Fluid Mechanics
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Lecture - 40
Flow of Ideal Fluids Part – II

Good morning, I welcome you all to this session of fluid mechanics. Today, we will be continuing the discussion on ideal fluids; it is ideal fluids part 2. Now last class if you recall, we discussed about the stream functions, and the velocity potential functions and the stream lines and the velocity potential lines, equipotential lines for source and sink flow.

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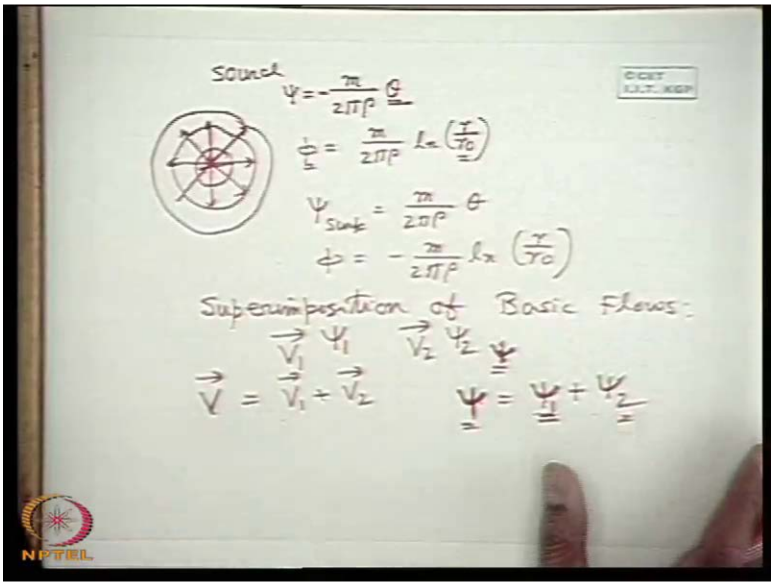

If you recall that; now we can see here, that is source flow is a flow which in a two dimensional plan can be thought of as a line source, which is seen in done as a point where from the fluid is coming out uniformly in all radial location. This is a source and sink is the reverse of these at the fluid is approaching towards a point uniformly from all radial directions. So, in this kind of flow we derived earlier the stream function was given by m; where m is the strength of the source, which is the mass flow rate. Total mass flow rate coming out from the source, by twice pi rho into theta with a minus sin and the velocity potential function was recognized as m by twice pi rho, l n, r by r 0, so where this r 0.

So, therefore, first of all let us see that the constant silence that is the stream lines, is psi is equal to constant where theta is constant; that means, the radial lines. So, radial lines are the stream line, similarly equipotential lines are the lines were r is constant if you make phi is equal to constant. So, these are concentric circles. And r 0 implies a radius where, we are considered and arbitrary value of 0 for the phi function. Now the for a sink, this is for a source. So for a sink, the stream function for sink is just with a negative sin because source and sink are physically same; only with a negative sin similarly phi for a sink will be that velocity potential function, for a sink will be like this.

Now today we will see the superimposition of certain basic flows. Now superimposition may, let me write superimposition of basic flows, superimposition of basic flows. Now one thing you must know that when two flow fields are superimposed, if the velocity vector of one flow fields this v 1 and the velocity vector one another flow field this V 2; that means, V 1 and V 2 are the velocity field of the two flows which are who, which are superimposed it one another, when the velocity vector the result and velocity of the flow field will be the vector radiation of this two velocity vectors.

Since psi is a linear function of the velocity vector, then if psi 1 represents this stream function for one flow and psi 2 for another flow. Then the resulting flow will have a psi function or a stream function of this stream function of the resulting flow will be the some of the stream function will have two flows. This is because stream function is a first order derivative of, not first order other I will tell first power; we are say first power relationship with the velocities, because it is simply the derivative of the velocities, velocity gradient. So, therefore, the basic conclusion is that if psi 1 is this stream function for one flow and psi 2 is this stream function for other flow; if this two flows are superimpose the result and flow psi function can be obtain by the addition of the psi function of this two.

Now with this method of, with this basic we can now, see first the combination of source and sink, source and sink, combination of source and sink. Now let us consider a flow where a sink source and sink located like that, let A is the source and B is the sink, A is the source B is the sink; now if we consider a point P in the flow field which is generated by the combination of source and sink, now we can write like this that if we just join this two points, let this is theta 1; theta 2; in a polar coordinate system we can write that for source this psi, that is psi 1, that is psi A is minus m by 2 phi rho theta 1.

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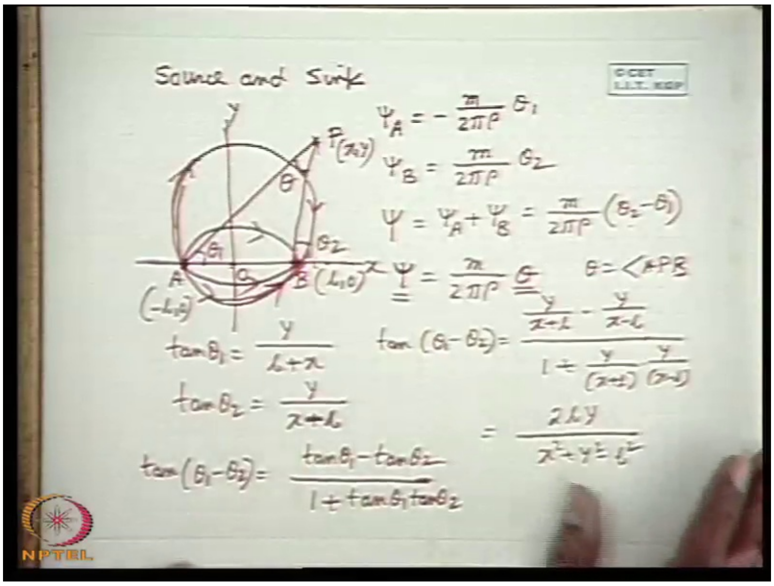

