical value depends on flow to flow situation, then, the flow velocity, at any point, irregularly fluctuates with time. And, even if the flow is one dimensional, that means, the bulk flow takes place in one dimensional, there occurs this irregular fluctuating velocity components in all the directions; that means, actually, a turbulent flow is three dimensional in practice.

So, therefore, a turbulent flow is basically, unsteady and three dimensional; that means, always, the motion or the velocity of a particle is fluctuating with time; that means, at any point, if you want to measure the velocity of a particle, we will see with time, the velocity is fluctuating irregularly; this is the random fluctuation. So, the flow field is associated with a random fluctuations with time.

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So, therefore, we can express the flow field of a turbulent flow like this, that, in case of a turbulent flow, for example, if we consider u , v , w as the three velocity components in case of laminar flow, we know, we define for a laminar steady flow, the velocity components are like this, alright; like this; and for an unsteady flow, we will introduce a time t .

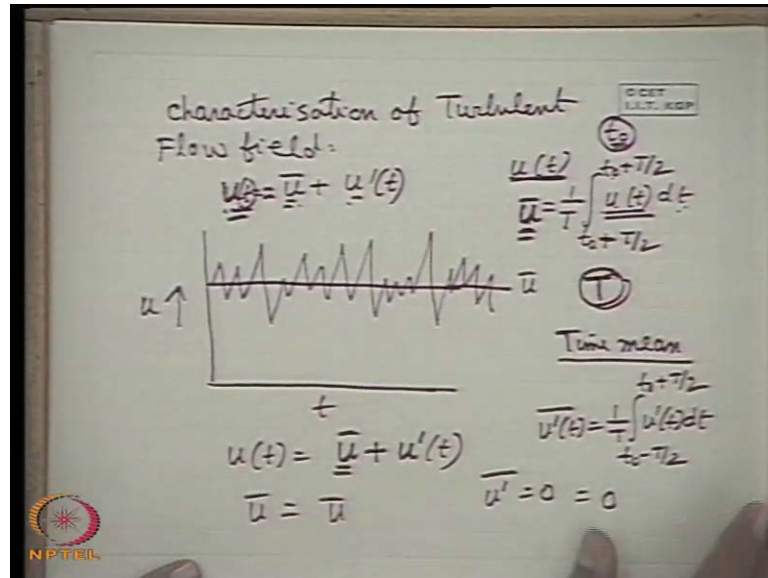
But for a turbulent flow, at any condition, the velocity component u is a function of space coordinates x , y , z , I am telling, and also a time; and, this function of time is an irregularly fluctuating function. So, v is also s and t , and w is also a function of s and t .

Even for a steady turbulent flow, this is an irregular fluctuation with time. Velocities are functions of time. But the concept of steady flow, I will come afterwards. Now, let us consider an one dimensional flow; for example, u is a function of x ; one dimensional steady laminar flow; but in case of one dimensional turbulent flow, it will be a function of t ; and therefore, velocity and v , w , there will be a fluctuating component of v , that is also, you can say, that is a s ; that means, space variable s ; we can better write, s and t .

So, this, we can show like this, that, there is a, if we... So, we want to show u versus t ; we will see that, an irregular fluctuating component. Similarly, if we plot v with t , we will see the same irregular fluctuating component; that means, at any point, the velocity components exhibits an irregular fluctuations with time.

But, these irregular fluctuations can be described statistically. This is random; this is random fluctuations; this is random fluctuations so that, they have a correlation; random fluctuations and they have a correlation.

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Now, according to Reynolds, the characterization of a turbulent field. Now, we, it is characterization of turbulent flow field, of turbulent flow field. Now, it was Reynolds first, who told, or who had a guess and found from experiments that, these random fluctuations are such that, it can be expressed as a mean, plus a fluctuating component; that means, for example, the u component of velocity can be expressed as a mean component, plus a fluctuating component.

