Fluid Mechanics

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Lecture - 29
Incompressible Viscous Flows Part -1

Well, good morning and welcome you all to this session of fluid mechanics. As I told in the last class, that today we will be discussing incompressible viscous flows. Now, first of all we have to know, what is incompressible viscous flow? The flow is incompressible means, when the change of density is not taken into account. That is true.

Now, viscous flows means that flow of viscous fluids; all fluids has viscosity. All real fluids have viscosity and flow of all real fluids is therefore viscous flows. Now, what happens? For real flows, the property viscosity is manifested in terms of generating shear stresses or the frictional forces between the fluid layers and fluid to solid surface. As a consequence of which we know, that the velocity of the fluid particle adjacent to the solid surface is 0; relative velocity is 0. That means, if the solid surface is at rest, the fluid velocity is also 0 there at the solid surface.

Now, earlier sections, we have discussed the flow of inviscid fluid or inviscid. These are hypothetical situation, where you considered the fluid viscosity is 0 and flow of such fluids are known as inviscid flows or flows of inviscid fluids. There we have found out, because of the absence of viscosity, the velocity at a cross section is uniform and which violates the practical findings that the velocity at the surface is 0; that means there is a velocity at the surface also.

So, now in this flow, we have seen the equation of momentum, which is known as Euler's equation, contains only the pressure force as the external surface forces. Of course, the body force is there. Body force depends upon the external forces imposed on it. That is not connected with fluid mechanics, so that there is a body force fluid or not. In general, we consider the gravity is the only body force fluid, if there is no external force imposed on the flow of fluid.

So therefore, the surface force on a fluid element, we recognized only the pressure forces, which is normal and we know which is same in all directions, which is known as hydrostatic or static pressures or we call it as thermodynamic pressure. So, this is the only force acting on it.

We know that the Euler's equations can be written in 3 coordinate directions along with the continuity equations. Now, the basic philosophy is like that, if we have 3 equations of motions, that means, in 3 coordinate directions x y z with 3 velocity components; you know what is Euler's equations, and then the continuity equations. Then we have 4 equations and at the same time, we have 4 unknowns u v w and pressure, because density is constant. The flow is incompressible too. So, these 4 unknowns and 4 equations we can solve for it.

But, we have seen in many situations, we do not have to solve directly the Euler's equations to know the velocity. Why? This is because if the flow is specified by the flow rate q, we know that if we divide the flow rate by the cross-sectional area, we get the velocity and that velocity is uniform at a cross-section. So therefore, Euler's equation is not solved for the solution of velocity. These are very important things. I am telling you this you will not be given in any books. Even not in my book also. I have not written like that. So therefore, sometimes it is not required.

So, why then Euler's equation is required or the Bernoulli's equation? The integrated form of Euler's equation to know the pressure variations. This is simply done in case of inviscid flow. We have done earlier. But in case of a viscous flow, you see again the equation of motions in 3 coordinate directions. We have got 3 equations along with the continuity equations, we have got 4 equations and we are with 4 variables, 3 velocity components in 3 directions are axises and pressure; taking density to be constant for incompressible flow. There we have to solve these equations.

All these 4 equations, equations of motions and the continuity equations to solve for velocity components and pressure, even if the problem is specified with a given flow rate. Why? If we know the flow rate and if we know at any cross-section, the cross-sectional area of the dark, if we divide the flow rate by the cross-sectional area, we get a velocity, which is defined as the average velocity of flow. That is, flow rate divided by the cross-sectional area. It is very important. But we do not know what is the velocity variation across a section and it is the viscosity of the fluid which makes a velocity variation across a section. This probably I discussed in the very first class, that is the manifestation or the cause of viscosity is to make a velocity distribution across a section. That is a normality division of flow.

Ultimately, the velocity at the solid surface will be 0, provided this solid surface is at rest, that is the no-slip condition. So, to know the exact velocity variation across a surface and also the variation of velocity along the direction of flow, along with the pressure variations, we will have to solve the equations of motions. So, it is not that easy that flow rate by area is the velocity and velocity is obtained, because it is not uniform velocity across a section. So therefore, the very first task that comes in learning or starting the viscous flow is to develop the equation of motion for viscous flow. The same equation of motion; that means, the Newton's law applied to fluid element. But here the difference is that, if we consider a fluid element, its surface forces, that is the forces acting on its surfaces that have been taken isolation from the entire fluid body are not consisting of only the pressure forces, but the shear forces. So therefore, the nature of the shear forces acting on the surfaces and their relationship with the shear strain or shear strain rate helps in developing the equation of motion in viscous flows. So, first part we will be concentrating our attention to that.

