Fluid Mechanics

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Lecture - 42
Flows with a Free Surface Part – II

Good morning. I welcome you to this session of fluid mechanics. In the last class, we were discussing the uniform steady, uniform flow through a straight channel. And we developed an expression for the average flow velocity in the channel.

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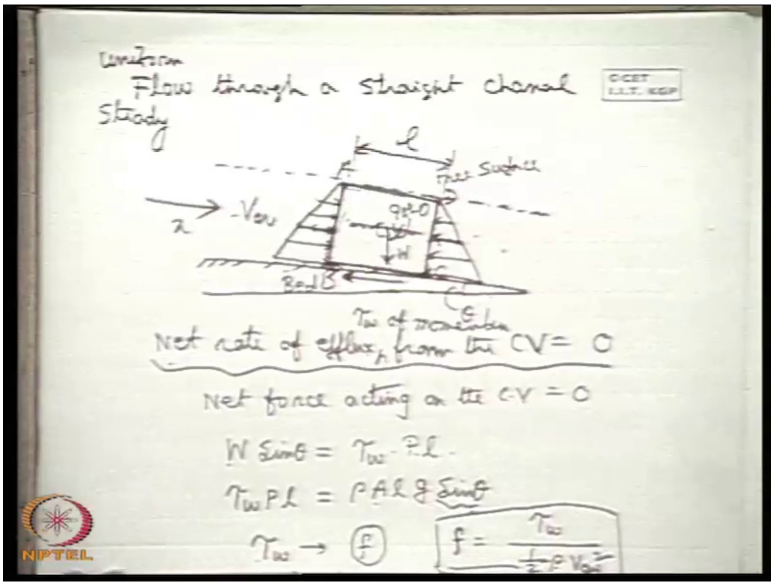

So, if we recall the discussion, we see that, we considered a straight rectangular channel with a slope. This is the slope with the horizontal; this is the channel and what we did? We took a control volume, where we have found that the pressure forces on this two surfaces balance each other. So, therefore, the net force acting on this is the difference between the frictional forces at the surface of the channel. That is the bed of the channel, and the wet. And since, the flow is uniform. So, we made the equation availably make use of the equation that the net force acting in the direction of flow. That means the component of the weight and the frictional force is to be 0. And ultimately we equated that, and we found an expression. Before that the wall shear stress was expressed in terms of the friction coefficient, and the average velocity of flow in this manner.

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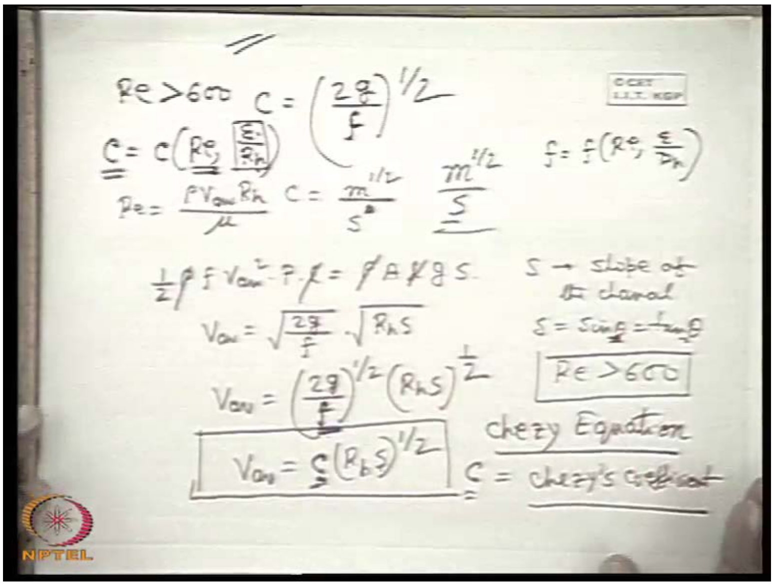

And finally, we arrived at the equation. V average is C R h s whole to the power half. Where this C is given by root over 2 g by a; this is c. So therefore, C is 2 g by f whole to the power half. So, this was defined as Chezy's coefficient, and this equation is known as Chezy's equation, where R h is the hydraulic radius and this is the slope of the channel, which is equal to sin theta or tan theta, for small values of theta that is the angle between this bed or base of the channel with the horizontal. Now, in this equation one thing is that C z, the Chezy's coefficient is unities. You see, f is free dimensionless. So, it is g to the power half; that means, meter; meter to the power half divided by second that is the unit of c. Now, we should know, what are the factors on which the C depends? So, you know C is the defined as 2 g by f as we have already discussed in case of pi flow. That is a friction factor depends on Reynolds number of flow and the relative roughness; that means, the roughness divided by the diameter or the hydraulic diameter of the pipe.

Here in the similar fashion, this Chezy's coefficient c is also depends. That means becomes a function of Reynolds number of flow, and the relative roughness defined as the roughness divided by the hydraulic radius. We have earlier discussed about the Reynolds number; The Reynolds number is defined in this case as rho times the average velocity times, the hydraulic radius divided by mu. In case of turbulent flow, when Reynolds number is greater than 600. The Chezy's coefficient depends on the average roughness coefficient. So, therefore, a dependence on Reynolds number is actually not there in case of turbulent flow in all practical cases flow is turbulent. So, therefore, the coefficient, the Chezy's coefficient depends only on the absolute roughness. You can tell the roughness and the hydraulic radius. So, these are the factors on which c depend.

