Fluid Mechanics
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Lecture - 23
Fluid Flow Applications Part – II

Good after noon, I welcome you all to this session of fluid mechanic well, last class we were discussing about the plane circular vortex flows and then we discussed further about a free vortex motion or free vortex flow. Now, if we recall it so, a vortex flow is described in a polar coordinate system with only tangential velocity, as the existing velocity component. That means there is no radial velocity fluid having only tangential velocity and immediately we have found that the consequence of continuity tells us the tangential velocity is a function of radial distance, radial coordinates r 1. If you consider a two dimensional case, then if there is no radial component of velocity only tangential velocity exist tangential velocity is a function of r. This is precisely the definition of a 2 dimensional or plane circular vortex flow.

Now, one of the vortex flows is the free vortex flow so, there is a generalize this is the generalization. A special case of this vortex flow is a free vortex flow and the flow is irrotational or total mechanical energy in the flow field remains constant, either of the two can hold good if one is correct, then other is automatically you know one holds good another automatically holds good. That means there is a plane circular vortex flow, where total mechanical energy in the flow field remains constant, means the flow becomes irrotational.

So, irrotational vortex flow where total mechanical energy remains constant throughout the flow field is known as free vortex flow. Where we have seen the tangential velocity bears an inverse relationship with r, that is v theta at any radial location r is some constant by r, and then we found the expression for pressure distribution from the equation of motion, and we found it is same, as if we derive it from Bernoullis equation. That means if we use the Bernoullis equation between 2 points, at 2 different radial locations then, p by rho plus v square by 2, here v is v theta, v theta square by 2 plus d z 1. If we consider the elevationate, if it is in a vertical plane, is equal to the p by rho plus v theta square by 2 plus d z to that means at the point 2. That means the sum of the 3 energy, 3 components of energy are same in all radial location because, the constant mechanic or mechanical energy is constant. So, from that also we can get the pressure distribution equation.

Now, after this we must know that there are certain practical situations, where the free vortex flow takes place but, one very important consequence of free vortex flow, you see that free vortex flow is that there is no energy added or no energy taken out, that means energy remains constant. Therefore, free vortex flow exist because, of some rotation which was previously imported on the fluid, due to some energy interactions and just it is maintaining its motion due to some internal action there is no dissipation of mechanical energy, no addition of mechanical energy, no withdrawal of mechanical energy, it remains constant.

So, this type of flow takes place usually, it is resembles to certain practical situations for example, if the water is taken out from a shallow vessel, with a from from a shallow vessel from a orifice at the bottom. Then the circulatory flow which is induced is more or less similar to a free vortex flow, which is often found in in our house when, the water comes out from a bath tub or from a washing machine. Another case is that if, in a whirlpool caused in a river at a far distance from the eye of the whirlpool, the flow field resembles more like a free vortex flow.

In a tornado, in an airflow, in a tornado also the flow of air resembles in some part, in some region to a free vortex flow. In certain cases like fluid machines, in a pump, the flow at the after the impeller casing, probably I have not read that centrifugal pump. In a centrifugal pump, flow coming out from the impeller casing, as it enters into the volute chamber, the rotary flow or the rotational flow is similar to a free vortex motion, free vortex flow.

Another example is that if, you start a liquid, mass of liquid in a vessel by a starrer or a paddle wheel. The flow field far away from the starrer, not very close to the starrer resembles to that of a free vortex flow. So, one very important and interesting thing in a free vortex flow is that if, you just recall the equation v theta is c by r, where r is measured from the origin, one thing you see mathematically, when r tends to zero v theta tends to infinity, which means mathematically or physically that there is a singularity at the center. That means at the center there is a singularity, this is a mathematical definition, means velocity is not defined, velocity becomes infinity. What does it mean, that means free vortex flow can never be extended physically after the origin, from where the r, the radius that is a radial coordinate is measured.

Which means in practice, free vortex motion is never defined, at the very near to the origin or at the origin, mathematically it is undefined. So, what happens in practice, whenever there is a free vortex flow, near the center or at the central core, either there is another type of vortex flow or there is a hollow core, you will see there will be always a hollow core. So, that free vortex flow is never extended up to the origin so, there is always a singularity as in mathematically found, that v theta is c by r. So, when r tends to zero v theta tends to infinity, these are all details about the free vortex flow. Now, we come to another type of vortex flow, that means this circular motion with only v theta a component of velocity, which is known as forced vortex flow. This type of vortex flow, the difference is that v theta is directly proportional to the radius not inversely proportional to the radius but, v theta directly proportional to the radius. For example, v theta is some constant into radius is very familiar, we are very familiar to it, since our school days this is because, we know this is a motion, which is exhibited by a solid body. If the solid body rotates then, at each and every point at different radial locations, the tangential velocity is the angular velocity into r, being omega is the angular velocity so, v theta is r into omega. So, you we know, that solid body rotates with same angular speed that means all points at different radial location rotates with the same angular speed and as we know the tangential velocity bears the relationship with the angular velocity, as tangential velocity is r times the angular velocity.

Omega is the angular velocity r omega. So, which means the tangential velocity is directly proportional to radial location, means the angular velocity remains constant in the entire motion, in the entire field of motion. So, in a liquid or fluid flow of fluid where v theta is directly proportional to the radius, means that fluid particles moves with constant or equal angular velocities which means that fluid body moves like a solid body That is why this type of motion described by v theta is some constant into radial location is known as solid body motion, solid body rotation or solid body vortex or forced vortex. So, this forced vortex generates is generated in practical situations, when a liquid is taken in a vessel, contained in a vessel if, vessel is given a rotation ,if a vessel containing liquid is rotated about its axis, then the solid, then the fluid fluid body which is being churned with this motion of the solid body, executes the rotary motion or rotational motion. That means exhibits tangential velocities, which follows a solid body rotation, that means the forced vortex motion. Where v theta is constant into the radial location, varies directly with the radial location sometimes if, certain practical applications, the rotation is important in the fluid flowing through a pipe and enhancing the heat transfer coefficient so, when fluid flowing to the pipe, pipe itself is rotated about its axis

Then, the fluid following a rotary motion is very close to that of a forced vortex motion. So, forced vortex motion is a solid body motion, where the tangential velocities directly proportional to radial location or radius, and the constant of proportionality is known as the angular velocity.

