Fluid Mechanics
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Lecture - 37
Principles of Similarity Part - II

Good morning. I welcome you all to the session of fluid mechanics. Last class we started the discussion on principle of similarity, and we have recognized that the physical similarity, complete physical similarity between two problems of same physics, but operating under different conditions to be obtained; we have to have three similarity criteria; one is geometrical similarity, kinematic similarity and dynamic similarity. While the geometrical similarity is where the similarities of as same; that means, the ratios of corresponding dimensions are same, the kinematic similarities at the ratio of corresponding velocities at similar points between the two systems to be the same.

The dynamic similarities tell that the ratios of corresponding forces at similar points in the two systems must be same. Then we recognized in general the different forces acting in a fluid flow or general forces causing the fluid flow. So, general forces causing the fluid flow are viscous force, pressure force, surface tension force, gravity force, elastic force and the vectors sum of all these forces give rise to the resultant force, which ultimately causes the fluid to flow. So, this is taken in concentration of D'Alembert's principle, that inertia force plus all the external forces to be 0. And one by one we discuss the ratio of the two forces governing a flow problem, which represents the similarity dynamic criteria of dynamic similarity, and this was expressed in terms of dimensionless parameter.

Because ratio of two forces is dimensionless and which comes in terms of different parameters; one is known parameters, conventional parameters one is the geometrical dimensions, another is the flow parameter like velocity and pressure, another is the fluid properties like viscosity and density. And evaluated for flows governed by viscous and inertia forces, the ratio of viscous to inertia force and pressure forces of course, ratio of viscous by inertia force or inertia force by viscous force is the well known Reynolds number, rho V l by mu; l is the characteristic length, V is the characteristic velocity, rho the density and mu the viscosity and known as Reynolds number.

Similarly, derived that the ratio, magnitude ratio of pressure force to inertia force, comes out to be delta p by rho V square, where V is the characteristic velocity and known as Euler number.

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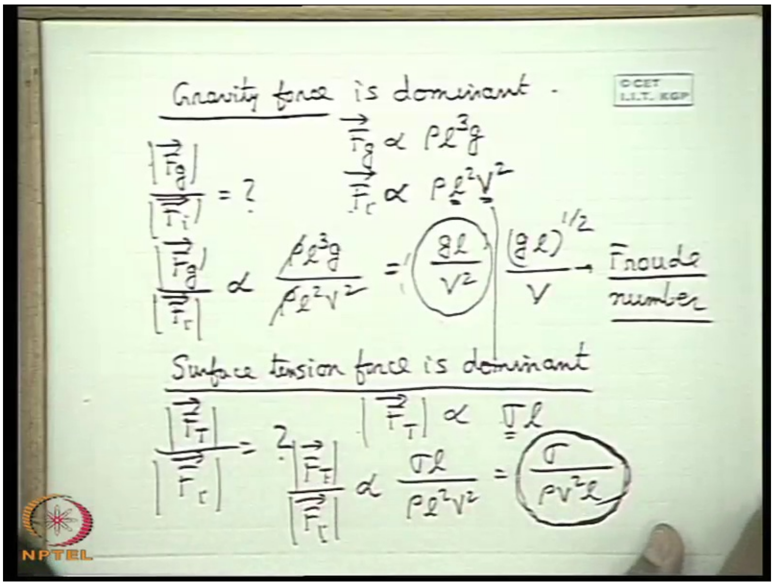

So, today we will see such things for flow, where the gravity force is dominant, where the gravity force is dominant, where the gravity force is dominant. Along with the other force, pressure force for example. So, when gravity force is dominant, then definitely the ratio, magnitude ratio of gravity forces to the inertia forces represents at similarity criteria, than for dynamic similarity the ratio of gravity to inertia forces to be same for two problems. So, how can you represent this?

Now, if you recall or you can just tell that the gravity force is proportional to mass into g, mass is proportional to rho into l cube, l is some characteristic length of the problem. So, rho l cube g, inertia force as we define is proportional to mass into acceleration and acceleration is proportional to some velocity by time and time is length by velocity. The way we derived it earlier and if you recall; it is rho l square V square, where V is any characteristic velocity of the problem, l is the characteristic dimension. So, therefore, if we now substitute this equation of proportionality, then the ratio of F g by F i better we should write this as a magnitude ratio, it looks well.

So, this is proportional to rho l cube g by rho l square V square and it becomes g l by V square. So, therefore, we see this dimensionless group, this is obviously dimensionless; if you put the dimensions you see the this is dimensions, this represent the ratio of this two forces magnitude ratio, proportional to this magnitude ratio of this two forces is proportional to this. And therefore, in two dynamically similar system. This as to be constant or which system, system dominated by gravity force, this happens for flow with a free surface like, flow through open channels. So, under two different sets of condition, this type of, this type of problems the for dynamic similarity to be maintain; this has to be maintained constant, this dimensionless term.

So, square root of this g l to the power half conventionally sometimes even this one is known as Froude number; “F R O U D E”. Froude is the name of a scientist, who first discovered it Froude number; in the study of ships, he was basically a naval architect g l by V square, this was sometimes this is referred as Froude number. There is no such hard and first rule, sometime the square root of this represents as Froude number. So, maintaining of constancy of this number means, the constancy of a square root does not matter.

Now, I consider another cases of fluid flow, where surface tension force is dominant, surface tension force is dominant, force is dominant. So, these happens in case of flow of free zor or flow of a thin sheet in ambient, whenever there is a separation or demarcation of two fluid surfaces; that means, there is a interface the surface tension force becomes dominate. So, flow of free flow of a thin sheet or the flow of liquid through a small capillary tube, vertical flow small diameter tube, the surface tension force is very important.