What is this mean? That means, if we express the, at any point, for example, the u component, or x component of velocity, we can express this as this; that means, these are fluctuating over a mean value; that means, fluctuations are such that, they describe a definite mean; so that, the velocity can be expressed in terms of a mean, plus the fluctuating component. This is the function of t. The value of mean is defined like that, u is a function of t. So, mean value is such that, $u \, dt$, if we start our investigation at any time t_0 , then, over a time period, for example, 1 by T , t_0 minus, well, around a time t_0 , this can be defined as t_0 plus T by 2 .

That means, if I want to find out the mean, this mean is time mean; this mean is time mean, time mean. Different means are also described in turbulent flow, but we will

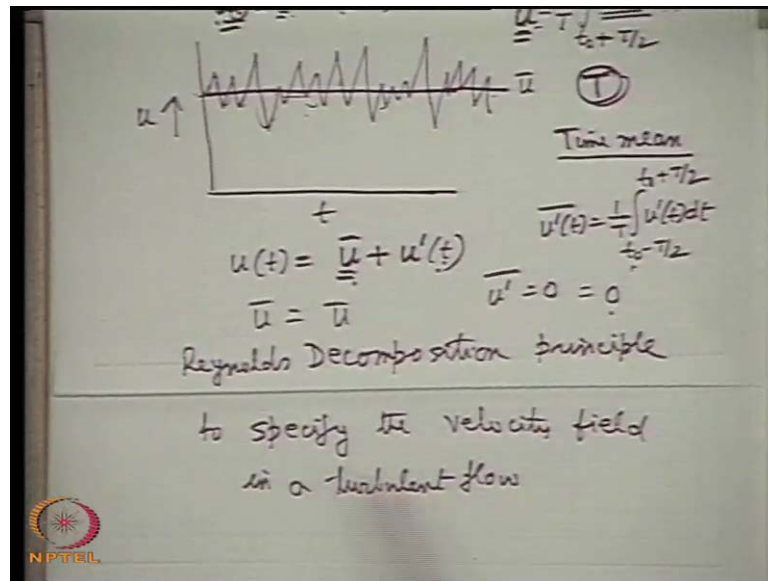
consider for this, plus time mean only. So, time mean value of the time varying component of velocity, fluctuating component of velocity is defined as $\frac{1}{T} \int_{t_0 - T/2}^{t_0 + T/2} u \, dt$. So, T is typically the period of time, and it has been found that, the velocity component, which is erratically changing with time, that is an erratic fluctuations with respect to time, this is function of time $u \, t$.

So, if we take the mean, that means, if we integrate this, over a time period T , where this time period T is sufficiently large to take care of a, more number of, to include more number of fluctuations, we see, we get a uniform value, steady value, which is irrespective of the time.

So, this is the entire philosophy; that means, if you integrate this time varying quantity, which is varying with time in an erratically irregular manner, its integral $\frac{1}{T} \int$; that means, if we take a time mean value, this way, over a time period T , which include many more fluctuations, number of fluctuations, we see that, this, we get a mean value, \bar{u} ; which means that, the velocity $u \, t$ can be expressed as $\bar{u} + u'$; that means, it is a, it can be expressed as a sum of one mean component which is independent of time, plus one fluctuating component, with this definition.

So, it obviously follows that, the fluctuating component mean value will be 0; that means, this will be $\frac{1}{T} \int_{t_0 - T/2}^{t_0 + T/2} u' \, dt$, T is any time period, is equal to 0, so that, if we take the mean from both sides, we get, \bar{u} is equal to \bar{u} . Why \bar{u} ? Mean is equal to 0; that is, the mean of the fluctuating component will be 0.

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This is known as Reynolds decomposition. This is known as Reynolds decomposition, known as Reynolds decomposition principle, to specify the velocity field or flow field;

To specify the velocity field in a turbulent flow; that means, again, I tell you that, in a turbulent flow, the velocity quantity, or any velocity component can be expressed as a mean plus a fluctuating one. This is because, the fluctuation is such that, this mean can be defined, which is irrespective of the time, by a mean quantity and the fluctuating quantity, so that, the mean of the fluctuating quantity is 0; that means, it is, the fluctuating velocity is superimposed on a mean value, so that, the mean of this fluctuating quantity, over a time period T , which includes many more fluctuations, is ultimately 0, so that, it can define a mean value. So, this is the Reynolds decomposition principle to specify the flow field.