Now for the sink at B, psi B is equal to negative of that. So, therefore, it will be twice pi rho theta 2. So, therefore, we see the psi function for the resulting flow, that is the flow which resulting due to the combination of the source and sink will be psi A plus psi B; which can be written as m by 2 pi rho into theta 2 minus theta 1. So, this theta 2 minus theta 1 from geometry is this end. So, therefore, we see psi is equal to theta were theta is the angle A P B, where theta is angle A B P.

Now here we see one interesting thing that the constants psi function line; that means, stream lines are those lines along with theta is constant; because if we put psi is equal constant which will give theta is constant, which means these will be the circular arcs; that means, the stream lines will be the circular arcs with A B as the base cord. So, that this angle will always remind constant; that means, this will be the, this will be the for example, this is one circle which called is A B, this is source, this is sink. So, this will be the stream lines. So, other circular arcs can be thought up this stream lines like this. Now we can develop the expression for stream function in a Cartesian coordinate system; let us consider this x let us consider the y axis, let the co ordinate of any point is (x y).

That Therefore, we can write tan, this theta 1 and theta 2 is there. Now we can write tan theta 1 is what y, the y coordinate of the point divided by b plus x, b plus x where we take distance thus B; that means, the coordinate of A where the source is kept is minus b

0 and the coordinate of B on the positive x axis this where the sink is kept is b 0; which means if this be the origin O A is equal to b, O B is equal to b, so b plus x.

Similarly tan theta 2 is equal to theta 2 is this angle we drop a perpendicular from the point P on the axis, we will get y is e into x plus b I am sorry x minus b, x, this is x minus b; it is x plus b, it is x minus b. So, therefore, we can write 10 theta 1 minus theta 2 which is equal to, you know the formula; for this. Tan theta 1 minus tan theta 2 divided by 1 plus tan theta 1 tan theta 2. So, therefore, we can write tan theta 1 minus theta 2 is equal to if you replace this y x plus b minus y x minus b divided by 1 plus y x plus b into y x minus b. So, if you simplify it, it will come 2 b y divided by x square plus y square minus b square.

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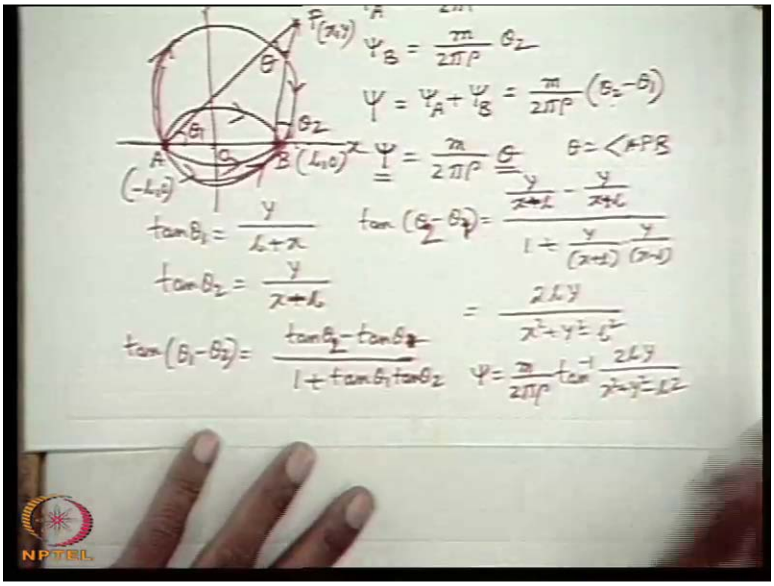

So, therefore, we can write with this that the psi, here we can write therefore, psi is m by 2 pi rho theta 2 minus theta 1. So, well it is theta 2, I am sorry will have to make it theta 2 minus theta 1. So, theta 2 is x plus b. So, theta 2 is x theta 2 is x minus b; it is x plus b. So, it as to be theta 2 minus theta 1, I am sorry theta 2. So, if you see theta 2 minus theta 1 is tan inversely; that means, tan inverse x square plus y square minus b square.

