Fluid Mechanics

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Lecture - 22
Fluid Flow Applications Part - I

Good morning, I welcome you all to this session of fluid mechanics. Today, we will start a new chapter, a new section, that is fluid flow applications. We have already discussed the conservation equations; that is the conservation of mass equations for conservation of mass, conservation of momentum and conservation of energy, and we have solved some problems related to finite control volumes, both industrial and non-industrial control volumes.

Now, in this section, chapter, we will discuss few practical problems of fluid flow applications, which will be solved on the basis of the equations for conservation of mass momentum and energy. The applications of those equations, better we can say that it is the application of conservation equations to fluid flow problems.

At the outset we will discuss little bit about the Bernoulli's equation or Bernoulli's theorem which we have discussed earlier, a little elaboration on that. If we recall we earlier recognized the Bernoulli's equation as the equation of mechanical energy or mechanical energy equation in case of a fluid flow which tells about the conservation of energy in a fluid flow. We have recognized that if the fluid is inviscid, obviously the mechanical energy remains constant. If there is no energy added from outside, no mechanical energy is taken out of the fluid. If the fluid is inviscid, that means viscous action is not present that fluid is inviscid, no viscosity that means no friction. So, mechanical energy cannot be dissipated in terms of intermolecular energy or heat as you can tell. So, in that case total mechanical energy remains constant, and we have seen that in general, these mechanical energy, total mechanical energy remains constant along a streamline. We have recognized different forms of mechanical energy as the pressure energy, kinetic energy and the potential energy.

If you recall the equation, you can tell that p by rho which represents the pressure energy per unit mass plus v square by 2, which represents the kinetic energy per unit mass, we have to always recall in this form plus g z where g z represents the potential energy per unit mass. Here of course we consider only the earth's gravitational force field as the only body force field. Therefore, the potential energy per unit mass will be g z, where g is the acceleration due to gravity and z is the elevation of the point concerned, where we are considering the energy per unit mass from an arbitrary datum.

So, p by rho plus v square by 2 plus g z thus constitutes the total mechanical energy and which is equal to constant for an incompressible steady flow, because this form of the equation comes only for a steady incompressible flow, that is of course important. Otherwise integral d p by rho will come for a compressible flow, but p by rho this term comes for an incompressible flow. So, for an incompressible steady flow, p by rho plus v square by 2 plus g z, simply the sum of the three three components of the mechanical energy is constant along a streamline. If you recall the derivation of these equation, we did it by integrating the Waeler's equation along a streamline. Probably you can recall this that integrating the Waeler's equation along a streamline, therefore the constant which came out of these integration was a constant. That means it was valid only along a streamline. Therefore, essentially the mechanical energy varies from streamline to streamline in general.

But, now we will extend this in case of an irrotational flow, that means if we add a further constraint in the flow that if the flow is irrotational, already you have started what is an irrotational flow in fluid kinematics, that a flow is irrotational when the rotation at each and every point is 0. You know the rotation of a fluid element at each point is defined as the Carl of the velocity vector Carl v in different co-ordinate system we can expand this Carl v. Physically the rotation means the arithmetic average of the angular velocity of the two linear segments meeting at that point which was initially perpendicular.

So, rotation is 0 means the flow is irrotational. If we add a separate or additional constant of irrotationality, that means if flow becomes irrotational in which its steady incompressible, then these mechanical energy is constant throughout the flow field. That means p by rho plus v square by two g plus g z is equal to constant not only along a streamline, but at any point in the entire flow field. This we will derive first which is a very simple derivation. Let us do it.

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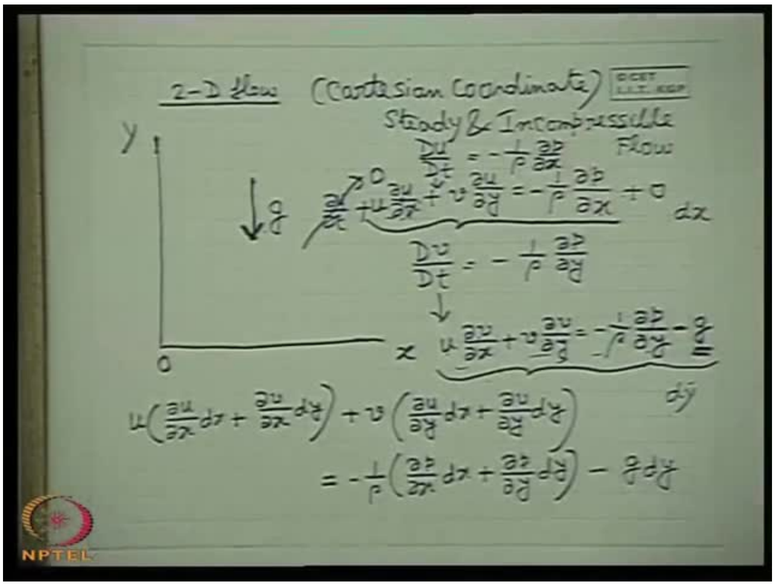

Let us see this, let us first take a 2 dimensional case, a two dimensional flow, a two dimensional flow in a Cartesian coordinate system, in a Cartesian co-ordinate system, in a Cartesian rectangular, that is Cartesian co-ordinate, a Cartesian co-ordinate system. In this case let us consider only x and y no z. So, why we consider and this we consider the y axis is in the vertical direction. Y directed positive upwards vertically upwards, so that g is acting like this. So, if this we consider as the frame of co-ordinate and also we consider a steady and incompressible flow, if we consider a steady and incompressible flow, steady and incompressible flow, then if we recall the equation of motion, that is the Waeler's equation, we can write that for a steady the temporal that u will not come u del u del x plus v del u del y is equal to minus 1 upon rho del p del x if you recall.

So, this is D u D t, big D u D t, big D u D t, that means this is equal to minus 1 upon rho del p del x. This is the x direction. So, this has been expanded, del u del t is 0 for steady state, so I have not written, del u del t is 0 for steady state and rho is minus 1 upon rho then this is the form x direction equation, y direction equation similarly will be D v D t if you recall, is equal to minus one upon rho del p del y.

So, if you expand this in case of steady flow that means del v del t is 0, then you get u del v del x plus v del v del y is equal to minus 1 upon rho del p del y. So, this two equations are the equations for incompressible steady flow with respect to a two dimensional Cartesian co-ordinate system. Of course, here I am wrong. Minus rho del p del y here g is here acting, so we have considered the y axis such a way that vertically upwards directing positive. So, g is in the negative direction, so body force term will come which will be here minus g, because these are the force per unit mass, because here these left hand term is the acceleration per unit mass. So, first term on the right hand side is the pressure force per unit mass. So, per unit mass the body force will be minus g, why minus? Per unit mass it will be g m g divided by m g and it is in the negative direction of the positive y axis, therefore minus g. This comes as the body force if you recall, the body force, here body force is 0 in the direction of x there is no body force.

So, we have chosen the y axis in such a way that it is directed upwards, vertically upwards, so that the gravity force come as the body force. So, per unit mass basis it appears as minus g. I am sorry. So, this term will come. Now, what we will do? We simply know that the basic approach for finding out the energy equation is to multiply the momentum equation with the displacement. So, what we do? If we multiply these equation x direction equation by d x and multiply the y direction equation by d y both this sides and add it. What we get? U del u del x d x, we first take the u term common then from this equation it will be del u del x d x and from this equation it will be u del v del x d x.

U del v del x d y. sorry. u del u del x d x plus v del v, u del v del x d y plus v del u del y d x plus del v del y d y is equal to minus 1 upon rho. If you take common then del p del x d x plus here del p del y d y you are multiplying this equation by d x and this equation by d y and adding up. u del u del x d x plus u del v del x d x del v del x d y, I am sorry. similarly, v you take common del u del y d x plus del v del y d y minus 1 upon rho del p del x d x plus del p del y d y minus g d y.