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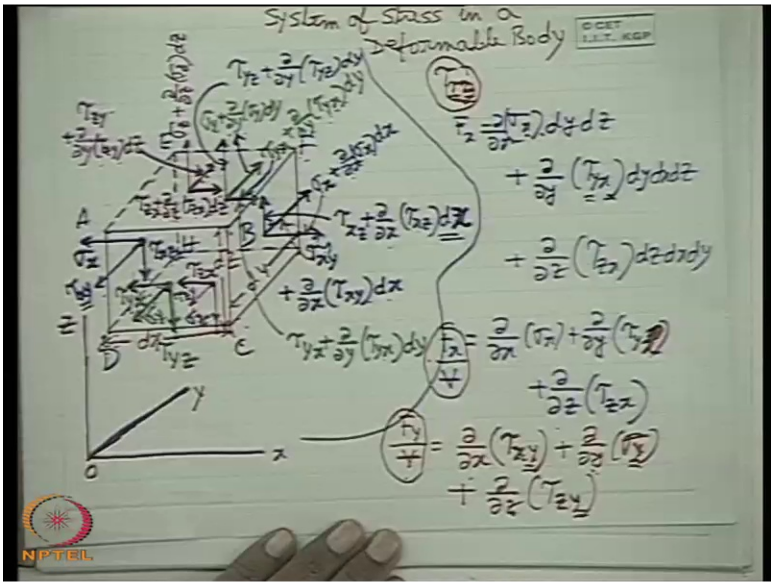

So, in doing so we first recollect it. It is a bit of recollection because we have already done it in your applied mechanics class. The stress system, system of stresses in deformable body. Better you write it. System of stresses in deformable body. System of stresses. System of stress, either you write, in a deformable body. This is any deformable body. Please, any question you can ask me. Deformable body. So, just it is a recapitulation.

Now, if we consider a, well, Cartesian coordinate system x y z, rectangular Cartesian coordinate system and consider a fluid element in a parallelepiped fluid element. We see that, there are 6 planes. As we have already recognised that, x plane y plane z planes. x planes means, the planes whose perpendicular is the x direction. Accordingly, the y plane, whose normals at the y directions and z planes whose normal are the z directions. So, you see, we define the stress like this.

Already you know, the stress is defined by 2 subscripts. One subscript stands for, for any stress, let tau with two subscripts 1 and 2. One subscript stands for the plane on which it acts and another subscript stands the direction; that means, on a plane, there may be a different direction. Direction is compatible with a co-ordinate axis. So therefore, to represent stress, we require two freedoms, two degree of freedom; that means, the stress requires two specifications. That is why stress is not a vector. It is a denser of order two. Those things are known to you. So, with this notation, we just fix.

Now, when a stress acting on a plane and along the direction normal to the plane; that means, this stress is on a plane and along the normal to the plane; that means, along the normal to the plane means, along perpendicular to that plane. Then that stress is normal stress and this is usually denoted by sigma, where this stress acting along the plane and not normal to the plane, but in different directions and known as shear stress, as you know and it is denoted by sigma. So therefore, when we write, denoted by tau, sorry. So, when we write sigma on any plane, its direction is fixed. So therefore, sigma is not denoted with two suffixes.

So therefore, if we see that, this is one x plane and this is another x plane, which is separated by a distance. I am sorry. This I have not done. This is d x, we consider a parallelepiped of d x d y and this one is d z. That means, this one is d z. This high, d z. So, another x plane separated by a d x. So, here the stresses are alike. This is the sigma x, sigma x plus del del x sigma x d x. This will be the sigma x. I am sorry. This will be the sigma x. This is the sigma x. Similarly, you see that there are two stresses, tau x y and tau x z. It is the x plane along the y direction. This is the x plane along the z direction and they are appearing in this way as a couple. That means, pair. So, this is the tau x z, but it has a increment for d x del del x of tau x z del del x of tau x z d x. I am sorry, del del x of tau x y, I am sorry, del del x of tau x y. I am sorry. I am sorry.

So, this should be del del z. But this is with the del del z. This is different. But this is sigma x is this one, tau x y is this one and tau x z is this one. So, this will be tau x y. So, this will be tau x y. So, I am sorry. This will be tau x y plus del del y of tau x y d x. So, this will be this. Sorry. I am sorry. So, this will be tau x z and this will be by mistake del x, sorry, del x. That means, tau x z and tau x y as we increase due to a change in the distance d x. So, this will be, so these three stresses appear like that.

Similarly, if you see the stresses in the y plane; that means, this y plane; this plane and this plane, if I give a name, then that will be better A B C D. Well, E F G H; this is simple thing. So, if I consider the plane A B C D and E F G H, then this is sigma y and this is tau y x. First suffix is the plane and the second suffice is the direction along x axis and tau y z. Similarly, they appear like this. You see, that is sigma y and this sigma y has changed for a distance of d y. So that means, this is the plane E F G H. I have given the green colour. So, sigma y del del y of sigma y d y.