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Handwritten notes on a whiteboard:

$$\begin{cases} u = \bar{u} + u'(t) \\ v = \bar{v} + v'(t) \\ w = \bar{w} + w'(t) \end{cases} \quad \bar{u}' = \bar{v}' = \bar{w}' = 0$$

Specification of turbulent flow:

Turbulent intensity $\bar{u}' = \frac{1}{T} \int_{-T/2}^{T/2} u' dt$

$$\bar{u}'^2 \neq 0 \quad \bar{v}'^2 \neq 0 \quad \bar{w}'^2 = 0$$

$$T.I. = \sqrt{\frac{1}{3} (\bar{u}'^2 + \bar{v}'^2 + \bar{w}'^2)}$$

Isotropic turbulence

$$\bar{u}'^2 = \bar{v}'^2 = \bar{w}'^2$$

NPTEL logo is visible in the bottom left corner of the whiteboard image.

So, now, if we specify the flow field this way, then, we can now write that, u is equal to \bar{u} plus u' ; v is equal to \bar{v} plus v' , and w is equal to \bar{w} plus w' . So, these are all functions of time, for which these are the functions of time.

So, a steady turbulent flow is a flow, where the mean quantity is independent of time. So, for a steady turbulent flow, the mean quantities are independent of time, but on them, there are superimpositions of fluctuating components, which are erratical fluctuations, or random fluctuations, over the mean, whose mean over a time period which include more fluctuations, many more fluctuations, is 0. So, this is the typical expressions of the velocity fields, in general, in a three dimensional coordinates, for a turbulent flow.

Now, specifications of turbulence; specifications of turbulent flows; specification of turbulent flows. Now, one specification is that, turbulent intensity. What is meant by turbulent intensity? Now, turbulent intensity signifies, or specifies the degree of turbulent in the flow field. And, as you know, the turbulent intensities are due to that, if this is exactly 0, this is exactly 0; this is exactly 0. So, this is a three dimensional laminar flow.

So, turbulent flow is, because of the presence of this. So, therefore, turbulent intensities can be expressed as some average values of these; these are changing with time; but unfortunately, u' , v' and w' is 0; this bar, I am using to express the average. This bar means, for example, \bar{u} means, the time mean; that is the time mean, $u dt$. So, some time interval t_0 ; this is the way I like to replace t_0 plus T by 2; that means,

whenever I put bar, that means the time mean value. So, time average value of this fluctuating quantity is 0.

But, because of this random fluctuations, it is so, that, this bar is not 0; $\overline{v^2}$ is not 0; $\overline{u^2}$ is not 0, like our trigonometric functions; that means, a periodic functions, whose mean value may be 0, time mean value; but if you square it and mean, that is not 0. So, therefore, the turbulent intensity will be represented by this value, $\overline{u^2}$, $\overline{v^2}$, $\overline{w^2}$; that means, this average time mean value of the square of the fluctuating components of velocities.

So, it is usually defined as, turbulent intensity, T I as the square root of the mean values of these, $\overline{u^2}$, plus $\overline{v^2}$, plus $\overline{w^2}$; that means, the mean values of the arithmetic averages of the mean values of the square of the fluctuating component. So, this is not 0; this is not 0; this is not 0; but these three are not necessarily to be same.

