Fluid Mechanics

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Lecture - 4
Fluid Statics Part – I

Good morning to all of you, and I welcome you to this session of fluid mechanics. Today we will start a new section that is fluid statics, but before I start this section I like to give you a closer, that is the summary of the earlier section that we discussed. That is the fundamental concept and introduction in the fundamental concept.

Well, the closer is like this, what we have discussed there, first we identified a fluid as distinct from a solid from the view point of mechanics is that a fluid goes on continuously deforming under the action of even an infinite small tangential force. Well, whereas a solid resist a tangential force under static equilibrium, fluid cannot do, so it resist tangential force only under dynamic equilibrium. Then we discussed the concept of continuum, which gives a continuous description of matter within a substance without any empty space, so that any property can be defined as a continuous function of space and time.

Then we recognize that in case of the flow of a fluid a shear stress or a shear force is developed at each and every layer in the fluid flow or each and every point in the fluid flow, through which the fluid offers resistance to flow. And this property is known as viscosity, by which the fluid offers a resistance to flow, and the shear stress is proportion to the rate of shear strain, which in case of a simple flow one dimensional flow it is the velocity gradient. For a class of fluid this proportionality is linear that means, linearly proportional the shear stress is proportional linearly proportional to velocity gradient. Those fluids are known as Newtonian fluids, because this law is known as Newton's law viscosity.

While a class of fluids which do not obey these linear relationship between the shear stress, and the rate of shear strain or the velocity gradient are known as non-Newtonian fluids. Then we recognize a definition of property or property definition of a property compressibility of a fluid, which is the measure of its change in volume or density with the application of pressure. So, fluid is more compressible when it has got large change in volume or density with an small application of pressure or small difference in pressure, and the characterized by the property known as bulk modulus of elasticity.

For liquids the bulk modulus of elasticity is very large, so that it is almost incompressible while for gas it is other way the fluids are compressible, its bulk modulus of elasticity is very low. Then we recognized the compressible and incompressible flow which is the measure of the change in density or volume of the flow due to a change in pressure brought about by the flow itself. That means it depends upon the flow velocity and the properties of the fluid so that the pressure difference which is caused is sufficient to cause a considerable change in density of the fluid. And it has been found that if the flow velocity is less than 0.33 times the acoustic speed or acoustic velocity in the fluid at that condition, then the change in density is less than 5 percent, then we consider the flow to be incompressible. The ratio of these two velocities that is the flow velocity and the acoustic velocity in that medium at that condition is defined as mach number. Therefore, when the mach number of flow is less than 0.33, then the flow is considered to be incompressible that means change in density is negligible.

Then we recognized another property of the fluid as surface tension, that it is because of the interplay of molecular cohesion and molecular adhesion we have found that work is done in creating a free surface of the fluid. Therefore, a free surface of the fluid always stores mechanical energy and the surface is always under a stretched condition and the force acting on a free surface per unit length of an imaginary line drawn on the surface is defined as the surface tension.

We have then recognize different phenomena like capillary rise, capillary depression as an interplay between the cohesion and adhesion. Cohesion is the force of attraction between the molecules of the same kind where adhesion is the force of attraction between the molecules of different kinds. It is because of this surface tension phenomena we have recognized that a curved interface creates a difference of pressure between the two sides of the interface. One side may be a gas or a miscible liquid and another side is the fluid or liquid itself to which we are paying our concentration. That means there is a difference in pressure between a between two sides of a curved interface, the interface is defined by the demarcation of two immiscible fluids.

So, in all we discussed this thing in our section fundamental introduction and fundamental concepts. So, today we will start fluid statics, now at the start I will just tell you this fluid statics will discuss the force field which is generated in a fluid at rest, in a fluid at rest, in an expanse of fluid at rest. As a consequence of which you will see the definition of hydrostatic pressure and then this section we will include all the manifestations of hydrostatic pressures in a fluid at rest. The measurement of hydrostatic pressures, then the forces exerted on a body submerged on a fluid at rest the concept of buoyancy and the stability of bodies submerged or floating in a fluid at rest. So, this will be in fact the contents of this section fluid statics which we will start today.

Now, at the beginning let us consider that in case of a fluid at rest if we consider an expanse of fluid at rest, if we find out a fluid element, which is an infinite small region of the fluid continuum, a fluid element in isolation from the entire fluid body and see it as a free body of the fluid element. You know what is a free body diagram that means we consider a fluid element in isolation from its entire surrounding of fluid and see that then we recognize two types of forces, two types of forces are acting on this fluid element in isolation in the free body.

One force is body force, one kind of force is known as body force. What is that? This body force is an external force body force is an external force which is acting throughout the mass of this fluid element, throughout the mass of this fluid element. It acts about throughout the mass it may be constant over the entire mass it may not be constant, but it is acting on the mass of the fluid element. And this body force is caused by an external agencies, there is nothing to do with the fluid it is an external force. For example, the gravity force is a body force, the force of gravitation and we know any substance or any matter on the surface of the earth is experience by the gravitational force or gravitational attractive force of the earth and which is acting on the mass of the body. This is the typical body force, the example of the typical body force. There may be other body forces if the fluid body is exposed in a magnetic field, it is a magnetic force is acting on the fluid mass. It is exposed in an electrostatic body force field so there may be a number of external agencies which may cause this type of body forces.

So therefore, one thing may be very clear to you that the detail description of body force or body force field does not come from the concept of fluid mechanics. It comes from the concept of the physics which describes that body force. For example, what should be the variation of gravity force, what should be the value of the gravity force for a given mass is described by the law of gravitation. Similarly, the body force is the magnetic force or electromagnetic force it will be governed by the physics of that domain that is, electromagnetism. Therefore, gravity force is governed by the physics of that particular domain which defines the force.