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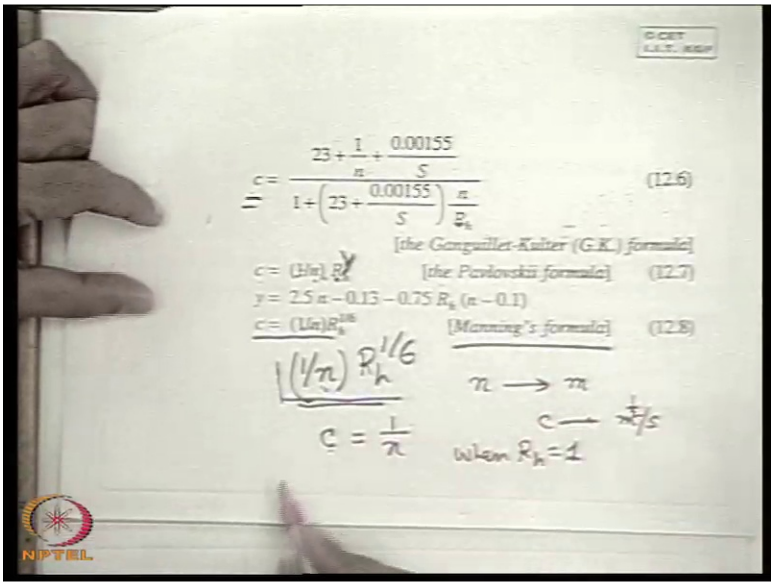

Now, I will show you certain formula, empirical formula for the Chezy's coefficient c, which is a function of the, n is the roughness factor here, and R h is the hydraulic radius. It is mostly the function of the roughness factor and the hydraulic radius. First one is that due to Ganguillet-Kulter G K formula. These are all empirical formula here of course, the slope of the channel S comes, but in other formulas this c that is due to Pavlowskii formula 1 by n R h to the power y; this is y R h to the power y, where y is given by this function. So, R h is the hydraulic radius. Another one is Manning's formula; Manning's formula 1 by n R h to the power 1 by 6. I am writing it again, it is very important 1 by n R h to the power 1 6. Now, these area all empirical formulas, where n is the roughness factor. If it is expressed in meter, we get the value of c in its dimension as we have seen earlier meter divided by meter to the power half divided by second.

So, amongst all these formula, it is found that this Manning's formula; this is the Manning's formula; this is written again here is the simplest one. And this is fairly accurate. So, therefore, Manning's formula is in popular use 1 by n R h to the power 1 6th. Now, one interesting feature can be observed from this formula. That all these empirical formulae gives the same value of Chezy's coefficient c as 1 by n, when R h is equal to 1. That means at unit hydraulic radius the value of c is 1 by n; that means, if we put R h is equal to 1 in this equation R h is equal to 1 either in this equation, it is clearly seen 1 by n. So, all these 3 equations show the value c is equal to 1 by n. Now, if we use the Manning’s formula, which is the most popular one because of its simplicity and fairly accurate in nature. We can write with this Manning’s formula 1 by n R h to the power 6.

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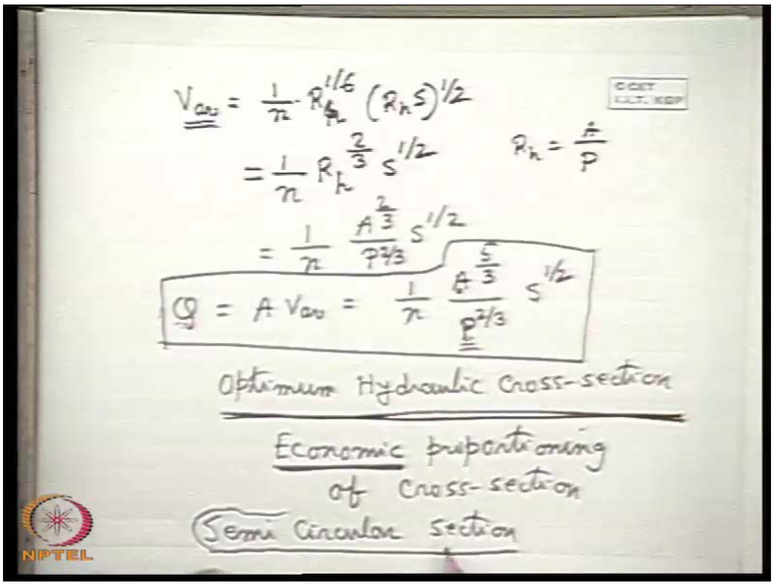

If we take this, we can write here, we can say that V earlier we deduced V is equal to V average is equal to, in place of c we write 1 by n into R h to the power 1 6th then R h S to the power half. What we are doing? We are using this expression V average is c R h s to the power half. And in place of c we are writing 1 by n R h to the power 1 6th from the Manning’s formula. So, therefore, we come to this value V average 1 by n R h to the power 1 6th, R h s to the power half. Now, this can be written in, if we now express R h, in terms of a by p, then what we can write. Before that I think R h 1 6th, and half this becomes; this is R h 1 6th plus half; that means, 1 plus 3 4 by 6 that is R h 2 third S to the power half R h 2 third s to the power half and if I write A by p.

Then, we can write 1 by n A to the power 2 third by p into S to the power half. And, if we write the discharge Q, which is equal to A into V, average cross sectional area. So, A is multiplied, we can write 1 by n A to the power 5 by 3 divided by, P to the power 2 third divided by P to the power two-third S to the power half. So, this is the expression for the volumetric flow rate. So, you see the volumetric flow through the channel or the flow average flow velocity is a function of the cross sectional area the perimeter and the slope of the channel and the roughness factor.

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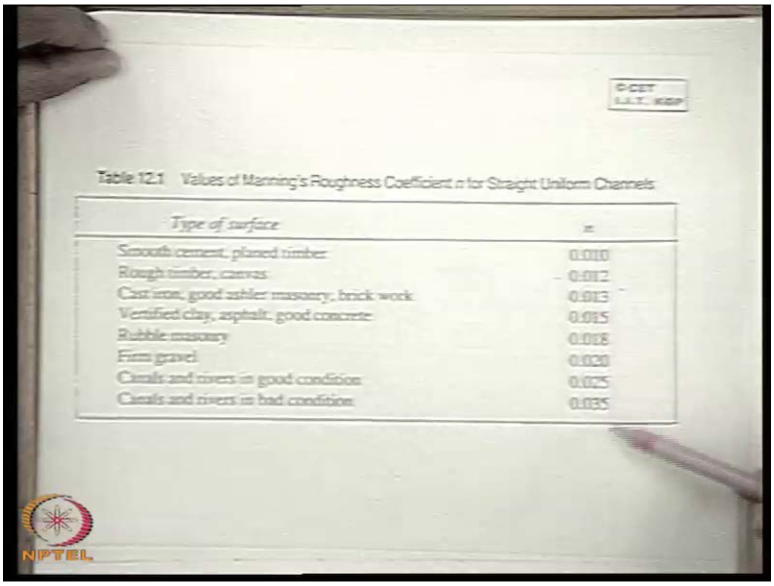