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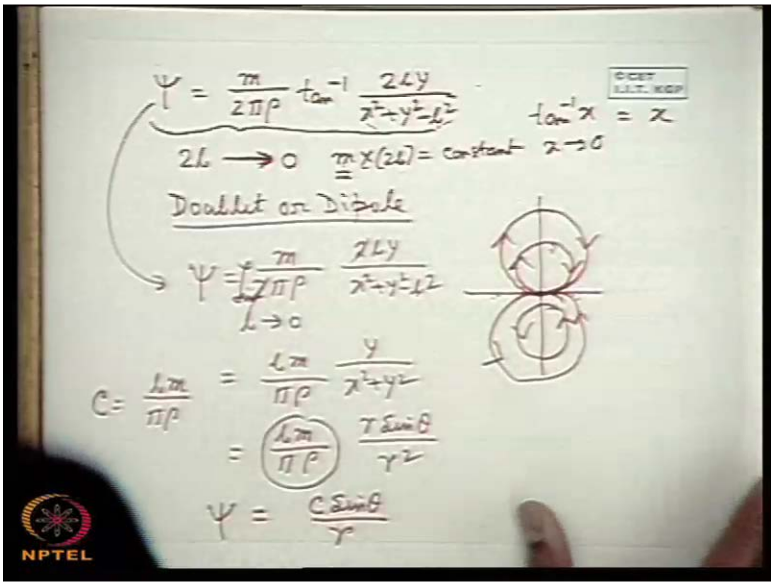

So, therefore, in Cartesian coordinate we get this stream function psi is equal to m by 2 pi rho, tan inverse in terms of the Cartesian coordinates 2 b y x square plus y square minus b square; write 2 b is the distance between the source and the sink, m is the strength of both the sources and the sink. Now we will consider a case, where source and sink are plus it such a way that the distance between them tends to 0 with the condition that, the strength times the distance remains finite or constant; that means, the sources and strength are kept that a distance an ultimately they are approaching towards each other the distance between them tending to 0; with the constraining in equation that the product of this strength and the distance of separation is constant.

So, these type the flow field which is arrived, this known as Doublet or Dipole; the flow field which is arrived by this way, combining source and sink in this way is known as a Dipole. So, what we can do this is the combine stream function for a combination of source and sink; from here we can see that if 2 b tends to 0; that means this tends 0. So, tan inverse of a quantity which is tending to 0 can be written as the quantity itself; which means that tan inverse x, at x tending to 0 is equal to x itself.

So, therefore, we can write under that limiting conditions for a Double or Dipole psi is equal to m by 2 pi rho with this mathematics into 2 b y divided by x square plus y square minus b square. So, limit of this as b tends to 0. Now when b tends 0, but b m is constant. So, that we can write b m by pi rho can selling this, 2 b m by pi rho into y b tending to 0;

that means, y by x square plus y square or simply we can write in terms of Polar coordinate; b m by pi rho into r sin in theta divided by r square. Considering this as a constants C, we defined a constant C as b m by pi rho because b m is constant according to the definition of this type of problem.

So, therefore, we can write C sin theta divided by r, this is the stream function of a Dipole. So, Dipole will be like that when you see this source and sink will approach each other, this angle will become 0 and ultimately this circular arcs with A B as the base cord will become tangent, will become circles with tangent at the x axis at the origin; that means, this will be the nature of the, this will be the nature of the, this will be the nature of this, now this we will be the nature of this Dipole; where Dipole is at the origin. Because they are plus symmetrically have from the original they are tending to that distance of separation tending to 0. This is the Dipole or Doublet stream lines of the Dipole or Doublet constant stream function.

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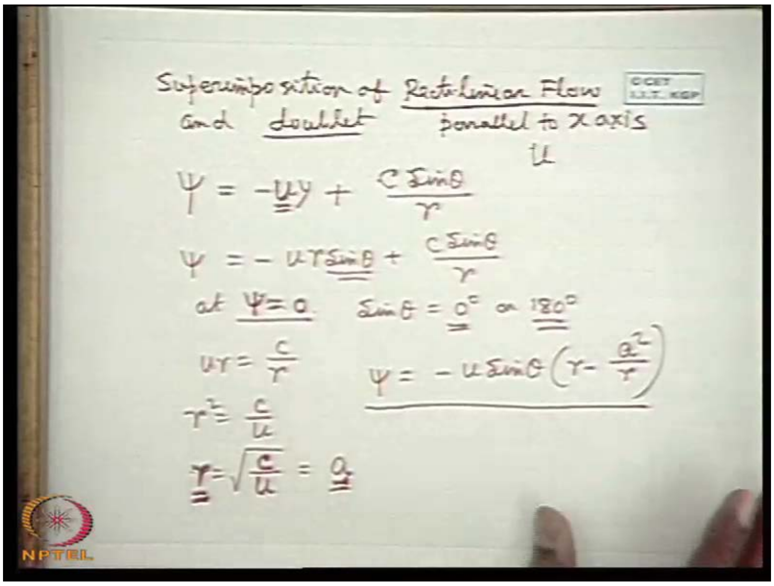

Now we will come to a very interesting phenomena or very interesting situation by superimposition of, superimposition of, superimposition of rectilinear flow, which we have discussed earlier rectilinear flow and doublet, and doublet.

Now, the superimposition of this two flow give this stream function for the result and flow, it is a some of this stream functions square rectilinear flow and that are doublet. Rectilinear flow if you recall thus stream function is given by u y; where u is the velocity of the rectilinear flow, but here rectilinear flow parallel to x axis. So, it is the superimposition of a rectilinear flow parallel to x axis and the doublet; that means, here if we see that there is a rectilinear flow parallel to x axis; if you take this as x this as y this is u. So, stream function for this rectilinear flow u y, we have already discussed.

And this is the stream function for the doublet. So, if you superimpose rectilinear flow with the doublet, then what you get? u y plus C as defined earlier for the doublet sin theta by r. Now if I write y in terms of r and theta in Polar coordinate system. So, it will be r sin theta plus C sin theta by r. This the typical stream function for the combination of rectilinear flow parallel to x axis and the doublet.

