Fluid Mechanics
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Lecture - 3
Introduction and Fundamental Concepts – III

Well, good afternoon to all of you to this session of fluid mechanics. Well, last class we were discussing about the surface tension, and we ultimately came to the conclusion, when two invisible fluids define an interface, then the concept of surface tension comes. That means, the inter-phase is under a stretched conditions, and mechanical energy stored in the surface that we recognize from the fact that, if we create an interface; that means, interface of a liquid for example, separating a gas, then liquid molecules have to be bought from the interior of the bulk of the liquid to the surface, where work is done against the inner intermolecular force of cohesion. By virtue of which, a mechanical energy is stored in the surface.

So therefore, in an interface, it is in stretched condition and surface tension is the force exerted on an imaginary line on the surface; per unit length of the surface. This we appreciated in the last class. Now, it is because of the surface tension effect, a curved liquid surface separating a liquid and a gas creates a higher pressure in the concave side as compared to that in the convex sides. This is one phenomenon observed.

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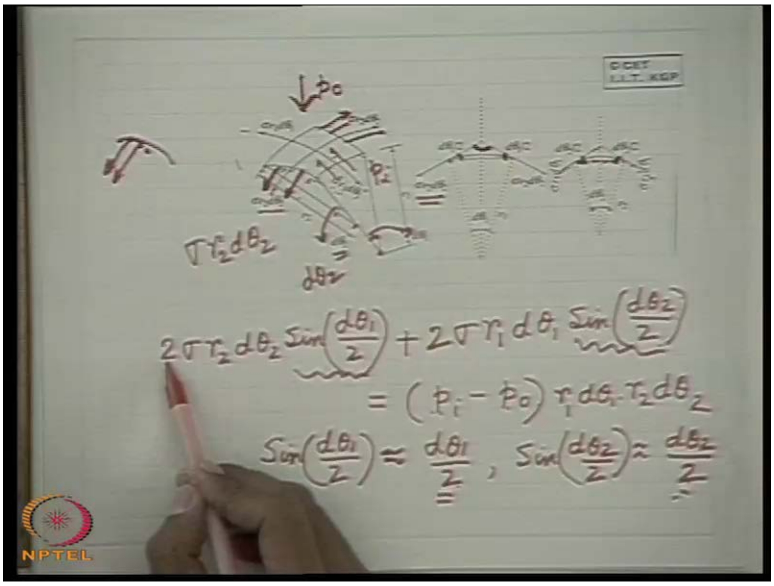

Now, let us see this thing here. Let us see here. This is a curve; elemental curves liquid surface, which distinguishes or separates the bulk of the liquid on its concave side. That means, this side is the bulk of the liquid and this is the convex sides and this side is the air. For example, air or any invisible fluid or any gas. So, this is a small curved elemental liquid surface.

Now, let us see that the surface is curved in both the directions, in such a way that the radius of curvature in this direction is r 2, which is less. That means, it is more curved and in another direction is r 1 and the lengths of the curved surfaces are such that they sustained angles of d theta 2; that means, this is d theta 2. This angle is d theta 2. This is d theta 2 and this is d theta 1. This is d theta 1. This angle d theta 1 and d theta 2 at the respective centre of curvature. These are the respective centre of curvature. This is r 1 and this is r 2. This is curved in both the directions; elemental curved surface. So, this side; that means, the convex side, this is gas and in the concave side, is the bulk of the liquid. We only consider a small elemental curved portion of the surface.

Now, we see the surface tension force acts on these 4 sides. Now, these 2 sides, which are of radius of curvature r 2, the surface tension force acts perpendicular to this linear element, you see, whose lengths are r 2 d theta 2. Therefore, this force is sigma r 2 d theta 2, which is written here. I think you can see. So, this is the surface tension force acting, which is perpendicular to this curved element and it is sigma r 2 d theta 2. r 2 d theta 2 is the length of this element.

Similarly, the surface tension force, which acts over these elements, this length, this is sigma r 1 d theta 1 because the length of this curved line is the r 1 d theta 1. So, according to the definition of surface tension, the total force is sigma r 1 d theta 1 in this direction. Now, if we look, this is a three dimensional view. If you look two dimensional view; that means, if you look from this angle, therefore, we see only this part of the curve like this, where this force is acting on like this. I am showing it by only 1 arrow. So, this is acting like this. So, this is sigma into r 2 d theta 2. All the forces sum up with this sigma into r 2 d theta 2. So, this will appear like that; sigma into r 2 d theta 2. So, this is d theta 1; and that means, you see these few.

Well, so from very simple geometry, you see this is d theta 1. So, this angle will be d theta 1 by 2, this angle. This angle, why? This is because this angle will be this is 90 degree, because this is tangent, this direction. So, this and this radius of curvature will be perpendicular. So, these angles are 90 degree, so that, this is 180 degree minus d theta 1. So therefore, these angles will be d theta 1 by 2.