Well, and similarly, this will be sigma y. I am sorry. So, this will be this. This is not properly shown. So, this will be again tau y x; that means, this is the tau y x; that means, this tau y x, and this will be tau y x d y. Well and this is tau y z. This is the tau y z. So, this is written as, well, tau y z. Where is it written? Have I written tau y z? No. It is not written. So, you can write, this one is, this one is tau y z plus del del y of tau y z d y. So, that means, these three stress components, normal stress and the shear stresses, they appear like this. del del y of, this is tau y z, tau y z. This is tau y x. This is tau y x and this is the sigma y and this is the sigma y. This is an incremental change. Similarly, if you see the z planes, please see that. D C H G, D C G H and another z plane separated by a distance d z along the positive direction of z, that is A B F E and if we see the red one, that is stress system, one is sigma z and another is tau z x, that is z plane x direction and another is tau z y.

Similarly, they are being shifted like this. They are being shifted to the plane A B F E. That is sigma z. This is the increment del del z of sigma z del z. Similarly, tau z x del del z of tau z x d z. Similarly, this one tau z y del del y of tau z y del z y. This way we can represent the stresses. Now, this is very simple. That, if we now want to find out what is the force in the x direction, what is the total force in the x direction, total force in the x direction, if we want to find out, we have to find out the force in the x direction. One is x direction force. Now, we have to see the planes. Now, for x planes, the contribution of x direction force is, that is sigma x only. That means, you see, therefore, sigma x, this minus this. So, this ultimately will come, del del x sigma x into d y d z plus x. Direction force will come from the y plane, contribution of y plane. That is tau y x and tau x y; that means, this one. So, tau x y minus tau y x plus del del y of tau y x minus this one. So therefore, this comes; del del y of tau y x d y and the area, it is d x d z d x d z. So therefore, we see this is due to the x direction force in x plane. So, this sigma represents the x plane.

So, this is the force in the y plane, but x direction. Similarly, the force in the z plane, but x direction, this is tau z x. That means, this one, tau z x plus del del z of tau z x d z minus this. Of course, this will be multiplied with the area; that means, d x d y; that means, this will ultimately give rise to del del z tau z x d z d x d y. It is very simple to remember that del del x of sigma x plus del del y; that means, change with y, what is that? The x direction force, but in y plane. So, whenever there is a differential del del z, it has to change with respect to z. That means, the force will be in z plane, but it is in the x direction. Always we are writing in the x direction. So, this will comprise the net x force per unit volume. Let F x by per unit volume, where v is the volume of the parallelepiped.

So therefore, we come to this, that del del x of sigma x plus del del y of tau x, sorry, tau y x is suffix plus del del z of tau z x is the x direction force per unit volume in this elemental fluid. Similarly, the y direction force per unit volume will be equal to, if you see that, first x direction force, if you find out what is the x direction force from y plane, if you start the y plane, y plane x direction force come only from tau y x and tau x y and this will be del del x of tau y plane.

So, this is the first plane and then this is the direction; that means, we are finding out x plane and y direction. We start first with x plane. That is better. We start with the x plane and the y direction force. So, x plane y direction force means, it is tau x y. It is tau x y. So therefore, x direction y plane. So, tau x y. Then del del y of sigma y. Then y plane. Then we come to y plane. So, what are the y direction force in y plane? This is only sigma y. So, this contribution will be del del y sigma. d x d y d z is coming here plus we will now consider z plane.

Z plane contribution for y direction force will be what? tau z y tau z y. So, this one, tau z y. This one tau z y. So, del del z of tau z y. So, we see that the force per unit volume in any direction, the direction is y and when del del x, it is tau x y. That means, the force in x plane y direction. del del y sigma y, that is the force in y direction in y plane. del del z, force in y direction and z plane; it is differentiated like that. So, this is the force per unit volume in the y direction and in the z direction. If you make the force analysis, you will come to this.

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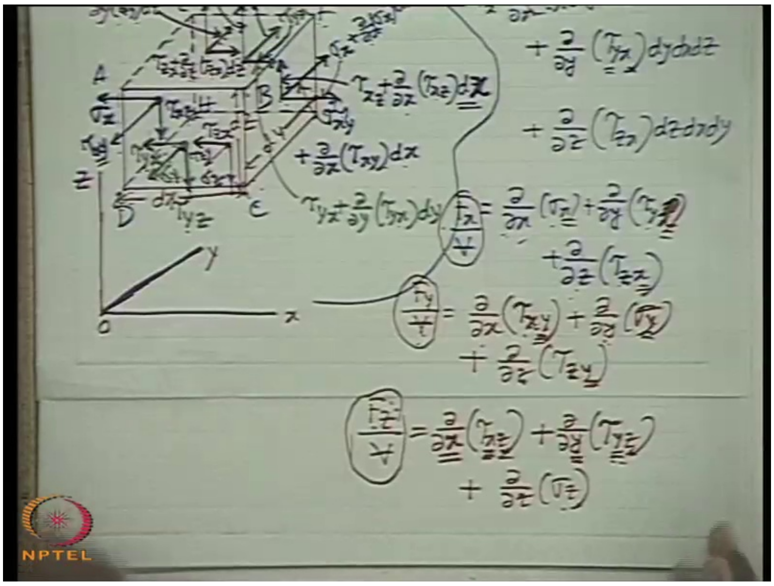