Table 12.1 Values of Manning's Roughness Coefficient n for Straight Uniform Channels

Type of surface	n
Smooth cement, planed timber	0.010
Rough timber, canvas	0.012
Cast iron, good ashler masonry, brick work	0.013
Vertified clay, asphalt, good concrete	0.015
Rubble masonry	0.018
Firm gravel	0.020
Canals and rivers in good condition	0.025
Canals and rivers in bad condition	0.035

Here I can show you the roughness factor for different materials of construction for the channels. The type of surface, values of Manning's roughness coefficient in for straight uniform channels. For smooth cement, planed timber 0.01, rough timber canvas 0.012. So, these are the cast iron, good ashler machinery, brick work 0.013. So, we see that tables are available from where we can find out the values of n depending up on the type of surface for the channel wet. Now, if we see this Q is equal to this formula. Now, this gives the rate of discharge as a function of the geometrical dimensions of the channel. Now, a one important application of this formula in the design of artificial canals is optimum hydraulic cross section. What is this optimum hydraulic cross section? Optimum hydraulic cross section is the cross section which gives for a given cross sectional area. The maximum discharge or for a given discharge and given cross sectional area the minimum perimeter.

The cross section of the channel which gives the maximum discharge for a given cross sectional area; that means, the minimum wetted perimeter for a given discharge and a given cross sectional area is known as the optimum hydraulic cross section. Here the discharge is maximum for the given cross sectional area. And also, we can say from other angle for a given discharge the perimeter is minimum. So, this is sometimes known as economic proportioning of cross section. Why it is known as economic? Because, if you see from this angle of the minimum perimeter, which means for a given discharge. If the wetted perimeter is minimum for a given cross sectional area, then we require minimum lining material to be added to the channel bed and the channel surface.

So, this part ends to the word economy. So, it is economic proportioning of cross section or optimum hydraulic cross section. That gives the maximum discharge for a given area or for a given discharge the minimum wetted perimeter. Now, out of all sections, we know that a semi circular section gives a minimum wetted perimeter or a minimum perimeter out of a given cross sectional area out of given area. But it is not used for different constructions from some practical difficulties. Though, it can be built from prefabricated sections, but it is not in use. Most popular section is a trapezoidal section.

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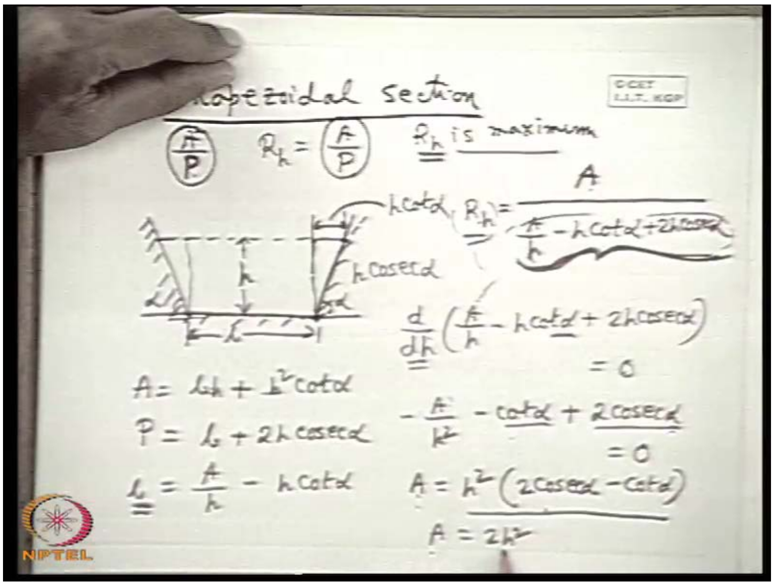

So, therefore, we will find out the condition for an optimum hydraulic cross section for a trapezoidal section. Now, let us review again the condition that A by P should be made minimum, maximum the wetted perimeter has to be made minimum. Now, if you define the hydraulic radius R h as A by P. Then, we can say that for a given area P is minimum means R h is maximum. That means, the cross section will be such, whose hydraulic radius will be maximum for a given flow. Now, let us find out what is this condition for trapezoidal section; that means, we consider a trapezoidal type of cross section for the channel. Let us consider this is the trapezoidal section. And is dimension is given by this b, with this b. Let us consider that the depth of flow is h, the central depth of flow is h. And the side slopes are alpha. The trapezoidal sections are the defined by these geometrical parameters with central depth of flow as h.

Now, what is this A area of cross section. Now, we see that if we make a perpendicular here. So, this is from the geometry. So, this is h. So, this is alpha. So, this is h cot alpha this much. Similarly, this length of this side is h cosec alpha. So, area is b into h; that means, the rectangle b into h plus. This one the triangle, the 2 triangles; that means, h square cot alpha plus. What is the perimeter P. Perimeter is this with b at the base plus; this length plus this length; that means, b plus 2 h cosec alpha. Now, from here, we can find out that R h is A by P, but we will have to eliminate this b from here. So, if I eliminate b from this equation S by h minus h cot alpha. Now, I can write R h is equal to A by P. Now, what is P? P is this one, but I replace b from this; that means, I have eliminated b from this equation. And now, the value of this b from this equation is replaced here in terms of in case of P; that means, A by h minus h cot alpha plus 2 h cosec alpha. So, this is the value of R h.

Now, you see that this is the value of R h. Now, for a maximum of R h for a given area; area is constant means the minimum of the numerator. So, we try to find out the minimum value of the numerator for the optimum trapezoidal section. Now, what are the variables? Now, if I find out these conditions for a particular depth; that means, we try to find out that at what depth of flow, these will satisfy the condition of maximum R h; that means, the minimum of the denominator. That means the maximum of the R h means the minimum of the denominator. So therefore, I differentiate the denominator A by h minus h cot alpha plus 2 h cosec alpha to be 0 to find out the minimum condition for the denominator. Let us do it, we will get now minus A by; that means, if we consider the geometry to be fixed alpha fixed differentiate with respect to h minus cot alpha plus 2 cosec alpha is equal to 0. From where we get A is equal to h square 2 cosec alpha minus cot alpha that is A.