Similarly, if we see a view from this direction, we will see the forces are acting which comes; which are the forces? These forces see, if you see this view, these forces are acting sigma r 1 d theta 1 sigma r 1 d theta 1 and these angles are d theta 2 by 2, because this is d theta 2. From simple geometry, this angle, this is d theta 2 by 2 and this is d theta 2 by 2. See, if you can recognize this, then a simple force balance. Now, let us write assuming that the convex side of this liquid surface is acted on by a uniform pressure p 0, that is the pressure of the gas, which demarcates this liquid. Similarly, the interior of the liquid, the pressure is p I, which is acting perpendicular to these surfaces; that means, in its concave side. Let, this is p i.

Now therefore, if we make a force balance in the vertical direction, then we get the surface tension forces acting sigma r 2 d theta 2. Two surfaces, so 2 sigma r 2 d theta 2 into sin d theta 1 by 2, sin component. This is d theta 1 by 2 vertical direction. Similarly, the force component due to sigma r 1 d theta 1 on both sides; that means, this side and this side, which is shown like this is twice sigma r 1 d theta 1 sin, well, d theta 2 by 2 and this net force in the downward direction will be balanced by the net pressure force in the upward direction. That means, p i minus p 0 times this surface area over which the difference of pressure; that means, each individual pressure is acting over a surface area.

So, pressure force is the pressure times the surface area. So similarly, p i into surface area minus p 0 into surface area p i minus p 0 into surface area, which is the product of these two lengths; that means, r 1 d theta 1 into r 2 d theta 2. Now, with a simple relationship from trigonometry, that for small angles, because this is a small elemental curved surface, where the sides subtends small angles d theta 1 and d theta 2 at the respective centers of curvature. We can write that for small angle, the sin of the angle is equal to the angle.

If you see this series, the series for sin function, you will see, we can neglect the higher order terms. So, you can approximately write d theta 1 by 2. So, you know this thing that sin theta is theta. Similarly, d theta 2 is equal to d theta 2 because the higher order term in this series can be neglected. So, if we now substitute these in terms, in place of sin;

that means, in place of sin d theta 1 by 2, I substitute d theta 1 by 2 and in place of sin d theta 2 by 2, I substitute d theta 2 by 2.

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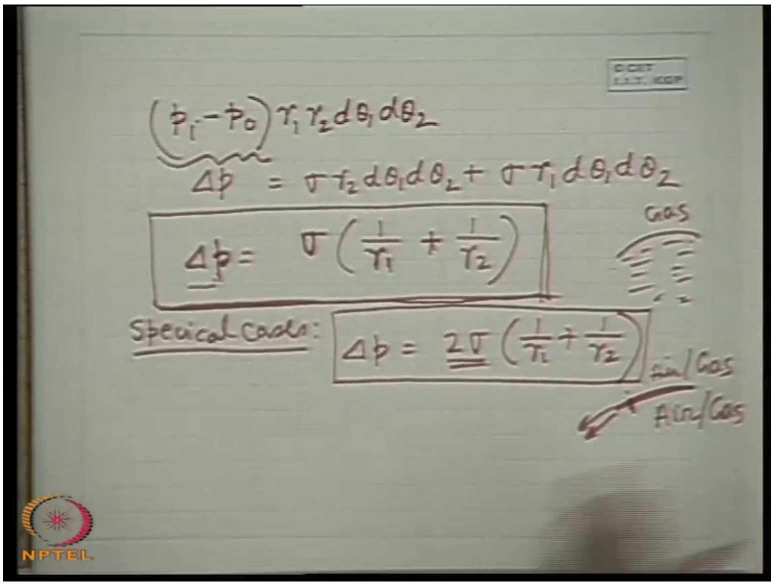

Then, by simplification, simple simplification, I get this p i minus p 0 into r 1 r 2 d theta 1 d theta 2 is equal to, what we get? sigma r 2 d theta 1 d theta 2 plus sigma r 2 into, what? This is sigma r 1 into d theta 1 d theta 2. So ultimately, if I replace this as delta p define p i minus p 0, we get sigma into 1 by r 1 plus 1 by r 2. So, this is a very very important formula. Now therefore, we see that there is a difference in pressure created. That means, the concave side is having a higher pressure p i, then the convex side, where the gas is there and concave side is the bulk of the liquid and this difference of pressure is given by the surface tension coefficient and the radius of curvature of the two sides. This is the general case of a liquid surface, liquid interface, which is curved in both directions.

Now, special cases are there. Now, first special case. Now, before coming to special cases, first I discuss that, if, now here, we have assumed a surface which demarcates a bulk of the liquid on this side. Try to understand, in the concave side and a gas on the convex side. But if the surface is a thin liquid surface, which has got in contact in both the directions, both the sides; that means, convex and concave side, the gas, then what happens? This surface tension force will be doubled because the surface tension acts from both due to top portion of the gas and due to bottom portion of the gas; that means, this is the surface and air is there at the top and air is there also at the bottom. For example, air. So therefore, from the bottom and the top, we get at the same location, the surface tension forces twice. That means, this will be multiplied with the 2.