Now, another force which acts on the fluid body in isolation is the surface force. What is the surface force? This comes from the picture that when you make the isolation of the fluid body from its entire surrounding that means, from a continuous expanse of fluid we take a fluid body in isolation as a free body, then the actions of its neighboring molecules or neighboring particles which you are in contact with it when it was in the continuous mass of the fluid gives rise to a force, which appears only on the surface and is known as surface force. You know from your elementary mechanics that this surface force which appear on the surface of a free body can be resolved into two components. One is perpendicular to the surface, perpendicular to the surface another is around the surface. The force perpendicular to the surface is known as normal force and the force along the surface, the component along the surface is known as shear force and the respective ratios with the area of the surface over which this forces act known as normal stress and the tangential or shear stress.

So therefore, we see there are two kinds of forces act, one is the body force another is the surface force and this surface force in fact in an expanse of fluid constitute the internal forces, that do not appear as an external forces, only the body force appears as an external forces. Now, first let us recognize what is the state of forces or stresses generated in a fluid at rest with this as the introduction.

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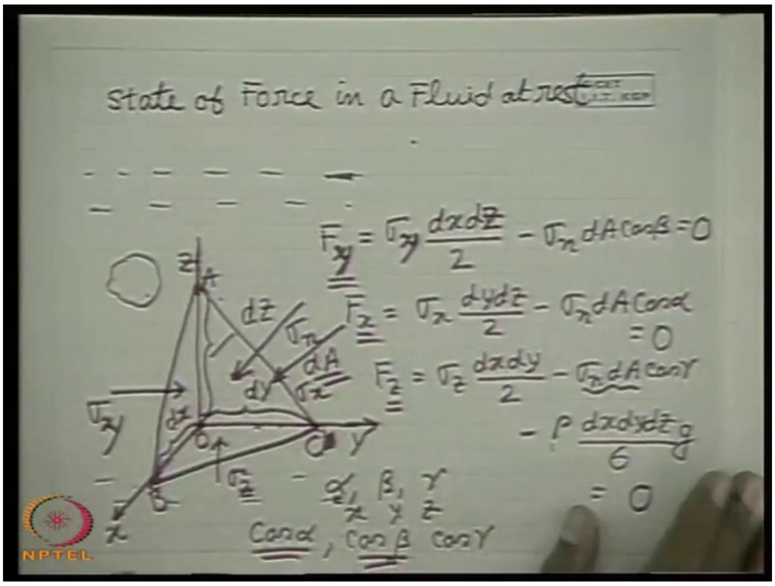

Let us now see here, state of force in a fluid at rest, in a fluid at rest, in a fluid at rest in a fluid at rest. Now, let us consider a fluid is at rest an expanse of fluid is at rest what should be the state of force. First, let us consider a fluid element a tetrahedral fluid element we consider in general. A tetrahedral fluid element like this, tetrahedral fluid element which coincides with the coordinate plane. Let us fix a coordinate axis, a cartesian coordinate axis x, y, z like this in this proper sense of rotation x, y, z and let us consider a tetrahedral fluid element, let this is z axis A, B let this is C, a tetrahedral fluid element A O B C A, which have distinct phases like A O B, A O C, B O C and another inclined phase is A B C, this phase.

This three phases are coinciding with the coordinate planes the phase B O C coincides with the x y plane, let x y. Similarly, the plane A O C coincides with the y z plane and the plane A O B coincides with the x y plane, x z plane sorry x z plane. If we take a very simple case of a tetrahedral fluid element whose planes are coinciding with the coordinate planes and now we try to find out the forces acting on this.

Now, first one thing you have to recognize then when the fluid is at rest there cannot be any tangential stress or tangential force on the fluid, it is 0. So, there cannot be any tangential force because fluid at rest can develop no tangential force similarly, a fluid at rest cannot develop any tensile force, so only force in a fluid at rest is the compressive force. Therefore, if we see the free body of this tetrahedral element then we see only the compressive forces are acting in this direction. For example, in this normal compressive force in this plane the normal compressive force is acting, if I denote the compressive stress as sigma x, this times. Now if I just this is d y write this, dimension as d y, this dimensions as d x and this dimension's as d z. that means I define the tetrahedral elements with the dimensions d x, d y and the height d z.

Then this is a small infinite small tetrahedral element it is actually this figure is made in an exaggerated way, so this is d x and this is d z. So the net force in the x direction on this tetrahedral element will be what? Will be the x, this is the x direction force on this due to the surface force sigma x, F x is equal to d x into d y by 2, this is a triangular surface. So this will be sigma x into d x d y by 2, d x d z by 2, very good. d x d x and d z by 2 into this will be sigma y, very good. So, this will be then F y so this will be then F y sorry. Then this is the y direction so I am writing this first as, if I let me write first F y. Then where from we will get another force, now you see the surface forces on this surface A O C, B O C will not be contributed in the direction of y.

So therefore, only surface force, let us define a surface force in this inclined plane A B C and let this trace is sigma n. Sigma n is the stress that is in the normal direction to the surface A B C and let us define that this normal to this A B C, to this plane A B C makes an angle of alpha, rather you write beta gamma with the x y and z axis. So that cos alpha, cos beta and cos gamma are the direction cosines or the direction cosines of this normal to the plane A O, A B C that means this inclined plane A B C has a normal which makes angle alpha with the x direction angle beta with the y direction and angle gamma with the z direction.

Now therefore, the forces acting perpendicular to this A B C plane that is, the surface force it will be what? Sigma n into the area of this plane A B C and its component in the y direction will be into cos beta. Therefore, it will be minus sigma n, let us consider d A is the area of this inclined plane A B C into d A into cos beta alright, sigma n d A cos beta.