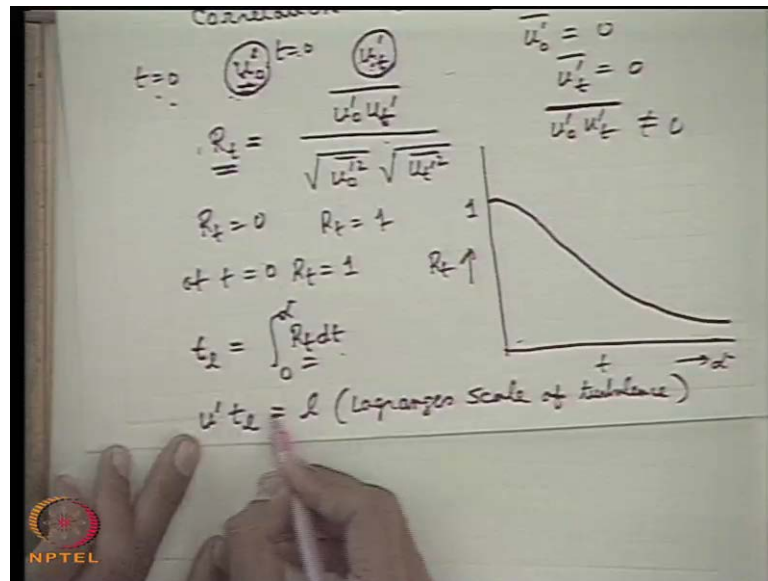
Where this three quantities are same, it is known isotropic turbulence; isotropic turbulence. Isotropic turbulent is the case, where this values are same; that means, this is independent of the space coordinates, direction; that, this magnitude of this average of time mean value of the square of the fluctuating components, that is their variations are, they are independent of the space coordinates, which means that, $\overline{u^2}$ is equal to $\overline{v^2}$, for isotropic turbulences; that means, if the turbulence is isotropic, that means, the value, this, this, this signifies the intensity of the turbulence for the x direction velocity component, intensity of the turbulence for the y direction velocity component. So, individually, they are equal; this is for the z direction velocity components.

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The image shows a whiteboard with handwritten mathematical definitions. On the left, it states $T.I. = \sqrt{u'^2}$ and $\text{Relative T.I.} = \frac{\sqrt{u'^2}}{u_x}$. On the right, it states $\bar{u}' = 0$ and $\bar{u}' = \bar{v}' = \bar{w}' = 0$. A small NPTEL logo is visible in the bottom left corner of the whiteboard.

So, for an isotropic turbulence, therefore, if we take this, here, so, we get turbulent intensity is equal to, what we, what we get, 3, 3 cancels; that means, we get u dash square bar square root; that means, square root of u dash; that means, it is the, root means square value for the fluctuating component. Since we know that, u dash bar is 0, u dash bar, for example, in case of an isotropic turbulence, they are identically 0; but this, is equal to this, is equal to this and they are non-zero. So, therefore, in isotropic turbulence, this is the turbulent intensity. When this is expressed as a ratio of relative turbulent intensity, as a ratio of the free stream velocity, divided by the free stream velocity, or average velocity in case of a duct flow, or a free stream velocity for flow past solid body; so, this ratio is known as, dimensionless parameter, as the relative turbulent intensity. So, what is the degree of the turbulence? When we tell that turbulent intensity is very high, that means, this value is very high; that means, the level of the fluctuating component is very high. So, this is the way we replace the turbulent velocity, a turbulent intensity.