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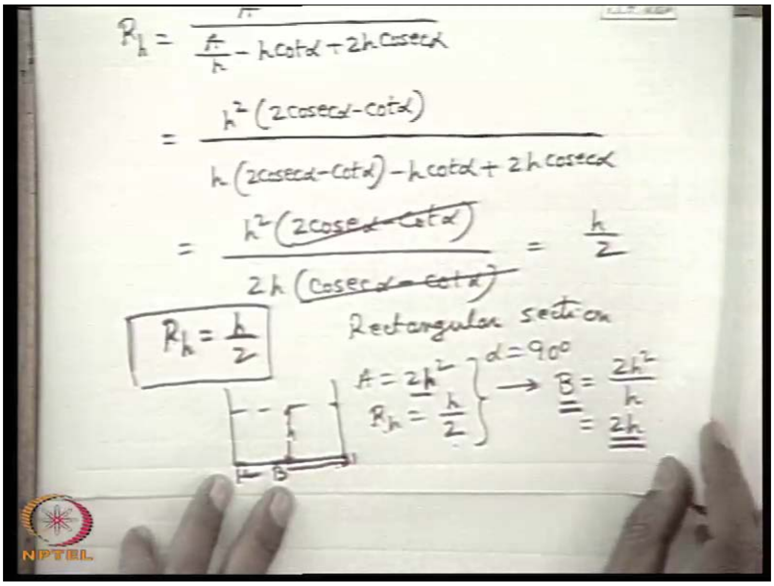

Now, if this value of A, we just put in this expression of R h. Now, let us again write this expression R h separately, otherwise difficult to follow. R h is A by h minus h cot alpha plus 2 h cosec alpha that is R h. And we have got A is h square 2 cosec alpha minus cot alpha. So, if I put that value of A which we have got h square 2 cosec alpha minus cot alpha. So, this is the value of alpha, we put here it will be h 2 cosec alpha minus cot alpha h then, minus h cot alpha plus 2 h cosec alpha. So, if you make it clear. So, you will see that numerator will be 2 cosec alpha make some algebraic manipulation. So, denominator will be 2 h. If you take 2 common then, it will be like this. So, this cancels.

So, this ultimately h by 2; so therefore, the result is that R h is h by 2; that means, the hydraulic radius will be equal to half of the central depth. This is the conclusion for the case that the optimum hydraulic section for a trapezoidal section. The hydraulic radius will be half of the central depth of the flow h by 2. Now, here we can develop a particular relation that in case of a rectangular section. You see that what is the rectangular section defined from a trapezoidal section is the special case, when alpha is 90 degree. That means, this is perpendicular to the base. In that case, what is the value of A from here for the maximum R h condition? That is A is equal to 2 h square, if you put alpha is equal to 90 degree.

One thing you must know, that we have talked about the minimum of that that is obvious. Because I have said this first differential is 0 and this is first differential with respect to it. So, second derivative is; obviously, positive because this 2 terms will not appear. So, this will give a positive term, which means this expression has a minimum with the variation of h. For h this is minimum one. So, that is proved. Now, A is equal to 2 h square for a rectangle. So, for a rectangle A is equal to 2 h square and again R h is equal to h by 2. So, this is the condition that R h is equal to h by 2 and A is equal to 2 h square. So, immediately from this one thing comes. If A is equal to 2 h square then the width of the rectangular channel will be 2 h square by h. Because h is the depth of flow which is the height; that means, this is the rectangular channel. So, if this is h, so therefore, if b is this width.

So, area is 2 h square. Area is b into h divided by this h will give you b which is equal to 2 h; that means the width of the rectangular channel will be twice the depth of flow alright. Now, if we see that now, we maximize R h or minimize this denominator with respect to h. Now, if we consider that the depth of flow remains constant. And we vary this angle for the trapezoidal section to find out at what angle the R h is maximum for the optimum hydraulic cross section. That means the denominator is minimum. In that case, we will have to differentiate this quantity with respect to alpha and is set to 0 instead of h. If we do it, then we will see if we do it then, we will see, if we differentiate it with respect to alpha then; that means this quantity without differentiating.

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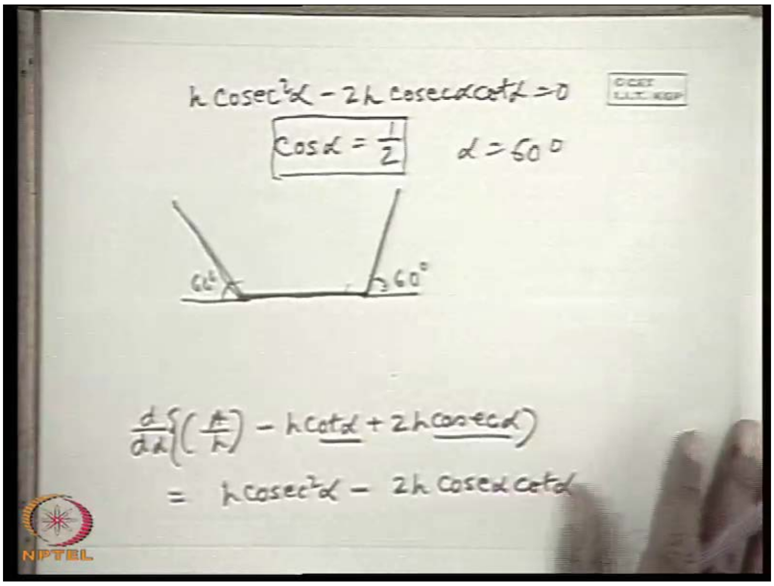

That means if we write the d d alpha of this quantity A by h minus h cot alpha plus twice h cosec alpha. Then, we find d d alpha. Then, we find that is equal to minus h cot alpha. So, h cosec square alpha minus 2 h cosec alpha cot alpha. Because the derivative of cot alpha is minus cosec square alpha and cosec alpha is minus cosec alpha cot alpha. And, if we put this is equal to 0; that means, h cosec square alpha minus 2 h cosec alpha cot alpha is 0. We get simply cos alpha is half, which means alpha is 60 degree. That means we get a trapezoidal channel with this angle as 60 degree, which is half of a regular hexagon. So, this is the angle. So, half of a regular hexagon, because hexagon regular hexagon the included angles are all equal to 120 degree. So, this is half of regular hexagon. So, this cross section given by alpha is equal to 60 degree half of a regular hexagon. For a given depth of flow pertains to the optimum hydraulic cross section for a trapezoidal section; that means, the maximum of hydraulic radius R h.