Now, here we assume another thing that gravity is the only body force field, that gravity is the only body force field and this axis z is along the vertical direction. Therefore, there is no other external force acting on this fluid element in the direction of y. So this is the only force acting in the y direction, net force. Similarly, if I write the x direction force F x, F x is acting in which way? That this is the x direction that means, it is acting on the plane A O C in the direction of x that means, this force.

So, this force will be due to the sigma x times the area of this plane, which will be delta y, d y into d z. So, sigma x into d y d z by 2 and similarly, the component which will come, the component in the x direction because of the force, normal force on the plane A B C will be this multiplied with cos alpha. That is the direction cosines alpha is the angle with the x axis of the normal to this inclined plane A B C. Similarly, if iI write F z, F z also it will be contributed first by the surface force, if sigma z I define this trace in the z direction ,so it will be acting on this surface of area d y d x by 2. That means sigma z, d x d y by 2 minus similarly, the contribution that is the vertical component of the force surface force in this A B C plane.

Another force will act which is the weight of the tetrahedron that means, the external body force is the gravity force that is, the weight, total weight the gravity force is the weight of this tetrahedron. That will be equal to rho times the volume, mass what will be the volume, volume of this tetrahedron from the simple geometry probably you know, I can write by 6 times the g because this is rho into v is the m and g, so this will be the net force in the z direction. This will be the net force in the x direction, this will be the net force in the y direction.

Now, what happens is that this tetrahedranal fluid element we consider for a for convenience such shape, so any fluid element in these static expanse of fluid is under equilibrium against this forces. That means we take a free body of the fluid element and analyze the forces we will see that these are in equilibrium with all these forces. That means the each and every component is equal to 0 for the static equilibrium of the fluid element, now if we make this 0 and at the same time, if we see that this has to be kept.

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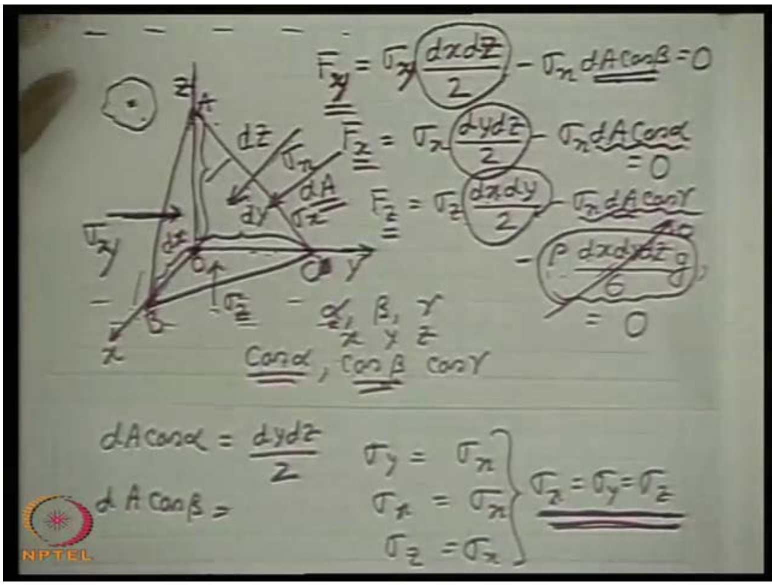

If we see that this d A cos alpha for example, here d A cos alpha where cos alpha is the direction cosines that means, cosines of the angle made by the normal with the x direction. So, this will be nothing but the what will be this? This will be clearly the projection on the plane which is perpendicular to the x direction that means, projection of the area on y z plane. That means this will be simply d y d z by 2 that means, this will simply be equal to this area d y d z by 2. Similarly, if we see d A cos beta, d A cos beta will be what? d A cos beta where beta is the angle of the normal with the y direction that means d A cos beta will the projectional area on the x z plane. That means which is perpendicular to the y direction that means this area d x d z by 2. That means d A cos beta will equalize this d x d z by 2. In the similar way this d A cos gamma will be equal to d x d y by 2 that means, this is the projectional area in the x y plane. That is the plane which is perpendicular to z direction.

So, if you put that and then you see from this first equation we get, sigma y is equal to sigma n, second equation we get sigma x is equal to sigma n and third equation we have to see one more thing. That since d x, d y, d z that is these d z, d x ,d y are so small that this is a relatively higher order term as compared to this two. Therefore, we can neglect this term that means, we can neglect the weight of the infinite small element.

So, if the infinite small element tends to be 0 that means, if we contract these element to a small one. So, the weight can be neglected because it is relatively at higher order as compared to other terms. So, here from also we get sigma z is equal to sigma n. So, out of these we get simply the things sigma x is equal to that means, which states that the stresses at any point because when we consider this tetrahedral fluid element, we know the fluid element is going to be infinite small. That means it comes almost to a point then this traces from all directions x y z they are equal in magnitude.

So therefore, we come to this conclusion that this traces in a static fluid at any point directed towards that point from all directions and they are of equal magnitude and they are of equal magnitude.