So, in case of a thin film, the curved surface as a thin film separating gases both on the concave and the convex sides, convex and the concave sides, then simply this will be doubled. That means, this is delta p will be 2 sigma into 1 by r 1 plus 1 by r 2. Again, I tell the difference between these two, this is the pressure difference between the liquid surface, a curved liquid surface, which demarcates a bulk of the liquid and a gas outside it. That means, there is a liquid surface, where this is the bulk of the liquid and this is gas. So, this is the pressure difference where. Where is a film of liquid, thin film of liquid, where both the sides air or gas is there, in that case what happens? The surface tension occurs both by the pull of the interaction between the air molecules and the liquid molecules and this side and air molecules and liquid molecules on the upper side. So, 2 times multiplication will be made here.

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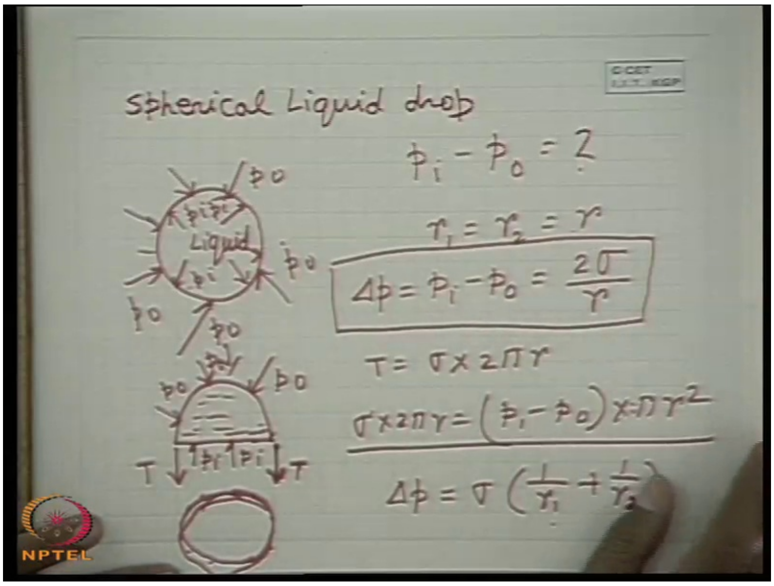

So, this is the formula for thin film or thin curved surfaces. Now, few special cases I now just discussed. One is the spherical liquid drop. One is the spherical liquid drop. So, spherical liquid drop means a sphere; that means, it is a sphere full of liquid, full of liquid, full of liquid. So, if outside pressure is p 0. For example, uniform out ambient pressure is p 0. Uniform ambient pressure is p 0. So, due to this curvature, because of surface tension, the inside pressure will be p i. So, in the liquid, the pressure generated inside is p i. So, how I know this p i minus p 0? So, simply I see this formula, sigma into 1 by r 1 plus, what is the difference here? Here, r 1 is equal to special case, r 2 is equal to r. So, delta p is equal to p i minus p 0 is simply 2 sigma by r, where r is the radius of the sphere.

This of course, come can come from the simple fundamentals without using the formula that you can also make, that you cut. You can see how the forces act that you can cut the surface. Then you take a half liquid sphere as a free body diagram. Then we can understand that here the surface tension force T acts; that means, throughout the periphery; that means, if you see from the top view, you see this is like this. Sorry. This is like, sorry, this is like this. That means, through the entire periphery, the surface tension force is acting and this value of T is sigma into twice pi into r, if r is the radius of the sphere. That means, it is in the entire periphery; that means, if you cut the surface, this is the liquid and then you get a surface tension force coming as an external force. So, these are the external p 0 as it is outside p 0. These are the p 0 as it is and pressure force p I, then comes as an external force like this.

So therefore, you can write sigma into 2 pie r is equal to p i minus p 0 into pi r square force balance. So, this will give you the same result. It is not necessary to do it from fundamentals. Since we know that the general expression that delta p for a liquid surface curved in both the directions as sigma into 1 by r 1 plus 1 by r 2, we simply, in case of a spherical liquid drop, put that r 1 is equal to r 2 is equal to r.