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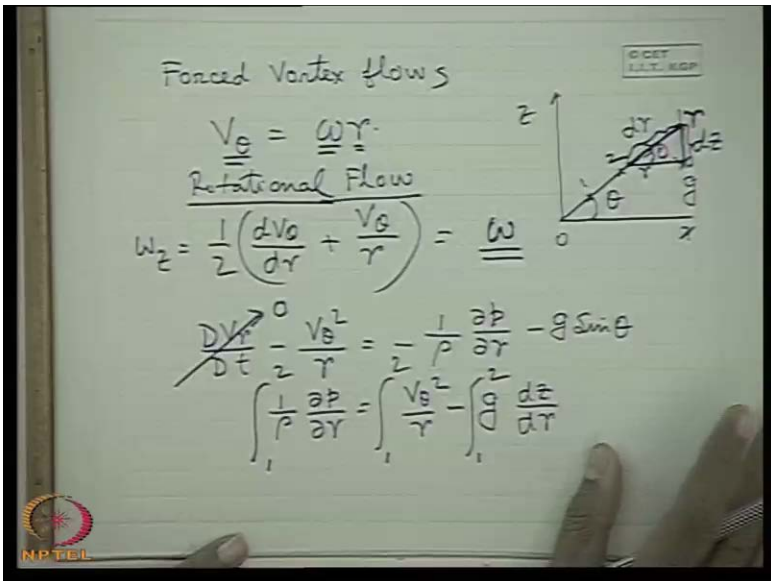

So, let us now, see that forced vortex motion with this in mind, forced vortex flows. Now, first by definition forced vortex flows is v theta is omega into r. That means omega is the angular velocity, that means at different radial location this will be v theta, very simple, v theta is equal to… so, forced vortex flow is not rotational irrotational. This is rotational, this rotational does mean this flow is rotational means, this fluid mechanical it is rotational, that means not irrotational, that means if you write the equation, you know d v theta d r, plus v theta by r, this is the half of this is the rotation, that is the angular velocity so, this is definitely not 0 because, d v theta d r omega v theta by omega 2 omega, that means it has got a rotation component, which is nothing but the angular velocity. So, it is not an irrotational flow, this is rotational flow now, next task is very simple, we can find out the pressure distribution.

So, now you see that pressure distribution always the ,simple thing is to find out from the eulers equation or equation of motion. Now, let us write the equation of motion in r direction, d v r d t ,minus v theta square by r ,the way we did for free vortex flow is equal to minus one upon rho del p, del r, we take in the horizontal plane or we take in the vertical-vertical plane, then of course, we can do that means we can take these g component, that means if, we consider this as x and this as z and this as r I have no objection if, we take this in a generalized case in a vertical, usually the vertex flows are the experienced in the horizontal plane but, you can take consider a vertical plane. Where if you like, there you can put g so, minus g sign theta in the r direction, the component minus g sign theta you can take, and since we have found earlier, that this is 0 because, v r is not existing so, that 1 can write 1 upon rho del p del r is equal to v theta square by r, minus g sign theta, sign theta is also if this is theta sign theta is this, divided by this, that means d r by d z this is the usual thing so, d z by d r is sign theta, this is a change in z, d z this is a change in dr. If, you make a small triangle with small elemental length d r d z so, this is the usual mathematical, terminologies g d z d r. So, now the rest part is this stereo calculations, that means if you integrate this equations, that means if you integrate this equations from 2 points, 1 and 2, similar way 1 to 2 if, you integrate this point 1 to 2 you get the required velocity distributions.

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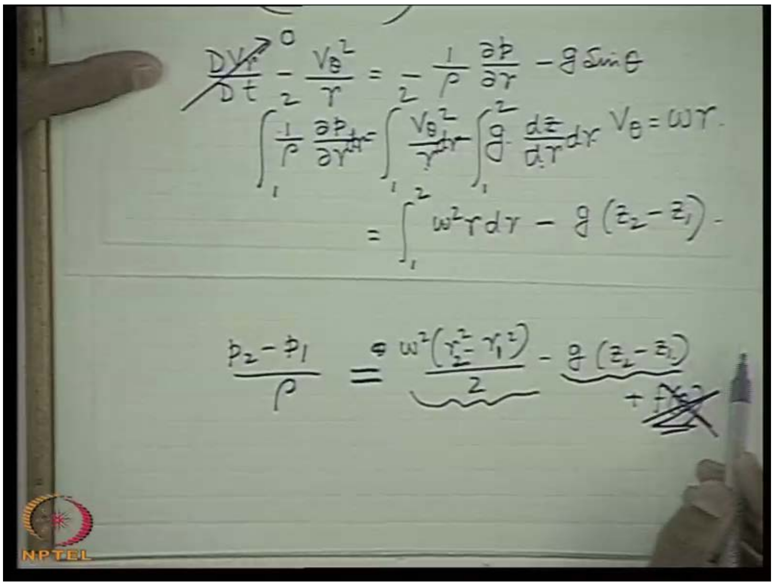

How to get before that, you have to substitute v theta as omega r, v theta is equal to omega into r, this is the definition of the free forced vortex flow. This is the basis, where from we started now, it becomes simple mathematics only, there is no fluid mechanics so, omega square r square by r that means, this becomes 1 to 2 omega square r ,d r. When I have given the integration sign d r so, omega square, r square by r omega square ,r d r minus g d z d r ,d r means I can straight way write g z 2 because, d z d r d r means z, z 2 minus z 1. Now, this part becomes equal to omega square r d r, that means omega square r 2 square minus r 1 square by 2 minus g z 2 minus z 1 and this becomes p 2 minus p 1 by p 2 minus 1 upon root, will be in compulsive.

So, here we see that, this is precise the plus a function of theta, we can induce a function of theta, a constant will be there, which will be a function of theta but, problem is that if we make an definite integral this function of theta may not come if, we integrate between 2 points at the same theta, that is why I am not taking the function of theta with the definite integral but, in an indefinite integral a function of theta may come so, you should not argue with that, that when you are making a definite integral with the thing that, r 2 and r 1, this points are taken in this fashion that, they will vary in z 2 and z 1 but, at a constant theta.

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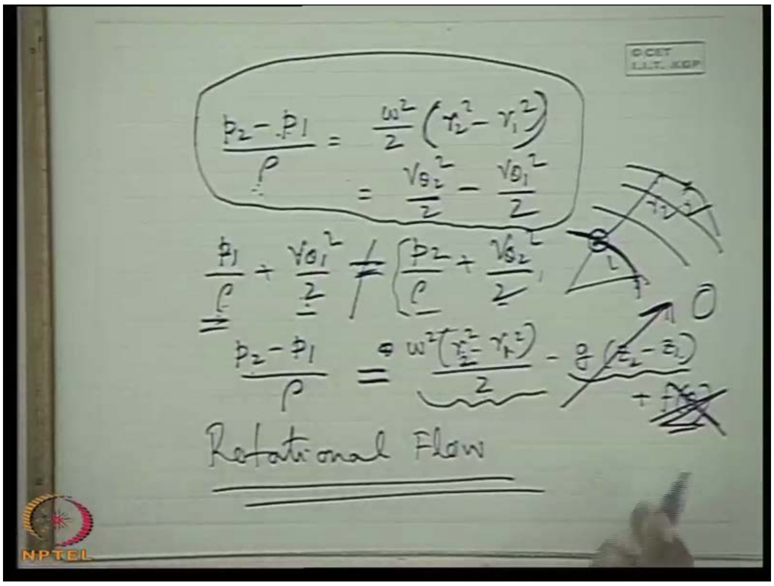

So, I am only interested in the pressure variation, with r 2 and r 1, r 2 square minus r 1 square by 2 minus g z 2 minus z 1. This is as simple as this now, for horizontal plane this becomes 0, so therefore, in a horizontal plane this thing becomes p 2 minus p 1 by rho is equal to omega square by 2, r 2 square minus r 1 square this is simple this.

Now, you see we can replace this as v theta 2 square by 2 minus v theta 1 square by 2 I can neglect this in a horizontal plane, see now you can see it is not following this Bernoullis equation, p 2 by rho plus v theta square by 2 is not coming, p 1 by rho the sign is not proper, that means I cannot write p 1 by rho plus v theta 1 square by 2 in the same horizontal plane, I cannot write this, that means this is not valid, otherwise this is not, this is a correct equation, this is derived, this is the correct pressure distribution equation so, from pressure distribution equation, that means the variation of pressure with radial location this is sufficient but, if I want to explore the relationship between pressure and velocity, I do not get this p by rho, that is why because, at 2 radial location the Bernoullis equation cannot be written with the same constant. That means p 1 by rho plus v theta 1 square by 2 is some constant, which is not same, for different stream line. The constancy of this, that means the level, the value of constant, that means the pressure energy plus kinetic energy sum of these 2 is constant only along a stream line at one radial location.