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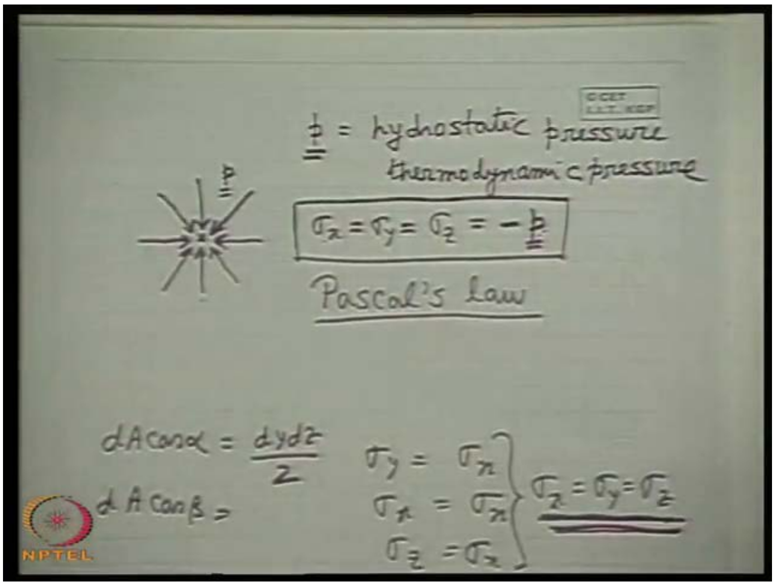

It may be from any directions and they are of equal you have read it at your school level and they are of equal magnitude. And this magnitude is defined as a scalar quantity p which is known as hydrostatic pressure, hydrostatic actually it should be fluid static, but conventionally it is known as hydrostatic. The word hydro is used hydrostatic pressure or thermodynamic pressure. When you will read thermodynamics you will see that this is also known as thermodynamic pressure. Here, specifically we will use hydrostatic pressure.

So, we define therefore, this sigma x is equal to sigma y is equal to sigma z with scalar quantity p, with a negative sign. Specifically, this is because to keep compatibility with the convention that tensile stresses are positive. So therefore, this stresses are compressive which is found or which is exhibited by these equation because with a minus sign and this is all of equal magnitude, whose magnitude is given by the quantity p. So, p represent the scalar quantity which defines the magnitude of the compressive stress at any point in the fluid element, which is same in all directions. This is known as Pascal's, Pascal's law, who first described this law, Pascal's law. These define this state of stress in a fluid at rest.

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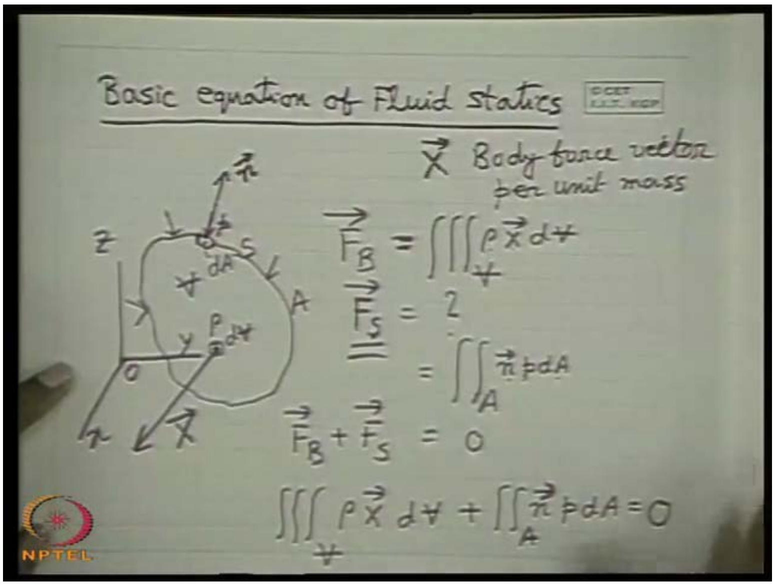

Now, after this I will describe the basic equations of fluid statics, you write basic equations, basic equation of basic equation of fluid statics basic equation of fluid statics. Now, we have recognized that in a fluid at rest the state of force or state of stress generated is such that, at each and every point the compressive stresses fluid at any point the stresses is directed towards the point from all directions. That any point is under a compressive stress, which is equal in all direction. That means the stresses are directed towards the point from all directions and have got the equal magnitude, which is known as the hydrostatic pressure.

Therefore, we recognize a pressure field is generated in a fluid at rest, but now we must know in a fluid at rest how this pressure field or the pressure varies with the space coordinates in the fluid. That means if we set a frame of reference axes, then how this pressure field behaves with the change in the coordinate axis. That means we want to seek a functional relationship between this pressure with the space coordinates x y z that means, to develop the analytical expression of the pressure field in a fluid at rest. This is what now you will do, which is known as basic equation of fluid statics.

Now, let us consider in a expanse of fluid, a fluid mass like this, a general fluid mass in as if free body. Consider this fluid mass is bounded by a surface S and the surface area is A and the total volume of this fluid mass or fluid element is v. Now, let us consider a small elemental volume at a point d v, where the density of the fluid is rho density is the point property, you know.

Now, let us first consider there is a body force field in the fluid or the general body force field. Let us consider a general body force field, which is so described that at this point let the body force vector acts like this, which is given by X bar as the body force vector per unit mass. This X bar is the body force vector, you write body force vector per unit mass at that point, at any point body force vector vector, body force vector per unit mass we define per unit mass. Let us have a reference coordinate axis like this x y and z. Now you see that this fluid element we know is under equilibrium against two force. One is the body force, another is the surface force, surface force as the pressure forces which is acting on its surface.

Now, I find out what is the total body force F B total body force vector, F B where X bar is the body force vector per unit mass at a particular point, where the point encompasses elemental volume d v and density is rho. So, from simple mathematics we can tell so the body force for this elemental volume will be, rho X bar into d v because rho d v is the mass and body force per unit mass. So, if you integrate this thing over the entire volume of this body, so we will get the total body force vector F B, which will be the volume integral of this F X bar d v over the entire volume.