So, now here also for dynamic similarity, the magnitude ratios of surface tension force to the inertia force have to be made constant for the two systems of the same physics; that means, dominated by surface tension force. Now again, what is the surface tension force magnitude? This is proportional to surface tension coefficient sigma into some characteristic length, you know the surface tension coefficient is defined this way; that we have read earlier, that is times the length characteristic length defines the surface tension force; F i is proportional to rho l square V square. So, this ratio ultimately that is again if I write, F T by F i the magnitude ratios of this two forces is proportional to sigma l divided by rho l square V square; that means, this is equal to sigma by rho V square l.

So, this is the dimensionless term, which represents the ratio, magnitude ratio of surface tension to inertia force has to be kept constant in two different systems. Where surface tension force is dominant; that means, in case of free z flow, in case of flow of liquid sheets, in case of two systems defecting the flow of liquid (( )) small diameter tube, but on the different at sets of conditions, but this ratio or this term combine term dimensionless term have to be same, have to be kept same; if the dynamic similarity between the two systems are short.

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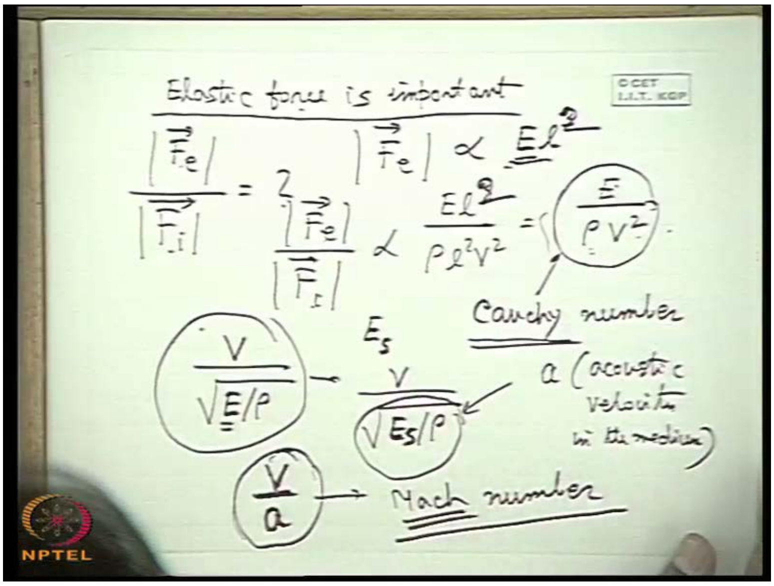

Then I come to another type of flow situations, where elastic force is important; of course this type of flow situation is beyond this scope of this book, this is important or dominant whatever you tell, dominant, earlier I wrote dominant elastic force.

In case of compressible flow, when the density of the fluid changes very fast to with respective pressure, this happens when the fluid velocity approaches sonic velocity or causes this sonic velocity; then the elastic force becomes dominant. Now in that case, the magnitude ratio of elastic force divided by the magnitude, sorry, magnitude ratio of elastic force to the inertia force; that means magnitude or elastic force divided by that of the inertia force is the dynamic similarity criteria; what is this? Very simple, the magnitude ratio of elastic force as you know is proportional to coefficient of elasticity, that is bulk modules of elasticity, times the volume; that means, l cube.

Some characteristic volume that is means cube of the characteristic length, F i we already know. So, therefore, this ratio between the elastic force and the inertia force is proportional to E l cube divided by rho l square V square. So, therefore, this becomes is equal to E; what is that? E l cube, sorry, l square because modules of elasticity is the force per unit area, I am sorry, E by rho V square. So, this is known as Cauchy number. I am sorry, I could not mention the earlier one, you have written it earlier one is known as Weber number. You must know this name; this is named after the German scientist Weber, the German scientist Weber, sigma by rho V square by l; that the representative magnitude ratio of surface tension to inertia force Weber number.

Similarly, representative magnitude ratios of elastic to inertia forces, which it is the dynamic similarity criteria parameter for the flow situation, the elastic force is dominant or important, this is known as Cauchy number. He is a mathematician, French mathematician, Cauchy. Now, Cauchy, Cauchy whatever you spell is proper noun, Cauchy number. So, now this number, this term can be expressed in a little different way, reciprocal of this and make a square root then you get; that means, equality of this number E by rho V square means, the equality of this number does not matter because you take reciprocal of this number with a square root we get this.

Now, you know the bulk modules of elasticity in case of gaseous system; we have to specify another constant for the path for which the pressure and volume changes. So, if we make the changes in an isentropic process E s, then this term will come V by route over E s by rho and exactly for the flow situation (( )) elastic force is important, the flow is so fast, the velocity is so high.

So, that elastic force becomes important and compressibility comes into consideration, any change in the density or volume due to pressure is considered very fast and so that, this is consider to be adiabatic carmo dynamically a first process means an adiabatic process because there is no time for it to be transferred and also with in very small zone, the change in density occurs. So, that it is friction less. So that, it is we always think this process as isentropic process. So that, the modules of elasticity, isentropic modules of elasticity is used and this can be shown by the theory of physics is equal to a. what is that a? Acoustics velocity, acoustic velocity or velocity of sound in the medium at that condition; this I told you earlier in relation to difference between incompressible and compressible rho. So, this is equal to the velocity of the flowing medium to the velocity of sound, that is the acoustic velocity in the medium at that condition and this ratio means the equality of Cauchy number or the magnitude ratio of F e by F i. This is proportional to this, that means one can tell that this ratio in terms of V by a, that is the velocity of the flow medium to the velocity of the sound in that medium at that condition. So, this number is known as Mac number, which is conveniently use to represent the dynamic similarity for flow situations were elastic force is important and known as Mac number, where Mac is the name of an Austrian physicist that Mac number.