Now you see that in this type of stream functions, we get some interesting thing that as psi is equal to 0; that means, if we consider one value of this stream function arbitrary to 0; that means, along a stream line where psi is equal to 0, we get some conclusion; that sin theta is either 0 degree or 180 degree; that means, if sin theta at sin theta is equal to 0 and 180 is the value of stream function is 0. We can ascribe the value of stream function of 0; another solution is that for any theta we can get a value of psi 0, provided u r is equal to c by r or simply r square is c by u or r is equal to root over c by u.

Let this route over c by u be denoted by a; that means, for all values of theta psi is equal to 0 is obtain along points whose radial coordinate are radius vector r is constant, because c is constant, for the doublet and u is the constant because rectilinear flow the velocity u is constant, it is parallel to the x axis. So, this is constant. So, if you replace now mathematically, root over c by u is equal to a then we get psi is equal to minus u sin theta into r minus a square by r; because c is x square u minus u sin theta r minus a square by r. So, this is the stream line which is obtained.

Now, if we write this stream line again, that psi is equal to minus u sin theta into r minus a square by r. This is the stream line which is obtained if a cylinder is placed at the origin; that means, this is the axis x this is the y. In an infinite expands of a parallel flow of an ideal fluid; that means, other way we can tell if in the if a parallel flow in infinite expands of an ideal fluid is cylinder is placed, the where this stream line will be deflected by the cylinder will be given by this similar functions; that means, this psi functions; that means, if we make this is equal to constant the low cos given by this theta and r coordinates will be similar to the case, when an parallel flow parallel to the x axis for an ideal fluid will be defected by a cylinder which is placed at the origin to the axis is at the origin; that means, it is the ideal flow passed a circular cylinder.

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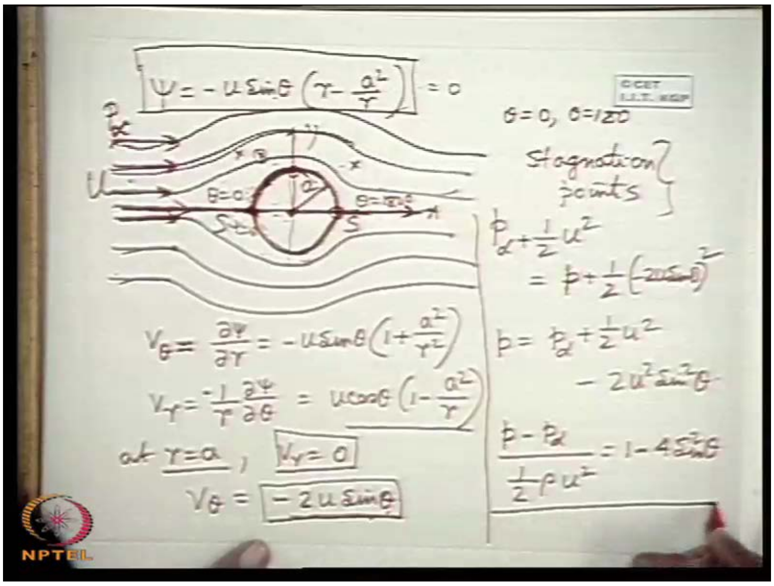

So, therefore, the stream functions or stream lines for an ideal parallel flow passed is cylinder like this can be generated by combination by the combination of doublet and a rectilinear flow. Let us find out two velocity component, one is the tangential velocity component; that means, we define for example, this is the direction. So, with this direction we define this is theta is equal to 0. Now another very interesting thing is that psi is equal to 0 is along the line theta is equal to 0 and this is theta is equal to 180 degree or pi and for all theta this is the surface of the cylinder where radius is constant.

So, a here in this equation will represent the radius of this cylinder. So, this is the stream function for a parallel flow passed parallel flow of ideal flow its passed a circular cylinder whose radius is a. So, therefore, the this line; that means, the fluid particle flow in this line theta is equal to 0 along the cylinder and then going by this line and another is diverted in this way this is corresponds to size equal to 0; these are the lines corresponding other values of sin.

Let us find out V theta and V r at any point here are any point here what is the definition of tangential velocity in terms of the stream function if you recollect this is del psi del r; that means, this becomes is equal to minus u sin theta if you differentiated 1 plus a square by r square. Similarly if we find out the radial velocity which is connected with this stream function as 1 by r del psi del theta and with a negative sin; this is positive, this is negative.

So, this becomes is equal to u cos theta into 1 minus a square by r. Now at this surface of this cylinder r is equal to a. So, at r is equal to a, we clearly see V r is equal to 0; which is obvious. At this solid surface they are cannot be in a radial velocity of the fluid because solid surface is impervious to the fluid flow of fluid until then unless it is porous surface. So, is solid surface they are cannot be any component normal to the surface. This is one of the very important and universal boundary condition, that for an ideal flowing this it flow, they are cannot be even for a real fluid, they are cannot be any normal component of flow velocity on a solid surface until and unless solid surface is porous. So, therefore, this is 0.