Now, we want to know what is the total surface force vector of this fluid element. How to know it? To know it mathematically or to give a mathematical expression for this we will have to consider this way. Let us consider a elemental surface area and small elemental surface area on the surface of this fluid element defined by d A and we define a vector unit vector n bar, that is the not bar this is the vector. We tell that n that the unit vector n, which is positive in a direction outward from the surface, outward from the surface, so that we can define the body force vector in this way which is acting on this elemental surface.

If p is the pressure which we have already recognized that is the pressure acting on this surface d A, we can express this as for these elemental area this will be n unit vector p d A. So p d A is the actual force acting on this area in this direction towards the area because this is the compressive force. So it is normal to the area d A, pressure into d A is the total force and this is n is the unit vector which is positive in the outward direction. So, this is the direction of the force acting on these elemental area d A. So, if we make the integral over the entire area for this fluid element we will get the vectors of the surface for total surface for vector over the entire body.

Now, under equilibrium for the equilibrium of the body or the body under equilibrium we can write F B plus F S is equal to 0. Therefore, what we can write? We can write this, rho X bar d v plus p d A is equal to 0. I think there is no difficulty, so this two things this is over area A v, so this sum of this F B plus F S is equal to 0.

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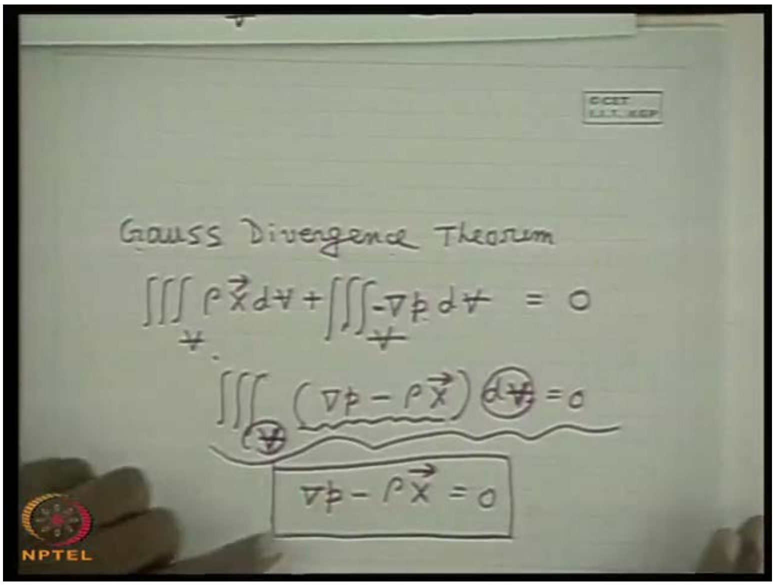

Now, we know from from our elementary mathematical knowledge by the use of Gauss divergence theorem. You know that Gauss diverge if we use Gauss divergence theorem we can change this surface integral to a volume integral. How?

Now, let me write this for the first term, please ask any question. X bar d v, now how to change the surface integral to a volume integral? This is a scalar quantity operated with the unit vector so this is a vector quantity, so it is just change in terms of the gradient gradient of the scalar quantity grad p, which is a vector d v. So, we know that we can change this surface integral to a volume integral in this manner, that means equality of this by this comes from the Gauss divergence theorem and that becomes equal to 0. Therefore, we can write that the entire thing like this grad p plus sorry a minus sign is there, I am sorry a minus is missing. So grad p minus rho X into d v is equal to 0, alright?

I think there is no problem now, again we see that this is valid, this expression is valid irrespective of the value of d v or d v. That means it is valid for any small elemental control volume or any volume of the fluid element. Therefore, we can write in general this part is 0 that means, in fact it is independent of d v or v. So this integration may be done for any volume v of the fluid element, so we can take this this is arbitrary. Therefore, this part is 0 therefore, we see this is the most important equation and this is the equation of fundamental equation of fluid statics.

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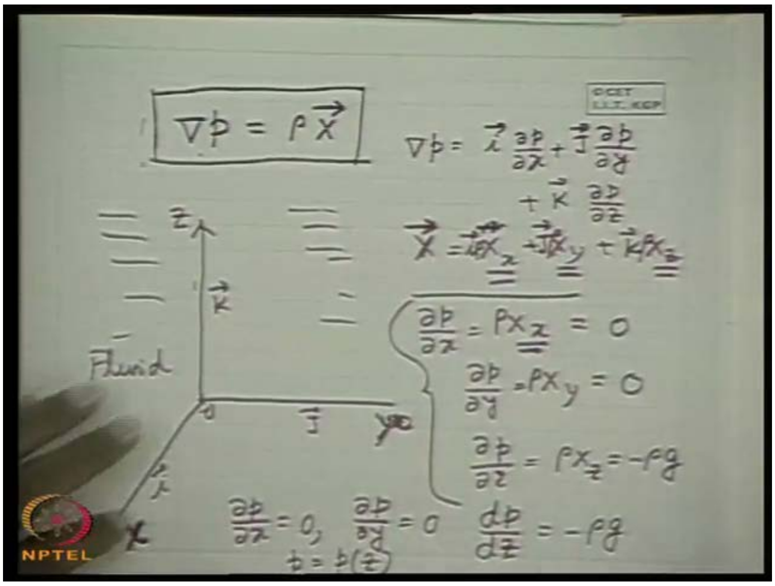

So the fundamental equation of fluid statics which describes the variation of pressure with the space coordinates or which gives the variation of or gives the pressure as a function of the space coordinates all like this. This is the expression in the most compact and vector form grad p is equal to rho into X bar, where p is the pressure and X bar is the body force vector.