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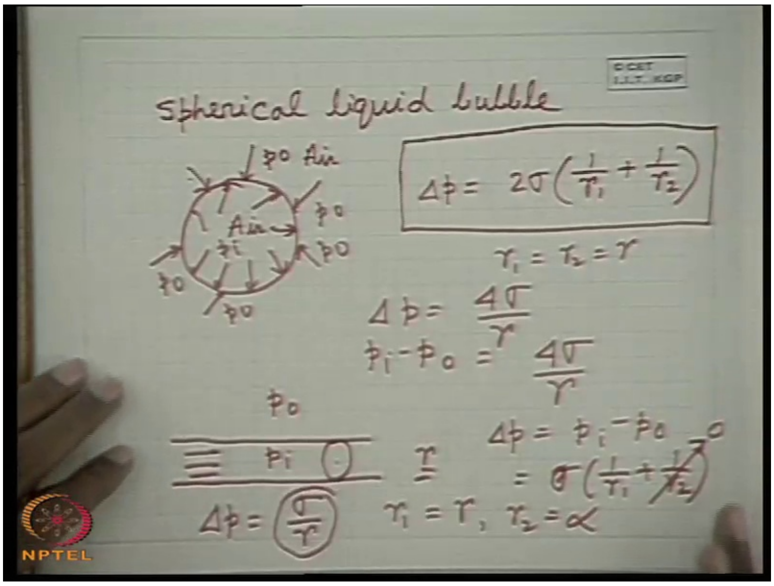

What happens in case of a spherical liquid bubble? Difference between the two is that spherical liquid bubble. In a spherical liquid drop, air is on one side and inside the liquid drop, the full is water or the liquid. Here, bubble contains air inside and outside; that means, it is this special case of this 2 sigma; that means, in case of a curved surface, which separates air on both the sides or gas on both the sides, concave and convex sides, the relationship is like that. So here, if I put r 1 is equal to r 2 is equal to r, we get delta p is equal to 4 sigma by r. So therefore, delta p p i minus p 0 becomes equal to 4 sigma by r.

That is, p i is the, well, p i is the inside pressure and p 0 is the outside pressure. p 0 is the outside pressure. Another limiting case, special case is z. What is z? It is a circular cross section. If z of a circular cross-section of radius r, then what formula we will use? It is a solid z; full of liquid. So, you will use the formula, so if the internal pressure is p i and outside ambient pressure is p 0, then p i minus p 0 will be what? 2. Not 2. It is sigma into 1 by r 1 plus 1 by r 2. You will use this formula. Where we will use? r 1 is equal to r and r 2 is equal to infinity. Very good.

So, this becomes 0. So, delta p simply becomes equal to sigma by r. So, this is the value by which the inside pressure or internal pressure in a straight liquid z is more than that of the outside ambient pressure.

Now, well, I come to another phenomenon of surface tension known as capillarity. Probably you know this. You have already read at school level that, if in a beaker or in a container of liquid, if we dip a tube, we see that liquid rises in that tube. Sometimes, liquid rises in that tube from the level of the liquid in the container in that tube. Who is responsible for rising of that liquid in that tube or who holds this extra amount of liquid height in that tube? That is purely the phenomenon of surface tension or the surface tension force is responsible for that.

In some cases, we have seen that liquid, if there is a container containing liquid and a tube is dipped into it, the liquid should not or does not come up to the surface of the liquid at the container. It goes up to a height, which is lower than the height of the liquid label in the container. So, these two phenomenon known as capillary rise and capillary depression.

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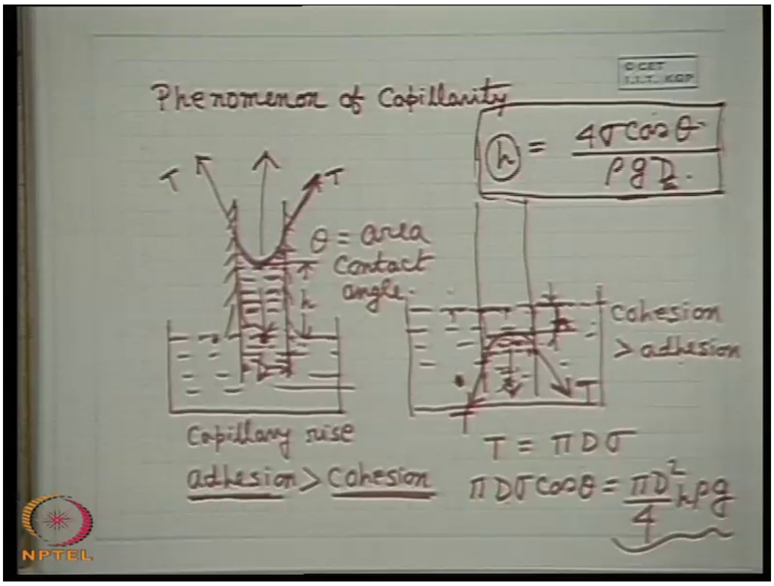

Let us look see that phenomenon of capillarity. What is this? Let us see this. If we take for example, water. Let this is water or any liquid, I can tell and if we dip a tube, you will see the water comes. Take an interface like this. An additional height of water comes like that. Is raised like that. Now, who is responsible for holding this additional height of water? For example, here the pressure is atmospheric pressure, but here, the pressure is the height of this water column. Let this height be h. So, who is responsible to make this dis-balance of pressure? That means, there must be some upward force, which is counter balancing this pressure. Or, other way we can tell, there must be some upward force, which can hold the column of water, which has been raised and this force is the prime cause of raising the liquid through that tube from the label in the container. So, this is known as capillary rise. This happens to those liquids where adhesion between the liquid and the solid tube; this is the solid tube, is more than the cohesion, this happens.