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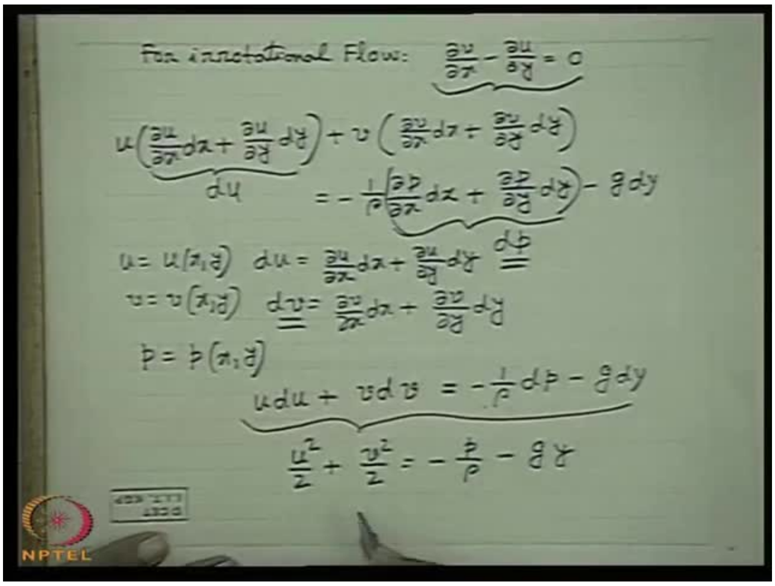

Now, after this we put the constrain for, constrain for irrotationality. Now, now for irrotational flow, for irrotational flow, irrotational flow, that is the condition for irrotationality is del v del x minus del u del y very simple is 0. Now we pose these irrotationality condition mathematically on this. So if you pose this now we just substitute del v del x as del u del y here and here del u del y as del v del x here the purpose is very simple you will understand that this will become the first terms in the bracket del u del x d x del v del x is del u del y plus del u del y d y well, so it becomes del u del x d x plus del u del y d y. Similarly, here in place of del u del y we substitute del v del x, so del v del x d x plus del v del y d y well is equal to minus 1 by rho del p del x d x plus, sorry this will be in the bracket del p del y d y minus g d y.

Now, it is simple mathematics, therefore we can see that we have described the problem in two dimensional in x y coordinate. Therefore, this we can write as d u, why? U is a function of x y if we consider then d u is the change in u is caused by a change with both x and y. So, mathematically by Tailor series expansion neglecting the higher order term we know that this we can write. Similarly, if we consider v as a function of x y in two dimensional system, so del v del x d x plus del v del y d y, del v del y d y can be written as d v. Similarly in the case of p that we can write this as d p, I am not writing it again. So, d p, that means this equation can be written now u d u plus v d v is equal to minus 1 upon rho d p minus g d y.

Now, this is the equation which you obtain, now if we integrate this equation what we will find out? We will find u square by 2 plus v square by 2 is equal to, now if we consider the incompressibility the condition of incompressible fluid rho is constant. So we can take out of these integral, so integral d p means p, so minus p by rho minus g is constant plus y.

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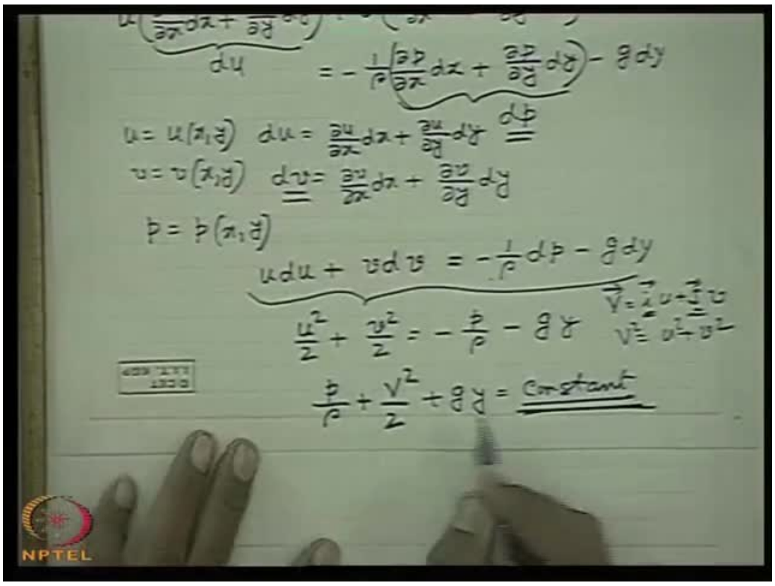

That means we can write now u square plus V square, means the resultant velocity V square and we take p on one side V square V is the resultant velocity, where v is the resultant velocity i u plus j v where i j are the unit vectors in x and y direction. That means V square is u square plus V square. So capital V is the resultant velocity plus g y is equal to constant. Now, you see that this is a constant where there is no restriction along a streamline, why? Here these equation means that the summation of this two.

Now, you know if we have a force vector f and if we make a dot product, that means the scalar product with a position vector in polar coordinate d r these implies the work or energy these d r is d x and d y component. Similarly, what we have done that equation of motions that the equation of motions, if you see that the equation of motion, because force into distance is the work. Similarly the equation of motion have been multiplied with the respective component displacement component d x d y and this has been added. That means this component d r is arbitrary there is no restriction that whether it will be always along a streamline or so.

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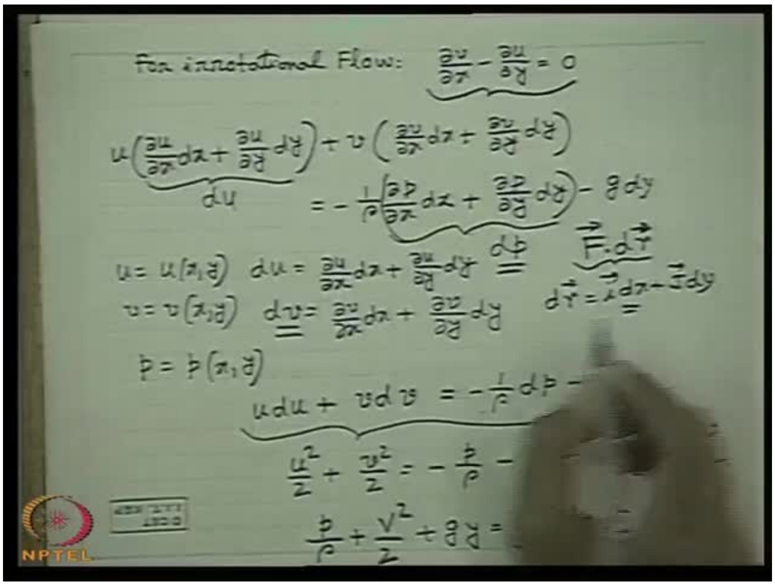

Therefore, in the entire deductions we have not meant any restriction regarding this choice of d r the arbitrary displacement which we have multiplied with the force vector or the equation of motion or the equation for the conservation of momentum, so that this constant is a constant throughout the force field. Therefore, now we derive or we come to a very general conclusion which is very, very important for you in the entire lifetime those who will be dealing with fluid mechanics that, the total mechanical energy that is the pressure energy it is per mass basis written kinetic energy and the potential energy is constant throughout the flow field in case of an irrotationality.

So therefore, for an irrotational flow the mechanical energy is constant throughout the flow field which physically implies that the irrotationality to be maintained in a flow field, energy should not be added, energy should not be taken out from the fluid. So, total mechanical energy remains the same. Therefore, irrotationality and the constants you have mechanical total mechanical energy in the, in the flow field is a consequence of one another or each other.

Now, after this we come to the definition of certain flows, certain important flows. First we start with plane plane circular vortex flow definition of certain flows, plane circular vortex flows. what are meant by plane circular vortex flows? Usually the word plane circular are not used many times simply vortex flows, but it is always better to use plane circular vortex flows.