Now, we come to a very important query. Now, we see that thing practical problems; what we have concluded that in practical problems for dynamic similarity to be obtain to obtain the dynamic similarity, we require the ratios of forces, ratios of corresponding forces at similar points to be equal between the two systems at different operating conditions and we that, thus we arrive that a number of dimensionless terms arises. Now, in a particular situation of fluid flow, sometimes it is difficult to find out from the force concept, that the ratio of forces proportional to certain non-dimensional term, combination of the dimension is variables, but it is easier to recognize the problem by their dimensional, by its dimensional variables. Now, there must be some easier way to find out what are this non-dimensional terms, which cover the similarity criteria of the problem or which ultimately define the problem. Now, let me put the question like that, if there is a problem defines by a number of variables, some m number of variables.

So, if that particular problem has to be made similar with a problem at different conditions; that means, there are several same class of problem, but under different set of conditions, then definitely certain dimensionless terms have to be kept fixed for both the problems, which represents the criteria of similarity, mainly the dynamic similarity. Now, question is that, how do you find what are such combine or such non-dimensional parameters? Which are the combinations of dimensional variables, it is not always easy to recognize the different forces governing that flow of fluid and there magnitude ratios, number two as we have already appreciated that in pi flow problem. For example, that the pressure drop is a function of so many parameters like, velocity of flow, diameter of the pi, density viscosity, but we have finally recognize that, the same problem is can be describe by the two dimensionless parameter; one is friction factor, another is Reynolds number. So, therefore, the two things come into one; that means, if problem can be made similar provided the dimensionless parameters, few dimensionless parameters are get fixed for problems at different alter conditions of the same physics and at the same time, this problem of a particular physics can be describe by this dimensionless parameters, less number of parameters rather than the dimensional parameters.

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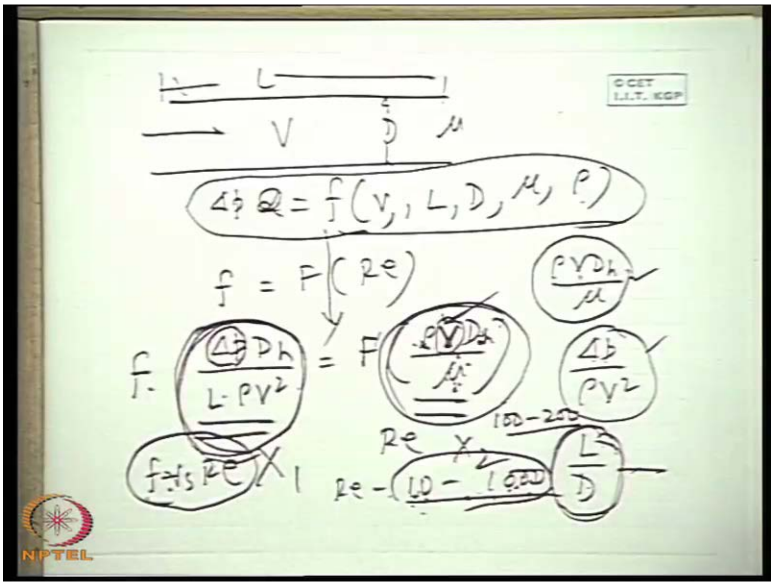

So, now the question comes, how can we find out this dimensionless parameter, but for an example, let us just see this thing; let us see that a pi flow problem. We know that delta p is proportional to depends upon what? delta p depends upon the velocity of flow; depends upon the length of the pi; depends upon let this is length; depends upon the diameter of the pi; depends upon the viscosity of the fluid; depends upon the density of the fluid, but at the same time we know that for similarity concepts. Now from our previous knowledge, we know that friction factor becomes a function of Reynolds number only.

What is friction factor? If we define friction factor, friction factor is delta p D h by L rho V square, I am not doing all with the half thing and this is a function of rho V D by mu. Now, from the dimensional from the concept of similarity, we will see that this two terms are the terms which define the dynamic similarity parameters, dynamic similarity parameter is define by rho V D h by mu as Reynolds number. Then delta p by rho V square as the violon number and definitely L by D for the domestic similarity, but these three dimensionless parameters representing the dynamic similarity, may be also clubbed in this form. So, that the entire problem may be described as a parameter, this is a function of parameter, this let this parameter is X 1 as a function of X 2.

Now, question is that, if it is so obtainable then it is very fine. This is because if we feel that the only dependent independent variable is this group, then we can vary any one of this and we can make, we can predict the variation of other parameters; for example, by varying V, we can cover a range of R e. For example, R e from 10 to 1000, we cover the range of R e and we can predict the friction factor as a function of this R e by a single curve and by a limited number of experiments only varying V from 10 to 1000, but if this is possible to express the same phenomena by this two dimensionless parameters.

That means, that it is possible to credit the variation of other variables to the variation of V only because a change in R e may be effected by change in any of the parameters. If I change tell that, R e is doubled 100 to 200; it may mean that, it is doubled either by doubling velocity or by doubling density or by doubling diameter or by having the viscosity. So, it does not matter which one has been change; that means, by varying one parameter it is simple to vary velocity of flow, we can cover a range of Reynolds number and we can predict the friction factor from the similarity point of view.