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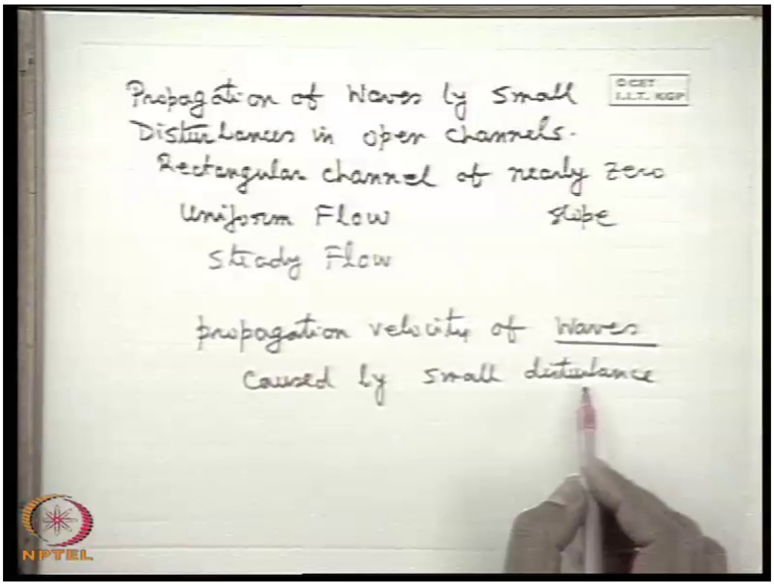

Now, we will discuss a new topic that disturbance of waves, small disturbance; propagation of waves by small disturbance in open channels. Now, let us see, what is meant by that? Now, you know in a pool of water. For example, any pool of water, if you make a disturbance that disturbance propagates in the form of wave. That means, a wave is generated and the disturbance propagates from the source of disturbance, where it is created in both upstream and downstream directions in the form of wave. Just an example, we can see that if in a river or a pond. If we put a stone or drop a stone, then you see the waves are created, the stone where it is dropped creates a disturbance. And automatically waves are generated, and by which the disturbance wave is ultimately propagated to the entire liquid mass.

Similar is the case, when in a river, if you open or close a swiss gate. Then, you will see a disturbance is created which is propagated both upstream downstream in the form of a wave. So, therefore, a small disturbance created in a pool of liquid, we either it is stagnant or it is flowing a disturbance creates a wave which is propagating through this fluid medium. Fluid medium may be stagnant or may be moving. Now, our first job is to find out what is the propagation velocity of this disturbing wave or the propagation velocity of the wave created by the disturbance? Let us consider that propagation velocity of the wave. For that we consider that a rectangular channel, we consider uniform flow and steady flow. We consider a uniform steady flow through a rectangular channel of nearly 0 slopes.

That means, whose bed is almost horizontal nearly 0 slopes. And we want to find what is the propagation of our main objective is the propagation main objective is to found is to find the propagation velocity of waves caused by small disturbance. In this regard I tell you that one terminology that a wave is considered positive, if it increases the height or the depth of flow. If it increases the depth of flow, it is considered to be negative. If we reduce the depth of flow, we consider a positive wave for our analysis.

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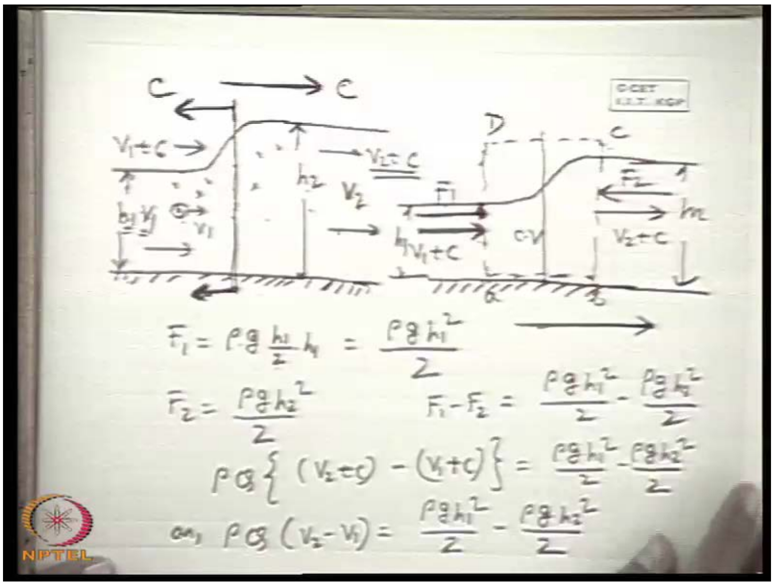

Now, let us consider a rectangular channel like this. Now, you see when the wave is moving through this flow the channel. Then, what happens? The downstream conditions with respect to the flow or we can tell the region of the flow through which this wave has already travelled has changed its hydro dynamic parameter like the flow velocity and the depth. Where the upstream flow velocity, and depth which was the undisturbed flow velocity, and depth is yet to suffer a change. Let us consider this is like this is h 1 is the upstream depth, which is the undisturbed depth and V 1 is the velocity. And, let us consider that this is the downstream depth h 2 and the velocity is V 2. So, the velocity is changed as the wave propagates.

