Fluid Mechanics

Prof. S. K. Som
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 41
Flows with a Free Surface Part – I

Good morning, I welcome you all to this session of fluid mechanics. Last class we were discussing the superimposition of the basic flows for an ideal fluid. First we discussed the superimposition of a rectilinear flow parallel to the x axis with a doublet, which generates the stream lines which could have been generated if an parallel ideal flow parallel to this x axis, rectilinear ideal flow parallel to the x axis is deflected by a circular cylinder placed at the origin whose axis is at the origin.

Then next we discussed the superimposition of a rectilinear flow parallel to x axis plus the doublet at the origin plus an irrotational vortex motion, which creates the stream lines that could have been obtained if there is a circulation, there is a circulation or circulation is added in the flow past a circular cylinder.

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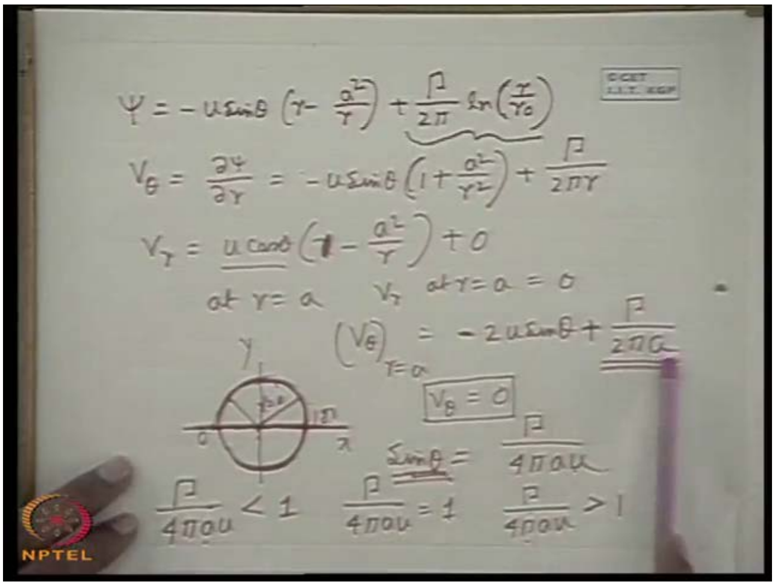

And in that case we show that the stream function is superimposition of the rectilinear flow and doublet that is flow past a circular cylinder plus the circulation that means the stream function for a free vortex. And here we have seen as a consequence, if we find out this term creates the asymmetricity in the velocity field, that if we find out the velocity V theta V r, then we will see that this V theta and V r is like this that at the surface V theta is this minus 2 u sin theta plus gamma by 2 pi a. So, therefore, we see this is the term which creates the asymmetricity; that means if we see this expression we will see the tangential velocity at the surface, because surface there is only tangential velocity. There is no radial velocity which is 0 at the surface which is not symmetrical about this axis, because the sin of sin theta changes it is positive from 0 to 180 and it is negative from 180 to 360.

So therefore, this V theta is symmetric about the y axis, but not symmetric about the x axis that means the velocity field in this half is different from that in this half. And another very interesting thing we found that the stagnation point is not occurring like a flow past a cylinder that means superimposition of only the rectilinear flow and the doublet that means this is equation for that, where that stagnation points occurred at theta is equal to 0 and theta is equal to 180 degree.

So, unlike that here the stagnation point occurs at different positions depending upon the cases that the sin of this quantity. That means for stagnation point we can write V is equal to 0 and we arrived at the equation sin theta is gamma by 4 pi a. So, depending upon the sin of this quantity, the existence of the stagnation points depend, the existence of the stagnation points depend on the sin of this quantity. There are three possible situations one is gamma by 4 pi a u less than 1, another is gamma by 4 pi a u is equal to 1, another is gamma by 4 pi u is greater than 1.

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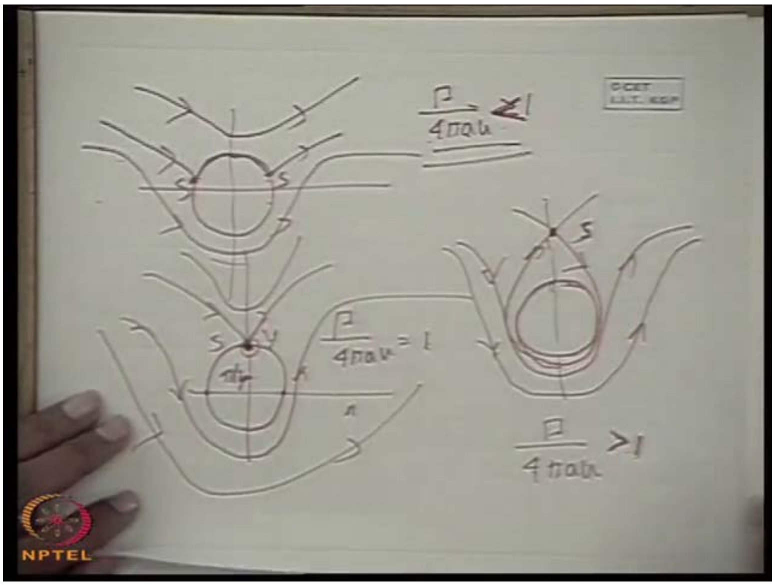

And depending upon those cases we have discussed earlier that if gamma by 4 pi a u is less than 1 then this stagnation point occurs on the cylinder, but it is shifted upwards like this, then the stream line patterns are like this, if it is exactly equal to 1 so this stagnation point takes place at theta is equal to pi by 2 instead of theta is equal to 0 or 180 degree unlike this parallel flow in a cylinder, it occurs at theta is equal to pi by 2. So, gamma by 4 pi u greater than 1, it is impossible to define the sin theta by this equation. Well, it means there is no stagnation point on the surface of the cylinder. So, stagnation point is shifted here in the flow field.

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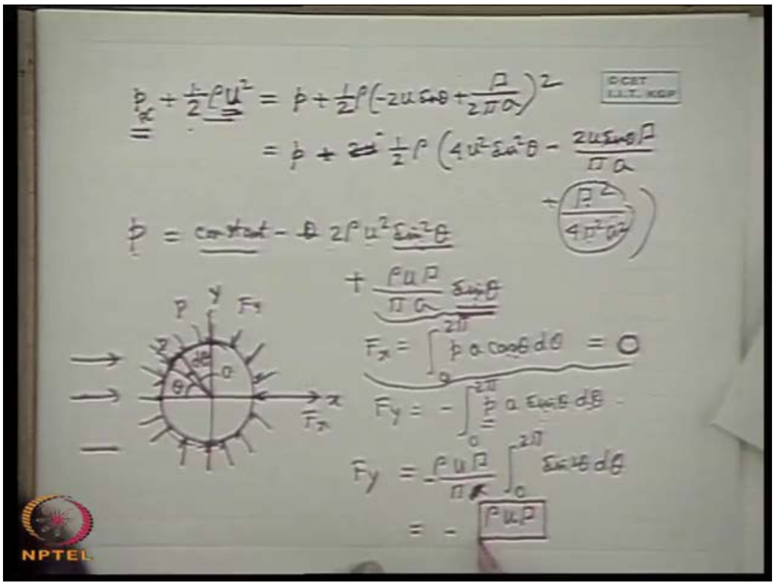

Now, we will find out the pressure expression. Let us see what is the pressure distribution? Now, if I write the pressure distribution, write the Bernoulli's equation at far upstream point that means at some point far upstream where the undisturbed pressure is p infinity and the rectilinear flow velocity is u square as we did in case of parallel flow past a cylinder that means superimposition of rectilinear flow and the doublet. Here also we are doing the same thing then we write p plus half.