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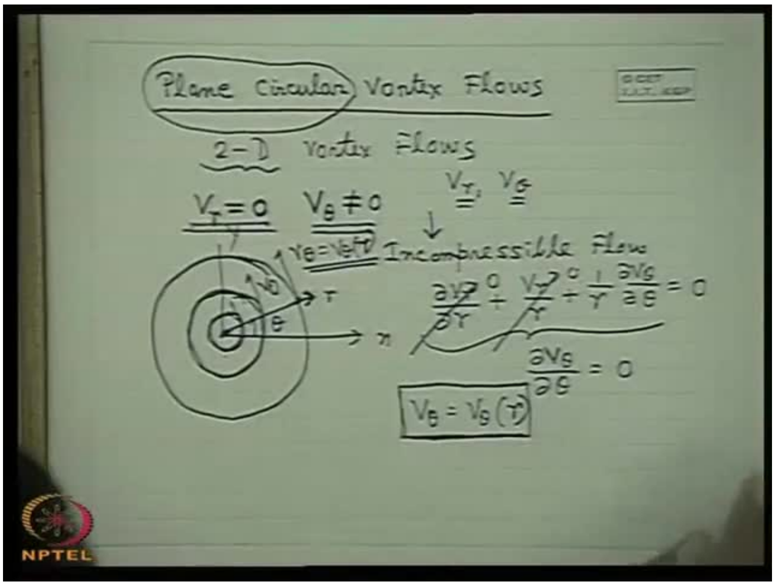

The first line of definition of vortex flows, plane circular vortex flows, that is a two dimensional vortex flows, rather plane circular is substituted by two dimensional vortex flows, this flows are flows where first line of definition in a polar co-ordinate system the velocity field can be represented like that. That means no radial velocity only tangential velocity. That means, a two dimensional flow field described by with respect to a polar co-ordinate systems as V r V theta its velocity components. So, there is no radial velocity, but it has got only tangential velocity. That means the flow where the tangential velocity is only present, but without any radial velocity are the vortex flow. Therefore, the streamlines will be concentric circles, because the fluid particles moves with circular path only with tangential velocity.

So, if any particle has got only tangential velocities with respect to any polar co-ordinate system r and we can for example, this is x y this is r this is theta, so always there is a V theta, always there is a V theta at different radius, but there is no V r there is no radial component of velocity. So, now if this be the first line of definition of a two dimensional vortex flows or plane circular vortex flows, then immediate consequence of it comes from the continuity equation, let us write the continuity equation. Another thing is that we considered the flow to be incompressible that is implied, so incompressible flow, so incompressible flow.

So, if you recall the continuity equation for incompressible flow in a two dimensional system like this if we write in terms of the polar co-ordinate del v r del r if you recall this you will have to recall always because it is not proper always to derive everything from the basic. so 1 upon r del v theta del theta is 0. This is the form for a incompressible two dimensional flow in a polar co-ordinate system del V r del r plus V r by r plus 1 by r del V theta del theta 0.

Since by definition V r is not there, that means it is a situation of flow where only tangential velocity exist. So these two term is 0. So, the immediate consequence from the continuity for this type of flow is that del V theta del theta is 0. That means V theta is a function of r, only that means del V theta del theta 0 means V theta cannot be a function of theta. That means V theta is the function of r which is a very important conclusion.

Therefore, we come to the next step concluding that in two dimensional vortex flow where only the tangential velocity exists. But radial velocity is 0 in that case the tangential velocity is a function of r only, that means the tangential velocity at different radius the fluid particles moving in a circular path for the tangential velocity at any radius is a function of this radius itself. Radius is defined from its origin. That means the tangential velocity is a function of radial coordinate.

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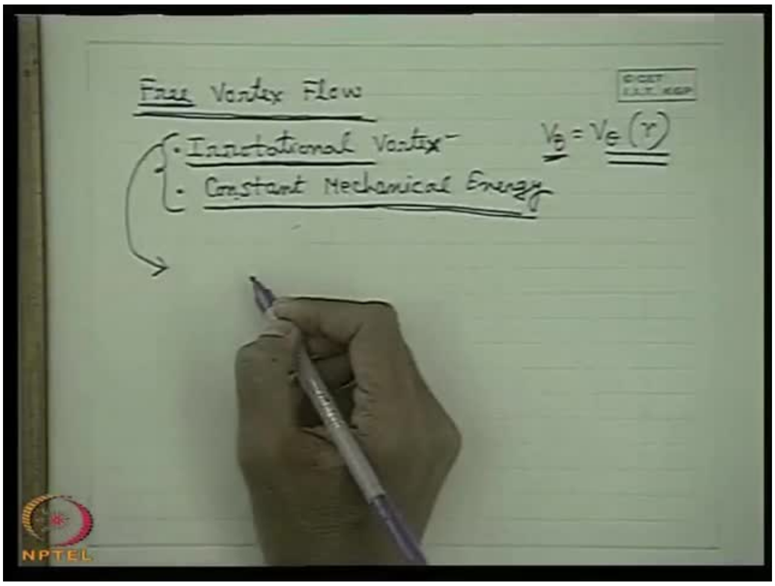

Now, the next question is that what is the form of this explicit functional relationship? What sort of functional relationship with r does the velocity component the tangential velocity component V theta bear? For that we will have to recognize two types of vortex flow. For that first we specify a two dimensional vortex flow. One of those types is free vortex, free vortex flow that means it is one of the vortex flow, free vortex flow. The word free poses a different constant, constant is that it is irrotational, irrotational.

So, these are the synonymous, free vortex flow means irrotational vortex flow or irrotational vortex, free vortex means irrotational vortex all or you can say because already we have discussed that the consequence of irrotationality is the total mechanical energy constant. That means constant mechanical energy, constant mechanical energy. Therefore, it means that a vortex flow where the total mechanical energy remains constant within the flow or the vortex flow which is irrotational. Just now we have seen that one is a direct consequence of the other, if we consider a flow to be irrotational mechanical energy is constant in the entire flow field or we consider the mechanical energy to be constant in the entire flow field this will be irrotational.

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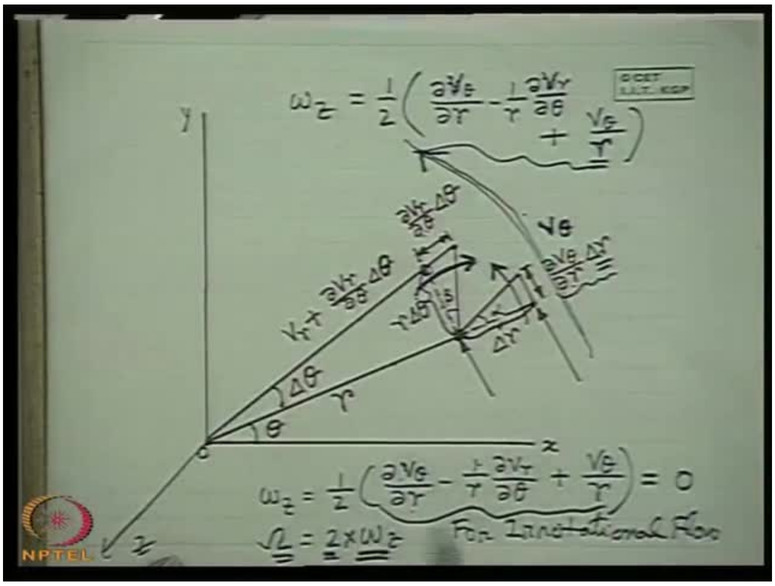

Therefore, either of the two will be valid, that means either of the two we can take automatically another one will be valid. That means the vortex flow if it is irrotational or its constant mechanical energy is known as free vortex flow. And with this constraint we can find out this explicit form of the function r that V theta is a function of r, anyone of this. I will show you both the things first. It is very easy to derive from these irrotationality condition. Now, what is an irrotational flow? Now, for a, in case of a polar co-ordinate system you know that rotation is 0. Better we can recall it, I do not know whether I have discussed it earlier or not.