So, when we change the radial location, at r 2 the sum of these 2, that means the mechanical energy is changed. That means mechanical energy varies from stream line to stream line because, the flow is rotational so, the mechanical energy constant in the entire flow field is a consequent of a irrotational flow, not for a rotational flow. This has to be made clear all right.

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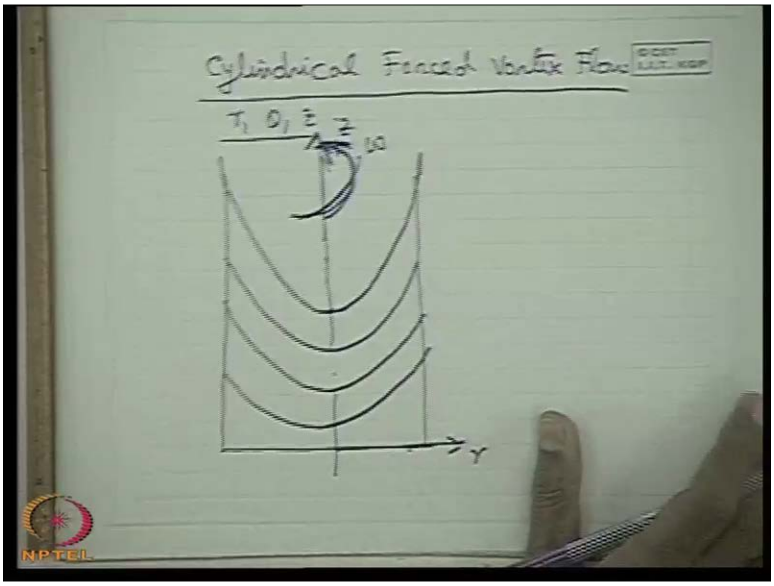

Now, next I tell you a cylindrical a cylindrical forced vortex, which is little important thing in from practice. A cylindrical forced vortex flow, which is cylindrical forced vortex flow is a forced vortex flow, which is realized in a cylindrical coordinate system, that means r theta z 3 dimensional that is known as cylindrical forced vortex. This is generated like this and a practical example is like this. If, we have a vessel, let this be its center. And if, we have a liquid in it, which is being rotated, let this a liquid within it and the vessel is rotated with some angular speed omega, let it rotate with an angular speed omega about its axis so, what will happen the liquid if there is a free surface in the liquid so, free surface will take a shape like this, shape take a shape like this and the surface of constant pressures will take a shape like this, this is the surface of constant pressure. Let us see this is the r direction, this is the z direction and theta, r theta we can see if if we take a plan we can see r theta direction. Here, how can we find out the equation of constant pressure lines or constant pressure planes in 3 dimensional concept. Very simple what is the equation, if you just think the pressure distribution equation, you can see what is the pressure distribution equation just.

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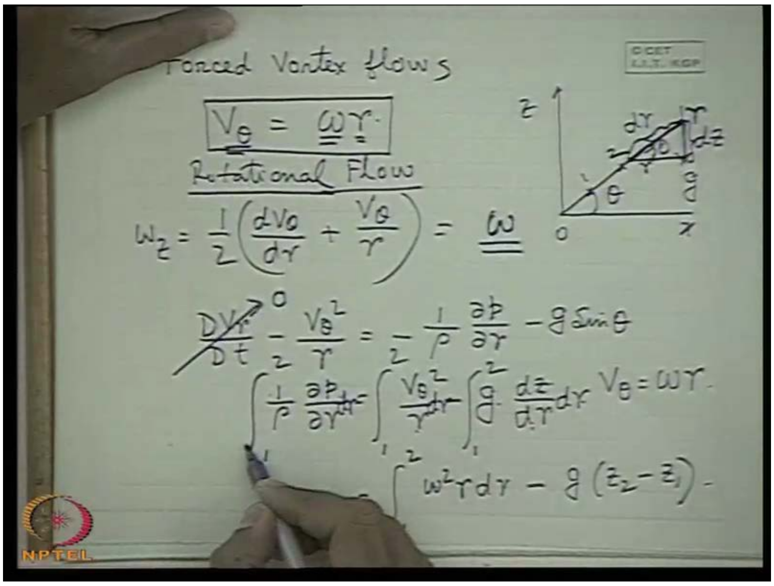

Let us see that what we have found out here, that is the pressure distribution equation, minus 1 upon rho del p del r, this 1 minus one upon del p del r is v theta square by r plus g sign theta.

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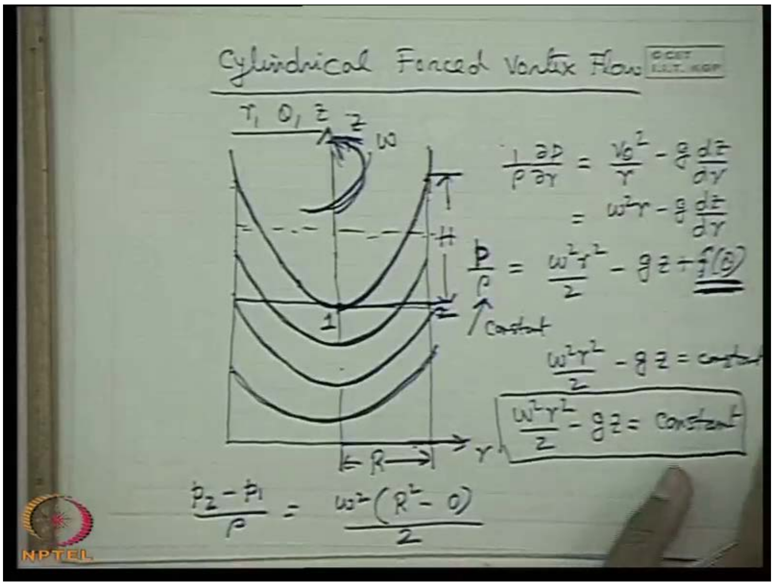

Let us start from here, 1 upon rho del p del r is equal to v theta square by r minus g sign theta, that means d z d r and we substitute v theta as omega r, that means omega square, r square r square r cancel minus g d z d r, without going for a definite integral from 2 points 1 and 2. We shall make a indefinite integral p by rho, that means just integrate it with respect to r, then this will be omega square r square by 2 minus g z plus some constant, there may be a function of theta. So, therefore in r z plane if, we look in r z plane, that means that a constant theta so, theta does not come into picture if, we look in r z plane because, the the symmetrical flow, flow is symmetric about the theta.

You understand therefore, a constant pressure line, equation of a constant pressure line if, we find out in r z plane, equation of a constant pressure line, that means this is constant, this is constant so therefore, one can write omega square r square by 2 minus g z is equal to constant. Omega square r square by 2 minus again I am writing g z is equal to constant so, it is simply the integration of the Eulers equation, to find out the pressure and then pressure is constant, that means this is the equation of constant pressure lines in r z plane or a constant pressure surface, which is the equation of a parabola or a paraboloid or a paraboloid in the three dimensional space. So, this is the equation of the constant pressure line.