Now, what is the velocity at the surface of the cylinder that means if we take at any point on the surface of the cylinder, this is the velocity at the surface of the cylinder V r is 0 only velocity is the tangential velocity. That means minus 2 u sin theta plus gamma by 2 pi a whole square. Now, if I write this then we can write p now, if I make this, clear this expression then it will be 4 that means 2 u square half rho you can take common, you can take half half rho as usual half rho as usual rather 4 u square sin square theta minus 2 u sin theta gamma by pi a plus gamma square by 4 pi square a square, square of this term.

Now, you see the p infinity is constant, this u is constant, this is defining the rectilinear flow, gamma the circulation strength is constant for the free vortex flow pi a, these are constant. So, we can write taking all constants together p is sum constant, only the sin functions minus half if I just take it 2 it will be 2 rho u square sin square theta minus rho u gamma by pi a sin theta.

So, p can be written as some constant. So, we are taking this, this, this, this we are taking this side p infinity plus half rho u square minus this quantity as constant minus 2 rho u square sin square theta minus of course, this is this minus this. So, this becomes plus so this becomes minus so this becomes plus plus what we get rho u gamma rho u gamma by pi a sin theta.

Now, it is very clear that this is the term which is responsible for the asymmetricity means what that let us consider the cylinder here. Now, let us consider this is the free stream velocity. So, this is the free stream velocity. Now, if we want to find out, now this is the pressure distribution as I have told the only force on the surface of the cylinder is the pressure force because the viscous force, that is the force along the tangent at this surface along the tangential direction at the surface is 0 because of the fact that fluid is inviscid.

So therefore, if we again want to find out the force in x and y direction then what is F x and F y. In the similar fashion we take that if this be the p for an elemental sector or circular arc of the surface which sustains d theta angle at the center and this angle is theta, we see that we already recognized F x is p a is the radius, this is a, this radius is a p a cos theta d theta from 0 to 2 pi. Earlier, also we did like that and F y with a negative sin y is directed a positive directed upwards so p a sin theta d theta.

Now, here if you see that if you multiple this with cos theta this term gives 0, this becomes the symmetric terms from 0 to 2 pi. That means integral of sin square theta cos theta from 0 to 2 pi, if you perform this integration it is very simple from the trigonometric function that it will be 0. Similarly the integration of sin theta cos theta which gives sin 2 theta from 0 to 2 pi also will be 0. So, which means that in F x direction there is no force, 0 which physically means that the pressure force is a symmetrical about the o y axis that means the force exerted on this half is exactly equal to that on this half in the x direction. So, that ultimately this gives 0 force.

But what happens in F y direction if you see carefully, if you multiply this expression of p with sin theta then you will see this will give a sin square theta term here and this will give a sin q theta. Now, sin q theta d theta 0 to 2 pi if you make the integration this will give 0 because this will give an integration of sin square theta sin theta d theta. So, this you will get a 0 term that means the integration of sin q theta d theta will be 0.

Whereas this even terms sin square theta d theta will not give a 0 term. So, this is because of this term we get F y a non 0 value and if you just perform this equation now we write only this term. Therefore, F y exist only for this term rho u gamma by pi a with a negative sin into 0 to 2 pi sin square theta d theta. And this becomes equal to with a negative sin rho u gamma o a cancels because a is there, a will not come simply rho u gamma.

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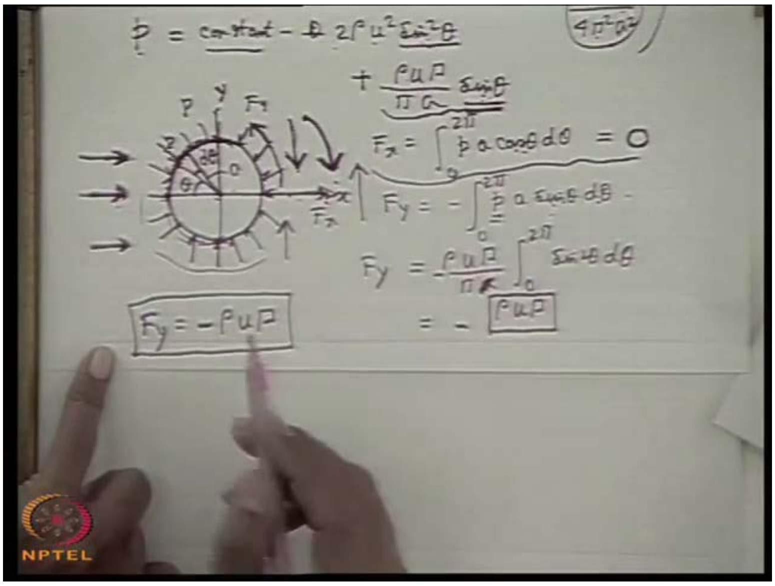

So therefore, we see that the force in the y direction, force per unit length of the cylinder in that per unit length of the cylinder the net force in the y direction is minus rho u gamma. This minus sin means the force is on the downward direction. So, the sin convention comes from from the relative direction sin of the rectilinear flow and the irrotational vortex. Here, we have considered the irrotational vortex positive in anti clockwise direction.

So, for a rectilinear flow which is positive in this fashion parallel to positive x direction means I can tell that for a rectilinear flow from left to right, and for a rotation or a circulation or a vortex motion in an anti clockwise direction the pressure forces on this half upper half of the cylinder is more than the lower half. So, that there occurs a net downward force in this transverse direction to the flow direction whose magnitude per unit length of the cylinder is minus rho u gamma.

So, therefore, we see in this case if you add a rotation or a circulation by imposing a vortex motion along with the rectilinear flow and the doublet motion then we get a net force either downwards or upwards in the transverse direction to the, that means the perpendicular direction to the direction of the flow.

Now, if we make a clockwise direction positive if we make a clockwise direction positive or if we add a vortex motion who in a positive in a clockwise direction then the net force will be in the upward direction.

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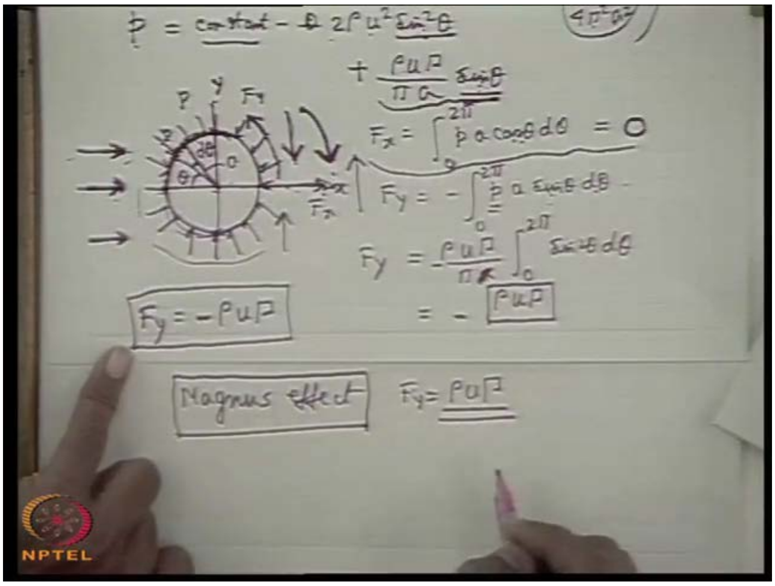

So, this phenomenon is known as Magnus effect. Magnus effect that means if we add a circulation in a cylinder flow past a cylinder, parallel flow past a cylinder then a transverse force is observed that means the cylinder is deflected transversely because of this transverse force is observed. That is the force in the perpendicular direction to the direction of flow which is known as the Magnus effect.