For water in a glass tube, the adhesion is more than the cohesion. So, this is marked in case of water in a glass tube. It is found in some other cases for example, mercury in a, other, here you can mercury in a glass tube; that means, if you have mercury in a truf and put a tube like that. You will see the mercury will not come up here. So, mercury will, the mercury interface will go like that. So, there will be a mercury like that. So, this place, in this place there is no mercury. So therefore, the mercury label here in the container up to this, but mercury will not come up to this; that means, what is the force that is responsible to create these dis-balance; that means, to make up the, here the pressure is atmospheric pressure plus this height of the liquid, but here, pressure is atmospheric pressure plus this height of the liquid. Let this is the depression. This height is the height depression of the liquid. Here, it is the rise. So, that means, this is the depression. So, here the pressure is less. So, there must be some additional force in the downward direction to make a balance between these two. What is that? That is the surface tension force.

What happens physically, now, let me explain. Because of this adhesion being more than the cohesion, the liquid particles are dragged along the solid surface, by virtue of which, the liquid is pulled. Depending upon the surface tension force, the liquid height will be maintained. The liquid highly will depend or depending upon the surface tension force, the amount of liquid will be raised. Let us consider this amount of liquid raised up to the height h. So, under this condition, if this is the interface, we can show that, in this interface here, the surface tension force T acts, surface tension force. This acts throughout the periphery, throughout the periphery in the linear element with an angle.

Let this angle is theta; that means, surface tension, the direction of the surface tension, this is tangent to this surface with these vertical surface of this tube. Let this angle be r theta. This angle is known as area contact angle. So now, this is the force; that means, you see, that if we take the component of these force in this vertical direction, that balances the weight of this column. You write this now, T into, if we consider the diameter of this pipe is D, big D, diameter of the pipe is D. So, T is equal to pi D into sigma.

So therefore, pie d sigma cos theta must balance the weight. If we neglect this portion and only take this cylindrical portion of the liquid weight, so this will be pi d square by 4. That is the area times the height h into rho in volume into g. So, this is the weight of this column of liquid. This is being balanced by the vertical component of the surface tension force. This gives rise to the value of h as, what is h? 4 sigma cos theta, 4 sigma cos theta, 4 sigma cos theta divided by what? By rho g D. Similar expression you will get for the capillary depression. The physical phenomenon is that, in this case, cohesion is more than adhesion. So, for liquid, where cohesion is more than adhesion, the adhesion is less than cohesion, than what happens? The liquid molecules will crowd within each other.

If their inter molecular forces, drag them out from, to come out from the free surface. That means, they come below the free surface. Then what happens? The surface tension force will act in this direction. So therefore, you see the vertical component of this surface tension force act in this direction, which is added up with the pressure of this less height and will make the balance of the pressure here. If this be the h, the difference you can find out is the weight of this column of liquid, which is an imaginary. It was not there. If this column, this liquid could have come up to this column, so this weight could have balanced. So, from that balance, you get the same expression h. So, here one interesting phenomenon comes, that the capillary rise or the capillary depression h is inversely proportional to d for a liquid of a particular value of sigma.

So therefore, this is observed only in small diameter tube. As the diameter of the tube becomes larger, so value of h becomes smaller, so that, we cannot observe it. We do not observe it. That is why we sometimes tell, the capillary rise takes place in a small tube. Or, if you do an experiment in a very small tube, you have to give a correction for capillary rise. It does not mean that physically capillary rise or capillary depression is attached to the small tubes only. Capillary depression and capillary rise is a phenomenon, which is attached to any diameter tube. So, liquid will always rise from this level in the container and liquid will always goes with a depression from the level of the container, depending upon the fact, whether adhesion is more than cohesion or adhesion is less than the cohesion. But the fact is that, this rise is inversely proportional to the diameter.

If diameter is bigger, this rise and depression is so negligible. We cannot observe it. We see that the meniscus in the tube is at the same level of the liquid in the container, where the tube is dipped and the bulk of the liquid (( )). Well, so this is the phenomenon of capillarity. I think that I end it here with the surface tension. Before I close down the lecture, let us go through two very interesting problems.