If, there is a free surface which was initially horizontal, which was initially horizontal will take the same shape like this, this is the free surface. Now, another interesting thing is that, what is the value of this h. That means the free surface will go like that and it will touch like that, this head h is very important, how to find out this head that means, what will be the depression of the free surface, that is the minimum point of the free surface and that is the point where the free surface touches the vessel.

So, this height will be found out from very simple calculation. If, we take a point 1 here and take a point 2 here, what is the pressure difference between p 1 and p 2, from it is a same horizontal line, this is 1 with this is the r then r is 0, here r is r, let capital R is the radius of this vessel. Then I can write the pressure distribution equation p 2 minus p 1 by rho is equal to what is p 2 minus p 1 by rho is equal to omega square, just now we have derived at 2 r is equal to r square, at 1 what is r, 0 divided by 2, just we have derived p 2 minus p 1 just now.

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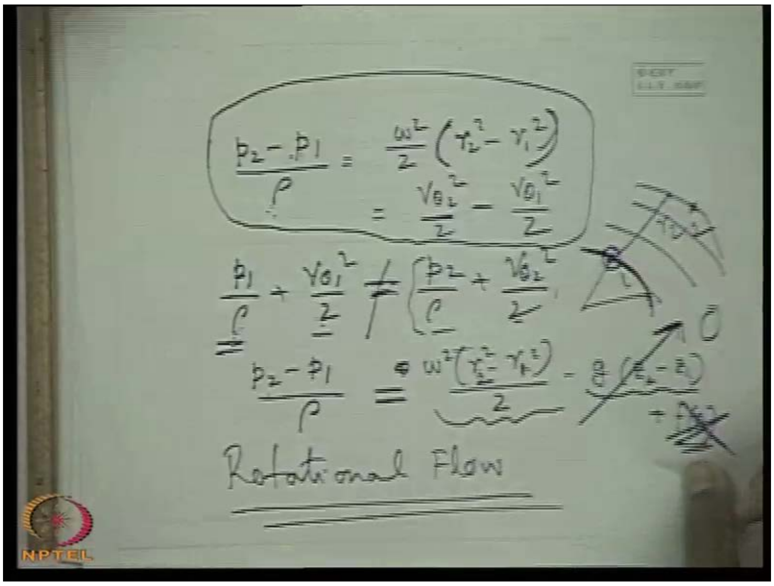

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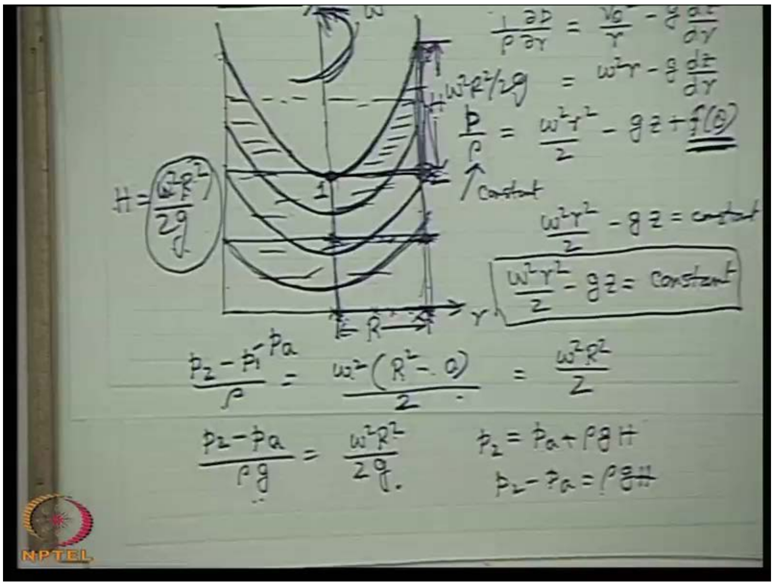

We have derived p 2 minus p 1 by rho omega r 2 square minus r 1 square by 2, that means v theta 2 square minus v theta 1 square by 2. So, this is simply equal to omega I am this is simply, is equal to omega square, r square by 2. Now, what is p 1, in a free surface this p 1 is atmospheric pa atmospheric pressure. So, p 2 minus p a by rho g rather is equal to omega square r square by 2 g. Between 1 and 2. Yes please, please all right very simple, this is, any problem, I am writing the Bernoullis… pressure difference between 2 and 1, p 2 minus p 1 by rho this is in the same horizontal plane 2 points 1 and 2 omega square r square minus 0 by 2.

What is d z, this is the same horizontal plane. Do not ask silly questions, you are IITians outsiders will laugh so, 1 and 2 I am telling again and again at the same radial location at the same horizontal plane, again and again I am telling it is at the same horizontal plane, no d z so, p 2 minus p 1 by rho omega square r square by 2 omega square r square by 2 g. All right, again p 1 is the pressure at atmospheric pressure now, if I find out the pressure 2 p 2 with in terms of this height h. We can write p 2 minus, what is p 2 here, the pressure is atmosphere throughout the surface pressure is atmospheric pressure, this is the free surface so, p a plus because it is full of liquid full of liquid so, what is the pressure here, p a plus rho g h so, p 2 minus p a is rho g h understood. Therefore, we can write p 2 minus p a by rho g is h, that means h is equal to omega square r square by 2 g that means, this point is omega square r square, that means this height is omega square r square by 2 g. So, at any 2 points in the same horizontal line the difference in pressure is nothing but this. Because, this height cancels difference between the 2 points is the pressure here, this is the maximum difference of pressure which is equal to omega. This is known as the maximum centrifugal head impressed due to rotational flow or forced vortex flow.

Any two point in a any horizontal line for example, in the base, the pressure at the difference at the two points if you consider from the fluid statics here, pressure is the atmospheric pressure plus this height here, pressure is the atmospheric pressure plus this height. So, if you neglect the difference make the subtraction 1 from other so, this is cancelling out so, this pressure at this point at r is equal to capital R with the pressure at the center is equal to rho g times this height. This static head and this static head is omega square r square by 2 g. So, the pressure difference between the center point and the radial at a point, at any radius, it is at any radius, the maximum takes place at when the point is at r is because of the rotation.

If omega is 0, there was no rotation static fluid, there is no pressure difference between one point to other point in the same horizontal plane, at different radial location, pressure only varies in the horizontal vertical direction. But, here the pressure varies in the horizontal plane, from one radial location to other radial location and becomes maximum the difference becomes maximum, when the point is at the one is at the center, other at the wall and this difference is omega square by r square by 2 g where r is the radius of this vessel. This is equal to this head, this is known as the maximum centrifugal head impressed. So, because of this centrifugal head the pressure difference takes place from one radius to other radius, this is also true for free vortex flow all right, clear, well understood.