Now, let us consider a simple case how to resolve it with respect to a frame of reference. Now I am interested to know a description of the pressure field in terms of x y coordinates that means, I sorry I think this will be x in proper sense of rotation x y and z. That means I am interested to know the variation of p in terms of x y and z, a simple rectangular cartesian coordinate system in case of fluid at rest, this is the fluid at rest. So, then what I will do from this general vector form of the equation I will write what is grad p grad p I can write I, where i j k as you know are the general nomenclature for the unit vector in x y z coordinate, i del p del x plus j del p del y, well plus k del p del z and if I write X bar sorry X vector X vector X that the body force per unit mass at X x plus X y i sorry plus j, not this one, j x y plus k x j, where X x is the component of the body force per unit mass in the x direction.

It is the component of body force per unit mass in the y direction it is the component of body force per unit mass in the x z direction and if I write this we get that del p del x equalizes X x, del p del y equalizes X y. I am sorry I am sorry rho is there, very good rho is there, very good rho X x, rho X y and del p del z is yes rho X z. Please please what is the problem please you tell me rho is there, rho X x, rho X y, rho X z. Please any problem, any difficulty please ask me. What happened? It is very simple there is nothing complicated rho X x, rho X that means this is precisely the expression in terms of a cartesian coordinate system.

Now, if we consider gravity is the only body force field, if we consider now gravity is the only body force field what will be the value of X x? If we consider z axis is in the vertical direction what is X x, 0, very good. What is X y and what is X z, if we consider z positive in the upward direction minus, so minus rho g.

So, if we consider only gravity is the only body force field that means there is no such external body force or external force acting on the fluid body, gravity will always be there. Then we finally, arrive these equation that del p del x is 0, del p del y is 0 which means that p is neither a function of x nor a function of y. That means in a horizontal plane pressure is same everywhere it is not a function of x and y well that a horizontal plane pressure is same everywhere, it is only a function of z that means p is only a function of z. Therefore, we can write del p, del z as d p d z, we can write in terms of the total differential or ordinary differential is minus row g.

So therefore, we come to a conclusion in a simple case that where there is no external force acting as the body force gravity will always be there.

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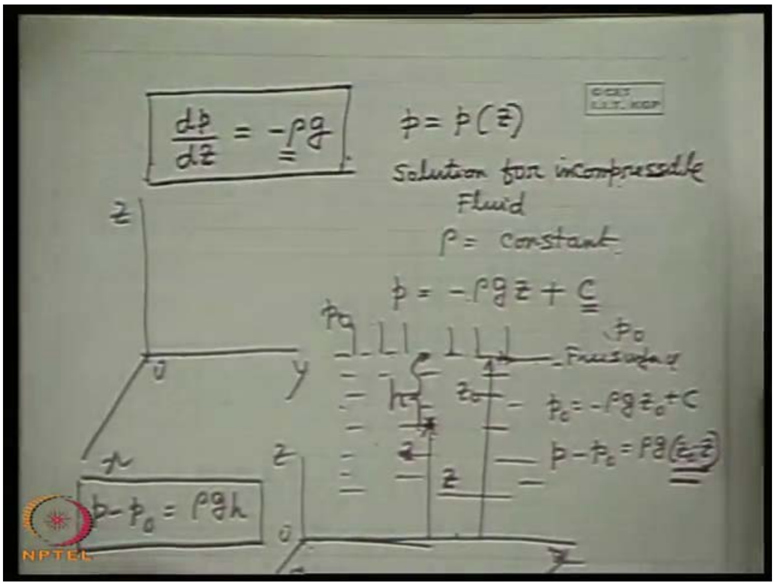

p is a function of only z, which is clear from these mathematical symbol that d p d z means, p is a function of z so that the ordinary differential form we write is equal to minus rho g. That means p varies only in z direction that means, if we consider a frame of reference like this x y and z, so p is neither a function of x nor a function of y, it is a function of z only. Therefore, we see the basic equation of fluid statics comes in this form which is a differential form, but if we want to find out an exclusive relationship of p with z, what we have to do? We have to integrate this equation, but integration of this equation will can cannot be done until and unless we know the nature of variation of rho that means, we cannot integrate it until and unless we know the nature of variation of rho with either z or p.

Let us consider first a solution for incompressible fluid, solution for incompressible fluid, solution for incompressible fluid solution for incompressible fluid solution for incompressible fluid when rho is constant, that means when rho is constant that means, in case of a liquid rho is constant. So, simply integrate it you get p is equal to what we will get minus rho g z plus a constant C, this is simply the equation. So C has to be found out from a suitable boundary condition, from a suitable boundary condition. Let us define a liquid with a free surface, let us consider a static mass of liquid with a free surface. Let us consider these x y z coordinate axis, let us considered these datum plane where from the z is being measured. So, we can tell that pressure at any point whose vertical coordinate is measured as z from a reference x plane, x coordinate axis, y coordinate axis. Let a x y z like that it is minus rho d z plus C.

If we define a free surface, if we define a free surface so that whose vertical coordinate is z 0, we can find out and the pressure is p 0. At the free surface where p 0 is the ambient pressure then we can find out that p 0 minus rho g z is 0 plus C. So, C we can find out as p 0 plus rho g z 0. So, we can put these value of C and we can show it is is equal to rho g z 0 minus, z 0 minus z. That means at any point so z 0 minus z, if I define as h I get p is equal to p minus p 0 is rho g h that means, it is rho g times the depression of this point from the free surface or depth of this point from the free surface.

So, this gives a very simple equation that in a fluid in a liquid or incompressible fluid at rest the pressure at any point, if there is a free surface where the pressure is the ambient pressure p 0, then at any point which is at a depth h from the free surface the pressure exceeds from that of the ambient by this quantity rho g h. This is a consequence of the solution of the basic equation of fluid statics in differential form for an incompressible case, where rho is constant. So, thank you next class we will discuss the other cases,

Thank you.