Whereas V theta r is equal to a, we get if you put it there we get minus 2 u sin. So, there is a tangential velocity. So, no slip condition will not prevail here because the fluid is ideal there is no viscosity; so fluid slips or flows pass this cylinder in a direction surface of this cylinder in a direction along this surface. So, it is minus 2 u sin theta; now from here one thing is very clear. So, this function is symmetrical, this function is symmetrical about this axis; that means, the velocity over this zone from 0 to 180 is same as 180 to 360, but it is not symmetrical about this axis, if you seeing this the velocity first become 0 here. Theta is equal to 0, then it goes on increasing at theta is equal to pi by 2; because the sin function attend this maximum value they are and then again it is reducing from pi by 2 to 180 and the same continue in this are into negative sin; that means, the numerical value of the velocity is 0 here, and then increases to the maximum again comes to the 0 and same thing is repeated here. So, therefore, velocity field is symmetric about this axis.

Now at this two point theta is equal to 0 and theta is equal to 180 degree, this two point the velocities are 0 and this two points are known as S; stagnation point that you know, we have discussed earlier. Where the velocity there in a particle approaching along this line theta is equal to the straight this surface and is velocity totally destroyed means it is converted only into the pressure energy because there is no other destructions; that means, the kinetic energy cannot be convert into inter molecular energy because fluid is frictionally. So, velocity is converted into pressure and they are the fluid particles at rest.

Similarly, fluid particle is at rest it reaches the maximum velocity, here this two points are known as stagnation, stagnation points. Where the in a flow fields stagnation points are those points where the flow velocities become 0, these are the points at theta is equal to 0 and theta is equal to 180 degree; this is known as forwards stagnation point and this point is known as rayers stagnation point. Now if you look the pressure field, what happens? Now if we describe the pressure at the for upstream or for downstream as the undisturbed pressure known as fresh stream pressure, P infinity; then we can write the Bernoulli's equation at any point in the first stream at any point near the cylinder, where the flow flied is develop due to this cylinder in this fashion according to this stream functions or this velocity functions that the velocity expressions of the velocity.

Then we can write P infinity plus; here the velocity field is only u, plus half u square is equal to pressure; that means, we are interested to find out the pressure, at any point the pressure, distribution equations p plus half velocity square, but now if we take the point, any point that the cylinder surface then what will be the velocity at the cylinder surface? The radial velocity is 0. So, the only velocity is the V theta. So, this is the velocity at the cylinder surface, minus 2 u sin theta whole square. So, we can write p is equal to p star plus half u square minus this is 4 this is 2. So, therefore, this will be becoming minus half twice u this is 4 twice, u square sin square theta. Now, this is the pressure distribution equation.

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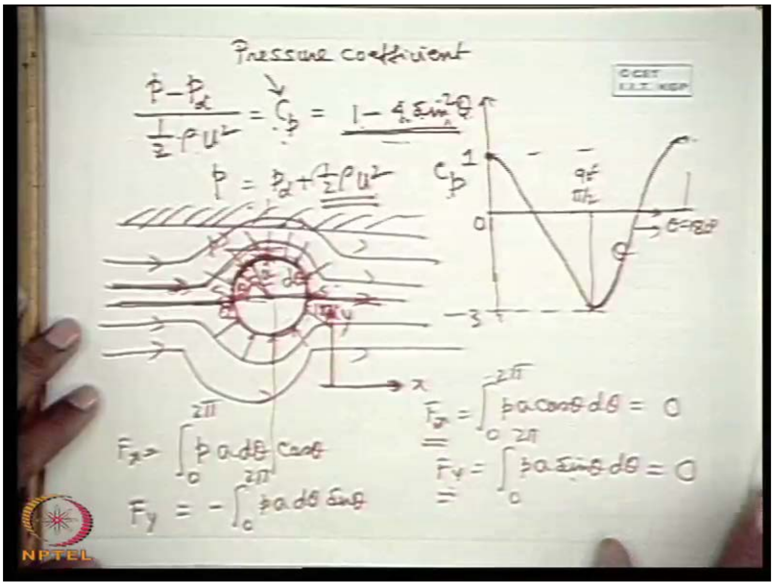

Now we make it in a condom national fashion express p minus p infinity at difference of pressure at the surface from this first stream pressure divided by half rho u square as 1 minus well 4 half rho u square 4 sin square theta. So, this is the pressure distribution equation. So, now, if we write the pressure distribution equation again, we see that the pressure distribution equation becomes p minus p infinity in terms of this dimensionless quantity, this is the dynamic head based on the pristine velocity, this is the difference between the pressure at any point and the pristine pressure; this is known as C p pressure coefficient, this is known as pressure coefficient, coefficient this become 1 minus 4 sin square theta.

Now, if we plot this thing we will get a curve like this, if we plot this thing we see will better plot here we will get a curve like this, now here if we see that punk theta is equal to 0; this is theta, this is 0. The at theta is equal to 0 this is 1; that means, p minus p infinity half; that means, p is p infinity plus; that means, this is the stagnation pressure. Here if you see here the V is 0; that means this velocity is 0. So, pressure here is stagnation pressure; that means, the dynamic head based upward this free stream parallel fluids convert into pressure. So, this is the stagnation pressure, so C p is1 here.

So, then pressure goes on decreasing as the velocity goes on increasing, I had shown you earlier velocity increases over this part. So, therefore, pressure continuously goes on decreasing and you see this function reaches if this is C p in this direction, this is the theta this function reaches a minimum value at theta is equal to pi by 2 which is 1 minus 4 minus 3. So, therefore, we see this reaches a minimum value at pi by. So, this is let minus 3 and this is pi by 2 or 90 degree. Then again it goes up and takes the value of 1 again this is 1 the same value, when theta is 180 degree.