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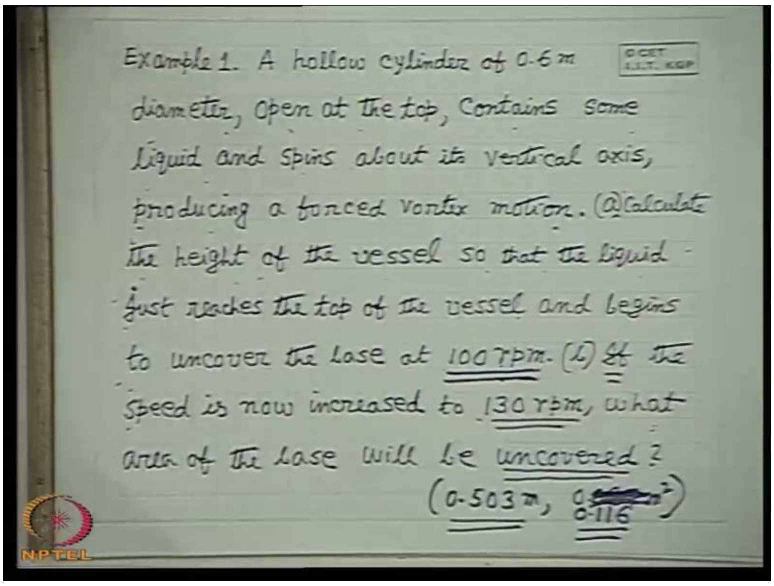

Now, I will be solving 2 problems, then things will be still better understood. A hollow cylinder, let us solve a problem on forced vortex then combine forced and free vortex. All right, do you have any query, please tell me as far as the theory part of free vortex and forced vortex flows example 1, a hollow cylinder of 0.6 meter diameter open at the top, that means it is a cylindrical drum, a hollow cylinder means a cylindrical drum or vessel of 0.6 meter diameter open at the top contains some liquid and spins about its vertical axis, that means it creates a forced vortex motion or forced vortex flow of the liquid in it. It is written, even if it is not given we should assume this but, producing a forced vertex motion, that means the problem is very explicit producing a force vortex motion. Number 1 a calculate the height of the vessel, so that the liquid just reaches the top of the vessel and begins to uncover the base at 100 r p m that means 100 r p m is the rotational, that means the angular velocity. The revolution of the revolutionary speed of the vessel angular speed of the vessel r p m, next one is that if, the speed is now increased to 130 r p m what area of the base will be uncovered

Well, I will again repeat the problem, a hollow cylinder of 0.6 meter diameter open at the top contains some liquid and spins about its vertical axis, producing a forced vertex motion. All right, calculate the height of the vessel, so that the liquid just reaches the top of the vessel and begins to uncover the base at 100 r p m if, the speed is now increased to 130 r p m, what area of the base will be uncovered, the answer is 0.503 meter, that is the height and this is the area 0.116 meter square 0.116 meter square.

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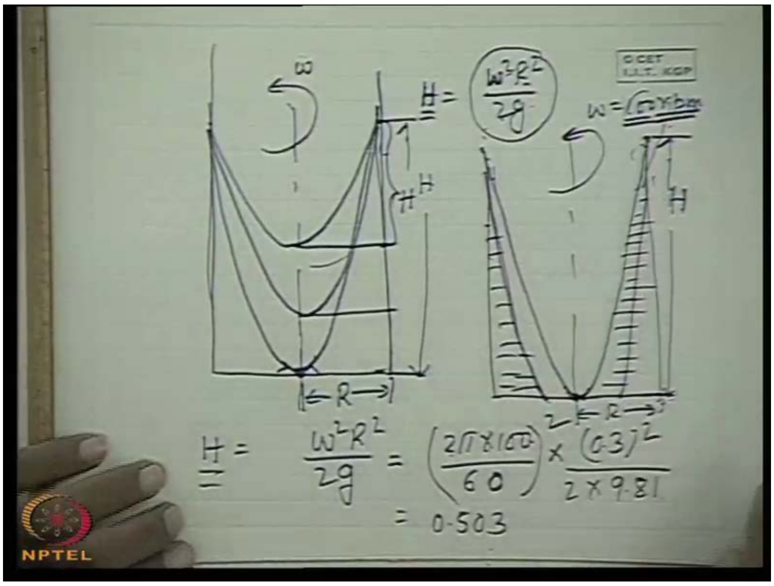

So, let me solve this problem, this problem is like this, let us consider a vessel, first part. Now, find out the height now, when this is being rotated with some angular speed omega, so what will happen, the liquid will do like this. If, it is further not rotated that means, you see this is h, you have to remember this is equal to omega square r square by 2 g now, r is fixed for the vessel r is fixed radial. Now, if we increase omega physically, what is happening, more and more centrifugal force is created and this head is more and more mathematically h becomes more, h increases when r increases. That means liquid will go like this, like this the surface will be generated.27:24

So, when you still increase omega 1, situation will come, when the liquid will touch the base. Let us consider this direction, this is very high, height is infinitely long so, it will touch the base. Now, first part is the is the of the problem is that find out that height of the vessel. When it will just touch the base, after that if you still increase the speed, it will detach from the base, liquid will spill out, that means without spilling out liquid will touch the base and will just touch the tip. That means what is that height, you have to find out that with this angular speed, the liquid what should be the height that if, we rotate with these angular speed, the liquid surface will just touch the tip while, the minimum point, that this point will touch the base, that means this is the point when the liquid will be just on the point of spilling. So, if you still increase the angular speed the liquid will spill out from this vessel and this will uncovered the base. So, this is the situation now, first part what should be the height of the vessel.

So, that corresponding to this omega 100 r p m 100 r p m. So, that liquid should touch this tip of the vessel and at the same time just or try starts on covering the base. That means the minimum point touches the base so, this is simply the now then what formula we will use, we will use h is equal to omega square r square by 2. Because, h is equal to omega square r square by 2 g, that means now the simple part is that, find out omega, what is omega twice pi. Now, fluid mechanics is over by 60, 100 r p m so by 60 is r p h within twice pi that means radian per second, proper unit omega square, r square r is given, what is r, 0.3 whole square. Now, on rest part is school level, simple arithmetic so, that we find out r is 0.503, next point is that you have to understand physically the problem. That if you know what happens, if we know still increase the speed beyond 100 r p m what will happen, you tell me some of the liquid will go out. Now, what will happen in that case mathematical you try to recognize. When the speed is still increased omega square r square by 2 g will be more than this h

Obviously, at 100 r p m because r is fixed r is fixed. When at hundred only at 100 r p m this h is made equal to that, based on that I have calculate it, that means now if, I fix the h and increase omega so, omega square r square by 2 g will be more than h. So, liquid cannot sustain cannot be sustain like that because, the centrifugal head is more than the static head. So, liquid column cannot stand like that. Then, what will happen liquid will this split outs, so liquid will go out from this vessel and this point will come here, that means liquid will uncover the base liquid will uncover the base.