Now, if I write the force in z direction, but per unit volume, please, you come with the x plane. What is the force in the z direction in the x plane? This is contributed by tau x z. This is contributed by tau x z at these two places and if you make it multiplied with the area and subtract, then we will simply get del del x of tau x z. That means, at the x plane it is appearing, but it is appearing in z direction. That is why del del x, we are differentiating with respect to x, because it is appearing in the x plane. Similarly, after x plane, if we go to y plane and we recognize that z direction force in y plane is contributed by tau y z. So, tau y z. So, this at the z direction force in y tau. Difference between this and this of course, multiplied with the area and that will be giving in del del y of tau y z. The first one is y and this one y; that means, differentiation with respect to y, when the force appearing in the y plane and second suffix is z direction. This is consistent, because it is the force in the z direction. Similarly, in the z plane, the force in the z direction is this. So, there is a very much good rhythm that the net x force in x direction is this. del del x sigma x tau y x plus del del z tau z x. All x direction force appearing in x y z plane.

Similarly, net surface force per unit volume in y directions or all y direction force appearing in x y and z plane. Similarly, the net z direction force per unit volume is contributed by all z direction forces appearing in x y and z plane. So, it is simple force analysis. We will keep this thing.

Now, after this, probably I am not going to prove it. You know that you can prove the another thing. Before that, I just tell you that, with this in mind, so we see that, there are three quantities. Again let me write this. Again let me write this.

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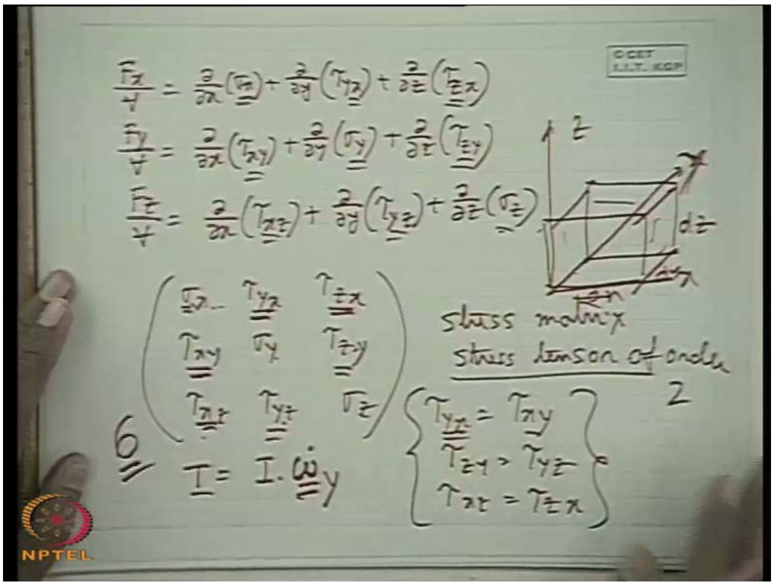

So, F x per unit volume is del del x of sigma x. Well, plus del del y of tau y x plus del del z of tau y z tau z x. So, all x direction force. Similarly, F y per unit volume is x plane. We start with the x plane first. So, y direction force; that means, x plane y direction force plus y plane y direction force means, sigma y plus z plane. That means, force in z plane, but y direction z y. Very simple to write. F z by v is again x plane z direction force; that means, x z plus y plane force in z direction; that means, it has to be a tau, y plane z direction and z plane z direction, it has to be a normal stress. So therefore, we see there are 6, 1 2 3..9. These 9 stresses, which define the entire state of stress. So, entire state of stress is therefore, defined by 9 stress quantity. So, I write this stress quantity x y sigma y tau z y tau x z tau y z and this forms a matrix, where the rho of the matrix represent the stresses in particular direction. These are all stresses in x direction, y direction, z direction. These are all known things. That is why I am little fast. The columns represent the plane, that is the stresses acting in x plane, y plane and z plane. So, ultimately these 9 stresses, this is known as stress matrix or stress tensor matrix. The elements arranged in an array known as matrix and tensor means the quantity, which requires apart from its magnitude more than one specification.

Vector requires one specification more than its magnitude, but tensor requires more specifications; more than one specification apart from its magnitude. It requires two specifications actually. It is tau x x tau y y tau z z, but we are writing in sigma. That is why two suffixes we are omitting. So, that is why it is tensor of order 2. This thing probably you know. Now, you also know from the rotational equilibrium of the element, that is the parallelepiped, if you consider the rotational equilibrium of parallelepiped about any of the axis; that means, about y or x or z, we can see, we can show that this is equal to this; the symmetrical element about the diagonal. These are the diagonal element and tau z y is tau y z; that means, tau y x is tau x y. This probably you know. I am not going to prove it. tau z y is tau y z and tau x z is tau z x. This can be proved, if you consider its rotational equilibrium.