So, this function you see is symmetrical about this axis, about this axis, but the variation takes place it is not symmetrical about this axis. So, we show only the variation from 0 to 180 degree. So, it shows that it is justify draw this thing again, this stream lines are like this, stream lines are like this. So, therefore, so this is the stream lines. So, this is the stream lines, so this is the stream lines. So, this is the axis. So, therefore, you see that this pressure distribution over this line from 0 degree to 180 degree; that means, this two stagnation point, this goes on decreasing. So, this for the fluid is accelerating and pressure is decreasing. So, this is the maximum velocity and minimum pressure point. So, this is C p the pressure coefficient; again this for the fluid is retarding velocity is decreased because we have seen earlier the velocity function; then the fluid pressure is increase then we recover here same pressure. There is no dispersion of pressure to the intermolecular energy because fluid friction is absent.

So, this part is the accelerating flow, this part is the retarding flow. This can be well understood that if we consider a surface here, as certain distances away from the cylinder you will see that for flow in this part this gives a convergent duct. So, curvatures is such as if it is flowing through a convergent duct whereas, flow in this per the curvature is here it is flowing through a divergent duct and the pressure is therefore, increase and velocity is decrease. Now one important thing comes that if we consider the force net force acting on the cylinder. So, it is an ideal fluid. So, cylinder surface the only force acting is the pressure force, this is the pressure force only force acting is the pressure force because there is no force along the tangent to the cylinder at it surface because there is no viscous force. So, this is only force p.

Now if we want to evaluate the force acting on the cylinder along x and y direction, x is the direction that is the direction of the flow the axis flow, axis parallel flow and y is the transverse direction, the direction perpendicular to this parallel flow direction. Then we can write if F x is equal to, if we take a small element here at an angle theta of radius this radius is a, and this subtended angle is d theta; the radius is a. So, we can write the force on this element is p if the pressure is p. So, the a b theta, this is the length of this r. a d theta per unit length of this cylinder in this direction that is in a direction perpendicular to this plain of paper, so per unit length this is the force on this element p a theta. So, if you take its component cos theta, cos theta will be its component, in this x direction because this is the angle theta. So therefore, cos theta if we integrate it over the entire cylinder, you will get similarly F y will be y is taken positive upwards minus 0 to 2 pi p a d theta, sin theta.

That means simply we can write F x is p a cos theta d theta integrated over 0 to 2 pi and F y is integration of p a sin theta d theta, integration of 0 to 2 pi. Now this you see if the pressure in involves a function like this, then if you multiply this is an even function of theta sin theta. So, this is independent of the sin of the sin theta. So, therefore, the pressure variation is symmetric about this axis. Now if we multiply this function with cos theta and sin theta and if you integrate to 0 to 2 pi in both the cases we will get 0.

That means in this case what happens is that, the entire cylinder does not experience any force in x and y direction; which means the pressure distribution is symmetrical over the entire cylinder. So, that it gives neither a, neither a force in x or y direction because here you see the sin p minus p infinity by half rho is the independent of the sin of the sin theta. So, that it is almost symmetrical; that means, it is symmetrical about both this and this axis. So, therefore, no force is exacted in neither in either x are y directions.

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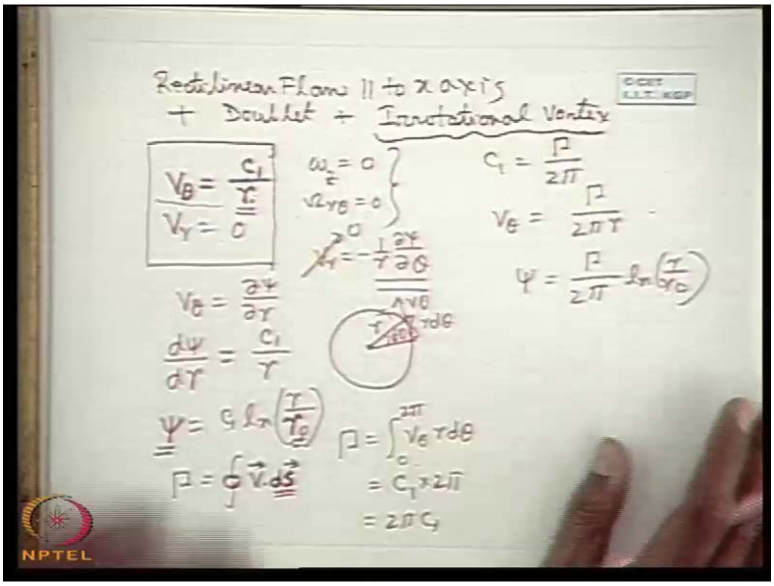

So, alright next we will discuss a combination of, a combination of rectilinear flow, we will discuss the combination of rectilinear flow, rectilinear flow the same flow parallel to y axis parallel to sorry x axis plus Doublet plus a irrotational vertex plus and irrotational vertex. You have already seen, let us first before that discussed little bit of irrotational vertex; we have already seen, that an irrotational vertex flow, that a vertex flow which is irrotational, the tangential velocity is describe by some constant by r, C by r; let this constant we denoted by c 1 by r, because we are already handling with one constant c in the stream function of Doublet and V r is 0; that means, vertex flows that the flows whose stream line are concentric circulars; that means, it has got only tangential velocity, where the velocity here say, functional relationship with the radius like this c 1 by r radial co ordinate and there is no radial velocity, this we have already seen and for this type of vertex flow the rotation is 0; that means, omega is 0, the rotation component is 0, omega z is 0; that means, the virtuosity in r theta plane is 0, that we have already recognized. So, this flow is known as irritation vertex flow.