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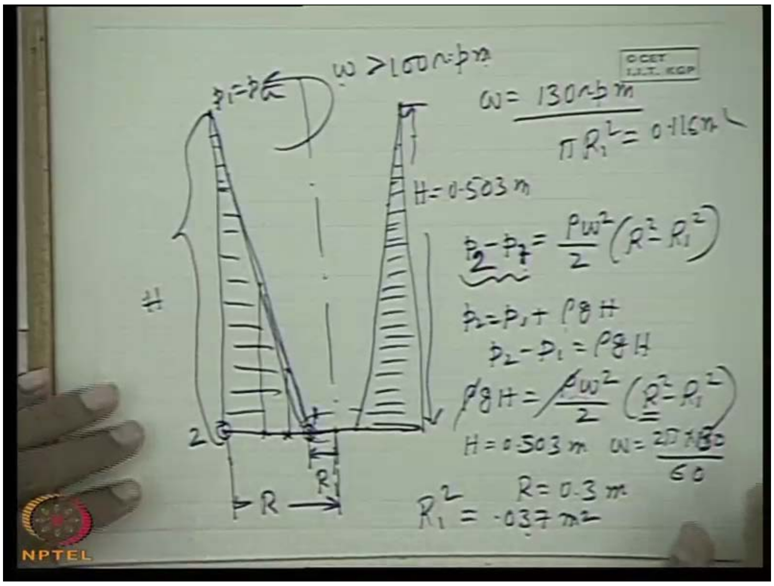

So, this situation I illustrate rather in a separate figure like this. When, we increase this speed what will happen, liquid will let this be the radius a central line so, liquid will then uncover the base, liquid will then things will be symmetric so, liquid will liquid will be spilt out. There will be spilling of the liquid from the vessel so, if it rotates at a velocity more than 100 r p m and with this same value of h, as I have calculated 0.503 meter, if it rotates at an r p m greater than 100 r p m. Now, I have been told to find out that uncovered part, what is that, what area of the base will be uncovered this, a that means to find out these radius detachment radius. Let this radius is R 1, this is R, total is R, that is given point 3 meter. So, I will have to find out this R 1, when omega is specified as 130 r p m, whenever omega greater than 100 r p m, this situation will occur. Let us consider this as R 1. Now, it is very simple, let us consider this point 1, this point 2.

So, find out p 1 minus p 2, what is p 1 minus p 2, p 1 p 2 minus p 1 or p 1 minus p 2 rather you write p 2 minus p 1 is equal to rho omega square by 2, R square minus R 1 square. That is p 2 minus p 1. Now, what is p 2 minus p 1. Tell me what is p 2,p 2 is atmospheric pressure. Because, this is the free surface this is the free surface so, throughout the free surface pressure is atmosphere, this pressure is at but, here the pressure is not atmosphere, atmospheric pressure plus this height, here therefore, from hydrostatics the pressure p 2 any side we can do it, it is symmetric pressure p 2 is atmospheric pressure plus this height, that means p 2 minus p 1because, p 1 is the atmospheric pressure p 1 and p 1, here is same that is atmospheric pressure so, this point p 1 plus p 1 plus rho g h, that means p 2 minus p 1 is rho g h, that means difference between these 2 pressure from the hydrostatics is rho g h therefore, rho g h must equal to rho omega square by 2 R square minus R 1 square. Now, here I will not do anything because, everything is given h is 0.503 meter, omega is given twice pi into 130 num in num, it is 130 by 60 and R 0.3 meter, you find out R 1, actually straight way R 1 square is found so, you better keep it in terms of R 1 square 0.037 meter square.

So, pi R 1 square will come, pi into this that means 0.116 meter square, that means this is the uncovered area. This is as simple as this, all right very simple. Now, another problem you just please write down is there any difficulty, please tell me, all right.

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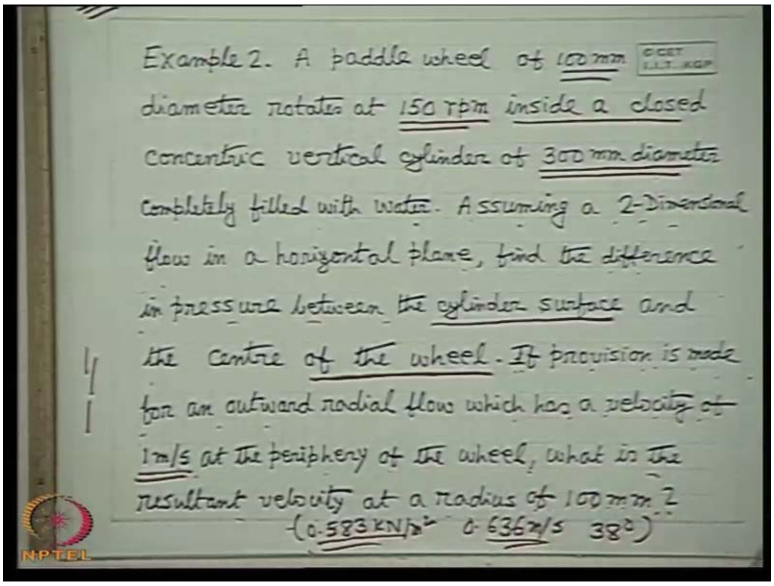

Example 2, a paddle wheel of 100 millimeter diameter paddle wheel of 100 millimeter diameter, rotates at rotates at 150 r p m, inside a close concentric vertical cylinder of 300 millimeter diameter, completely filled with water. That means a vertical cylinder again a cylindrical drum type of thing 300 millimeter diameter full of water inside it, there is a paddle wheel whose diameter is 150 r p m, paddle wheel means, a number of blades wheels starrer type of thing ,which is connected to a small shaft and this is being rotated. That means to start the water inside the cylindrical drum, assuming a 2 dimensional flow in a horizontal plane, that means horizontal plane 2 dimensional flow this is same at all heights. Assuming a 2 dimensional flow in a horizontal plane, find the difference in pressure between the cylinder surface. That means at the cylinder surface, at the extreme radial location and the center of the wheel center of the wheel means center of the cylinder if, provision is made for an outward radial flow, which has a velocity of 1 meter per second at the periphery of the wheel, what is the resultant velocity at a radius of 100 meter. So, first point is the pressure difference between the cylinder surface and the center of the wheel.

The second point is if, provision is made for an outward radial flow which has a velocity of 1 meter per second. This can be done if, we make wholes at the cylinder surface water will come out from the cylinder surface. This state that if provision is made for an outward radial flow then flow is coming out from the cylinder, which has a velocity of 1 meter per second at the periphery of the wheel. What is the resultant velocity at a radius of hundred millimeter, all right. This is the these are the answers but, let me tell you how to do it, all right taken it, a paddle wheel of 100 millimeter diameter rotates at 150 r p m inside a closed concentric vertical cylinder of 300 millimeter diameter, completely filled with water. Assuming a 2 dimensional flow in a horizontal plane, find the difference in pressure between the cylinder surface and the center of the wheel. Well if, provision is made for an outward radial flow, which has a velocity of 1 meter per second at the periphery of the wheel. What is the resultant velocity at a radius of 100 millimeter, all right, let me explain this problem.

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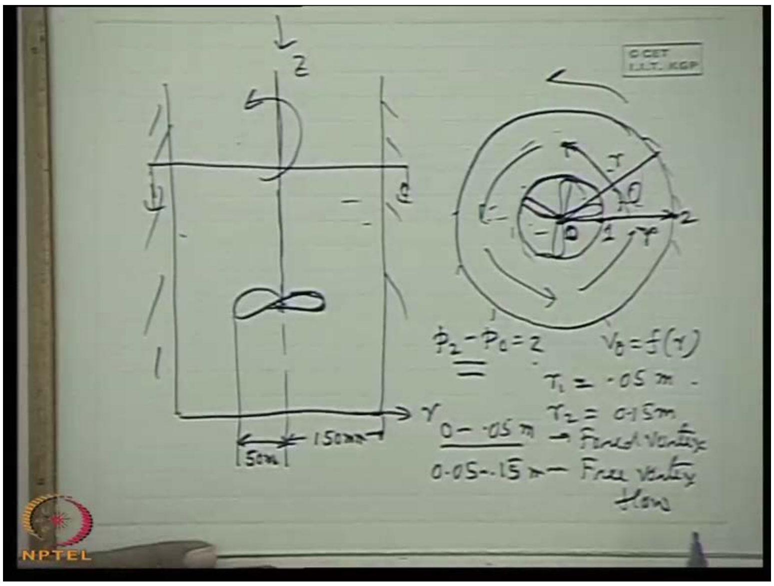