If you write its rotational equilibrium equation; that means, torque is equal to I into omega dot, where omega is the angular velocity and omega dot is the angular acceleration about any axis. Let us first consider about y axis. Then the shear stresses will appear on z planes and on y plane. x plane, sorry, x planes.

So, this will cause the rotational motion of the element about the y axis and if I write it, I am considering d x d y and d z to be very small. Take the limit and you will be able to prove that. This I left you as an exercise. Probably you have done it earlier. So, if you take this; that means, this is equal to this, this is equal to this and this is equal to this; that means, there are 6 unknown stress quantities, which define the stress system. So therefore, 6 unknown quantities are there.

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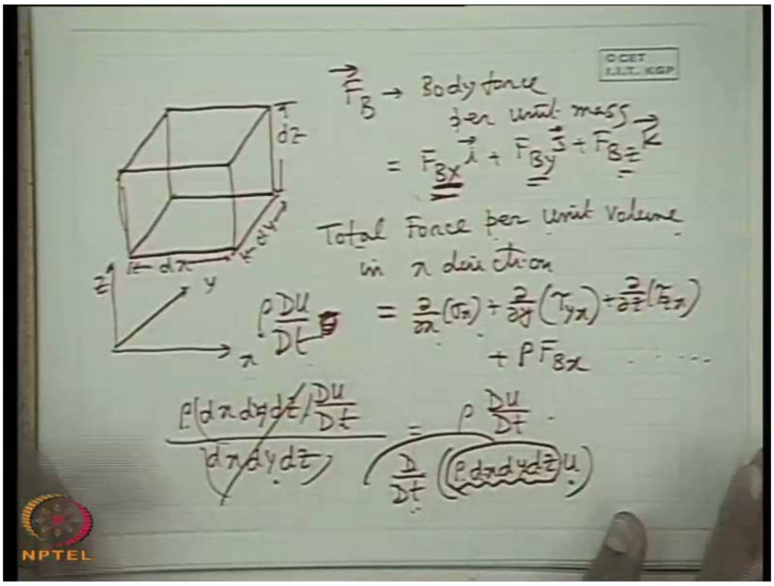

Now, we write that, therefore, if I come to the equation of motion for this fluid element, rather I again draw this element like this. I again draw this element like this. Well, I again draw the element like this. So, let this is x direction, this is y direction, this is z direction. So, this is d x. Well, so this is d y, well and this is d z. Then I know that the force per unit volume F x by v for this element is this, force per unit volume in the y direction, force per unit volume in the z direction. Now, what is the total force acting in x direction? Let us now consider the F b is the body force. Body force per unit mass. Body force vector per unit mass, per unit mass, which can be represented as F b x I plus F b y j plus F b z k. That means, this has got three distinct components in x y z. Therefore, total force per unit volume in x direction will be what? Total force, sorry, per unit mass, per unit mass. No, per unit volume in x direction.

What is total force per unit volume in x direction? You tell me. Total force per unit volume in x direction F, so I am already using F. So, better you write total force in per unit volume in x direction is equal to del del x sigma x plus del del y tau sigma x del del y y x. But now it is not very essential, x y or y x, del del z in z plane, but x direction, but x z and z x are same plus rho into F b x, if I define F b x as the x component of body force per unit mass and the rho is the density. Total force per unit volume in x direction and that must be equal to its mass into acceleration per unit volume. That means, the inertia force, negative of the inertia force per unit volume; that means, mass into acceleration per unit volume. So, what is this mass? rho d x d y d y d z. What is the acceleration? d u d t. So, per unit volume d x d y d z. So, this d x d y d z cancels out. So, this becomes is equal to rho d u d t. Alright? Or, one can write the rate of change of momentum. One can write d d t of momentum. What is the momentum? mass rho d x d y d z into u, but rho d x d y d z being the mass of a system, which by definition is fixed. It comes out. So, it becomes rho d u d t and this d x d y d z. It cancels from left hand and right hand side, if we equate this with this, then this has to be written as total force.

So, either you write the rate of change of momentum with the total force or you write total force per unit volume in one direction with the mass into acceleration divided by volume in that direction, which ultimately makes rho d u d t. Or, you write total force in the x direction; that means, multiply this with d x d y d z and multiply this with d x d y d z; that means, entire right hand side is multiplied with d x d y d z is equal to rate of change of momentum in x direction according to Newton's law. So there, remember it. Sometimes even in the viva voce, we ask that, why rho comes out? Is it for incompressible flow only? No, it is for compressible flow. There is no restriction for incompressible flows. So, rho comes out. This is because in the deduction, we see that rho d x d y d z as a whole mass of the system comes out, so that, d x d y d z get cancelled. So, rho d u d T is del del x of sigma x del del y.