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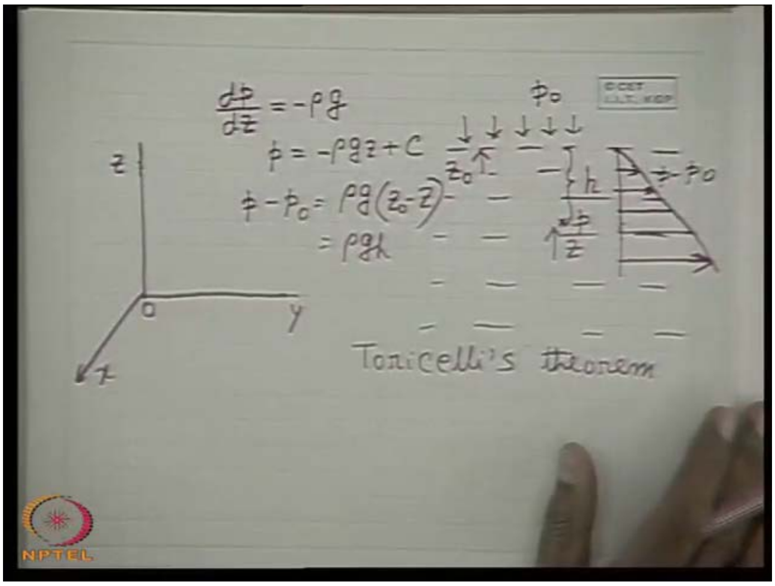

And if we now recall this mathematically then we see that if we recognize this x, y and z coordinate, we have seen that the basic equation is like that d p d z that means, p is a function of z only and its slope is given by minus rho into g provided, the body force is the only gravitational force. That means in this expanse of fluid at rest no other external force is acting that means only external force acting or the body force acting is the gravity force and then the pressure varies only in the z direction by these equation. And also we recognize that in case of an incompressible liquid the solution for this is p is equal to that means, when rho is constant minus rho g z plus a constant. And we recognize that if we have an expanse of fluid at rest with a free surface, if this be a free surface where the pressure impressed is the ambient pressure p 0.

Then we can find these value of C and if we define the z coordinate of the free surface as z 0 from any frame of reference. And that any point where we are finding out the pressure p the z coordinate is z then these equation can be written that p minus p 0 is simply, rho g into z 0 minus z, alright?

Alright, z 0 minus z is simply the vertical depression of this point from the free surface. If we define it as h it is simply rho g h. Therefore, we see that if we express the p minus p 0, the difference in pressure from that of the ambient pressure exerted on the free surface of an expanse of fluid, it is a linear variation p minus p 0. It starts from 0 and linearly varies with the depth linearly varies with the depth and this formula was first found by the scientist Toricelli and that is why it is known as Toricelli's theorem.

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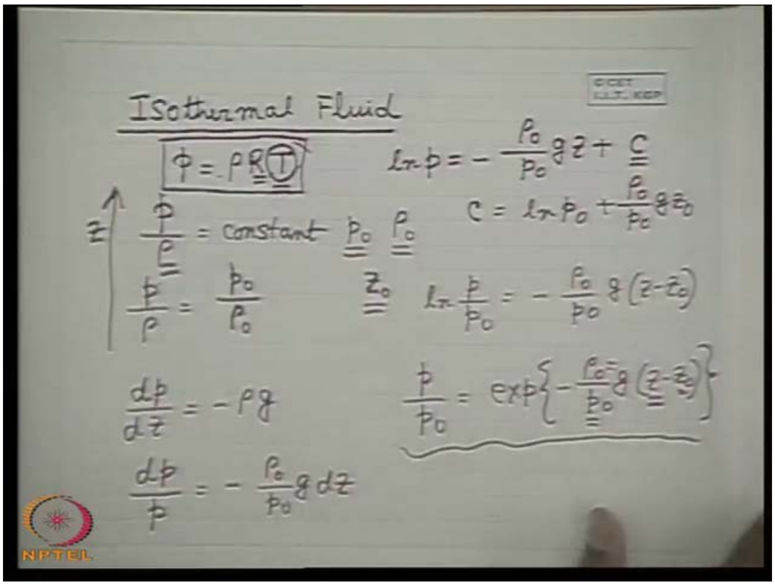

Now, we will found out the similar exclusive relationship of p with z for compressible fluids, this is simple mathematics not much fluid mechanics is involved. Now, we first consider an isothermal fluid, compressible fluid means whose density changes, a isothermal fluid that means a fluid at rest a fluid at rest where the density changes, but temperature remains constant. Now, you know for any system the equation defining the pressure density and temperature is known as the equation of state. Equation of state defines a functional relationship between pressure, density and temperature.

So, if we consider the compressible system or the gas as the perfect gas, this functional relationship between pressure, density and temperature equation of state is given by p is equal to, probably you know this things from your physics knowledge in physics or thermodynamics where R is the characteristic gas constant. So for an isothermal fluid T is constant. Therefore, the relationship between pressure and density is that, p by rho is constant. Now, it becomes simply school level mathematics, we can write the pressure p, let us write p like this p by rho in terms of a referring pressure p 0 by rho 0, where p 0 and rho 0 is at reference state. That means at some location if pressure is p 0 and density is rho 0, which we take as reference state then p by rho these are the variables is p 0 by rho 0.

Now, the simple task is to solve this simple mathematics d p d z, is very simple minus rho g by substituting the value of rho. If you substitute the value of rho from this equation here, you get d p by p rho is p rho 0 by p 0 is minus what rho 0 by p 0 rho 0 by p 0 g d z, alright. Now, if we integrate this we get l n p is equal to we get integrate very simple rho 0 by p 0, these are the constant defined by the reference state plus some constant.