Let us see here that in case of a polar co-ordinate system how do you define the rotation. Now, this a point the similar way as we discussed in Cartesian co-ordinate system. The rotation is like that here there is the co-ordinate r and this is theta. Now, at a delta theta this is the position. Now, if we consider a point here and the fluid element, this dotted line. Now, this fluid element rotates in this direction, this is the constants of rotation or sense of angular velocity, just like in Cartesian co-ordinate. This is because of what, so this point let this elemental length is delta r. So, this point has got a more tangential velocity than this point. This is because that V theta is the function of r. So, this value is del V theta del r into del r. That means this point has a the tangential velocity relative to this, that means this point displaces with respect to this point in the tangential direction by this amount because the theta is the function of r.

Similarly, this point which is separated by this point by a distance r delta theta or by the angular displacement delta theta has got a more radial direction displacement or a more radial velocity by an amount del v r del theta del theta, well because of the angle delta theta displacement. So this point has got some radial velocity this point also has got some radial velocity, but the radial velocity at this point is more than this point because the radial velocity is a function of theta and that is del V r del theta into delta theta, so this is the displacement.

So, by virtue of which this dotted element has which has got an angular rotation in these direction. Now you see this angle is alpha and this angle is beta and taking this as the positive direction anti-clock wise, so the rotation the omega z if we consider this as the z that means the perpendicular to this the z.

Let us consider the z axis omega about z we will be half of the angular velocity if we take del V theta del r into del r divided by this. So these becomes simply del V theta del r well and these direction del V r del theta del theta divided by r del theta that means minus 1 by r del V r del theta or a capital V, I am using this capital v del v r del theta.

So this is in these direction, but there is an additional component which is very important here. This is because apart from this rotation about this point of this two linear fluid element, the entire fluid elements anywhere if you take a fluid element, it has got a rotation about this point because of this value V theta tangential velocity whose magnitude is V theta by r. That means in any circular motion or curvilinear streamlines which is always represented in Polar co-ordinate system where there is a tangential velocity V theta. It implies always an angular velocity about the origin o whose value is V theta by r, which has to be always added and this sense is the positive sense is the positive V theta direction, so which has to be always added.

Therefore, the conclusion is that the omega z that is the angular velocity. Sorry, the rotation the average of the angular velocity the rotation becomes del V theta del r that cross differential just we have seen del V del x minus del u del y in case of a two dimensional Cartesian co-ordinate system del V del x minus del u del y here del V theta del r minus 1 by r del V r del theta, but only the additional term V theta by r is there. This is the rotation. And similarly the vorticity is twice the rotation as I have told u the vorticity is defined as twice the angular rotation, so two times a this is equal to vorticity.

Now, for an irrotational flow, therefore this will be 0 for irrotational flow for irrotational flow, so for irrotational flow this will be 0. So for a recap I just tell you that how we derive the irrotationality condition or the rotation the expression for the rotation.

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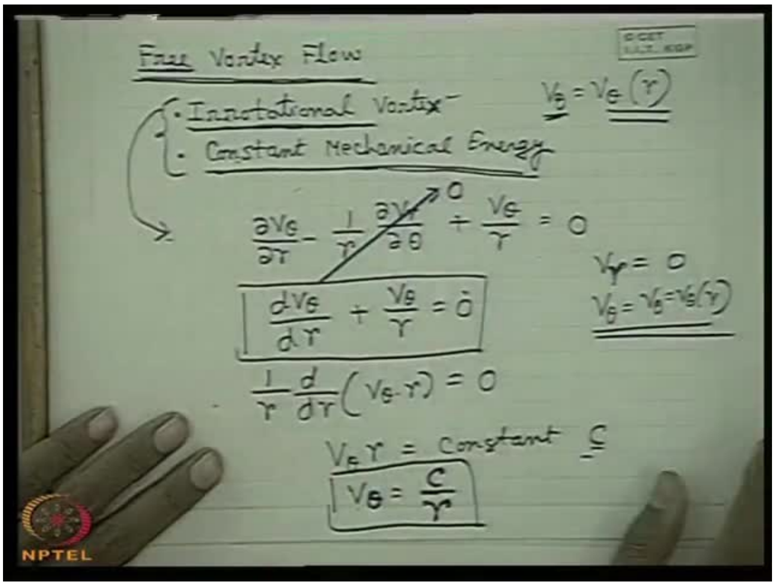

Therefore, for a free vortex flow if we start from irrotational vortex, therefore in a two dimensional flow, the concept is del v theta del r that is rotation has to be 0 minus 1 by r del V r del theta I write the full form first V theta by r is 0.

Now, since for a two dimensional vortex flow this is 0, and V theta is a function of r, so we simply write in terms of the total differential or ordinary differential nomenclature. So del we do not write because already in our hand it is like that V r 0 and V theta is a function of r these are already in our hand.

Beyond this we are giving an additional constant that irrotational vortex, so d V theta d r plus V theta by r 0. So, this is the equation immediately gives the, if you integrate these equation it gives the function V theta as r. So, how it can be written as 1 by r d d r of straight we can integrate it, but we can write this way also d d r of V theta into r is equal to 0, so if you integrate it V theta into r is equal to constant.

Let this constant is C, that means v theta is equal to C by r well V theta is equal to, therefore this is the equation for the tangential velocity distribution that is of tangential velocity with other that is inversely proportional to r in case of a free vortex flow. This can also be deduced from constant mechanical energy. I will show you that this can also be deduced from a constant mechanical energy consideration, so what is the total mechanical energy?

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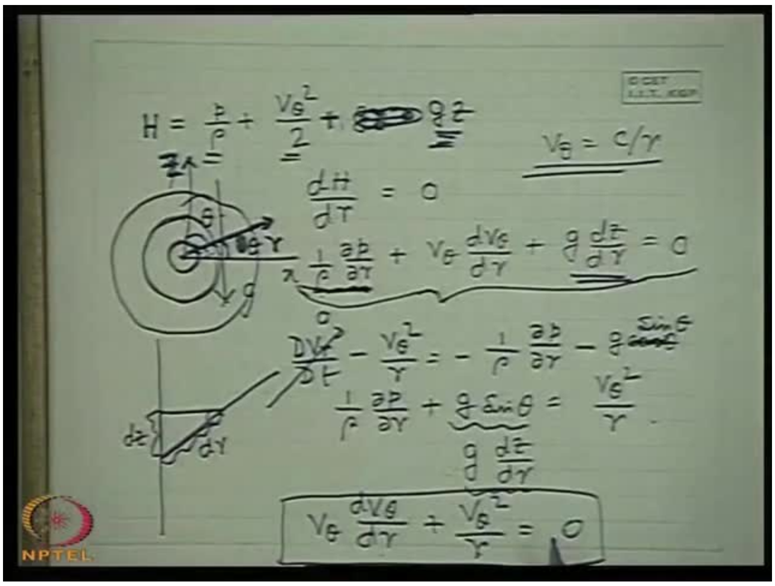

p by rho plus V theta square by 2 to the only velocity is V theta plus g y is equal to, let this is the total mechanical energy let this is equal to H.

So, this is the mechanical energy per unit pressure, energy per unit mass, this is the energy, what is this? Kinetic energy per unit mass and this is the g y this is the potential energy per unit mass. Here you can take y or you can take z better our general notation g z that is the potential energy per unit mass so these are the components of the mechanical energy, this is the total mechanical energy H.

Now, we know that the mechanical energy remains constant, that means if we take the vortex flow circles, that means for different r r with r the mechanical energy remains constant. That means mechanical energy at this streamline at this streamline at this streamline remains constant. That means simply we exploit the mathematical thing d H d r is equal to 0.