And the deduction of this force per unit length is rho u gamma and it has been shown that this force is equal to rho u gamma. So, I am not taking the sin here it depends upon the relative direction of the rectilinear flow or parallel flow and the circulation or the vortex motion. So, the magnitude is rho u gamma per unit length it can be proved and it was proved that for anybody anybody he must in an ideal flow experience a transverse force per unit length rho u gamma, if a circulation is added along with a parallel flow past this body.

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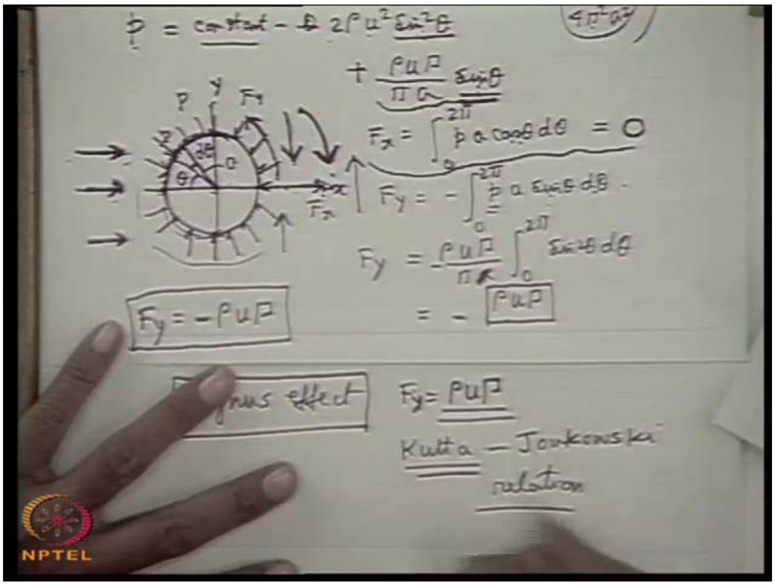

And it was first deducted by two scientist Kutta German scientist and Joukowski and that is why this is known as Kutta Joukowski, Kutta Joukowski relation this is known as.

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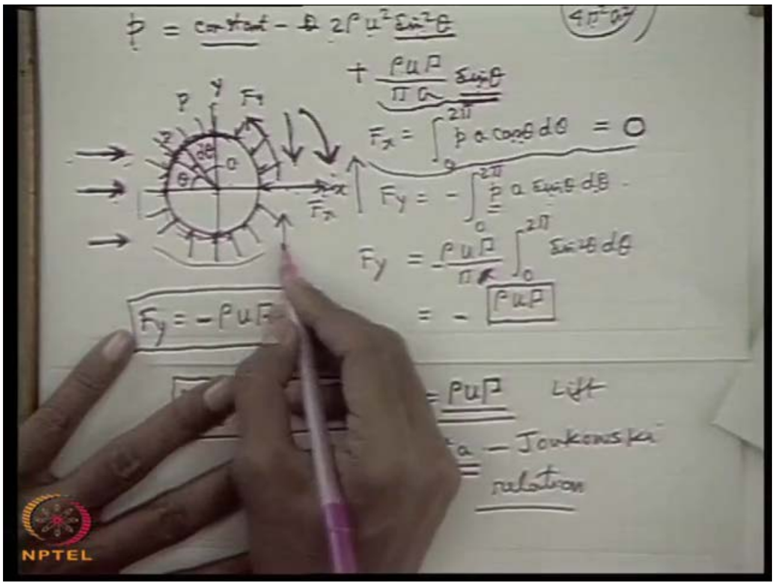

And this force is known as lift force, if it is in the upward direction. So therefore, you see that depending upon the direction of the vortex motion there is a direction of the transverse motion is created depends.

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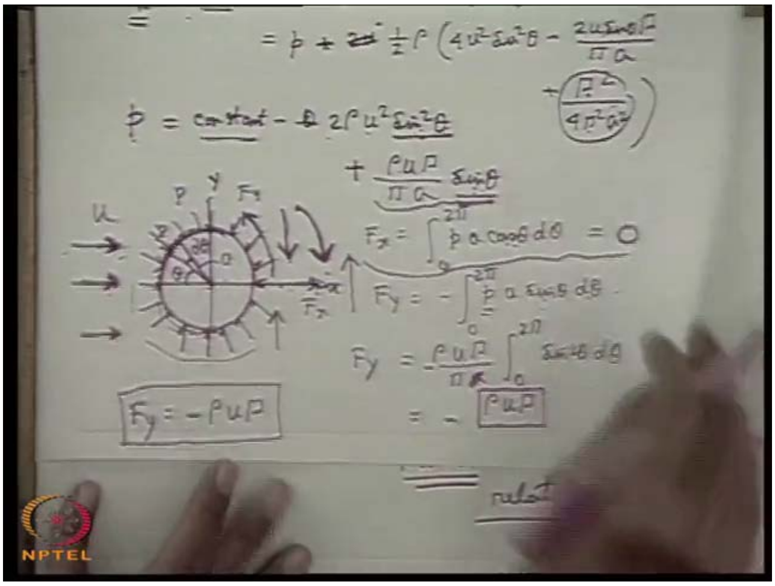

So, the net result is that if there is a circulation, how this circulation is added? Now, I tell you that this circulation is added if we have seen that if we combine rectilinear flow with the doublet we get the parallel flow past a cylinder.

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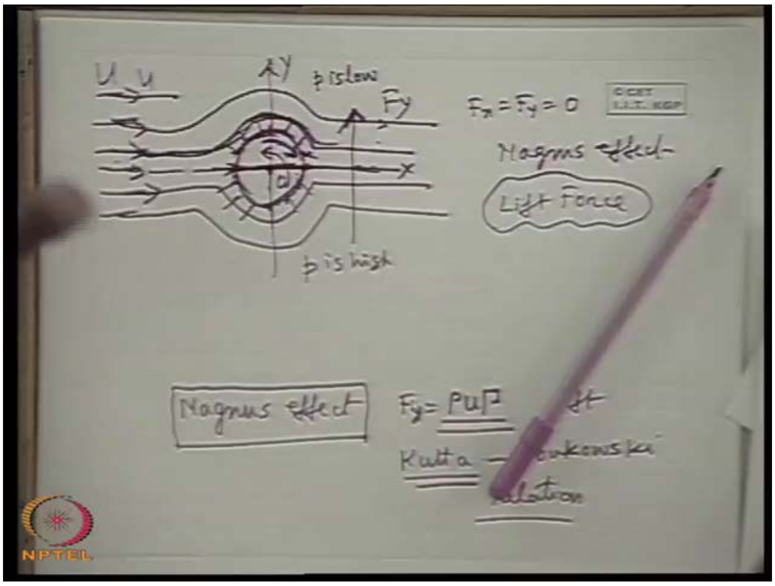

Let me show this thing that parallel flow past a cylinder that means a simply parallel flow past a cylinder which is the combination of rectilinear flow and the doublet. That means if a parallel flow u which is approaching from a far stream undisturbed condition this is u, if it is deflected by a cylinder which is placed at the axis then this stream line is like that, this is the static cylinder. It is the combination of rectilinear flow and the doublet.