Let us consider at any instant, this is the position of the wave which is propagating upstream with the velocity C. At any instant, this is the position of the disturbing wave which is propagating with the velocity c in the upstream direction. And. So, that the with respect to these waves; this is the downstream; this is the upstream. With respect to the flow this is the upstream and this is the downstream. So, with respect to the flow, we can tell the upstream conditions for depth of flow. And a velocity is undisturbed, and remains as h 1 V 1, which are changed to h 2 V 2. For a positive wave, the depth is increased to h 2. And the velocity becomes V 2 velocity will be reduced, because the depth is increased h 2 and V 2. So, this is the condition, we just can see at any instant.

Now, if you want to analyze the situation through the application of the momentum theorem. Then, what we have to do? We have to take a control volume surrounding this wave. So, if I take this control volume separately in this, we will show the control volume like this. If this is like this, we take a control volume like this surrounding this wave. Now, we have already discussed that for this type of case, the control volume is considered to be fixed with this moving object. Here the moving object is the disturbing wave, why this is because? If we consider that the entire thing with respect to this analyze, with respect to this wave. Then, what happens? The things become steady, otherwise the things become unsteady.

Just for example, you see at any point in the fluid here. Now, the velocity is V 1, when the wave passes through this point of velocity will be changed to V 2 or the depth will be changed from here h 1 to h 2. So, with respect to time a point suffers a change in its hydrodynamic parameters; that means, in unsteady flow. But what we do instead, we impose a velocity C in opposite direction. That means, in that case, if we consider the fluid is moving with velocity V 1 plus C in this direction and V 2 plus C in this direction. And wave is stationary at particular location. Then what happens? The flow becomes steady. That means, for all points here the velocity is V 2 plus C which is very end with time. And for all points in this region velocity is V 1 plus C varying with time, which is varying with time. So, therefore, for this purpose we make an analysis with respect to the standing wave. That means, the coordinate axis is attached to this moving wave, the coordinate axis is attached to this moving wave or in other words we tell that wave is made stationary by super imposing a velocity C in the opposite direction to the entire system. So, in that case, we see that if we designate the control volume as A B C D. So, a fluid comes through the phase A D with a velocity V 1 plus C. And the fluid leaves the phase B C with a velocity V 2 plus C. This is simply the picture. Now, you see what happens, what is this; this is h; this is h 1 and this depth is h 2. Now, you see the flow is uniform. So, there no there, the flow is not uniform here V 1 plus V 2 plus C.

Now, you write the control momentum theory for the control volume. That means, the net force is acting on the volume is equal to the net rate of momentum effects in a specified direction. We take this direction, that direction of flow to be in the specified direction. What is the force acting in this direction? One force is the force which is acting on this surface due to hydrostatic pressure. Similarly on this surface the force acting in this direction is the hydrostatic pressure. Let this force is F 1, this force is F 2. And if we consider this control volume to be small; that means, this distance to be small enough. Because the wave is very thin. So, here it is shown in the diagram in a very exaggerated way. Actually, this width is very small. So, that the frictional force can be neglected as compared to this other forces hydrostatic forces.

But the thing is that the hydrostatic force here is due to the static pressure variation in this fluid during the throughout this side h 1. So, what is that value? We have already discussed this in fluid statics this F 1 will be rho g h 1 by 2 in to h 1. If we take a unit dimension in the direction perpendicular to the plane of the paper, then, we can take that this h 1; that means, width is unit width of the channel. If you take unit then h rho g h 1 by 2 is the pressure intensity at the centroid and then area is h 1. So, simply rho g h 1 square by 2 is the hydrostatic pressure force on this surface A D of the control volume. That is due to this part of the width; that means this part. Similarly, if 2 is the hydrostatic pressure force in this part of the liquid which is equal to rho g h 2 square by 2. So, therefore, the net force acting in this direction will be F 1 minus F 2 is equal to rho g h 1 square by 2 minus rho g h 2 square by 2. Now, what is the change in momentum in the direction or in the momentum a flux for a control volume? The statement will be net rate of momentum a flux in this direction. It is mass that is rho Q into the change of velocity that is V 2 plus C minus V 1 plus C. And that must be equal to according to the momentum theory h 1 square minus rho g h 2 square by 2. Or we can write rho Q V 2 minus V 1 is equal to rho g h 1 square by 2 minus rho g h 2 square by 2. Now, we see that what is Q? Q can be written as Q is equal to area into velocity.

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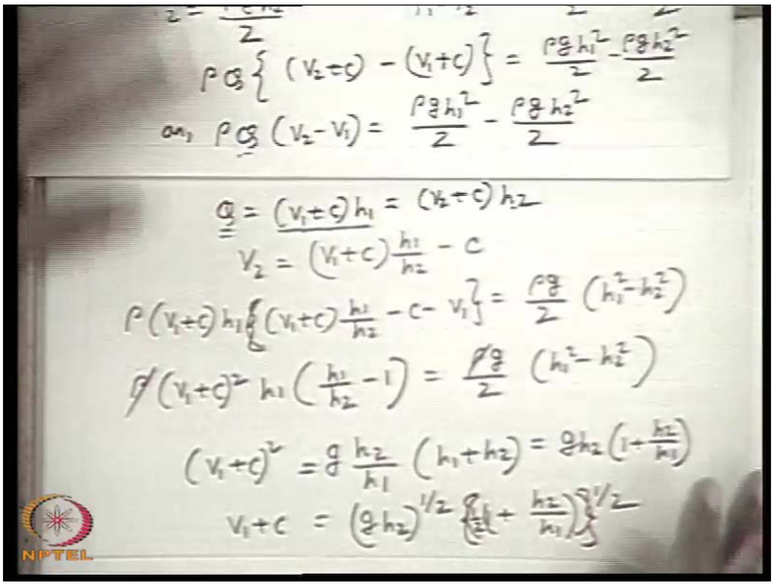

So, therefore, Q can be written as V 1 plus C into h 1 or is equal to V 2 plus C into h 2. So, if we now want to express everything in terms of V 1, what we will do? We will eliminate V 2 from here in terms of V 1 which is V 1 plus C into h 1 by h 2 minus C h 1 by h 2 C and replace Q in terms of V 1. V 1 plus C into h 1 in this equation; that means, then we get rho into V 1 plus C into h 1 into V 2. What is V 2? V 2 is V 1 plus C. You can make a second bracket into h 1 by h 2 minus C minus V 1. And that becomes is equal to rho g by 2 h 1 square minus h 2 square. So, we can write this rho V 1 plus C whole square. If you take V 1 plus C common, then h 1 by h 2 minus 1 is equal to rho g by 2 h 1 square minus h 2 square from which we get that V 1 plus C whole square, if we take this h 1 minus h 2.