So, the irritation vertex flow, if you want to derive this stream function, who can you derive this stream function? You can write, V theta is del psi del r and V r is minus 1 by r del psi del theta. Since we are is 0 by the definition of any vertex flow so psi is not a function of r. So, psi theta sorry psi is a function of r only; that means, we can write d psi d r instead of del psi del r and V theta can be written as some constant by r, if we integrate it you get psi is equal to c 1 l n r by r 0. So, r 0 is a constant comes after the integration, which physically signifies that are r is equal to r 0 psi is equal to 0; that means, we define and arbiter value of psi is equal to 0 at r is equal to r 0 and also you know that, if V vertex motion can never we describe of to the centre are the origin of the coordinate system were r is 0, because here v theta is undefined in finite that gives a point of singularity.

Now, if you recall the definition of circulation gamma, we now this is the defined as the close canto integral of V dot d s where d s is the elemental, d s is vector if you take along the element along an elemental length, along a path and closed canto integral, along a close canto v is the velocity vector. So, in a free vortex flow in that way, if we define the if we evaluate this circulation, we will get that d s means, let the radius is r at over any circular part and this is, this is the theta so this d s is r d theta and V dot d s V is only in the tangential direction V theta. So, v theta r d theta and v theta is c 1 by r; that means, it will be 0 to 2 pi along the close canto it will 0 to 2 pi.

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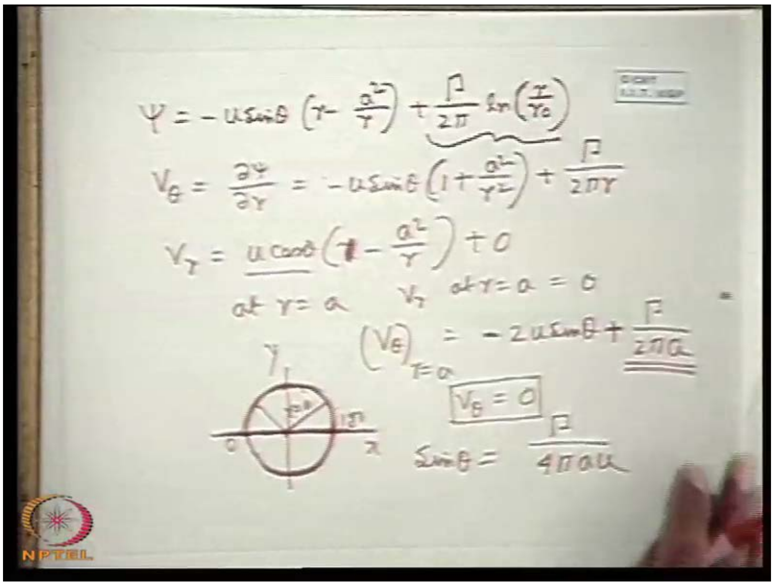

So, c 1 by r r are cancels it is c 1 and d theta is 2 pi into 2 pi. So, therefore, 2 pi c 1 therefore, this constant in the velocity function defining the tangential velocity in a vortex flow is nothing but gamma by 2 pi. So, therefore, the theta can be written usually is value written in terms of the circulation constant like this and psi also can we written as gamma by 2 pi. So, because c 1 is gamma by 2 pi this we have to l n r by r 0.

So, if we know now this thing the now the combination of the rectilinear flow parallel to x axis is Doublet and irrrotational flow gives a stream function, which is first of all the rectilinear flow and Doublet gives this stream function which already we are derived as, minus u sin theta into r minus a square by r only with this we add gamma by 2 pi l n r by r 0. Now here we will notice always that when we add a circulation term r n irrotational flow vertex, irrotational vertex motion are a free vertex motion, then what we get, we get an asymmetric because of separate ambitional functional add a deity.

Let us see now, if we just try to find out the v theta is del psi del r; let us see, what is V theta? That is del psi del r; that means, minus u sin theta 1 plus a square by r square, if you differentiate this we get gamma by again twice pi r and if you write V r it will be minus 1 by r del psi del theta; that means, u cos theta same thing r 1 by r del psi del theta; that means, it is 1 minus a square by r and this part will be 0 this is independent of theta. So, therefore, on this surface of the cylinder; that means, mathematically at r is equal to a; that means, if we plot the cylinder.

Here on the surface of the cylinder r is equal to a V r is 0, but what is V theta? V theta V r at r is equal to a as usual 0, but V theta at r is equal to a does not give fully minus 2 u sin theta in this case, because this first star will give the similar term the same term minus 2 u sin theta, but added with another term twice pi a, so minus 2 u sin theta plus twice gamma by twice pi a. So, therefore, it is distinct that this term 2 s and azimuthally; that means, velocity vector now, is not symmetric about this axis about this x axis; which is because sin theta changes it sin from 0 to 180 degree and 0 to, so 0 to 180 degrees sin theta is got one sin and the sin of sin theta changes from 180 degree to 0. So, therefore, this sin is changing and this is superimposed with a constant term gamma by 2 pi a.