This problem is like this, there is a cylindrical vessel there is a cylindrical vessel, let these be this axis cylindrical vessel whose radius is what 150 millimeter 150 millimeter, all right and inside this there is a paddle wheel there is a paddle wheel

This paddle wheel radius is how much what is this, this is 50 millimeter, very good 50 millimeter, that means if you see the plan view at so, liquid is moving so, paddle wheel is cylinder is fixed paddle wheel is rotated that means if you see a plan view you cannot draw here I am drawing at this place. So, you will see a view like this, this is the cylinder surface this is the cylinder surface this is the cylindrical surface and there is the paddle wheel, that means this is the paddle wheel, this is the center of the cylinder and also the wheel, that means this is the paddle wheel has got the same this vein type of thing, this is the paddle wheel. So, therefore this can be described and r theta, plane 2 dimensional any plane if we see that water is being churned if, it is rotated in this direction so, water is being churned so, in tangential is a vortex flow but, first problem is that find out the pressure difference between these 2 point, let this point is 1, that is at the let this point is 0, at the center beta 0 origin and let this point is 2. So, I have to find out p 2 minus p 0, this is the centre so, this thing prevails at all height so, about the 3 dimensionality of the problem we do not have to consider we consider any 2 dimensional plane that means at any section.

All of you have understood so, this is full of liquid so how to find out this. So, what you will assume, this is full of liquid we cannot assume this as the entire free vortex motion because, free vortex motion cannot be extended up to origin, you know that there is a singularity at the origin and free vortex motion does not come up to origin at r tends to 2 0 v theta becomes infinite. So, what we will do here can you tell me, what we will do nothing is told but, what model we will do to find out the pressure difference. First of all we will have to model the flow field, that means we will have to take the model of the velocity field that means v theta is a function of r. So, what type of model we will take, we will consider the forced vertex flow up to this radius, let this point is 1, that means r 1 is equal to 50 millimeter that means 0.05 meter and r 2 is equal to 0.15 meter that means from 0 to 0.05 meter within this region, we will consider the flow is a solid body rotation. Where the energy is directly imparted by the paddle wheel, where from this point, this is an approximation, we will have to tell we may argue sir how do you know exactly here the flow will be if, we actually measure the velocity distribution we may see the velocity varies directly with radius up to some distance little away from the paddle wheel. Then far away from the paddle wheel near the wall it may the free vortex.

But, we do not know exactly the point but, we approximately like that up to this point 1 that means the radius of the paddle wheel radius of the paddle wheel the flow will be forced vortex. forced vortex forced vortex and from 0.05211 no 0.15 that means 2 it is free vortex, all right free vortex flow. Then it is very simple now, all right now, it is very simple I find out then it is very simple what I have to do.

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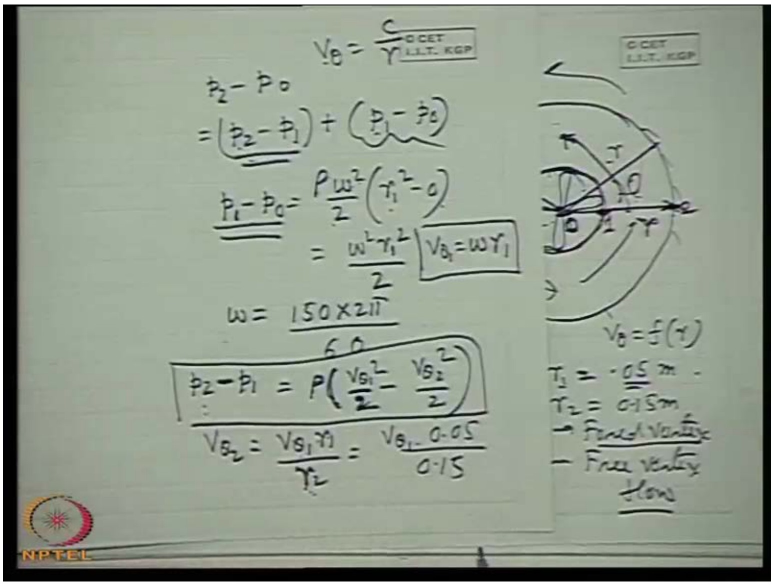

Now I have to do like this, then I write p 2 minus p 0 is equal to p 2 minus p 1, plus p 1 minus p 0. What is p 1 minus p 0, that is a forced vortex equation, what is p 1 minus p 0, you can calculate this p 1 minus p 0 if, I find out can know the omega is omega square by 2 into what is p 1 minus p 0 ,is omega square by 2 into this is r 1 I am writing in terms of r 1 square minus 0 ,that is omega square r 1 square by 2, that means this is p 1 minus p 0, is omega square r 1 square by 2 you know the value of omega omega. You know what is the value of omega, paddle will rotates at 150 150 into 2 pi by 60 omega. This r 1 you know r 1 is 0.05 then from p 2 to p 1 you equate with free vortex flow. What is that p 2 minus p 1 you can write here, I am sorry some rho will be there so, here you see, the you need rho so this will be rho into just the Bernoullis equation. It is the Bernoullis equation v theta 1 square by 2 minus v theta 2 v theta 1 square by 2. All right because, p 2 by rho plus v theta square by 2 is equal to p 1 by rho this already discussed.

That the radial distribution of pressure means that you can write the bernoulli's equation. Same thing so, I know v theta 1 how to know v theta 2, v theta 2 is what because, v theta 2 and v theta 1 is related by the free vortex equation, where is free vortex equation v theta is c by r that means v theta into r is equal to constant. That means v theta 2 is v theta 1 r 1 by r 2 that means if, I know v theta 1 that is r 1 by r 2 or what is r 1 0.05 r 2 is 0.1 but, you have to know the v theta 1. How to know v theta 1, v theta is omega into r 1 because, v theta 1, here it can be found out by the forced vortex equation. So, v theta 1 is equal to v theta 1 is equal to omega into r 1 all of you have understood this problem that means I will evaluate p 1 minus p 0 part from the forced vortex equation. Where everything is known I can find out and p 2 minus p 1 from the free vortex equation because my flow model assumption is that from 0 to 0.05 is the forced vortex flow.

From 0.05 to 0.15 is the free vortex flow therefore, this is the free vortex equation. This is the pressure distribution for the free vortex equation, to know the v theta 2 from v theta 1 I use the free vortex tangential velocity distribution v theta 1 r 1e v theta 2 r 2, if you do it, you will get these values.