If a system occurs is perverse under two different conditions. So, that the range of Reynolds number is same, then the friction factor versus Reynolds number curve will be equally apply, will be equally applied in both the cases. Only thing is that, one has to read the friction factor from a given Reynolds number and accordingly he as to decoct for his pressure value depending upon its value of diameter is value of velocity, for his prevailing in each set of conditions, but the range of Reynolds number has to be made same. So, that we can make the dynamic similarity and the same time the L by ratio as to be same because the geometric similarity has to be maintained.

So, this is very simple and we understand this, but the question comes in such complicated problems, different problems. How do you know these combined dimensional variables? As the dimensionless parameters, which define the problem which is other way explained by a number of dimensionless variables, that is very simply given by two mathematical theorems. One is due to Buckingham, that is the name of the scientist; another is due to Rayleigh and this two theorems are known as Buckingham’s theorem and another is Rayleigh’s theorem and this two theorems, the analytical theorems or analytical procedure is based on one condition, that condition is the dimensional homogeneity of the physical variables involved in the problem and how it is done you see.

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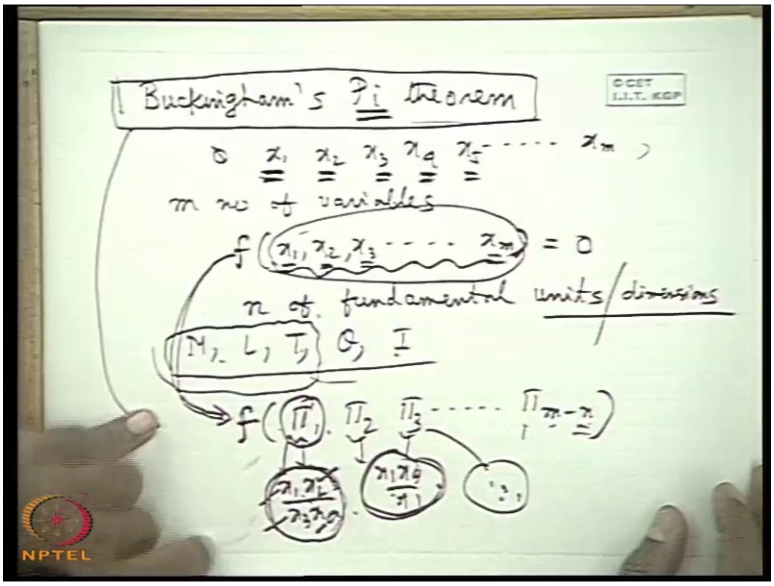

The two problems, one is Buckingham problems and this is known as Buckingham's pi theorem. Why the (( )) what pi comes Buckingham's describe this dimensionless terms has pi theorem. What is this? Then Buckingham's pi theorem is the most or the easiest method to develop the non-dimensional terms; now, our basic objective is like that for example, if a problem, if any physical problem is described by m number of physical variables for example, if you know pi flow problem, if described by pressure drop, length of the pi; diameter of the pi; density of the pi; viscosity of the pi. For example, if free z problem is define by the diameter of the z, defined by the density of the z, viscosity of the z, surface tension between the z and the ambient medium the velocity of the z.

So, we can definitely recognize first, this reorganization comes from our brain; that from our understanding of the physical problem. No theorem before and will tell me and no note book will tell me that open note examination, that no note book will tell me, that what are the variables? That defines a particular problem. If we failed to recognize very important here, that surface tension is one of the dominant variables, which plays the role in case of flow of a free z or flow of a liquid through very small diameter in the order of

1 millimeter or less that is our fault. So, we will have to find out that, what are the variables? Physical variables that control the flow.

So, let us consider there are some m number of physical variables controlling or defining or describing a particular physical problem, then analytically one can express the physical problem like that; that means, if m number of physical variable like x 1, x 2, x 3. This are viscosity, density, velocity, pressure like that, if m number of physical variables are involved in describing a problem, then we can express the problem analytically like this, function of x 1 to x m is equal to 0; in by an implicit functional relation m number of physical variables.

Now, Buckingham's told that, if these m variables are expressed as n number of fundamental, n number of fundamental units, that if m variables are expressed as n number of fundamental units or fundamental dimensions, rather I write dimensions. What are fundamental dimensions as you know? The fundamental dimensions are mass length, time temperature, current intensity or laminous intensity; these are the fundamental dimensions. In fluid flow, we are only involved with mass length and time not even the temperature.

Please do not talk why you talk. So, much I do not know it is very shameful affair for I it students I am telling. So, much you talk those you talk you please go out do not create a disturbance others are also listening to this lecture. So, they will know that I it students are always talking like a school boys even if their teacher tells like that this is the scenario that happens I am sorry to other listeners who all listening to this course that it is only main for IIT students who are attending the course inspite of telling. So, many times they are go on chitchatting in the class and probably this is the usual train that you can see that we will have to tell. So, many times you people are IIT that you please keep silence and really I am sorry and I have to be excused by other listeners that I have to do that because simultaneously I have to conduct a class of IST students who are continuously gossiping and having a chitchat in the class.