There what we get? We get h 1 minus h 2 that is h 1 plus h 2 into h 2 by h 1. So, therefore, we get V 1 plus C whole square is h 2 by h 1 into g is the there into h 1 plus h 2. Or we can write g h 2 into 1 plus h 2 by h 1. Well. So, V 1 plus C is equal to g h 2 to the power half into 1 plus h 2 by h 1 to the power half. So, this is the expression for V 1 plus C g h 2 to the power half into 1 plus h 2 by h 1 to the power half. So, there is a 2; 2 were missing. So, 2 will be there. So, divided by 2 to the power half or we can write here half into 1 plus h 2 by h 1 whole to the power half. And we can make a second bracket fine.

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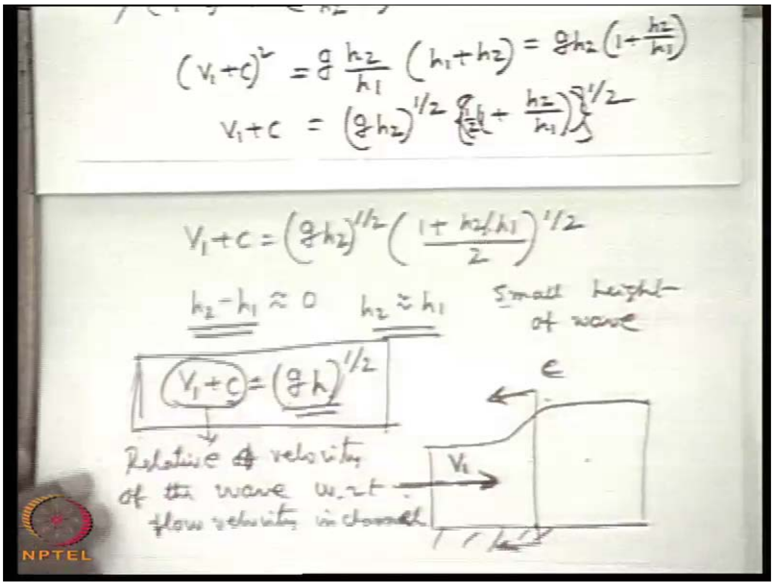

So, we can write it again that V 1 plus C is equal to g h 2 to the power half into. We can write half into 1 plus h 2 by h 1 by 2; that means, 1 plus h 2 by h 1 by 2. You can write in this fashion whole to the power half this is the expression. Now, if we consider the height of the disturbing wave is small, which means that h 2 minus h 1 is very less is almost 0. Or h 2 is very close to h 1 that is a small height of the wave for a small disturbance, small height of wave. That means, wave height is very small where the difference is depth of flow is very less, h 2 is h 1. So, then we can put this is equal to 1. So, this becomes 1in that case V 1 plus C becomes the depth of the. That means for a limiting condition that small height waves that h 2 is approximately equal to h 1, we get this expression. This is a very very important expression, what does it then physically signifies. This signifies that this is the propagation velocity; that means, if this is the channel. So, this is the velocity c.

That is the propagation velocity of the disturbing wave. So, V 1 is the flow velocity in the channel. So, therefore, V 1 plus C is the relative velocity of the wave with respect to channel velocity, with respect to flow velocity of channel, flow velocity in channel. That means, this V 1 plus C is the velocity of the wave related to the velocity of the fluid in the channel which is equal to g h to the power half. So, this is a very important conclusion that in case of a wave generated by a disturbance. This propagates in the fluid in both upstream and downstream direction. Whose magnitude with respect to the flow velocity in the channel is given by g h to the power half, where h is the depth of flow.

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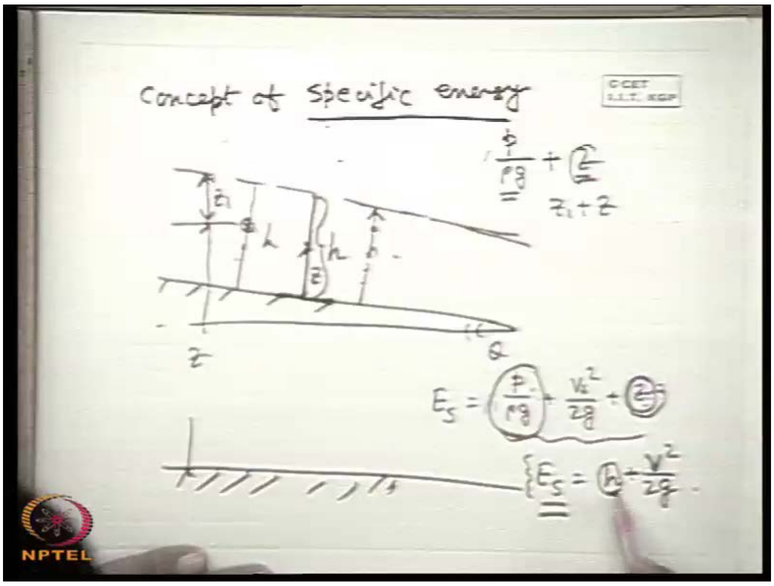

Now next I will start the concept of specific energy. Now, you know the total mechanical energy in a fluid is the sum of the kinetic energy plus the potential energy, because of the exposure of position in a conservative body force field. If you consider gravity is the only body force filed. If you consider gravity is the only body force field it is the gravitational potential energy or the pressure energy. And energy per unit wave is the total mechanical energy per unit or the total mechanical head. Now, to measure the potential energy; kinetic energy is given by mass time velocity square by 2. The pressure energy is given per unit, where it is p by rho g or per unit mass is p by rho.