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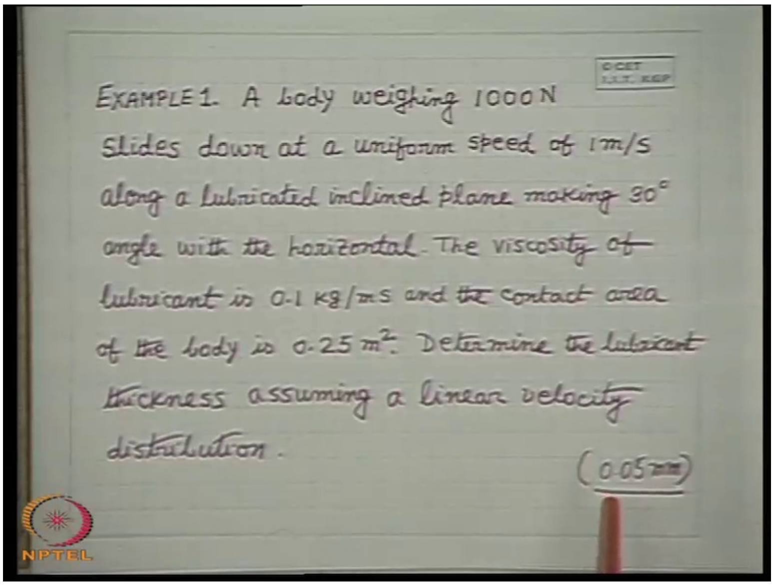

Let us see. First example, 1. Problem, please see. problem 1. A body weighing 1000 Newton slides down at a uniform speed of 1 meter per second. Again, a body weighing 1000 Newton slides down at a uniform speed of 1 meter per second. Along a lubricated inclined plane making 30 degree angle with the horizontal. Well, along a lubricated inclined plane making 30 degree angle with the horizontal. The viscosity of lubricant is 0.1 kg per meter second. This is the unit of the viscosity mu. You can see from the equation tau is equal to mu d u d y and the contact area of the body is 0.25 meter square. Well, determine the lubricant thickness assuming a linear velocity distribution. Well, so let me solve this problem. A body weighing 1000 Newton slides down at a uniform speed of 1 meter per second along a lubricated inclined plane making 30 degree angle with the horizontal. The viscosity of lubricant is 0.1 kg per meter second and the contact area of the body is 0.25 meter square. Determine the lubricant thickness assuming a linear velocity distribution. It is a very simple problem.

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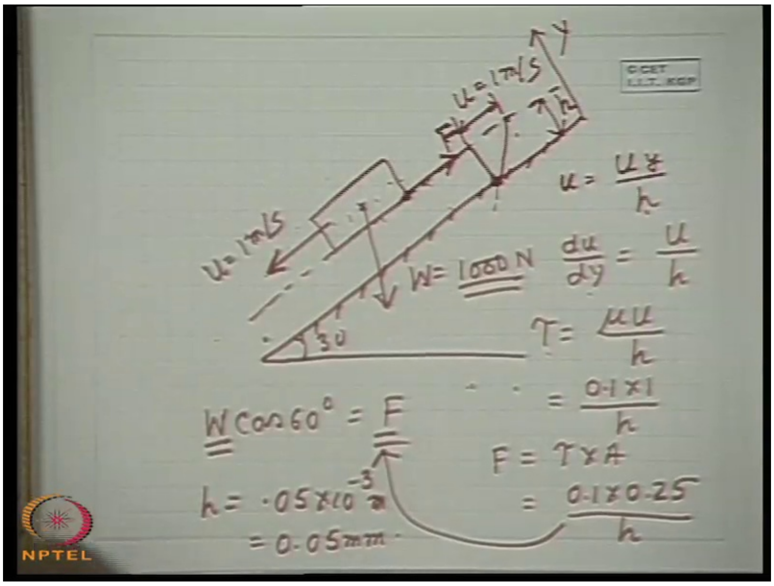

So, problem is like this. There is a, this is the area. This is 30 degree. Let this is the exaggerated view, thin film of the liquid and this is the body, which is moving with a velocity of U. This big capital velocity of 1 constant velocity of 1 meter per second. Now, what are the forces acting on the body? You recognize? One is its weight acting downward, and w is equal to 1000 Newton. Another is the drag force acting on the body, because body is sliding down, because of its weight along the liquid over a liquid film, whose thickness, let we have to find out h. So, this shear force, because of the flow, when the body flows or moves in this direction, this induces a flow in the liquid.

When a flow in the liquid is set of, the liquid shear stress field is generated. So, this shear stress gives in opposing force F on the body. So, body is under equilibrium between these two forces, W and F, because there is no inertia force and speed is uniform. So, the simple force balance of the body is W. So, this angle is 60 degree. So, in this direction, W cos 60 degree is equal to F. Simple case, because this direction, the dragging force is F and this is balanced by the component of the weight in this direction. So, only thing from fluid mechanics is to find F. So, because of this motion U, there is a motion which is being setup in the liquid. It has been told in the problem, you consider a linear velocity distribution. What will be the velocity distribution?

As I have told in the earlier class, that mostly condition demands that the velocity of the fluid on this surface will be 0. Velocity of the fluid of this layer, what will be the velocity of the fluid in this layer? 0. How? 1 meter per second that is U; that means, I am drawing the velocity variation here itself. For clarity, I am doing it here. It will be 1 meter per second, because relative velocity is 0. That means, it is moving with capital U. That is 1 meter per second. So therefore, the velocity profile will be u is equal to, small u is equal to capital U into y by h. What is y? I consider y in this direction coordinate and h is the liquid film thickness, which I have to find out. So therefore, d u d y is constant in this case, which is capital U by h; linear velocity distribution. So therefore, slope is constant. So, d u d y is constant. So, shear, so therefore, tau is mu into u by h. Very simple.