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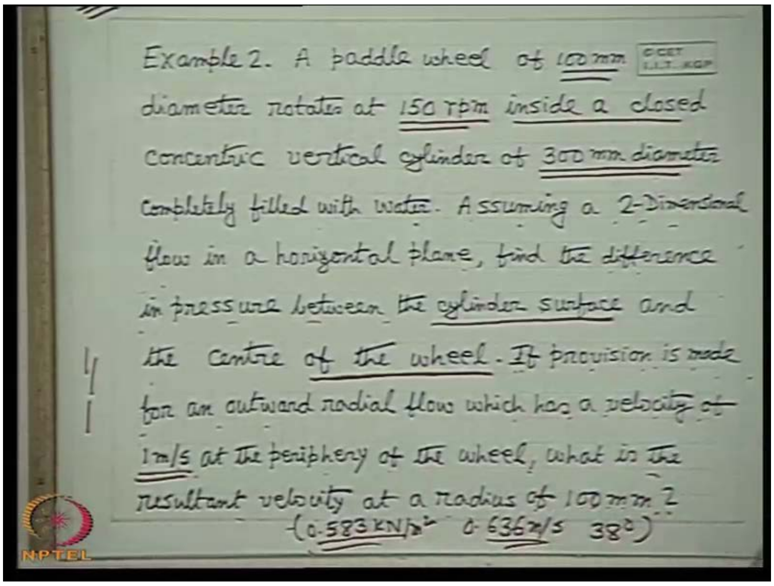

Of course, the second part I have not done, next class I will do time is up, this is 0.583 kilo Newton per meter square. If, you do it you will see 583 kilo Newton per meter square. This is the answer, cleared any problem, all right next part of the problem I will solve in the next class. Before I start the theory part or you can solve it, this is an exercise given to you, yes please time is up.

Thank you

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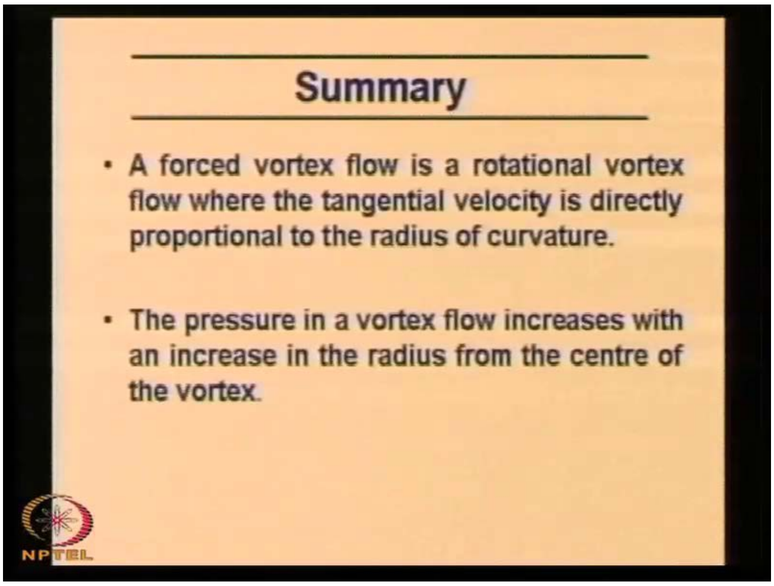

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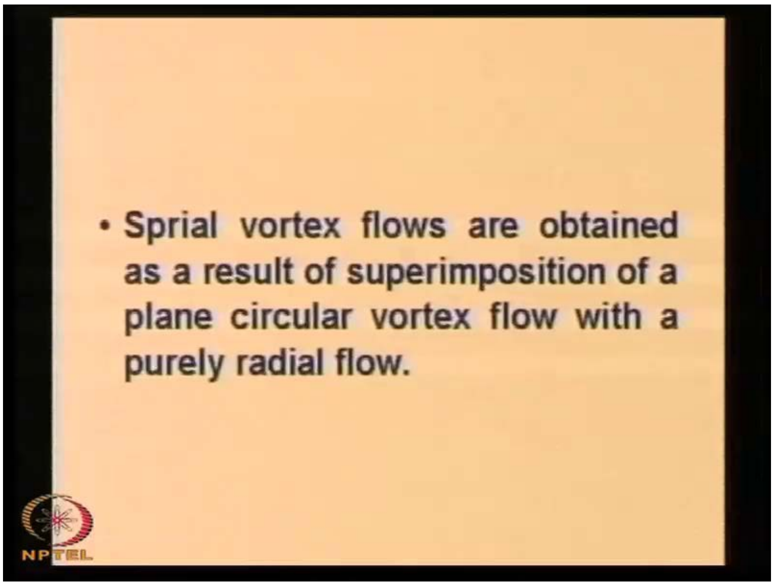

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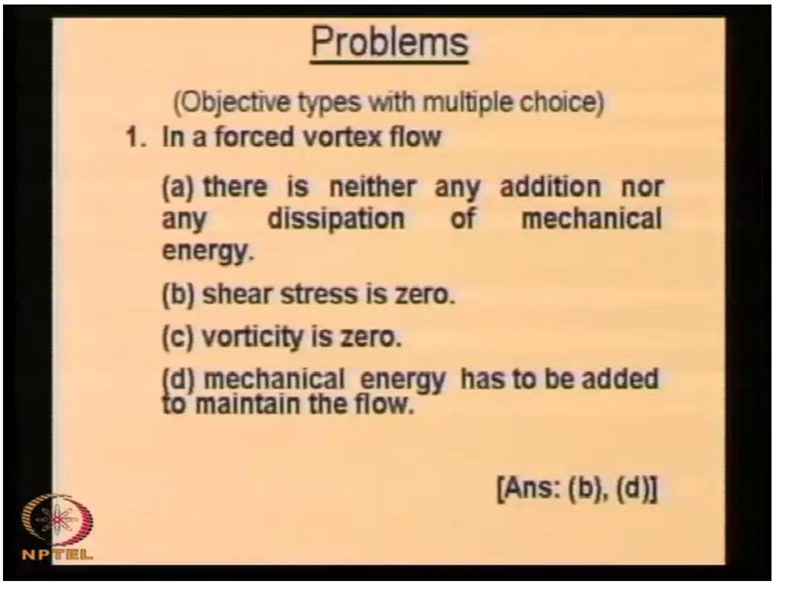

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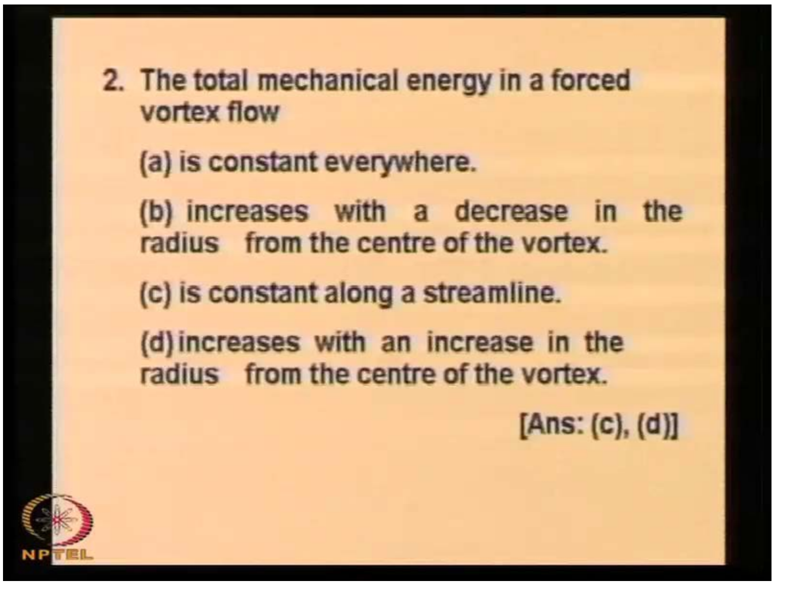

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