In that case flow is symmetrical about both this axis x and y. That means if we integrate, there is no force both the velocity and pressure field is symmetrical about both o y and o x axis that means F x is equal to F y is equal to both of them vanish identically. That means there is no net force in the x direction, there is no net force in y direction. That means the pressure distribution is throughout symmetric that means we get a spherical symmetric pressure distribution you understand.

But if you superimpose an irrotational vortex which means physically that if you give a rotation of this cylinder depending upon the direction of the rotation, if you give a clockwise rotation, then we will see a net upward force is acting on the cylinder in y direction and what happens is that if we give a clockwise rotation then what will happen the velocity field will be asymmetric with respect to this axis while it becomes symmetric with respect to y axis.

Let us consider a clockwise rotation, if you give a clockwise rotation to this type of rectilinear flow from left to right what happens? This increases the velocity in the lower half while this decreases the velocity in the upper half. So, this decrease sorry I am sorry if you give a clockwise rotation what happens the velocity in the lower half is decreased while the velocity in the upper half is increased.

So, if the velocity in the upper half is increased that means the fluid is accelerated in the upper half. So, the pressure is lowered. So, pressure is low on the upper half whereas, when the fluid is decreased you see depending upon the direction of the rotation the fluid velocity is decreased in the lower half so pressure is high according to Bernoulli’s theory. As a result there is a net pressure force in the upward direction.

So, imparting the rotation to the cylinder implies in some half upper half the fluid is for example, in this direction fluid velocity is accelerated in other half fluid velocity is decelerated. If we give a rotation in the opposite direction which we did first in that case what happened? It decelerated the fluid flow that means decreases the velocity in the upper half and increases the velocity in the lower half. Depending upon that the pressure will be decreased in one side and increased in other side and will create a net transverse force in the direction perpendicular to the direction of flow which is known as Magnus effect.

And if the direction of rotation and the rectilinear flow such that this transverse force in the upward direction that means in the direction opposite to the weight of the gravity, gravitational force on this cylinder then this force is known as the lift force. And this is the o a the lift force is generated. In fact, in practical cases the rotation is not given rather the shape of the body is made in such a way that when the fluid flows past it, it automatically creates a circulation and made a, makes a change in the velocity field in the upper and lower half.

Because of which the pressure field changes and a net force in the transverse direction upward is generated which is known as the lift force and which balances its weight or lifts this body in air and that is precisely now I can tell you the principle used in designing the shape of the aircraft wing, which is known as the aerofoil section. The wing of the aircraft or the aerofoil section is generated in such a way that when the flow pass this section the velocity in the upper half is accelerated that means pressure is is accelerated the pressure is reduced and the velocity in the lower half is increased, where the pressure is sorry velocity is reduced where the pressure is increased.

So, a net positive upward force is generated which is known as the lift force. Then alright this is the end of the chapter on or the discussion on flow of ideal fluids. So, today actually we will be starting a new section or chapter that is flow of ideal fluids or flows of sorry not flow of ideal fluids flows of, flows with a free surface that is the flow of fluids or flows with a free surface.

Now, so far we have seen that the flow is taking place in an confined passage where the boundaries of the flows are solid that means flows are bounded from all directions by solid boundaries. But it is not always necessary that in all classes of flow the flow will be bounded by solid boundaries. There are classes of flow where one of the boundaries are the free surface are the free surface, one of the boundaries is the free surface, the free surface of the liquid. This happens in case of flow of liquids. There are instances where the flow of liquids takes place with a free surface as one of the boundaries, other boundaries are the solid boundaries. These are the flows, they are known as flows in open channels flows in open channels, one of the examples is the flow in open channels where the top surface is the free surface.

Now, in difference between this class of flow and the flows in confined duct where the flow boundaries are all bounded by the solid surface is that in a close duct the flow takes place because of the energy gradient and mainly it is a p geometric pressure drop which is responsible for a flow, for the flow a real fluids you get a p geometric pressure drop. It is the difference in static pressure plus the elevation head which makes the flow responsible, but in case of flow with a free surface we see that it is the difference in the elevation part that means the weight of the body rather than the static pressure which makes the flow possible. If there is a free surface in the flow for example, flow take place through a channel where there is a free surface is maintained.

Then from one section at upstream to another section at the downstream static pressure remains constant. This is because of the fact that the upper surface is exposed to atmosphere, the pressure is atmospheric pressure. So, if we find out the differences in p geometric pressure it is mainly due to the differences in the elevation head and it is due to the differences in the weight of the element, weight of the fluid element. So, flow is responsible by the weight of the fluid element in this case.

This flows are a flows with a free surface and an example of this flow is the flow through an open channel. The practical examples are a open channel is there in artificial canals in irrigation details. So, you can have this type of examples so first of all we will discuss the different terminologies for flows with free surface, then we will go to the basic flows and their deductions. Let us see the certain terminologies. Now, what are the terminologies in flow through open channels?

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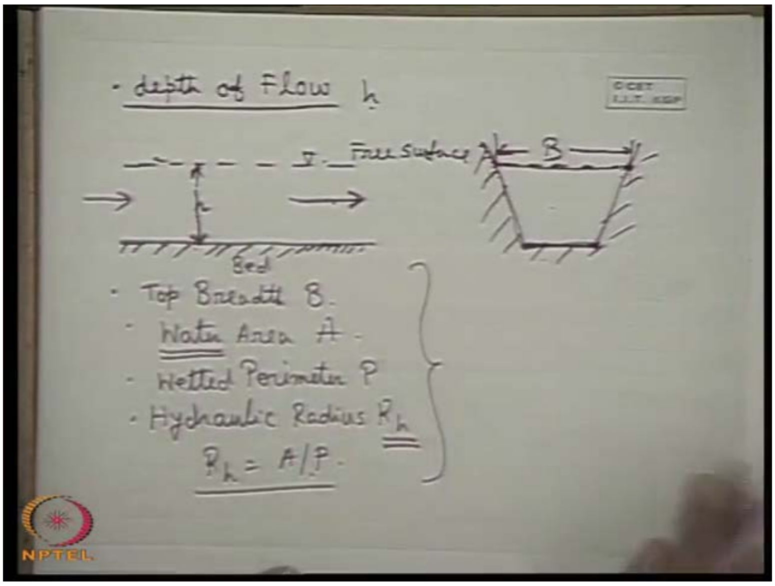

One is the depth of flow one is depth of flow, depth of flow h. How do you define the depth of flow, first terminology. Now, depth of flow is the, fine lets us consider the flow through a channel, channel flow. So, depth of flow is the like that this is the channel bed, this is the free surface. So, depth of flow is defined as the vertical distances of the bed of the channel, this is the bed of the channel from the free surface, this is the free surface of the liquid. So, this is the free surface flowing over a channel, let channel if you look from this direction you get that, let the channel cross section is that, that prismatic cross section. So, this is the free surface, this is the channel.

Therefore, this is the bed so it is the depth at any section, depth of flow is defined at any section, it is the vertical height of the bed or vertical distance of the bed from the free surface or the vertical height of the free surface from the bed, this is known as depth of flow. Another terminology is very important, it is known as top breadth top breadth. Top breadth is the breadth or width that is B at the free surface. That means this is the cross section if this be the cross section of the channel so this is the top breadth, that is the breadth or width at the free surface is known as top breadth. Another is water area. The word water is used because the liquid in practices water always so therefore, this is the cross sectional area perpendicular to the direction of flow. That means this area, this may or may not remain constant in the direction of the flow, depending upon the design, geometrical design of the channel.