So, if you make d H d r first term will be 1 upon rho del p del r, no problem here also we can get V theta d V theta d r plus g d z d r is 0. But immediately it appears that it is difficult to evaluate the V theta as a function of r which we got very quickly from the irrotationality condition. I will prefer this though I have done in this fashion in my book, but this is different because in my book I gave an expression for the total mechanical energy in a curve a liner streamline. But I think the irrotationality condition from the irrotationality condition to derive V theta is C by r is more simple. But I will show you from this to derive immediately after writing these. It is very difficult until and unless we can substitute this del p del r in terms of the velocity. How can we do? How can we substitute the pressure term in terms of velocity? Then we will have to take the help of which equation? Momentum equation, that is Waeler’s equation. If you recollect the Waeler’s equation, Waeler’s equation in r direction is d V r d t that means in r directions. Let again the x y like this, this is r this is theta, minus v theta square by r this I discussed earlier I am not doing it again, minus 1 upon rho del p del r.

Here of course, I should not show it y z because here I have taken z minus what we will the do r directions so z is positive out upward vertically, so this will be minus g cos theta, minus g cos theta. If we consider theta of course, here we consider theta with the vertical of course, you can consider with the horizontal in that case you will take that g cos theta means it will be g sin theta minus g sin theta. It depends upon the angle theta you take because g is acting vertically downward. So g sin theta if you take this as theta, so it will be g sin theta, all right? Now here d V r d t is 0 because no V r is there, so minus V theta square by r minus 1 upon rho del p del r is minus g sin theta.

So therefore, 1 up on rho del p del r, 1 up on rho del p del r plus g sin theta is equal to V theta square by r. Again this g sin theta can be written as d g d z d r because if this be r and this is this, so this we can write as d z change and this is d r. well, so this is sin theta which can be written as d z d r. If we define this as theta which I have done in my book, so this will be g cos theta that means the body force in r direction will be minus d cos theta. In this direction the component will be minus g cos theta then cos theta will be d z d r here sin theta will be d z d r finally, the d z d r will come, which means 1 upon rho del p del r plus g d z d r will be simply substituted by V theta square by r. That means we ultimately come up to the condition of the irrotationality, because irrotationality and the constant C in total mechanical energy is same. That means again this is same, that means if you take V theta common then d V theta d r plus V theta by r is equal to 0. That means the irrotationality condition, d V theta d r plus d theta by r 0 and we arrive as d theta is C by r.

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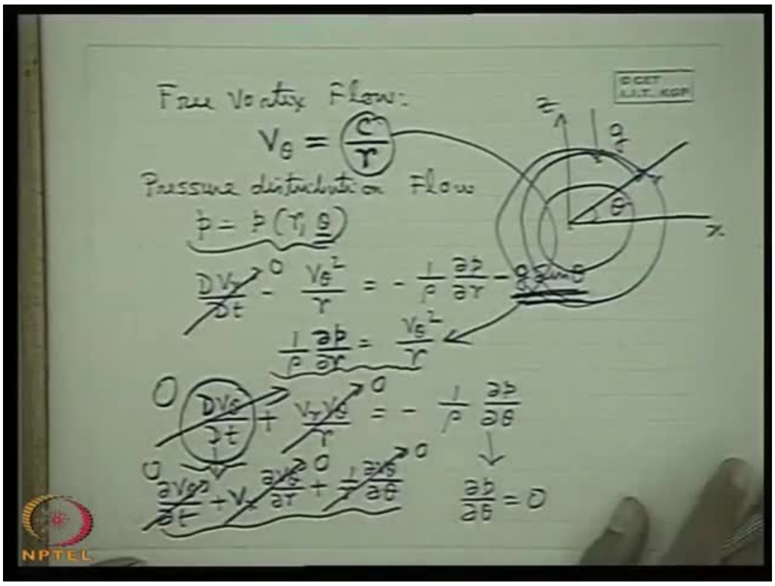

So, either from irrotationality condition or from total mechanical energy constant in the entire flow field, we get the free vortex flow equation is this where there is only tangential velocity component and which satisfies the relation with and it is the function of radius only a radial location and satisfies the equation c by r. That means it is an inverse function of the radial distance from the origin V theta is c by r. Now next task is the pressure distribution in a free vortex flow pressure distribution.

So pressure distribution in a free vortex flow if we have to find out you start from general that p be a be a function of both r and theta, so if one knows the velocity field and if he wants to find out the pressure the general rule is that you have to exploit the equation of momentum or the equation of motion conservation of momentum. That means you look to equation of motion where velocity and pressure are linked, so that we know velocity how can we find out this. Ok let us see, no problem, d V r d t minus V theta square by r is equal to minus 1 upon rho del p del r.

So, now here we see one thing that d v r d t is 0, therefore 1 upon rho del p del r, but we should not leave it here. We know that V theta is C by r very simple that we can integrate it, but we should satisfy ourself that these del p del r is equal to d p d r that means V should not be a function of theta. So, if we start from here and write del p del r is equal to v theta square by r you substitute this and integrate it it will be little incomplete until and unless we show that p becomes a function of r only. So to prove it conclusively we take the, consider the theta direction equation of motion or momentum conservation equation if you recall it, I discussed it earlier, is equal to minus 1 upon rho del p del theta.

I am sorry, in r direction the equation of motion minus g sin theta will come or cos theta will come sin theta will come if you take let this is x this is z and if we take this as the theta and z is the vertically upwards that means g acting downward, so this is r theta, so if you consider this vortex motion like this, sorry tangential like this, therefore what happens? These g sin theta will come, this is because this constitutes the body force per unit mass in the negative r direction, so g sin theta will come.

So, now here you see, what is that first V r is 0, so this is 0, what is the value of this? Very difficult to tell it from here, can you tell what is the value of d V theta d t V theta is not 0, but will it be 0? It is 0, why? Until and unless you expand it you cannot tell it. Just from here it will be confusing. It is confusing that whether it is 0 or not, though V theta is not 0, but V r is 0 it will be 0 why? Because if you expand it V r del V theta del r plus 1 upon r del V theta del theta in two dimensional you make it three dimensional V z is not there, so if you expand this term this is the expansion steady flow as because V r is 0, so this is 0, so del V theta del theta 0 earlier it is proved from continuity V theta is a function of r which means this term is 0. Therefore the theta direction equation of motion tells us that del p del theta 0. Now, we should integrate this.

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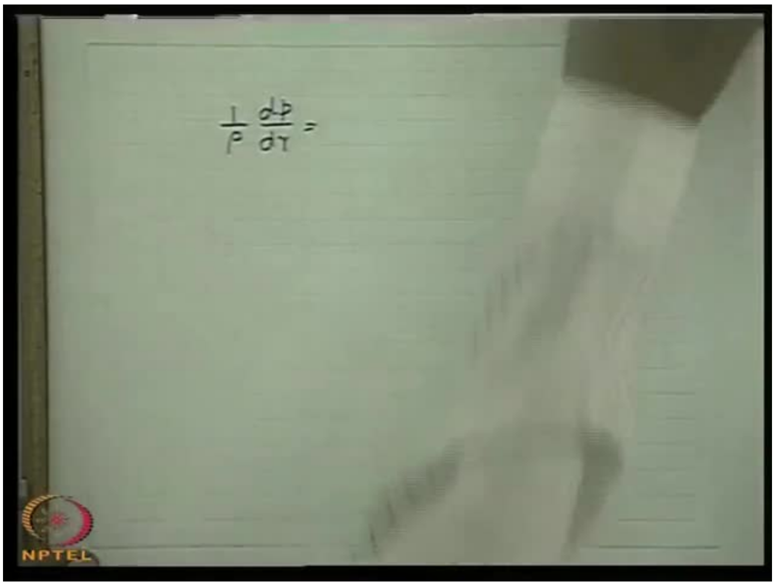