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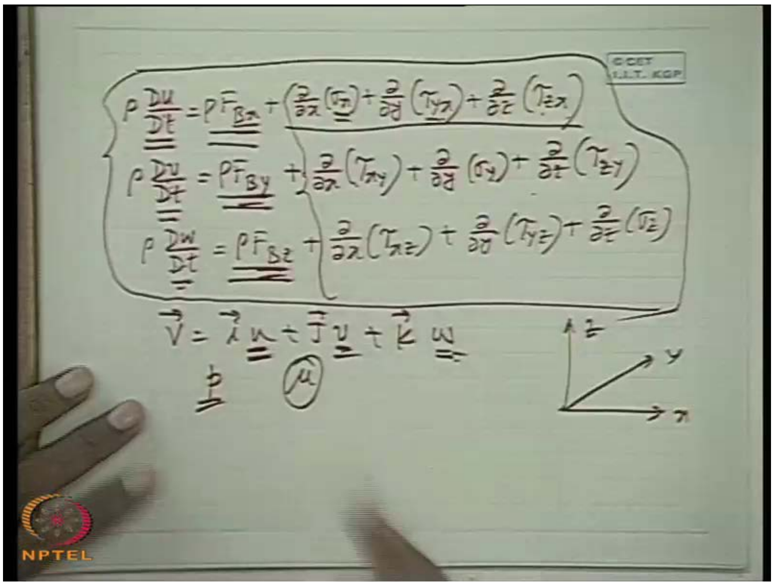

In this way, I can write the equation of motion in y and z directions accordingly and this comes out to be like that. In x direction, I write rho d u d t is equal to; I write the body force term first, the convention, plus the surface force sigma x plus del del y tau y x tau y x or tau x y immaterial, but still I follow the notation, z x. Similarly, this is the mass times acceleration per unit volume equals to the net force acting in y direction per unit volume. That is del del x of tau x y. These two are equal. tau x y or y x del del y of sigma y plus del del z of tau, that is the y plane z y. Well, rho d w d t is F b z plus z direction. rho d w.

Rho? Rho is missing? Oh! Very good. plus del del x of tau x z plus del del y of tau y z plus del del z of sigma z, because F b x F b y are the body force components per unit mass. So, these 3 constitutes the equation of motion of a deformable body or a fluid body in x y z directions. But here, you see the left hand side is the velocity. It can be expressed in velocity. This is the substantial derivative, which can be split in terms of its local and convective. But the forces are expressed in stress form.

Now, this part is beyond, it is not within the jurisdiction of fluid mechanics. It depends upon the physics of the body force field. Usually, body force field is the gravity force field, whose physics is very simple and we can find out, what are the components depending upon the choice of the coordinate axis. But the main task of deducing the Navier-stokes equation, because our main objective, you must know to develop the equation of motions. For example, in 3 coordinate directions x y z in terms of the velocity components. Here, velocity vector V is defined as that u v; that means, already you have recognized it. That, u v w are the x y z component of velocities. That is, this is the x, this is the y, this is the z; rectangular coordinate axis.

That means, our main objective will be to develop x coordinate, x direction equation of motion, y direction equation of motion, z direction equation of motion in terms of u v w and it may include pressure p, it may include viscosity, coefficient of viscosity mu, which we have read at the very beginning in our course. So, how to do it? So, that next task is to relate the stresses in terms of the strain rates, so that, the relationship between stress and strain rates will ultimately induce these variables to come in these equation and that is the most important and pivotal point of deducing the Navier-stoke's equations.

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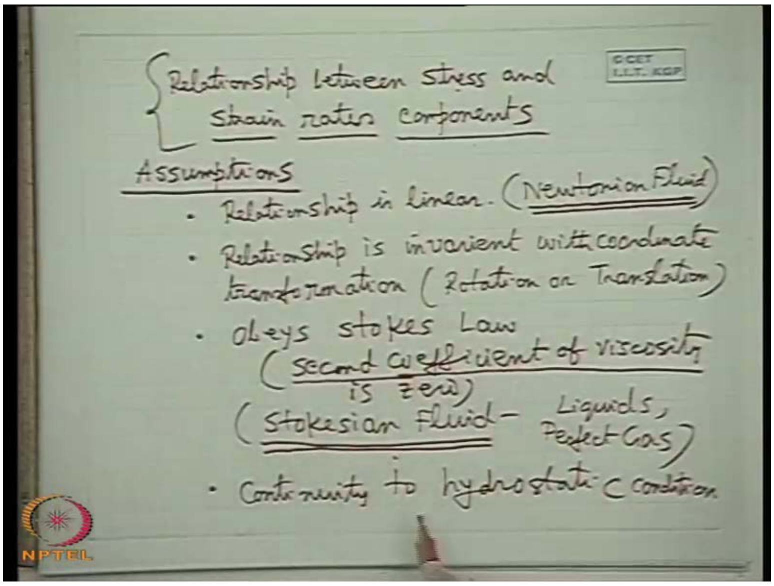