So this therefore, water area is the area perpendicular to the direction of flow, well another is this weighted perimeter weighted perimeter. What is weighted perimeter? It is the solid is the perimeter of the solid which is in contact with the water or the liquid. So, perimeter of this solid which is in contact with the liquid is the weighted perimeter P, this is A, and hydraulic radius here in the concept, hydraulic radius in this context is defined R h as R h is equal to A by P.

So, we see the terminologies which are defined and we will refer to frequently afterward. One is the depth of flow that is the vertical distance of the bed of the channel from the free surface, another is top breadth that is the breadth at the free surface in the channel, water area is the cross sectional area perpendicular to the direction of the flow. So, this is the water area, so this is the cross sectional area of perpendicular to the direction of flow, weighted perimeter is the perimeter of the solid which is in contact with the water or the liquid.

And hydraulic radius R h is A by P that means the ratio of the water area to the weighted perimeter. So, these are the terminologies defined, in free surface flow, flows with free surface.

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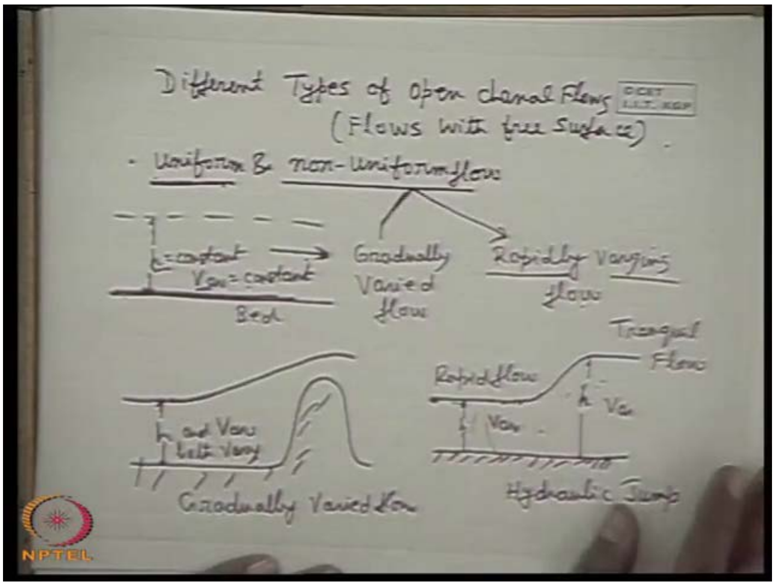

Now, we will discuss different types of free surface flow or different types of open channel flow rather you write open channel. Open channel flows or flows with free surface or flows with free surface or flows with free surface.

Different types, there are different types of flow. First definition is that uniform and non uniform flow, we are I am clubbing this different types depending upon the same category uniform and non uniform non uniform flow. Now, uniform and non uniform flow, uniform flow is defined where the average velocity and depth of flow remains constant in the direction of flow. That means if we write here, if you draw here that means if the depth of flow h remains constant throughout, that means in this case an obviously the average velocity will be constant sorry if the depth of flow remains constant in the direction of flow average velocity also will be constant.

This happens if the bed of the channel and the free surface runs parallel to each other which gives a constant depth of flow in the direction of flow along with a constant average velocity of flow, this flow means uniform. Non uniform flow is a flow where both depth of flow and accordingly the average velocity of flow varies in the direction of flow. Depending upon the degree of variation is non uniform flow is categorized in two types, one is gradually varied flow gradually varied flow where the variation in depth of flow, and average velocity of the flow is gradual not rapid or not abrupt and another is the rapidly varying flow. This are the again the terminologies, rapidly varying flow where the depth of flow or the average velocity of flow varies at any section very rapidly.

So, that is rapidly varying flow. So, gradually varying flow is like that the depth this happens because of some controls structure in a channel for example, flow over a speed lower in a dam so what happens the water level is like this h it gradually varies like that because of this speed lower. So, here h and V average both vary both vary, but vary gradually so this is the depth, this is going to vary. So, depth of the flow is varying and accordingly the average velocity V average. So, this is known as gradually, this is an example of gradually varied flow.

Under some situations of flow with a free surface, we see that there is a variation, let this is a channel there is a variation from a lower depth of flow to a higher depth of flow, certain variation. So, the depth of flow varies very rapidly so that the flow becomes the flow velocity V average and the depth of flow h varies very rapidly, from a very lower depth of flow it suddenly jumps to a very higher depth of flow and V average also changes very rapidly. This situation is known as hydraulic jump which we will discuss afterwards due to certain changes in the geometry of the channel the hydraulic jump takes place this happens when the flow changes from one kind to other kind. This flow is known as rapid flow in the upstream direction that we will discuss afterwards and the downstream flow is known as tranquil flow.

That means in a transition from a rapid flow to a tranquil flow certain situations, certain kinds of flow which happens because of certain changes in the geometry of the channel, then the depth of flow and the average velocity varies very sharp or abruptly. So, therefore, depending on that therefore, this type of flow is known as rapid varying flow. Both this flows are non uniform gradually varied flow a non uniformity is gradual and the rapidly varying flow at the non uniformity, the non uniformity is very rapid.

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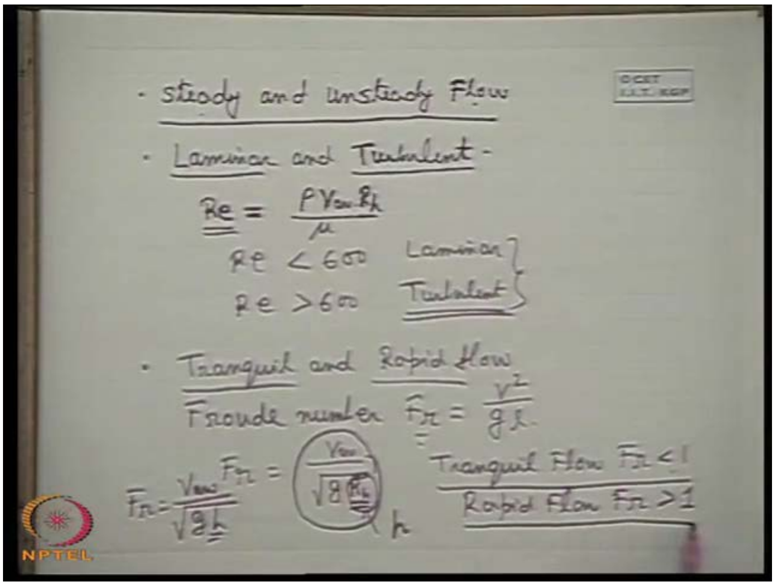

Now, the next demarcation is that steady and unsteady flow. Steady and unsteady flow we will not go for any further discussion already we have discussed that steady flow is a flow where all flow parameters are invariant with time that means flow velocity or any hydro dynamic parameter, depth of flow or anything, parameter at any point is invariant with time while it vary when it varies with time the flow is termed as unsteady flow. So, variation of flow parameters with time at any point is term as unsteady flow whether and when it is invariant with time this is steady flow. Another is the laminar flow and turbulent flow.

So, laminar flow and turbulent flow we will discuss afterwards. So, beyond a certain Reynolds number for any kind of flow, flow changes from a laminar to turbulent flow.