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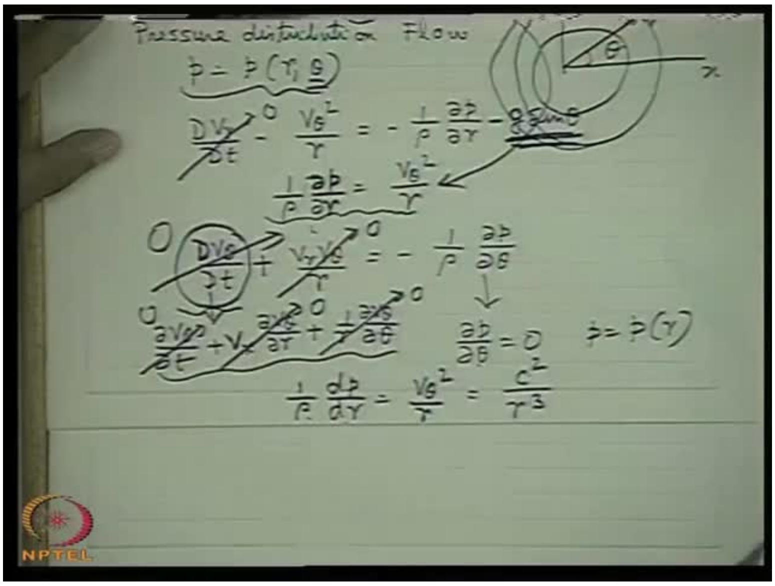

That means after this we can write 1 upon rho d p d r. That means now I can write this del p del r, this del p del r as 1 upon rho d p d r is equal to V theta square by r and V theta is C by r that means C square by r cube. So, 1 upon rho d p d r is C square by r cube, all right now this C square by r cube, now I, I can write 1 upon rho d p d r because del p del theta is 0, that means p is a function of r. Now I can write in terms of ordinary differential d p d r and V theta square by r, now V theta is C by r C square by r square and r r cube.

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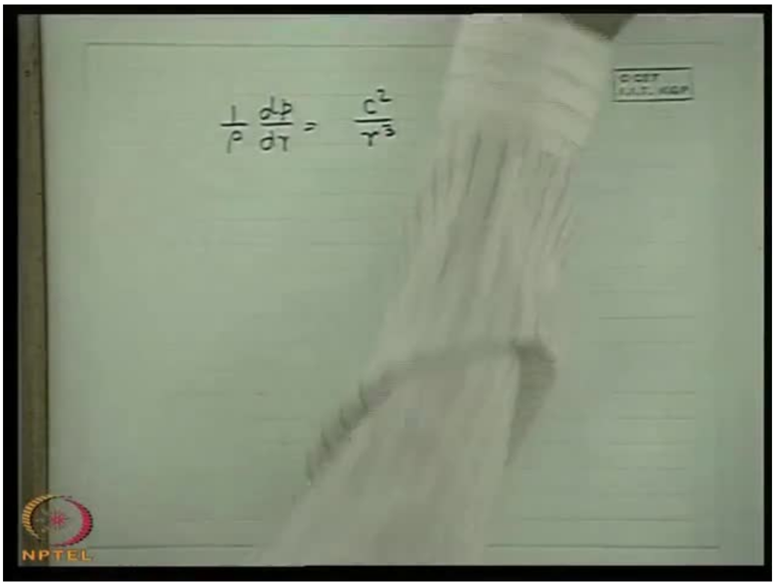

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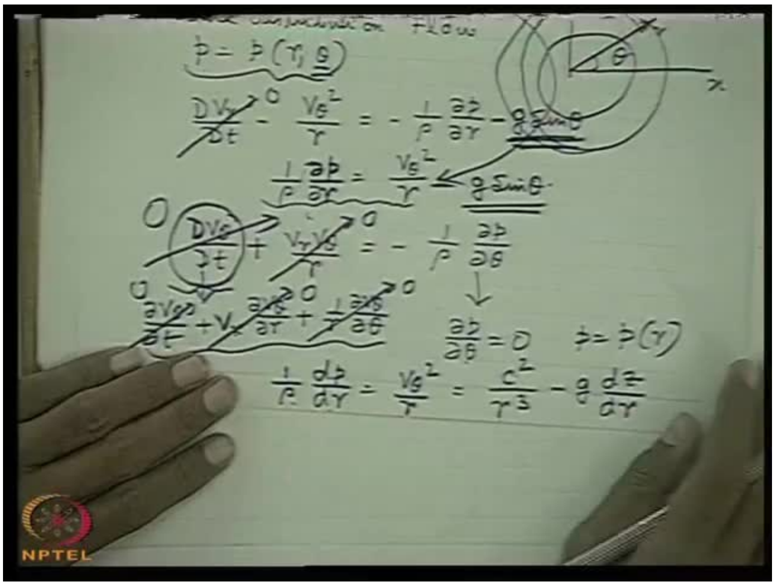

So, 1 upon rho d p d r is C square by r cube, sorry, I am again sorry, that this will be minus g sin theta, minus g sin theta and g sin theta sin theta is d z d r that means minus g d z d r, so this will be always added, yes please.

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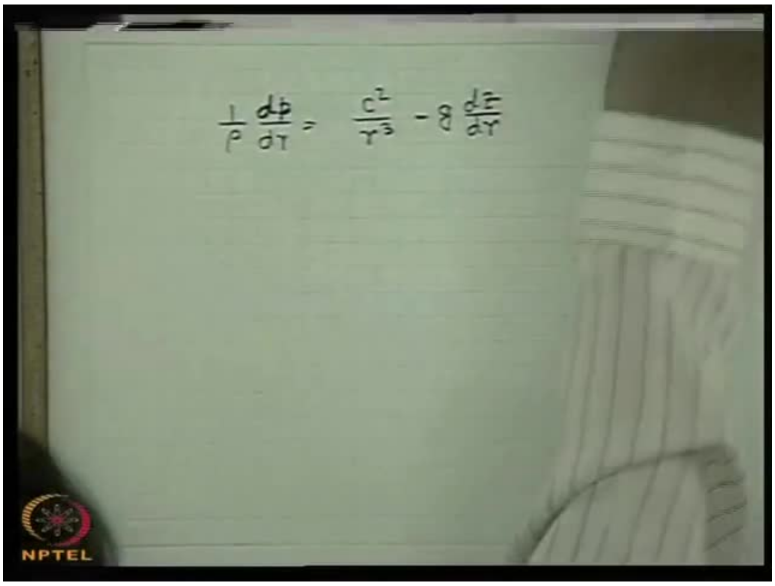

Theta direction; theta direction body force? What is theta direction?

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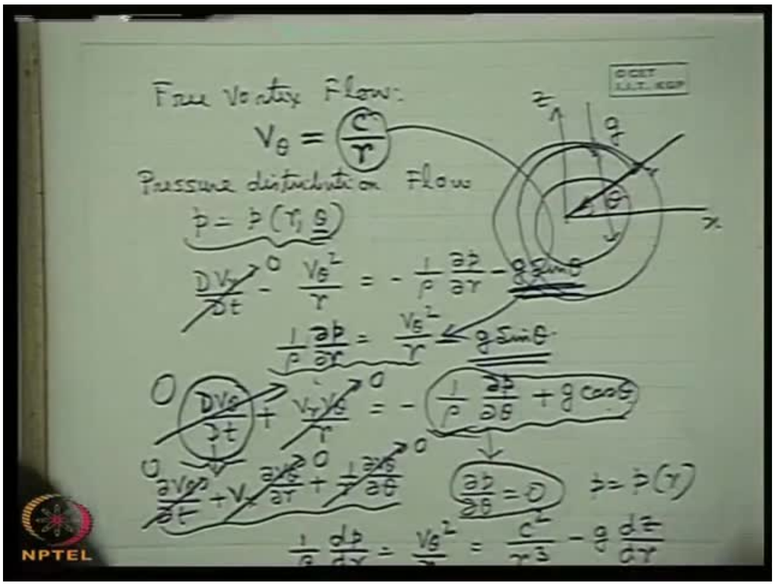

Well there will be theta direction body force, yes, g cos theta this will be balanced by del p del theta always this you can take in terms of p itself, so we do not bother it. del p del theta 0 you can write, but theta direction there will be a body force. Yes very correct. if g is the body force in the r direction, so theta direction there will be also a body force. You are correct. So, that will be taken care of g cos theta, so we cannot then exclusively if we take this we cannot exclusively tell del p del theta 0. We can take a function of theta which can we can make 0 here. But for this time being we take it as d p d r is C square r cube minus g d z d r, all right well.