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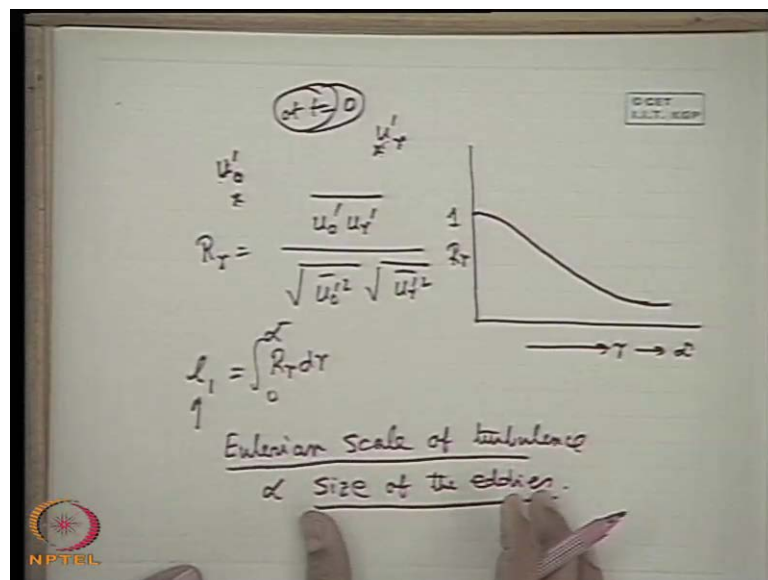
Now, next, I would like to tell you the scales of turbulence; the scale of turbulence. These are the specifications of turbulent flows, , scales of turbulence. Before that, what is correlation coefficient? Now, let us consider the velocity components, say, one directional velocity, u dash, fluctuating component, at any time t is equal to 0. Now, the correlation coefficient is defined like this, if we have a fluctuating component of velocity at the same point, at a different time u dash t , then, the correlation coefficient R_t is defined like this, u dash dot u t dash average, u 0 dash, square this; similarly, u t dash square average; this is the correlation; that means, we know that, u dash, fluctuating component at t is equal to 0, average value is equal to 0. So, fluctuating component at t is equal to t . So, time mean value is 0. But if we make this quantity, that u dash at 0 and u dash at t , product of this and make a time mean, this is not 0. And this, divided by this quantity, is the correlation coefficient. This physically signifies that, how the fluctuating velocity at an instant t is equal to 0, at any point, correlates the fluctuating velocity component at a time t ; whether there is a correlation between this two or not, if R_t is equal to 0, there is absolutely no correlation.

And, if R_t is equal to 1, that means, there is full correlation; that means, the fluctuating component at t is equal to 0, whether correlates, or has a correlation with the fluctuating component at time t at the same location. So, this is given by this correlation coefficient.

Now, at t is equal to 0, there is absolutely, correlation, full correlation at t is equal to 1. So, therefore, if we make a graph of this R_t with t , we will see that, at t is equal to 0 the correlation is 1, and as we go, with infinite time, as the time goes on increasing, the correlation between this two fluctuating components, at any instant, is just becoming less and less; that means, there is no correlation and at infinitely high time, so, there is no correlation between the fluctuating component; at time t is equal to 0, with an fluctuating component at the same location after an infinite long time, so that, at t tends to infinity, so, this correlation coefficient goes to 0. So, therefore, this, integral of this, that means, the area of the curve under this t axis, that is, the integral of this $R_t d t$, 0 to infinity, this gives a finite area, because of this nature of this curve. This becomes a temporal characteristics of the turbulent flow.

This is a time unit; R_t is non dimensional, $d t$. So, this t represents at $t l$ and if this is multiplied with, $t l$ is multiplied with u dash, this gives you a scale l , which is known as Lagrange's scale of turbulence. So, at any instant, at a point, the Lagrange's scale of turbulence is given by this quantity. Similarly, if we consider the correlation between the, not, here, we have considered the correlations between the two velocities at the same point, but at different instants.

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But if we consider the two fluctuating components of velocities, at a point r is equal to 0, and another point at r is equal to R , the two different point, but the same time, at t , or at t

is equal to 0, whatever may be the value of t ; that means, at the same instant, we consider the fluctuating components of velocities, at two, this two points, two spatial, this two points at two spatial locations. And, one is denoted by u_0 and another is u_r . Then, what is the correlation between this two? The correlation coefficient in this case, R_r , in the similar fashion, is defined as, the product of this time mean value and then, the root mean square. Well, u_r squared; so, this is it. That means, if there is correlation between this two velocity components, fluctuating velocity components, at a distance R apart, that means, two different spatial locations, then, R_r will be a value, which is not 0. But if R_r is 0 means, there is absolutely no correlation; if R_r is 1 means, there is absolutely correlation; that means, when R is equal to 0, that means, the two points, not just one point, so, they are totally correlated. This has got 1 and as R goes on increasing, that means, if the distance between the two points goes on increasing, that means, the correlation becomes less and less, and ultimately, there is no correlation between the fluctuating components, fluctuating velocity components for two points, which are infinitely distant apart; that means, r tends to infinity, this is 0. The same thing, this is R_r .