Turbulent flow is a flow situations where velocity varies random with time and this random fluctuating velocities occur in three directions which is the main characteristic feature of the turbulent flows for which an extra momentum exchange in the fluid takes place and which increases the shear stress at the wall. So, the flow changes from laminar to turbulent at depending upon certain Reynolds number. So, Reynolds number is the criteria for any flow we will discuss it afterwards when we will talk about turbulent flows. Now, Reynolds number is the criteria for the change of flow from laminar to turbulent in any class of flow.

So therefore, this Reynolds number should be well defined for that class of flow. In case of a channel flow Reynolds number is defined as the product of, we have come across this Reynolds number earlier in the principle of similarity and many other problems like problems in pipe flow that rho into average velocity, the direction of the flow rho V by mu and the this characteristic dimension is the hydraulic radius. So, the definition of Reynolds number in general is the rho times some characteristic velocity times the characteristic geometrical dimension by the viscosity of the working fluid. So, here the characteristic velocity is the average velocity in the direction of the flow and the characteristic geometrical dimension is the hydraulic radius.

So, by this way if we define the Reynolds number then the criteria for laminar flow that when Reynolds number is less than 600, the flow in an open channel remains laminar. When the Reynolds number is greater than 600 the flow becomes turbulent and all the characteristics for the turbulent flow is present which we will now discuss. We will now, which we will not discuss now. We will discuss in the context of turbulent flow. Now, another type, another differentiation is the tranquil and rapid flow. Now, this disk difference is that the differentiation between tranquil and rapid flow is that under certain situation of flow if you make an disturbance in the downstream thus disturbance can move to the upstream and can change the upstream condition and the flow situation is changed.

This one situation of flow that takes in one situation of flow the any disturbance in the upstream, in the downstream, if you create any disturbance in the downstream that signal reaches the upstream and changes the condition there and changes the entire flow situation, flow condition. This type of flow is known as tranquil flow and under certain situations of flow any disturbance in the downstream does not reach to upstream condition, that signal does not reach to upstream and the flow conditions at the up streams and are not altered.

So, depending upon these criteria whether a disturbance and the downstream, disturbance created at the downstream reaches the upstream or not the flow is divided into two categories, one is the tranquil flow another is the rapid flow. The flow is tranquil flow when the disturbance created at the downstream reaches the upstream and changes the upstream condition and rapid flow is a flow where the disturbance created at the downstream cannot reach the upstream and cannot change therefore, the upstream condition.

So this therefore, whether the flow will be tranquil or rapid depending upon simple condition that the relative magnitude of the flow velocity and the velocity with which the disturbance created at the downstream moves towards the upstream that means the velocity of propagation of the wave which is moving as disturbance as the signal of the disturbance in the upstream and if the velocity of the flow is more than that the velocity so this signal will not reach in the velocity of flow is less than the velocity of propagation of the disturbance wave then the disturbance will reach the upstream.

Therefore, it depends upon the magnitude of the flow velocity and also the magnitude of the velocity of propagation of the wave that is generated due to the disturbance at the downstream. And this depends largely on the gravity, local gravity therefore, local gravity and the flow velocity decides whether the flow will be tranquil or rapid and this characterized by a dimension less term known as Froude number as you have already seen earlier in the principle of similarity Froude number is the ratio of the inertia force to the gravity force.

Now, if you write the Froude number, if you write the Froude number well Froude number F r which is defined earlier we defined as V square by g some characteristics length l. Here, the Froude number for free surface flow is defined just the square root of this V by root over this is the average V average velocity by root over g into R h. R h is the hydraulic radius. Therefore, depending upon the value of this Froude number, we define whether the flow is tranquil and rapid. For tranquil flow for tranquil flow for tranquil flow Froude number is less than 1, for rapid flow Froude number is greater than 1.

So, tranquil flow Froude number is less than 1 and for rapid flow, for this R h sometimes we substitute it by the depth of the flow that means Froude number is defined as V average by root over g h simply the depth of the flow. So, tranquil will flow F r less than 1 rapid flow F r Froude number greater than 1.

So, we see the different types of flow that uniform and non uniform flow. Non uniform flow are two types gradually varied and rapidly varying flow depending upon whether the variations of h depth and average velocity is gradual or rapid, then steady and unsteady flow, laminar and turbulent flow, tranquil and rapid flow.

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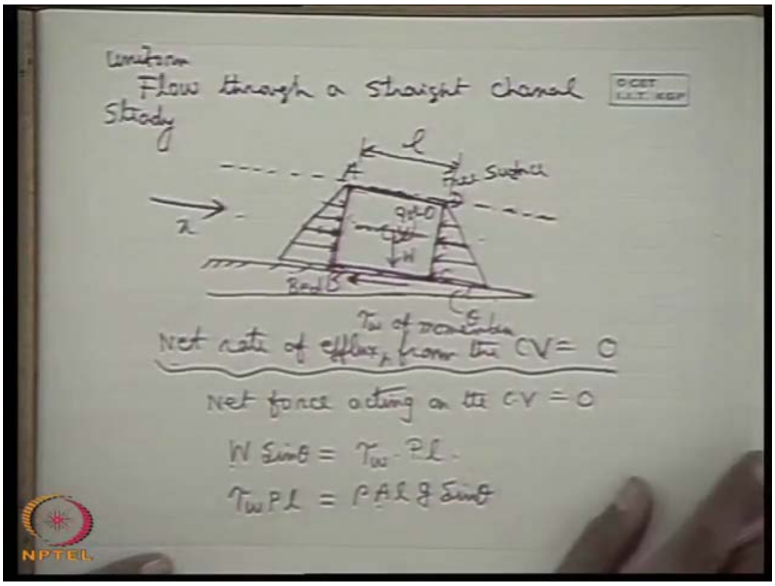

Now, I come to a derivation for flow through a uniform flow rather I must write uniform word, uniform flow through a straight channel through a straight channel, uniform flow through a channel. Let us consider a straight channel like this and let us consider uniform flow that means the free surface is parallel to the, this is the free surface parallel to the bed of the channel, this is the bed of the channel which makes with the horizontal, this is the horizontal line and angle theta that means an inclined with horizontal and this is the free surface and the flow takes place in the, uniform flow takes place.

What is the expression for the flow velocity and the rate of discharge? To do that we take a control volume, a simple control volume like this. Let the control volume like this we take A B C D. Now, what are the forces acting on the control volume? There is no force acting on the free surface of the control volume. On this surface of the control volume adjacent to this bed, solid bed of the channel the force acting is the shear force tau that is denoted by the shear stress, shear stress into the area that is the shear stress acting opposite direction to the flow.

At this surface the pressure force is acting, at this surface pressure force is. Now, in case of a flow in a channel that means flow with a free surface the pressure force over the area is the hydrostatic force. So, it is like this, the variation of the force is like this as you know the hydrostatic force is linear with the depth, so this is the hydrostatic force acting. Similarly, on this surface also the similar way the hydrostatic force is acting.

And there is no change in the static force that means what I told you earlier also at any height h from the bed the static force at this point and static force at this point remains constant because this is atmospheric pressure plus this height which is remaining constant. Therefore, in case of an uniform flow of this type the pressure force is acting on this surface A B and C D balance each other.

Now, since the force is steady we can tell that if you consider a fluid element as a system that is the net force acting on this fluid element is 0 or from the momentum principle we can see that if we define it with respect to a control volume that is the net rate of momentum reflects from the control volume because under steady state no momentum is being generated or is being accumulated in the control volume or is being depleted in the control volume.