This we have asked intelligent question that d V theta d t plus V r V theta by r. This is the acceleration in the theta direction must be equal to 1 upon rho del p del theta and g cos theta you will see afterwards that this body force term we can take it within the pressure variation. That means the pressure variation in the theta is such the takes care of this g cos theta term. You are correct, so that we can tell that the pressure variation in the theta direction will be taken care of by the gravitational field if we consider in a vertical plane the force vortex equations, usually we neglect this because the change in the gravitational force within the extend of the vortex motion in a gravitational field is negligible. So, either g cos theta g sin theta is very negligible, it does not appear.

So, usually in many books you will see, I do not know what I have done in my book that they do not take care of these. They initially tell that consider a horizontal plane, so only they do with del p del r del p del theta there is no harm wel. But this is true that g cos theta is there, so del p del theta takes care of g cos theta, so there we cannot tell that d p d r we have to tell only del p del r and they are a when you integrate it a constant will come which will be a function of theta. But for the time being we just discuss that we take d p d r is c square by r cube minus g d z d r, all right very good.

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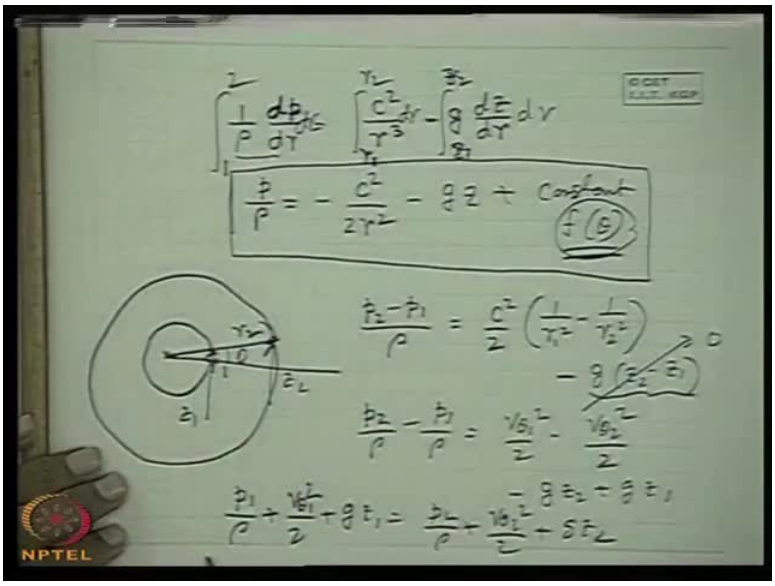

Now if we integrate this, we get P by rho is equal to, with respect to r that means we integrate this with respect to r.

Then what we get? What is the integration of this? Minus C square by 2 r square r to the power minus 3 plus 1 r to the power minus 2 divided by minus 3 plus 1 minus 2 minus g z plus a constant which in case the gravitational force field is prominent it will be a function of theta. I do appreciate it, so p by rho, so this is the pressure field pressure field in a free vortex equation. But most important thing is that if we make a change between two radial point at the same theta then, of course this function of theta will not come that means if we make a definite integral from 1 to 2 from r 1 to r 2.

So point one represents a radial location r 1 that means at a radial location r 1 to a radial location r 2 that means let this be the r 1 and this be the r 2, so from a radial location r 1 to r 2 that is r 1 to r 2 and r 1 to r 2 corresponds to z 1 to z 2. So if you integrate it with respect to that z 1 to z 2, so due to a change in r 1 to r 2 the elevation changes from z 1 to z 2 while the theta remains same, theta remains same. So if you make the definite integral you get p 2 minus p 1 by rho, rho is constant is equal to what do you get it c square by 2 that is minus, so 1 by r 1 square minus 1 by r 2 square minus g into z 2 minus z 1.

So, this is the equation, so in case of a horizontal plane this term will not come simply p 2 minus p 1 by rho C square by 2 one by, but in case of a vertical plane where the gravitational force are considerable then g z 2 minus z 1. If you see that we can write p 2 by rho minus p 1 by rho is equal to than C square by 2 r 1 square is V theta 1 square by 2 minus V theta 2 square by 2 because C by r 1 is V theta minus g z 2 plus g z 1.

This is simply p 1 by rho plus V 1 square by 2 V theta 1 square by 2 plus g that means writing the Bernoulli’s equation. That means an intelligent student will find out the pressure distribution not from integrating the equation of motion. He will simply write the Bernoulli’s equation at the two radial point because he knows for an irrotational flow the Bernoulli’s equation with same constant that means p by rho plus V square by 2 plus g z at one point on one streamline is same as another point in another streamline. That means between two radial locations he can write the Bernoulli’s equation, that means this equation is same as writing the energy equation that Bernoulli’s equation at two radial locations, all right.

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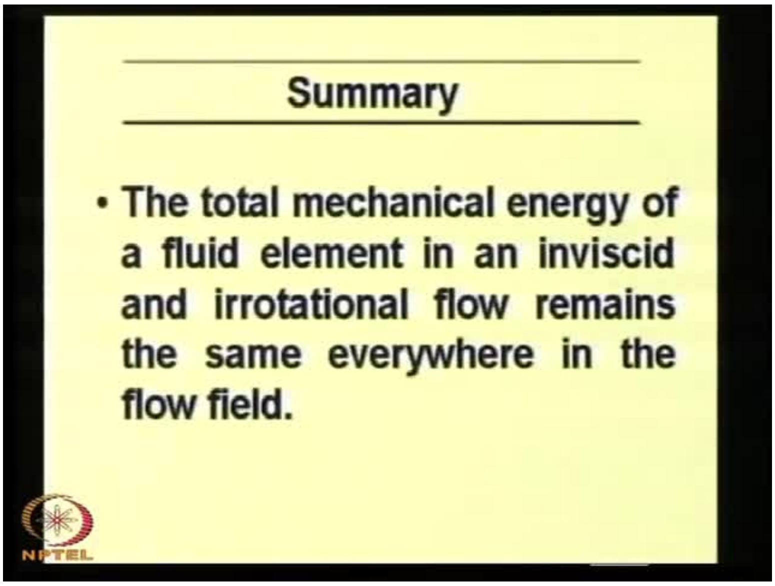

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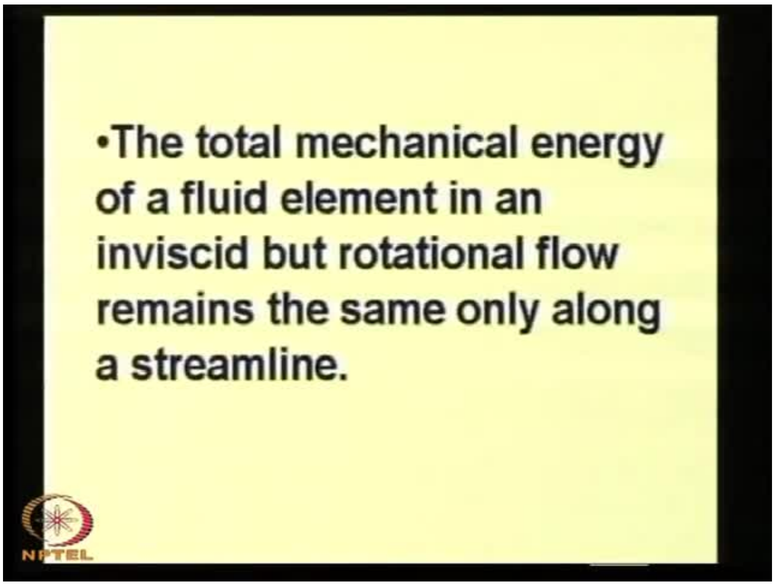