This constant we can found, we can find if we define that the z coordinate at these reference state of p 0 and rho 0, where pressure is p 0 and rho 0. We define the z coordinate for the reference states that z coordinates from any reference coordinate axis is z 0. Then we can find out the value of C as l n p 0 plus rho 0 by p 0 g z 0 that we can find out the value of C, and these value of C if we substitute here, simple school level thing. We get l n p by p 0 is minus rho 0 by p 0 into g z minus z 0, this is the simple expression or you can write in terms of the exponential function l n means that it is the exponential function that is, the exponential function of minus p 0 by rho 0. Sorry by p 0 g times z minus z 0, so we get so there is nothing fluid mechanics it is only a mathematical exercise through which we can find out the variation of pressure, with the vertical height h z, that vertical height z.

In case of a compressible fluid where density changes along with the change in the pressure or other you can tell the pressure changes because of the change in density, in a fashion that the temperature remain constant and we consider the system of compressible gas or a compressible system behaves as a perfect gas, where the equation of state. That means the relationship between pressure density and temperature has this form p is equal to rho R T. So that we can use that and therefore, the nomenclature p 0 rho 0 and z 0 are the reference state. That means at some location z 0, we know the value of pressure as p 0 and density as rho 0 so this is the expression. In the similar way we can find out the expression where the temperature changes, where the temperature changes rather we can write non-isothermal, non-isothermal case, non-isothermal case.

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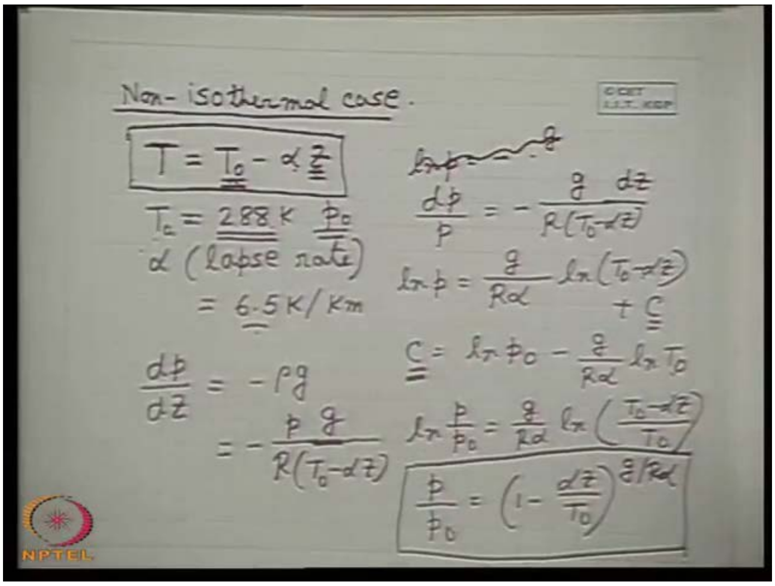

Now, this is very important where the temperature changes, but in this regard I like to tell you one thing that in our atmosphere the temperature up to a certain height changes linearly with the altitude. And if we express the temperature in that level of atmosphere from the earth surface where temperature changes linearly with the altitude we can write this way. This is the temperature at the sea level and z is the altitude from the sea level. In our usual atmosphere the value of T 0 at the sea level is approximately 288 Kelvin and the value of alpha, this is known as this is the terminology lapse rate that up to the altitude which this linear decrease in temperature takes place, the lapse rate that is constant in this equation. That is equal to 6.5 Kelvin per kilometer.

So, this way if we define the temperature change as a temperature decreases with the altitude. Then the rest part that means to find out the explicit relationship of pressure with the altitude becomes a simple mathematics. That means minus rho g, now rho is what from the equation of state along with that if we assume that the atmosphere varies sorry behaves as a perfect gas where rho can be expressed as p by R T.

And in place of T if I write T 0 minus alpha z that is times the g that means what I have done rho is p by R T and T is varying with z T 0 is a constant, which is the value when z is equal to 0. At z is measured from the earth surface at sea level and this is the temperature at any altitude z. So T 0 is usually 288 Kelvin it is a typical value and alpha is this one, so T 0 alpha are the constant parameter.

Now, rest part is simple mathematics that you go on integrating this, if you integrate this taking p here d p by p, then you get l n p is is equal to minus g by R, sorry I think we, I must write another line otherwise, it will be difficult for you. d p by p is equal to minus g by R T 0 minus alpha z d z. So, if you integrate it then here it will be l n p and if you integrate it, it will be l n T, l n T 0 minus alpha z with respect to d z and alpha minus alpha, so minus, minus plus and alpha will come, this coefficient in the denominator usual rule of integration, and simply T 0 minus alpha z as the whole argument ln.

So, this will induce a constant C and this constant you can very well determine as you say that T 0 is the temperature when z is equal to 0. That means I can write C is equal to l n p 0. If we considered the p 0 is the atmospheric pressure at sea level corresponding to T 0 and z is equal to 0. Then I can simply write C is equal to minus g by R alpha l n T 0.

So, if I substitute this value of C here then I get a simple expression l n p by p 0, is what l n g by R alpha, l n T 0 minus alpha z, minus l n T 0 I take inside l n this minus l n this is l n this by this. That means T 0 minus alpha z by T 0. This l n things I can remove and I can keep it in terms of I write it alpha z by T 0 comes within the bracket raised to the power R alpha. That means these gives a power law type of variation so this is nothing but a simple mathematics, well any difficulty, alright.