So, the net rate of a flux of momentum in the control volume must equal to the net force acting on the control volume and for a uniform flow the net uniform flow or the steady flow, the net rate of a flux steady uniform flow this is also steady uniform flow, the net rate of a flux from the control volume is 0. Therefore, net rate of a flux net rate of a flux if you control volume from the control volume is 0 for a steady uniform flow because for steady flow there is no rate of change of control volume within the flux. Only the net of change of momentum within the control volume and for uniform flow net rate of a flux from the a flux net rate of a flux of momentum sorry of momentum x direction momentum let we take this as x direction from the control volume is 0 for uniform flow. From a system approach we can tell that the rate of change of momentum of this system is 0 which ultimately gives the same thing that net force acting, that is the net force acting net force acting on the control volume or the system if you take a system approach is 0.

So, what is the net force acting on the control volume? If we take the net force acting on the control volume we will see that net force acting is the W weight and the shear stress with the shear force at this plane. So, if you make the force analysis in the direction of this, now, if we make this, now, we will see that this will be this angle is 90 degree minus theta because the angle with the horizontal is theta. So, this is a parallel line to the bed the direction of flow with the vertical will be 90 degree minus theta.

So, simply we can write W sin theta is equal to the frictional force that means tau W into the area will be that if we denote this length of the control volume in the direction of flow as l it will be P into l. Well, so now W is the mass times g. So, we can write therefore, tau w P into l is equal to W is the mass. Mass is density into A into l, A is the cross sectional area of the flow. So, A into l this is the mass of the control volume into g into sin theta.

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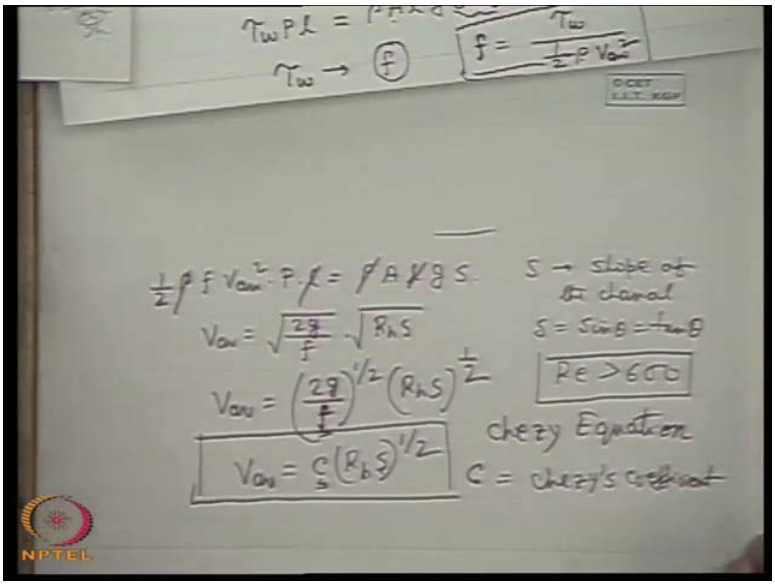

Now, we see that we now substitute this tau w in terms of a friction coefficient, in terms of a friction coefficient. We know that we defined the friction coefficient in a flow as we discussed in pipe flow problem as the general definition tau w the valve shear stress divided by the dynamic head and in case of this flow as similar to that of flow through a closed contour or duct the dynamic head is based on the average flow velocity.

So therefore, with this definition of friction coefficient I can replace tau w as f into half rho V average square. So, tau w if I write half rho f into V average square into P into l here we can write rho A l g and this sin theta is replaced by S, where S is known as the slope of the channel slope of the channel. S is sin theta and for small angle it can be equated to tan theta also.

Then what we see rho rho cancels, l l cancels. So, simply we can write V average the flow velocity is root over 2 g by f into now A by P is R h, that is the hydraulic radius we have already defined R h into S root over R h into S or simply we can write in a different fashion 2 g by f whole to the power half into R h S whole to the power half.

Now, usually all flows through the channel is turbulent flow that means Reynolds number is greater than 600 and in turbulent flow the friction factor as we have discussed earlier is usually independent of Reynolds number or the flow velocity. Therefore, friction factor works as a constant parameter g is constant so that this is R h S whole to the power half.

So, this equation gives is well known this a very popular equation that average velocity of flow through the channel that means this average velocity of flow through this channel is given by a constant times R h the hydraulic radius of the channel into the slope of the channel to the power half. This is known as well known Chezy equation depend on the after the name of the person who first derived this Chezy equation.

And this C is known as Chezy’s coefficient. This C is known as Chezy’s coefficient, C is known as this constant, C is known as Chezy’s coefficient. Now therefore, we see the Chezy’s coefficient depends upon the friction factor. So, we will discuss the Chezy’s coefficient in the next class.

Thank you.

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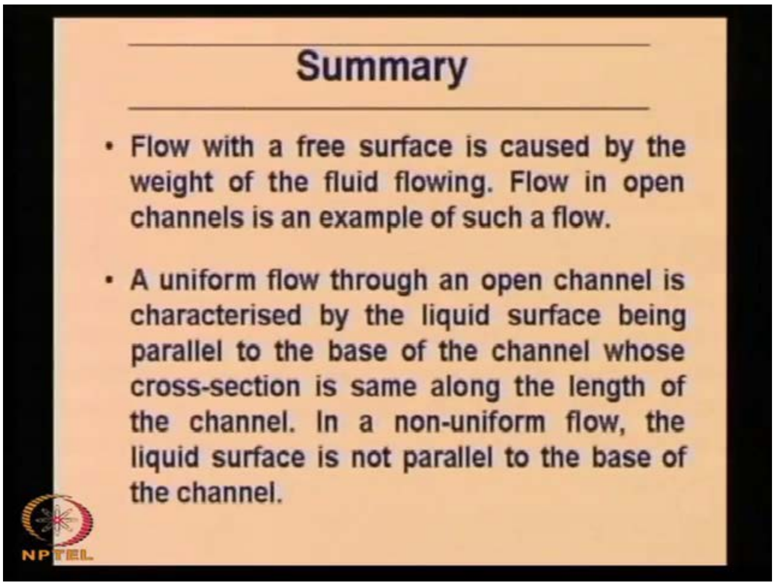

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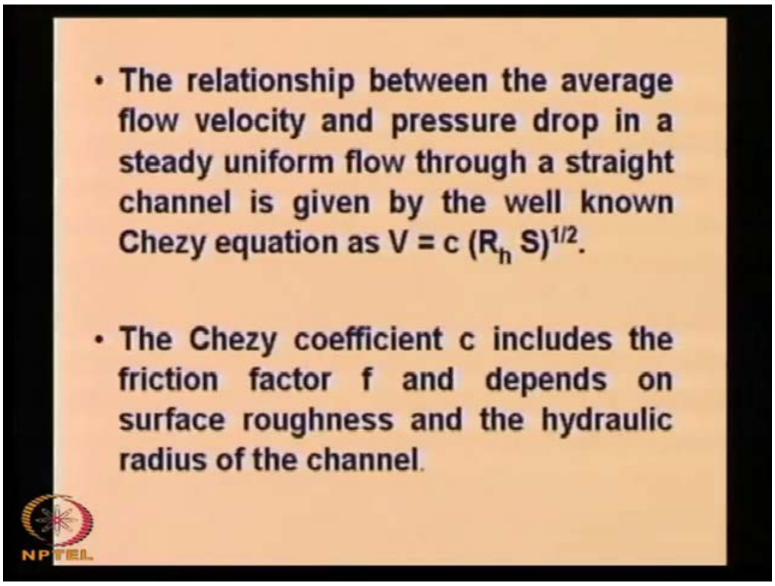