Inspite of telling. So, many times I feel sorry to tell them and remind them again and again, but please be serious in the class. n is the number of fundamental units of dimension in which this m variables have to be expressed. If this is so then Buckingham first proof that, this physical problem may be describe by dimensionless terms, which he expressed at pi terms; these are the dimensionless term by m minus n dimensionless term; that means, from m number of variables the number of dimensionless terms is reduce to m minus n. Where n is the number of fundamental dimensions in which the m variables can be express and this pi terms are all the combinations of the certain combinations, this may be x 1 x 2 by x 3 x 4; x 1 into x 4 by x 1.

So, this pi terms are the dimensionless term; that means, certain combinations of the dimensional variables and this as a physical meaning, all this dimensionless term represents the criteria of similarity; that means, again I am telling that a physical problem, which is describe or define by m physical dimensional variables is similar to the description or you can tell, can also be describe in terms of m minus n non-dimensional variables, instead of dimensional variable by m minus n non-dimensional variables. Where n is the fundamental dimensions in which the m variables can be expressed.

So, therefore, we see, we can recognize a less number of parameters, dimensionless parameters from the dimensional parameter to describe the same problem and we have already recognized or we have already seen the beauty of it, through the pi flow problem. Just an example, I have given that, if we reduce the number of variables from dimensional form to non-dimensional form; that means, we can vary one of the variables to vary the non-dimensional terms, any non-dimensional terms. So, that the effect of other variables defining the non-dimensional terms are clear or can be implemented. So, this is basically the Buckingham’s pi theorem.

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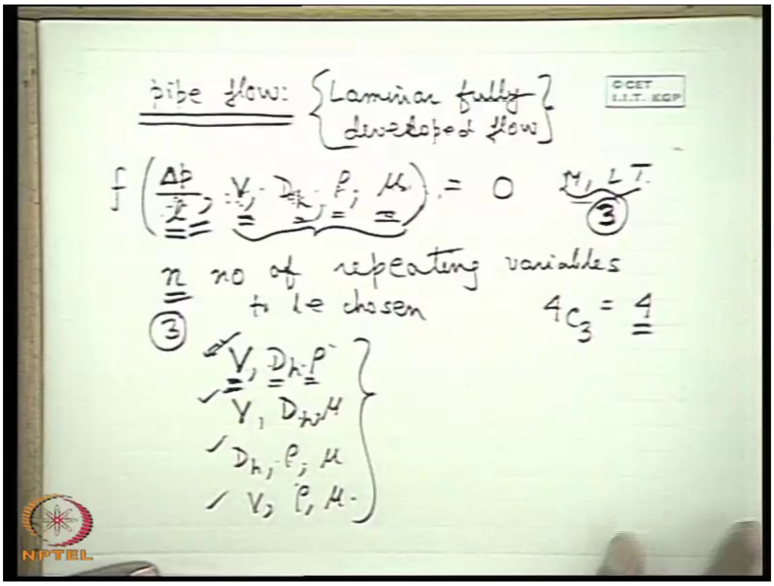

So, first we learn the Buckingham pi theorem. Now, let us see that, how Buckingham pi theorem is explained? In case of a pi flow Laminar, Laminar, Laminar fully developed flow, laminar fully developed flow, fully developed flow; now in case of laminar fully developed pi flow, it is true that we always consider the pressure drop per unit length. Why? Because, length is not a pertinent operating parameter length of the pi can change and accordingly the pressure drop will change and it has been found the pressure drop per unit length is the criteria because in a laminar fully developed flow pressure drop becomes the linear function of L.

So, pressure drop per unit length is the output parameter, which depends upon several input parameters or operating parameters of the problem, but what we think in this way, that a pi flow problem is describe or defined by what variables first is pressure drop per unit length. Another variable is velocity; another variable is pi diameter; another variable is rho; another variable mu. So, therefore, we see that these 1 2 3 4 5, this five variables define the pi flow problem. There is no other variable. So, pi flow problem is define by its flow velocity, pressure drop per unit length, diameter, rho and mu.

So, we can write, that a pi flow problem can be mathematically express by an implicit functional relationship of this, but what is the functional relationship explicit form? That is very difficult to know, either one has to find out from experiments or one has to make a theoretical analysis. Of course, laminar flow the theoretical analysis comes from an exact solution and one know the Hagen-Poiseuille equation, which exactly defines the relationship between this parameters, but in turbulent flow one cannot do theories so easily, tubules models have to be employed and they are dependent on experiment. So, that one can find out from experiment, that what are the functional relationship? In that case, what he as to find out first of all he has to recognize that, this is the output parameter.

So, this is a function of all these input independent parameter so has to vary this keeping all this constant, then influence of this on this. He as to vary this keeping other constant influence of this on this, but without doing that, if we know that, this phenomena can be reduced that the to describe the same phenomena. We can reduce the number of variables from so many to a lesser one in non-dimensional term, that is better; now, we will apply Buckingham’s pi theorem to that, how to find out the pi term? So, principle is like that you consider first three variables out of the input parameter.

First recognize which one is the output parameter, that is pressure drop per unit length. This is the predicted parameter to be predicted. So, out of this input parameters, you choose n number of repeating variables, what is meant by repeating variable is will be clear afterwards, just in the next step. So, n number of repeating variables to be chosen, first to be chosen; n number means, n is the fundamental dimensions in which the variables have to be expressed. Now, here the variables are expressed in how many fundamental dimensions, all the variables are expressed in M L and T.