So, in a linear velocity distribution from 0 to capital U, that is the speed of the solid body, because of no slip condition. Now, you write the value mu is 0.1 kg per meter second and u is 1 meter per second and h. So, f will be tau into contact area; that means, this is equal to 0.1 into 0.25 divided by h. So, if you put this value of f and the value of w as 1000 Newton and cos 60 half, you get the value of h as 0.05 into 10 to the power minus 3 meter. All are in m k s unit is equal to 0.05 millimeter. Alright? Very simple problem, school level problem.

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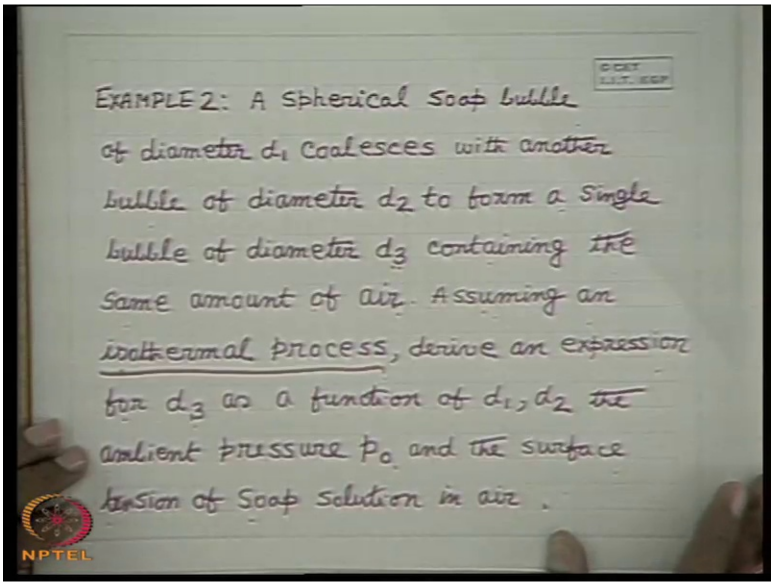

Another problem, which is a little tougher. Example 2. A spherical soap bubble, quick, a spherical soap bubble of diameter d 1 coalesces with another bubble of diameter d 2. One spherical soap bubble of diameter d 1 coalesces with another bubble of diameter d 2 to form a single bubble of diameter d 3. Single bubble containing the same amount of air;

that means, the 2 bubbles air is equal to the air of the third bubble after coalescence. Assuming an isothermal process, derive an expression for d 3 as a function of d 1, d 2, the ambient pressure p 0 and the surface tension of soap solution in air.

So again, I read, spherical soap bubble of diameter d 1 coalesces with another bubble of diameter d 2 to form a third single bubble of diameter d 3 containing the same amount of air assuming an isothermal process; that means, the process is isothermal of coalescence process. This is a process constant given. Otherwise, we cannot solve. Derive an expression for d 3 as a function of d 1, d 2, the ambient pressure p 0 and the surface tension of soap solution. Very simple problem.

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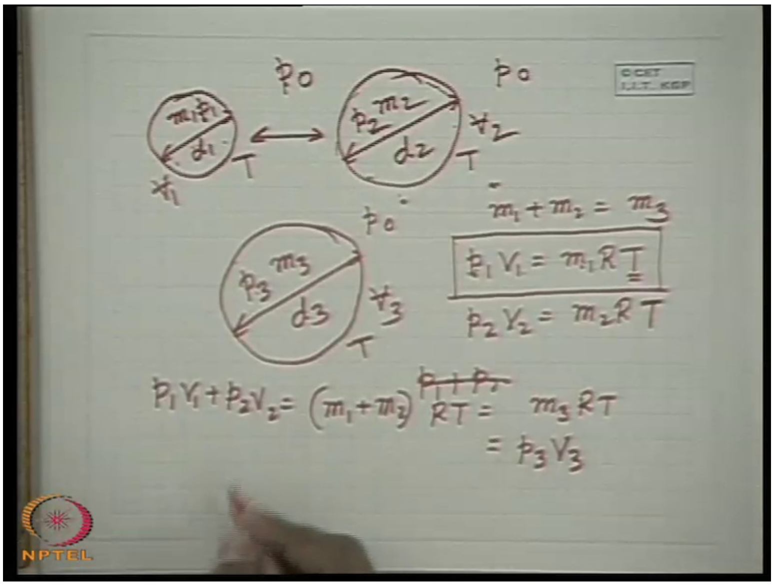

Let us see that. Let us consider a soap bubble of diameter d 1. Let us consider the mass of air inside is m 1. Let us consider another soap bubble of diameter d 2 and mass of air, let m 2. Two coalesces and after coalescence, they form a soap bubble of diameter d 3. Let the mass of the air m 3. Now, what physics will be followed? What are the conservation? What are the things which are conserved after coalescence? One is the mass of the air. It has been told. That means, m 1 plus m 2 is equal to m 3. Now, let the pressure inside the soap bubble is p 1, inside this soap bubble is p 2 and inside this soap bubble is p 3 and there is no such direct relationship at present we get from between p 1, p 2, and p 3.