So, therefore, V theta is never symmetric about this axis so; that means, of flow field becomes a symmetric; that means, flow field about this up and flow field about this up will be different. Now let us find out then this stagnation point, where are the stagnation point, where v theta is 0, where are the stagnation point; that means, V theta is 0; if we put hear we get sin theta, then we do not get theta is equal to 0, then we get sin theta is equal to gamma by 4 pi a u, now they are may appear, now gamma then we get sin theta is equal to here if we keep it here.

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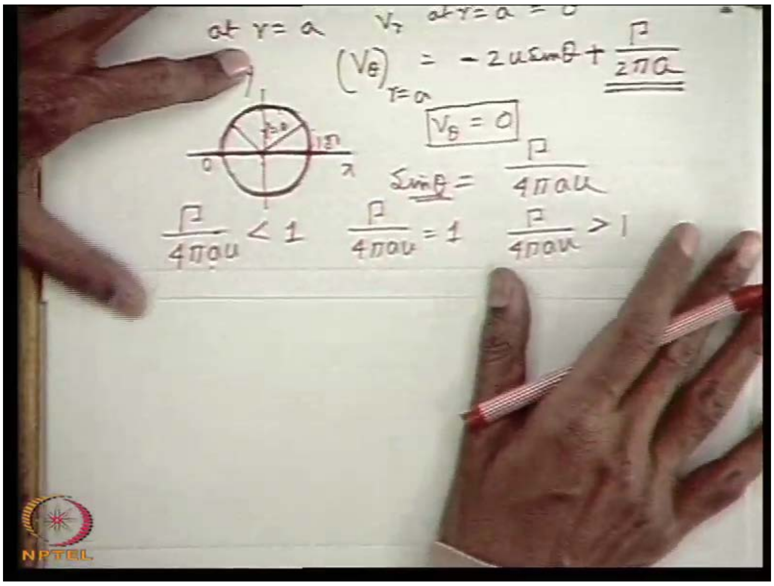

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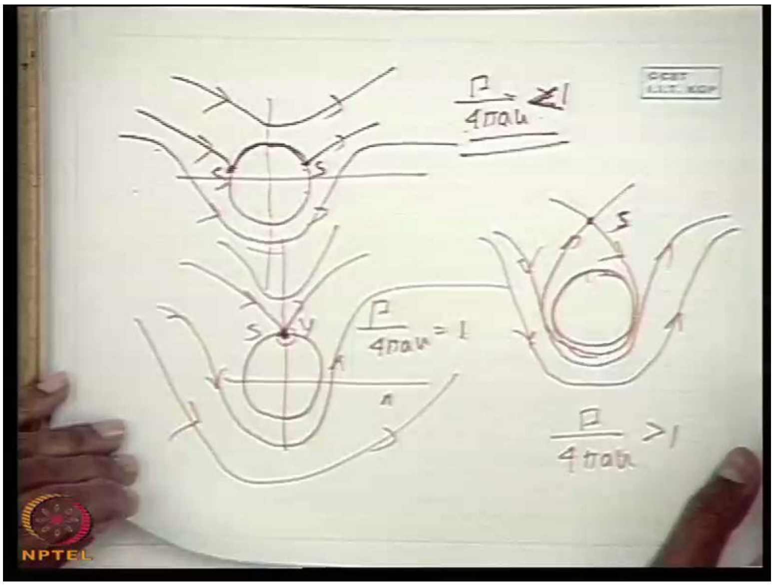

So, they are may appear three condition, when gamma 4 pi a u less than 1; it may appear gamma by 4 pi a u is equal to 1 and they are may case gamma by 4 pi a u greater than 1. Now it is less than 1; that means, it will occur on this cylinder surface, because sin theta is define for less than 1 and it will displaced up; that means, in that case, in this is the case when gamma by 4 pi u less than 1.

So, the stream lines will be like this. So, this will be this stagnation point. So, this will be this stream line. So, these are these stagnation points, this are this stagnation point. This is gamma by 4 pi a u less than 1, when gamma by 4 pi a u is 1; that means, gamma by 4 pi a u is equal to 1, then this stream function, stream lines is like this. Then this stagnation point, just margins here at pi by 2 line here x and y. So, this will be the stream lines, this will be the direction of this stream lines so this is the stagnation. But gamma by 4 pi a u greater than 1 gives an impossible case for sin theta; which means, physically that the stagnation point is never occur, is never occurs on this cylinder surface; that means, in that case there is no stagnation point in the cylinder surface, the flow velocity is like this and this stagnation point in that case, this is this stagnation point; in that case, this will be this stream lines pattern.

So, this is the case when in gamma by 4 three situation are their greater than 1. So, stagnation point will not occur on this cylinder surface, this is the case gamma by 4 pi u is equal to 1 and gamma by 4 pi a less than 1; sorry this is the case when, this is less than 1, sorry this is the less than 1. So, three distingue cases are there, the pattern of stream line depending upon the values of gamma by 4 pi a u is less than 1, that is stagnation points on the cylindrical surface it is lifted up; then it is equal to 1 it is at this point and it is greater than 1 and this source a stagnation point will not be occurring on the cylindrical surface. So, today I have to end here; so next time I will just complete it another part, that the velocity distributions and the pressure distribution for this type of flow, that is the combination of rectilinear flow the doublet and the irrotational vertex.

Thank you.