We require only three fundamental dimensions not more than three though in general physics, the fundamental dimensions are more than that, but here in this problem we require only three fundamental dimensions; mass, length and time to discuss the problem. So, therefore, three numbers of repeating variables we choose, let us first choose V, D h and rho. Let us first choose the parameter V, D h and rho, let us choose the V, D h and rho as the parameter; V, D h and rho. Now, Buckingham told that, now you can test there any three; we can choose their maybe 1 2 3 4, four combination, four parameters are there. So, combination of three repeating variables is 4 C 3 that is 4 combinations are there, they are V, D h and rho. D h is the hydraulic diameter; V, D h, mu; D h, rho, mu; V, rho, mu. So, we can choose 1 2 3 4. Four choices are there, now there are certain rules for choosing the repeating variables.

So, repeating variables should be chosen from the input parameter excluding the output parameter or performance parameter and repeating variable should contain all the fundamental dimensions not in a single one, but collectively; that means, if I choose V, D h, rho all the fundamental dimensions are there, if I choose V, D h, mu you see all the fundamental dimensions here; probably in all the repeating variables choice for all the three fundamental dimensions are there. So, we are little bit misled. Let us see, I will explain there is no misleading thing, but let us first try with V, D h, rho has the independent parameter.

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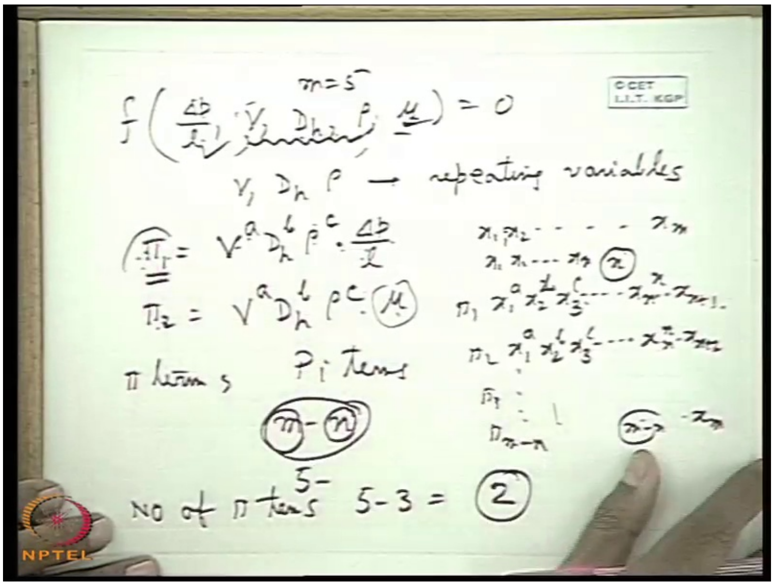

Now, let us start with that the problem is defined by delta p by l, V, D h, rho, mu. So, through a problem only I am defining; that means, first step of Buckingham's pi theorem is choose any three repeating variables, let us choose V, D h, rho as repeating variables. The rule is that, these repeating variables should not include the performance parameter and should include all the fundamental dimension. Then set the pi terms by raising the repeating variables to an unknown integer exponent and any of the remaining parameters; that means, V, D h, rho.

So, remaining parameters means, one is delta p by l and another pi 2 will be V to the power a, another pi term will be this a b c's are different, another remaining pi term mu; that means, if there are x 1, x 2 to x m variables are there, if there are n number of fundamental dimensions, choose n as repeating variables; that means, x 1 to the power a, x 2 to the power 2 V, x 3 to the power c dot dot dot x n to the power n times, the remaining variables; that means, x n plus 1 like that, x 1 to the power this is the general description x 3 to the power; that means, first we choose x 1, x 2 to x n, some n variables as the repeating variables to the power n x to x n plus 2. So, this way we will be getting only m minus n number of terms n plus 1, n plus 2 like x m. So, therefore, m minus n pi terms; that means, you choose three variables, three fundamental dimensions. We choose any three as the repeating variables by the rule, that is should exclude the performance parameter. It should must contain the fundamental dimension, then when I choose three here m is equal to 5. So, two variables are left. So, one pi term; that means, this pi terms are the, this is known as pi terms.

Pi terms; that means, non-dimensional term will be found out by raising these repeating variables to arbitrary unknown integer as index into multiplied by one of the remaining, one of the remaining variables delta p l, next pi term is another remaining variable. So, here will be two pi terms because the Buckingham pi theorem tells that, m minus n. So, number of pi terms first is fix. So, number of pi terms first number of pi terms is fixed according to Buckingham pi theorem. If m is the variables defining the problem, if n is the fundamental dimension in which it is expressed. So, number of pi terms is m minus n. So, in this case 5 minus 3. So, 2 pi terms, it is already decided.

How to find out two pi terms that, pi one is this; pi two is this, but we have to know the value of a b c to find out this pi one and pi two terms, expressively this is done by equating the fundamental dimension, this is dimensionless term.

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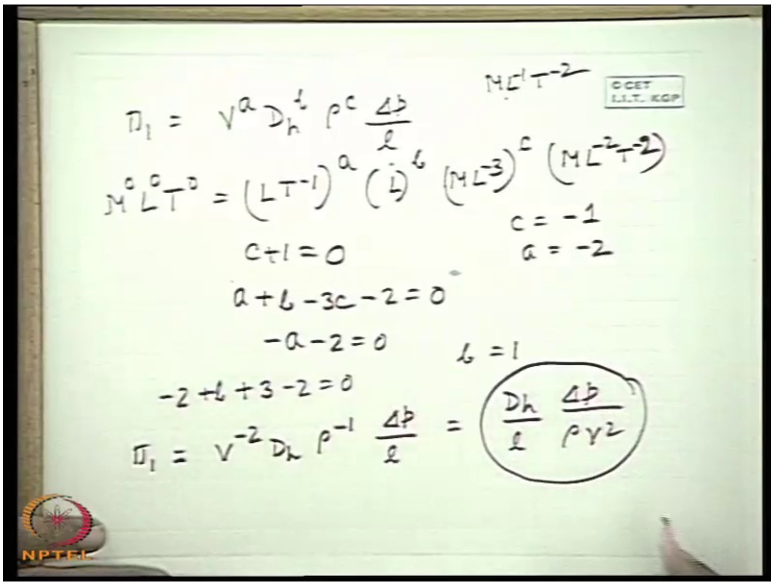