Let the volume of this soap bubble is v 1. Let the volume of this soap bubble is v 2 with a cut and let the volume of this soap bubble is v 3. Now, what we can do and now how can we relate this p’s? We can relate these p’s in two ways. Ambient pressure is p 0. One is the surface tension, which can determine the difference of p 1 from p 0, difference of p 2 from p 0, same ambient pressure and different of p 3 from p 0. But to relate p 1 p 2 p 3, we will have to take the help of the Ideal Gas laws.

We will consider the air in the bubble to behave as an ideal gas. We will consider the air in the bubble to behave as an ideal gas. This constant or this condition, this assumption, you will have to take. Then only we can relate the pressure and the volume. So, if we write this for the first bubble, p 1 v 1 is equal to m 1 and r is the characteristic gas constant and T is the temperature. Another condition given in the temperatures are same that means, the process is isothermal. Temperature of the air in this bubble and the temperature of the air in this bubble and after the coalescence, for the single bubble, remains same as T. You know the equation of state for an ideal gas states, that p into v, that the pressure and the volume of the gas is equal to the, if a mass of gas m occupies a volume v and the pressure exerted is p v is equal to m R T, where T is the characteristic gas constant.

These, I can write for the first bubble and I can also write for the second bubble, the same equation with the same characteristic gas constant, because the same here with the same temperature T, since the process is isothermal. If we add these two, we get p 1 plus p 2, sorry, we get p 1 v 1 plus p 2 v 2 is equal to m 1 plus m 2 into r T is equal to m 3 into r T. Because m 1 plus m 2 is m 3. Again, m 3 into r T can be written as p 3. What is m 3 into r T? It is equal to p 3 v 3.

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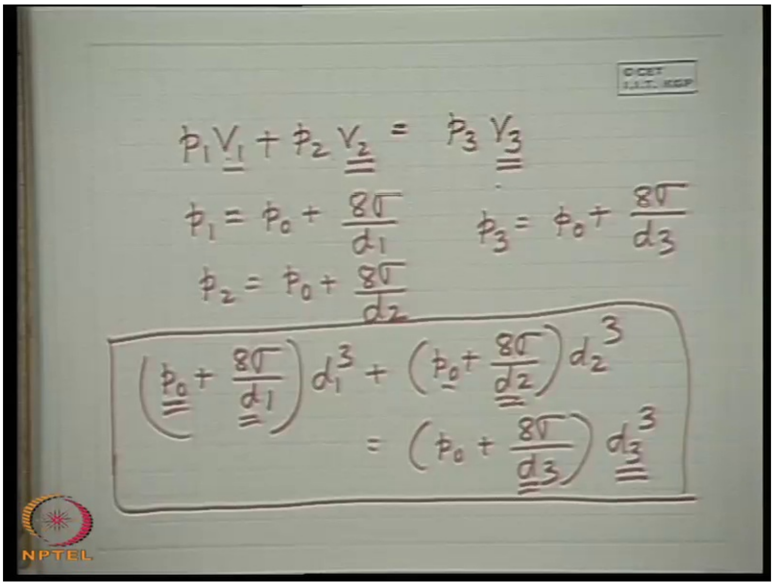

So therefore, we get from this p 1 v 1 plus p 2 v 2 is equal to p 3 v 3. Now, we can write p 1 is equal to, from the surface tension concept p 0 plus, if you recall this, 4 sigma by r; that means, 8 sigma by d 1, because it is a bubble. Similarly, we can write p 2 is p 0 plus 8 sigma by d 2. Similarly, we can write p 3 is equal to p 0 plus 8 sigma by d 3 and v 1 is proportional to d 1 cube, v 2 is proportional to d 2 cube and v 3 is d 3 cube.

If you write this, you get 8 sigma by d 1 into d 1 cube, pi will be canceling, so that, p 0 plus 8 sigma by d 2 into d 2 cube is equal to p 0 plus 8 sigma. So, this is the required expression. See, if I know the values for d 1, d 2, sigma and p 0, I can find out d 3. So, this the required expression for the final diameter of the final single bubble in terms of the diameter of the individual bubbles, the ambient pressure and the surface tension of the soap solution. Well, so with this I conclude this lecture on introduction and fundamental concept. So, next class I will start the fluids statics. So, introduction and the fundamental concepts, which covers the basic properties of the fluid and this concludes this section of the lecture. So, next class I will start fluid statics.

Thank you.