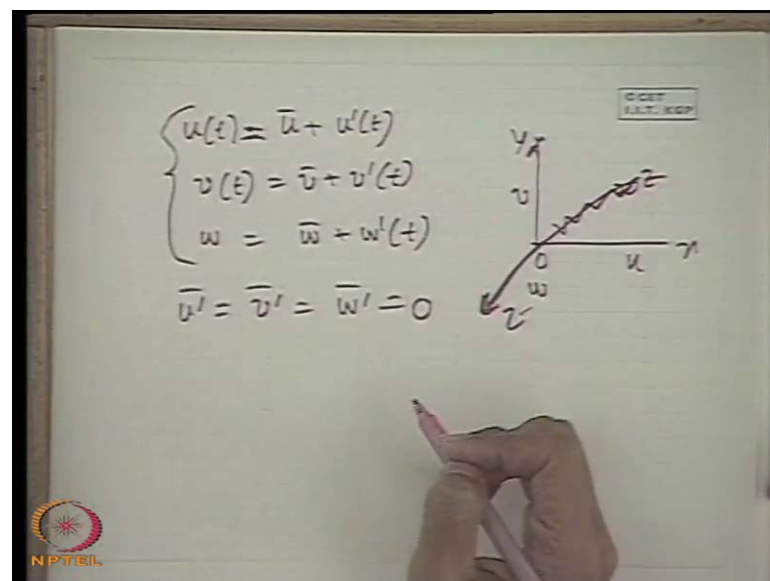
Now, this $R_r d r$, this integral, represents the area of this curve under this R axis, and this is known as another quantity l ; let l_1 , this is known as Eulerian, Eulerian scale of turbulence. Now, this scale of turbulence is proportional to the size, is proportional to the size of the eddies. This is because, the turbulent flow because of the fluctuations in the flow field, they develop eddies, and the size of the eddies is proportional to this Eulerian scale of turbulence. They determine the size of the eddies and it is the size of the eddies, like this, which determines the correlation between the fluctuating components at two points; that is, the size of the eddy which connects the two points and $(())$ the correlation of the fluctuating components of velocities between the two points.

Now, therefore, in short, we can tell now, the turbulent flow is a flow, where there is an irregular fluctuations of velocity components, in a random way, in all the directions at any point, with time. But this can be expressed as a mean velocity, as a time, invariant velocity plus a fluctuating component. Now, it happens because of high energy in the flow. Usually, the basic origin of the turbulent flow is due to some disturbances; some disturbances in the flow. It may be in the original reservoir, or in the pipe surface, usually, in the solid surface, or some solid materials, they are in the fluid itself.

So, from somewhere, due to some disturbances, this thing happens. So, what happens, any disturbance in the flow field, tries to make a disturbing nature of the flow, for which the flow becomes irregularly varying with time. But due to the viscosity of the fluid, this disturbance is dampened out; it is an inherent nature of the fluid; so that, below a certain Reynolds number, the flow becomes laminar; that means, any disturbances, the small disturbances originating from any disturbing sources are dampened out.

But what happens, when the flow velocity is very high, associated with a very high Reynolds number? So, fluid particles gets more energy from the main stream with high velocity, so that, the disturbances grow, rather being dampened out, and makes the flow fully turbulent. And, it was Reynolds first, who showed that, the flow field can be expressed as a time invariant mean quantity superimposed with a fluctuating quantity, so that, the time mean value of the fluctuating quantities are 0.

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So, therefore, we can write again that, the Reynolds decomposition principle, the flow field u is a function of time, can be expressed as u mean plus a fluctuating component. Similarly, v , in general, if we expressed in x, y, z coordinate system, x, y, z coordinate system, then, rather here, x, y , same x, y, z coordinate system, we can write, u, v, w is equal to...so that, u dash bar is equal to v dash bar is equal to w dash bar; that means, the time mean value is...So, this is the decomposition principle of Reynolds.

Now, we will modify the Navier-Stokes equation with the help of these relationships; or this way, these relationships, or these equations for the velocity field. And, we can now write the Navier-Stokes equations and see that, how the Navier-Stokes equation or the equation of motions look like, in case of a turbulent flow. I think today's time is up. So, we will discuss it in the next class.