Let us find out know this very good, pi 1 is V to the power a; D h to the power b; rho c; delta p by l. what is pi 1? It is dimension is M to the power 0, L to the (( )) dimensionless term. What is V? V is L T to the power minus 1 to the power a; what is D h? L to the power b; what is rho? M L to the power minus 3 to the power c; what is delta p by l? Please tell me, delta p is M L to the power minus 1, L to the power minus 2, T to the power minus 1 because delta p is what? p is M L to the power minus 1, T to the power minus 2, that is force by area.

So, divided by L, L to the power minus 2, T to the power minus 2 very good. So, you solve it and what you will get? If you solve it M 0, so what is the M? C plus 1 is equal to 0 C plus 1; L 0 equates the exponents, what is L 0? a plus b minus 3 c minus 2 is equal to 0, T is 0; that is minus a minus 2 is equal to 0, which gives C is equal to minus 1, a is equal to minus 2. So, minus 2 plus V, C is equal to minus 1, means plus 3 minus 2 is equal to 0; what is b? 3 minus 2 1, 1 minus 2 b is equal to 1.

So, pi 1 will be V is equal V to the power a, a is equal to minus 2, V to the power minus 2, D h 1 rho to the power C is minus 1 rho to the power minus 1 delta p by l. So, this will be D h by L delta p by rho V square, you see this is precisely the first pi term which is the friction factor.

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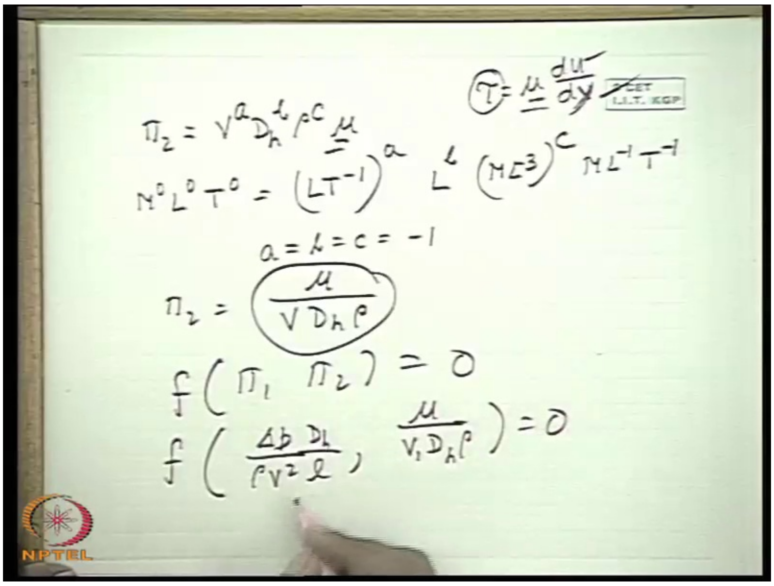

So, this is pi 1; second pi 2 quick second pi 2 terms is V to the power a, D h to the power b, rho to the power C and another term is mu. So, M 0 L 0 T 0 is equal to L T to the power of minus 1 a, Dh L to the power b, M L to the power minus 3. What is the dimension of mu? This you will have to know; M L to the power minus 1, this we will have to know or you will have to derive, where from you will derive the dimension of mu, tau is equal to mu d u, d y. You know the dimension of stress, you know the dimensions of u, you do not know the dimensions of y; this is done in school level, I know. So, now you cannot tell me that, I know this dimension of mu is done at school level, M L to the power minus 1; T to the power minus 1. If you do that and solve it by equating the exponent, you will get a is equal to b is equal to c is equal to minus 1. So, pi 2 is mu by V D h rho. So, therefore, the problem is described by pi 1, pi 2 0. So, this becomes is equal to delta p by rho V square into D h by L and mu by V D h rho as 0. So, this is pi 1 and this is pi 2.

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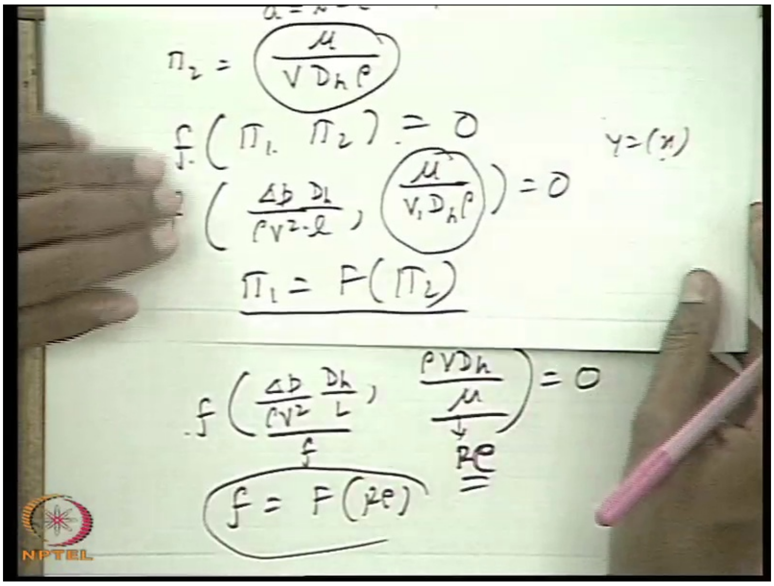