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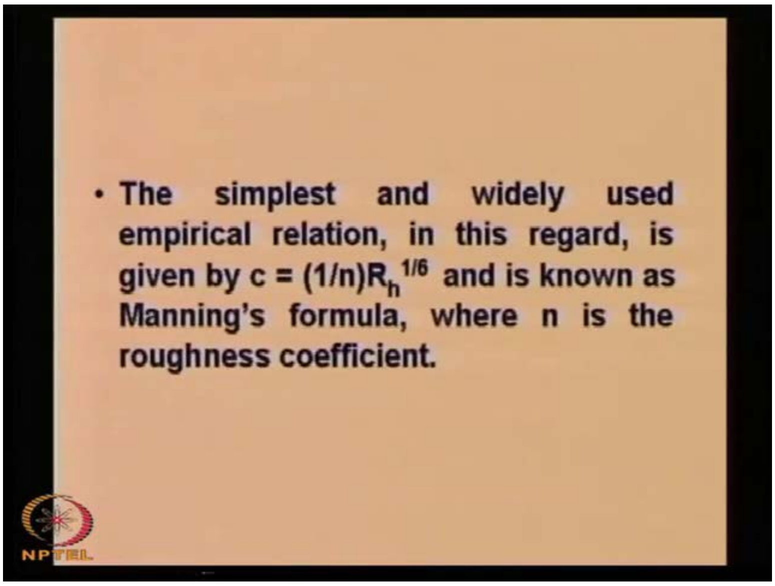

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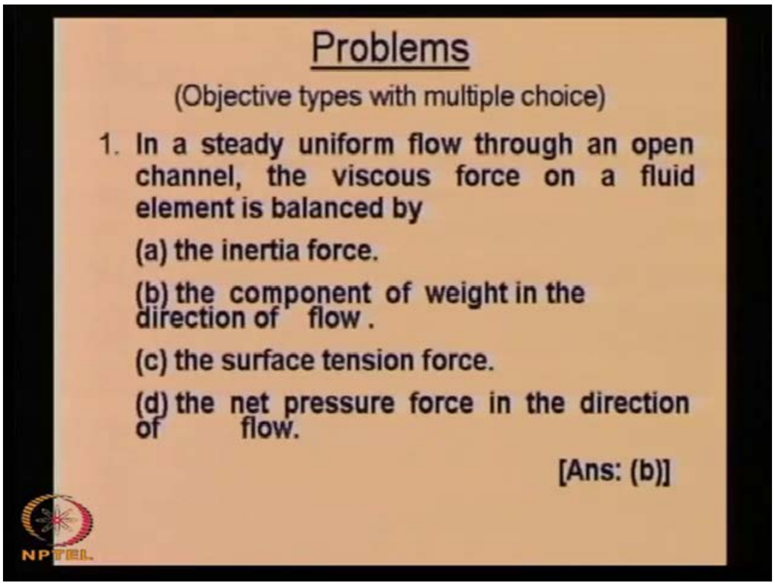

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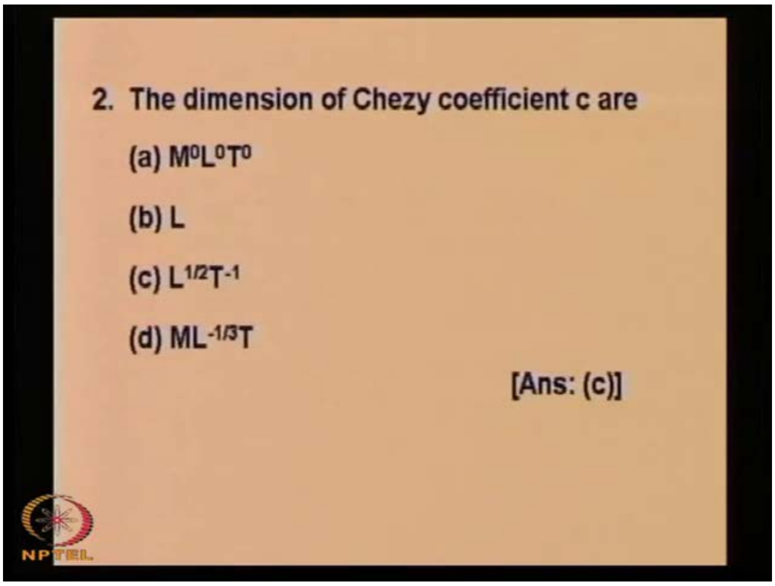

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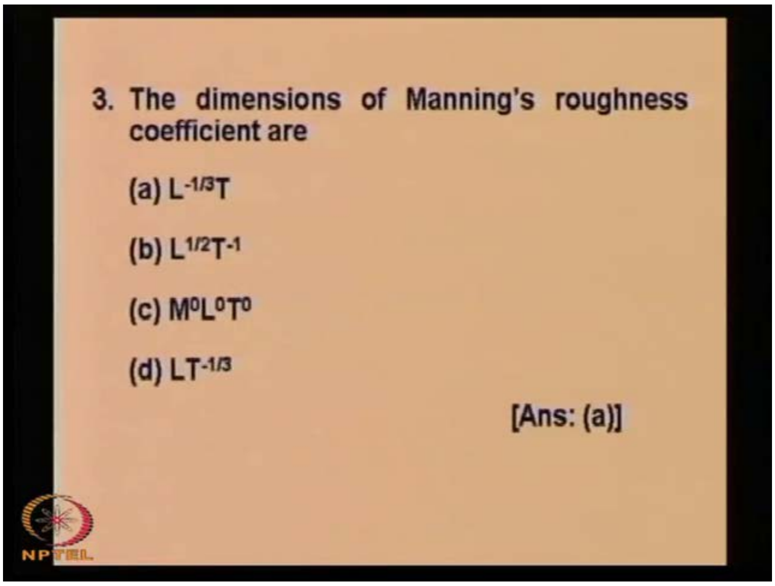

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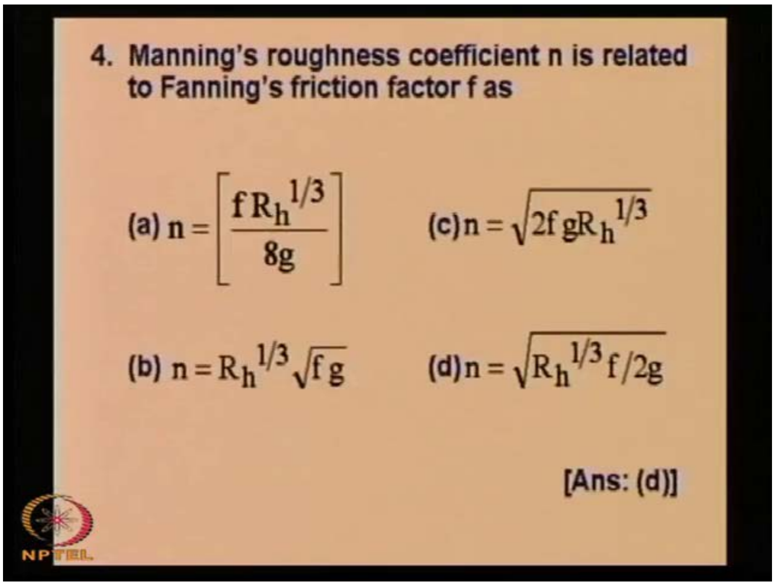

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