So, relationship between stress and; I have told you earlier the difference between a fluid and a solid is that strain rates. So, it is not the strain, but it is the strain rates. So, this is the most vital part; relationship between stress and strain rates components. So, how to do it? This is beyond the scope of your discussion, your study or your syllabus. This is usually done at the advanced level of fluid mechanics class. So, relationship between stress and strain rates component. Without going for this deduction between of this relationship between stress and strain rates component, I will tell you, which are very important to know how the relationships are deduced, final relationships and what are the assumptions. So, this is one of the toughest job and it requires a higher algebra to develop this relationship between stress and strain rate components for, in general for a fluid, but if we take few assumptions this becomes little easier.

So, what are the assumptions that are made? First assumption is that, this relationship is linear. Why this assumption is made? Relation, it is Newton first who found out that there are different, number of fluids or class of fluids, which obeys a linear relationship between stress and strain rates. These fluids are known as, according to his name, Newtonian. So therefore, the equation of motions, which will be developed for viscous flow, this is for Newtonian fluid; that means, this is a viscous fluid, but Newtonian fluid. If I ask you what is Newtonian fluid, it is where the relationship between stress and strain rates are linear. Another thing is that, this relationship is invariant with coordinate transformation; that means, either a rotation, both or translation.

If you give a translation or rotation of the coordinate system, this relationship, invariant with their coordinate transformation, what does it mean physically? If a relationship is invariant with coordinate transformation, what does it mean physically? Any intelligent student of the class? Any intelligent student of the class? If a relationship of certain variables is independent of the coordinate transformation, what does it mean? This is a physic? This is an identity. This is a physical law. Identity is another name of an equation. There is a difference between equation subtle difference. It is a physical law. Any equation defining a physical law is invariant with the coordinate x transformation.

Number 3 is that, obeys Stroke's law. Of course, this it is very difficult for you to understand. Stoke's law. Stoke's law means, second coefficient of viscosity is 0. Second coefficient of viscosity is 0. That, I will describe. Second coefficient of viscosity. The fluids which obey, these are known as Stokesian fluid. Stokesian fluid. Usually, liquids perfect gases obey Stokesian fluid. This is now just for your information. Not beyond this. The concept of second coefficient of viscosity arises when there is an exchange of momentum of energy between different degrees of freedom like translation to rotation, rotation to vibration. So, this second coefficient of viscosity arises in those cases, which is so small. Can be taken to be 0 for a class of fluid, which is known as Stokesian fluid.

So therefore, the equation of motion for a viscous fluid will be generated for Newtonian fluid and Stokesian fluid. Relationship is invariant with coordinate. Last assumption is that, continuity to hydrostatics. That means, the relationship should be such, the equation should be such that, if we put hydrostatic condition, the continuity to hydrostatic condition; that means, if you make all velocity component u v w 0, then these sets of equation should reduce to the equation of hydrostatic. So, with all these assumptions, we develop the relationships.

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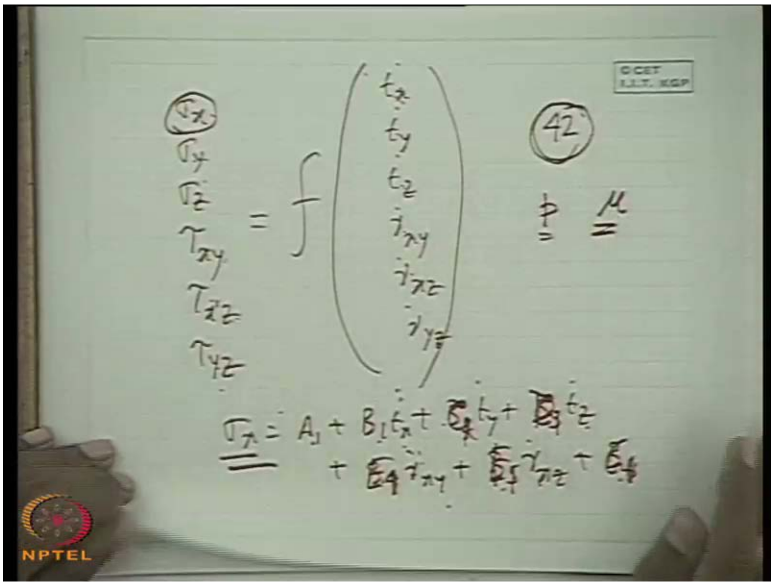

That means, stress, sigma x, then sigma y, I write first the normal stresses, and then the shear stress. There are 6 quantities. Now, we have recognized, out of 9, there are 3 x y y x, same x z z x y z. So, these are functions of, in general, what are the strain rates? Linear strain rates and angular strain rates. This is the linear strain rates. This is the, sorry, angular strain rate. So, each stress component should be in general function of 1 2 3 4 5 6 strain rate component. So, this way, 1 2 3 4 5 6. So, 6 equations with 1 2 3 4 5 6, 6 terms plus additionally 1 constant term, if we consider a linear relationship. So, this will induce 42 unknowns. Because for example, if sigma x is related as A 1 plus d 1 epsilon x, you will be astonished to see that, how it does. epsilon z in 3 dimension plus B 4 epsilon x. Now, you see, sorry, sorry, I am sorry, A 1 B 1 C 1, then oh sorry, A 1 B 1, then you write C 1, then you write D 1, then you write E 1, then you write F 1 and then you write G 1. All looks good.