If you make this reciprocal then we get that, this is delta p rho V square D h by L, reciprocal of this; that means, if it is governed by this parameter, it does not mean it is same that it is governed by the reciprocal. If y is a function of x means y is a function of 1 by x also because this is the conventional Reynolds number. So, therefore, this is friction factor f is equal to 0. So, this can be expressed as friction factor is a function of R e; that means, function of pi 1, pi 2 0 means pi 1 is a function of pi 2, pi 1 is a function of pi 2; that means, friction factor is a function of R e. So, friction factor is; that means, we get the relationship therefore, we say that this problem is define by this two function, this pertinent dimensionless terms. So, now after this I like to tell you that, the very important thing that we have several other choices for the repeating variables.

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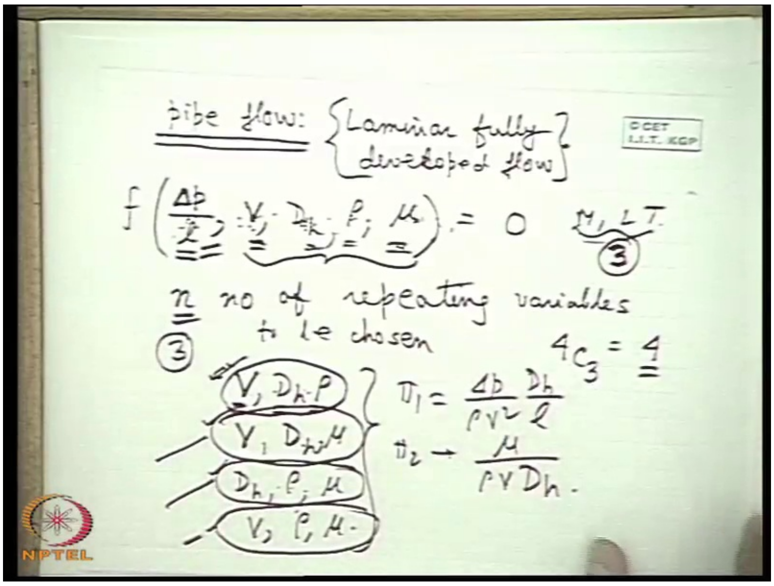

So, this choice give this two parameters pi 1 and pi 2 like that, delta p rho V square D h l and pi 2 as mu by rhoV D h, but what are the pi terms? We landed of with if we choose these three sets as the repeating variables, that I will discuss in the next class.

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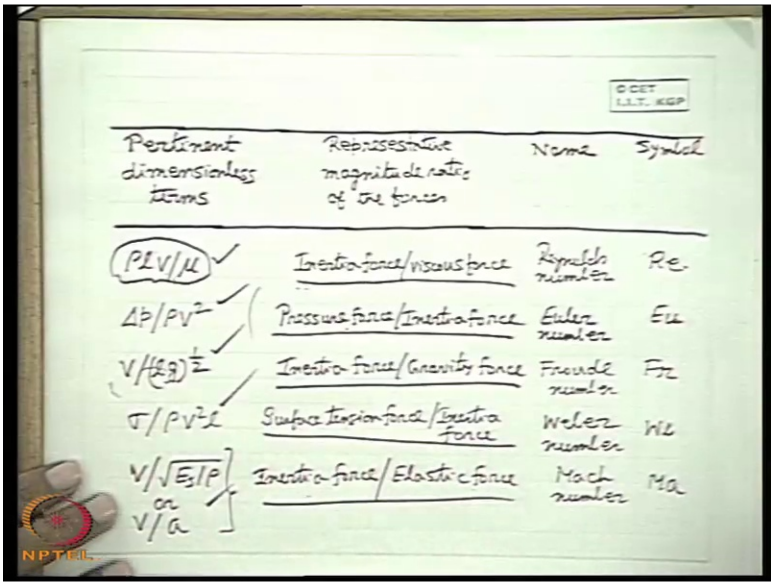

So, at the end today I just tell you that, while discussing at the beginning which is very important that pertinent dimensionless terms, which have come the representative magnitude of forces. You must remember this, rho l V by mu is the inertia force by viscous force that is Reynolds number. In fact, everywhere inertia force comes at the denominator, but here inertia force is numerator, but there is no such conventions sometime in some problems the reverse of this or the reciprocal of this is defined as Froude number; surface tension inertia force the elastic force inertia force by elastic force.

This is the Mac number, this is the definition of course, the inertia force is at the numerator this is because that all this numbers were not originated first from the principle of similarity. It came just from the intuition by doing an experiment, while Reynolds first started doing an experiment with the pi flow in a laminar region by seeing that, how the flow condition takes place from particular one to another type; that means, laminar to turbulent. We first found out the combination as the pertinent governing parameter. So, this was named after his name as Reynolds number ultimately from principle of similarity his intuition was there; that probably this combination physically signifies something and the principle of similarity came out. So, it was inertia force by viscous force; sometimes the Mac number inertia force by elastic force, but somewhere the (( )) number is pressure force by inertia force. So these are the conventional form, where this numbers are define and they are the representative magnitudes of certain forces as the criteria of dynamic similarity is very important.

Thank you.