Now, to measure the potential energy, we have to fix a reference datum from which the potential energy has to be measured. And that datum the potential energy is 0. So, potential energy is given by n g into a divided by z is the height from the reference datum from the system concern. Now, in a channel flow, you see certain things just like this. Let us consider a channel flow of some slope theta. Let us consider this a straight channel which is flowing with a free surface and this is the depth of the flow. Now, if we consider any reference datum. And then one thing is very clear that for any point in the channel. If at that section the depth is h. So, at this point, what is the sum of the pressure energy plus the potential energy from this datum? So, that is equal to if this is z plus z, the plus z.

So, therefore, we see that the pressure energy is equal to this a p by rho g plus h. Now, what is this P by rho g? P by rho g is this value this vertical distance. Because, if we consider this P by rho g that is from the atmospheric pressure. So, this vertical distance let this is called z 1. So, therefore, this becomes z 1 plus z. That means always this is the height from the reference datum of the free surface of the fluid. Which represents the sum of the pressure energy, and the potential energy of any point in that section? So, therefore, if I know the depths of flow at a particular section, then the vertical distance or the vertical height from the reference datum for up to free surface at that section represents the sum of the pressure energy, and the potential energy for any point in that section. Now, specific energy is the sum of all the energies per unit wet; that means, E s is sum of all the energies per unit wet.

That means P by rho g plus V square by 2 g per unit wet plus z provided the potential energy is measured from the bed of the channel, from the bed or base of the channel. Now, what happens that any point, if I measure the potential energy by this height? That means, let this is z, then this is z. Then V square by 2 g is. So, what is P by rho g? This is P by rho g. That means, therefore, we see that in this case, it becomes the depth of flow. So, therefore, this 2 p by rho g and z, this considers the depth of flow. So, therefore, we can write that E s is equal to depth of flow plus V square by 2 g. So, therefore, according to the definition, this specific energy is the sum of all the energy, mechanical energy, pressure energy, kinetic energy and the potential energy per unit wet.

That means, this is the total mechanical energy per unit wet. Where the component potential energy is measured by the height from the bed of the channel; that means, the bed of the channel with respect to slope is considered to be the reference datum. So, in that case, z the potential energy plus the pressure energy, which is measured from that above the atmospheric pressure is P by rho g. So, this 2 constitutes the depth of the flow at any section. So, therefore, the specific energy at any section is h plus V square by 2 g, where V is the average flow velocity at that section. Or for a point then it will be the velocity of that point in that section. So, therefore, at a section, if we represent the specific energy it will correspond to the depth of flow at that section and the average flow velocity at that section.

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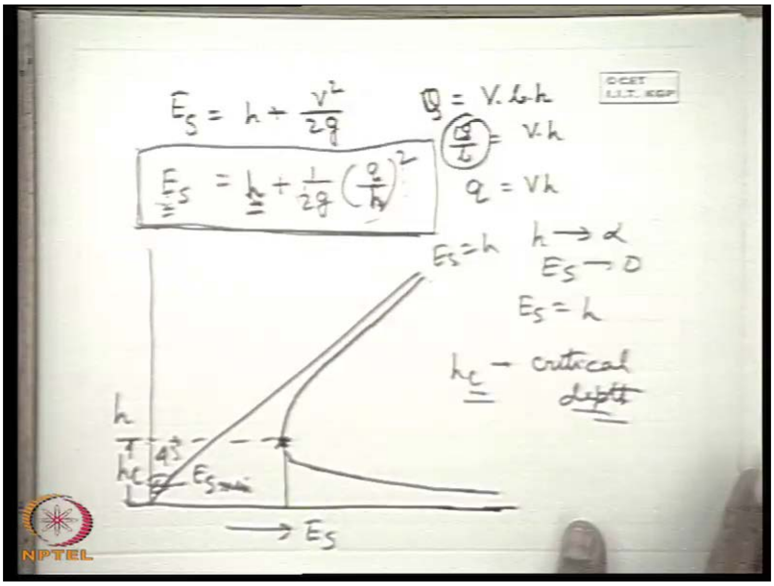

So, with that if I write E s is equal to h plus V square by 2 g. Then I can write one step further with the definition that Q the flow rate is equal to V into the width of the channel b into height that is the depth of flow. So, we can write that Q by V is equal to V h. If we represent Q by V as small q that is the flow rate per unit q, then q is equal to V into h. So, if you replace V in terms of q by h. Then we can write h plus 1 by 2 g into q by h whole square. V square is q by h whole Square. So, E s can be written as h plus. So, this is a very important relationship among the specific energy with specific energy with the depth of flow and the flow per unit with q.

Now, in this equation, we see the there are 3 variables E s h and q so; obviously, from basic mathematical sense that out of this 3 variable, 2 are independent and 1 is automatically fixed by this equation. Now, if we find try to find out, what is the now, if we try to find out or we focus our attention in a particular fashion that the discharge is constant. Now, how E s varies with h. If we see that how E s varies with h for a particular value of q; that means, if we plot this curve E s versus h with a particular q what you will get. Let us draw this curve E s versus h. Now, we see for very small value of h this term will be dominant. Where E s is inversely proportional to h and at the same time, when h tends to infinity E s tends to 0. So, this curve will be like this small value of E s.

So, that E s tends to infinity h tends to 0, but at the same time, when higher values of h. You see this term will be insignificant because, this term will be smaller and smaller with h being higher and higher. So, then E s becomes approximately equal to h, E s is equal to h. So, they are linear with the 45 degree line; that means, if we draw a line E s is equal to h. That means, this is 45 degree, then at the higher value the curve goes to the asymptotic value to this is line E s is equal to h, because the first term is dominant. So, therefore, we see the curve has a distinct minimum; that means there is a minimum value of E s for a given value of h; this is the minimum E s minimum. So, this is E s minimum. And the height corresponding to the minimum value of E s is designated as h c and h c is the critical height or depth critical depth.

So, therefore, we see the variation of E s with h for a constant q so, the distinct minimum. So, that at particular depth h c known as critical depth, the E S is minimum. If we decrease the depth E s increases which is because of this term, and if we increase the depth above this, then also E s increases because of this term. So, this is the typical variation of E s with h well the time is up. So, we will discuss further in the next class.

Thank you.

Summary; the optimum hydraulic cross section of a channel is characterized by the maximum value of the hydraulic radius for a trapezoidal section this condition is satisfied when the hydraulic radius becomes equal to half the central depth of flow.