So, you see, if we consider a linear relationship, so general linear relationship will be normal stress is a function, linear function of all these strain rates. A 1 plus B 1 epsilon x dot plus C 1 epsilon y dot plus D 1 epsilon z dot plus E 1 epsilon gamma dot x y plus F 1 gamma dot x z plus; that means, 1 2 3 4 5 6 7, 1 2 3 4 5 42 unknowns. You will be astonished that these 42 unknowns will be reduced by the assumption. These 3 assumptions; this is taken care of. If we express 6 equations like that, ultimately into 2 unknown, 1 is P and another is V. That is beyond the scope of this class.

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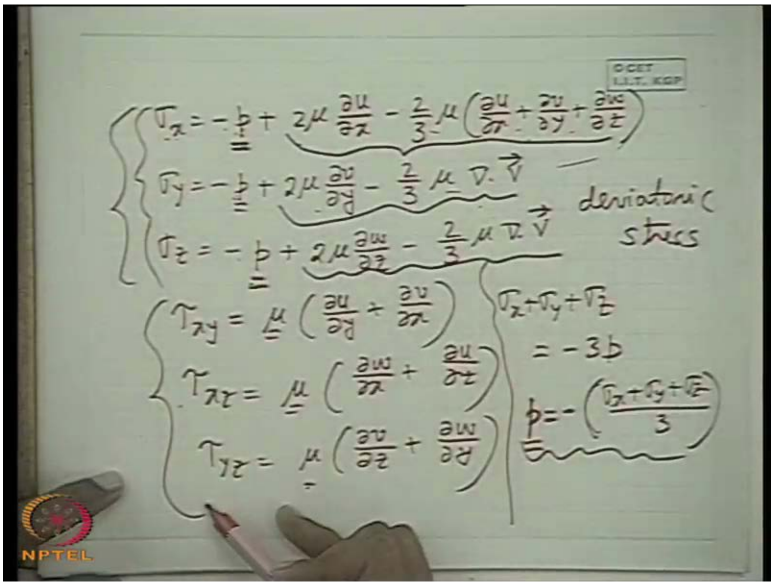

Finally, this relationship is like this. Sigma x minus p plus 2 mu two-third mu into del u del x. If you have any query you please ask me. Otherwise, if you just talk in the class, I will take it very serious. Without any query, if you talk, if you have any query, you ask me. Sir, you are fast. So, this I am writing as divergents of the same thing. Please ask me, sir, what is this? We do not understand. This deduction is ultimately out of the scope of this and tau x y is mu del u del y plus del u del x tau x z is mu del w del x plus del u del z tau y z is equal to mu del v del z plus del w del y, where mu is the coefficient of viscosity and p is the thermodynamic or static pressure.

So therefore, the sigma x, sigma y, sigma z, the normal stresses and the shear stresses are derived, are developed in terms of the velocity components like that. Here you see, this is the thermodynamic pressures. So, when the fluid is at rest or there is a frictionless fluid flows, the pressure, the normal stresses are only minus. That means, compressive stress, which is equal to the thermodynamic pressure. But this part is added when the fluid flows. So, this part, second and third is known as deviatory stress; that means, stress deviates from that of the thermodynamic pressure. That is the deviatory stress, because of the friction. So, this part comes because of the friction.

For incompressible flow, of course, this part will become 0. Now, if we add the 3 equations, first 3, you will see that, even if the flow is not incompressible, this becomes is equal to minus 3 p. Why? Because this becomes 2 mu. Common del u del x del v del y del w del z minus, if you add these 3, it will be also minus 2 mu into del u del x. So, this cancels, which means that thermodynamic pressure is minus of the arithmetic average of the 3 shears, 3 normal stresses in a flow of a frictional viscous stokesian fluid. So, fluid which obeys Stoke's law, second coefficient of viscosity is 0. So, thermodynamic pressure is the arithmetic average of sigma x sigma y sigma z.

Now, our next task is to substitute this into the Navier-stoke's equations. So, that means, in sigma x tau y x tau z x tau x y sigma y tau z y tau x z tau y z, all places, we just substitute this and finally, solve it for the velocity that the equation of motions in x y z coordinate directions. So, today after this.

Thank you. We are lagging behind. I am telling this takes time.