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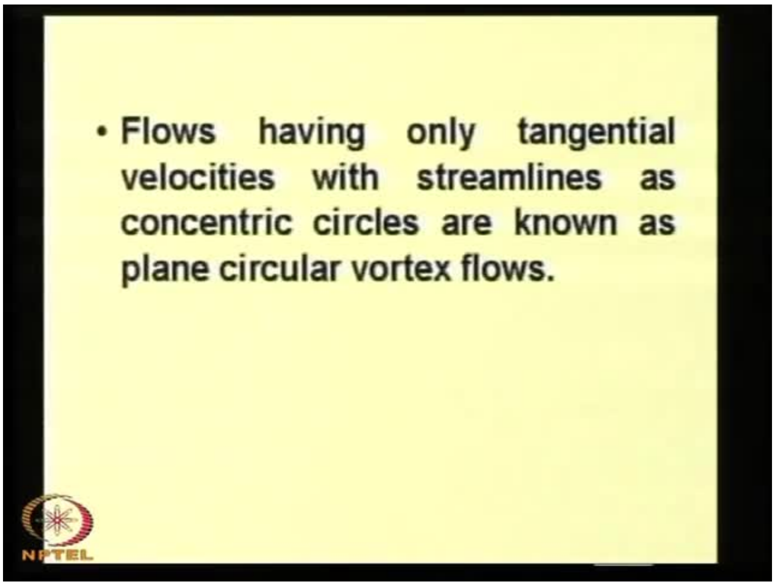

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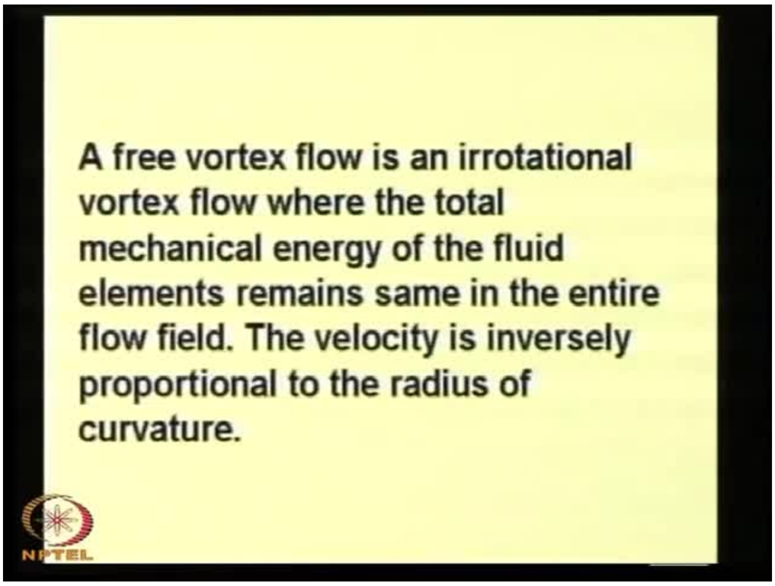

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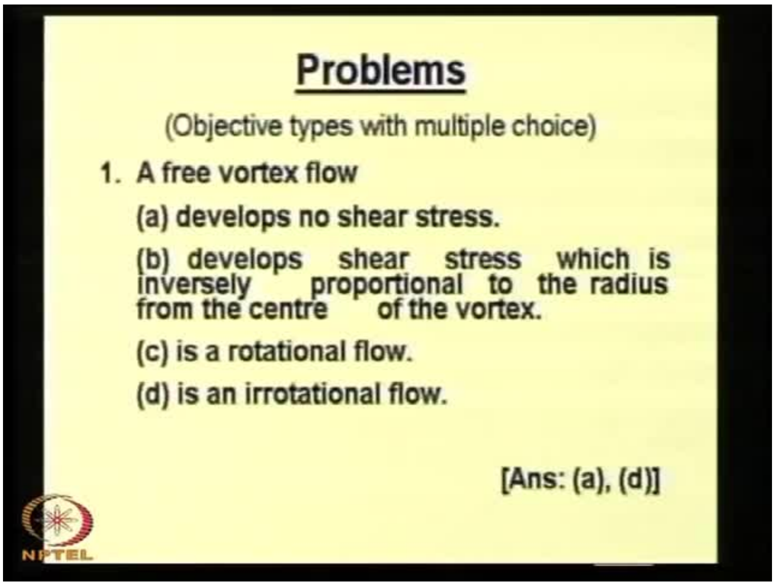

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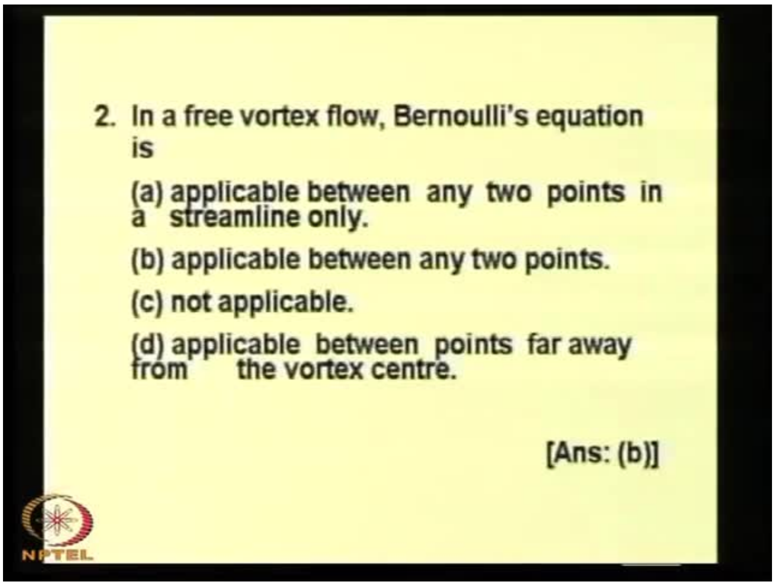

Fluid Mechanics
Prof. S. K. Som
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Indian Institute of Technology, Kharagpur

Lecture No - 23
Fluid Flow Applications Part-II

Good afternoon, I welcome you all to this session of fluid mechanics. Well, last class we were discussing about the plane circular vortex flows. And then we discussed further about a free vortex motion or free vortex flow. Now if we recall it so a vortex flow is described in a polar coordinate system with only tangential velocity as the existing velocity component. That means there is no radial velocity fluid having only tangential velocity. And immediately we have found that the consequence of continuity tells us the tangential velocity is a function of radial distance, radial coordinate r 1, if you consider a two dimensional case then if there is no radial component of velocity only tangential velocity exist the tangential velocity it is a function of r. This is precisely the definition of a two dimensional or plane circular vortex flow.

Now, one of the vortex flows is the free vortex flow, so there is a general, this is the generalization special case of these vortex flow is a free vortex flow when the flow is irrotational or total mechanical energy in the flow field remains constant either of the two can hold good. If one is correct then other is automatically, if no one holds good other automatically holds good that means there is a plane circular vortex flow where total mechanical energy in the flow field remains constant means the flow becomes irrotational, so irrotational vortex flow where total mechanical energy remains constant throughout the flow field. It is known as free vortex flow, where we have seen the tangential velocity bears an inverse relationship with r that is V theta at any radial location r is some constant by r. And then we found the expression for pressure distribution from the equation of motion, and we found it is same as if we derive it from bernoulli's equation.

That means if we use the Bernoulli's equation between two points at two different radial locations then p by rho plus V square by 2 here V is V theta V theta square by 2 plus g z 1. If we consider the elevation at if it is in a vertical plane is equal to the p by rho plus V theta square by 2 plus g z 2. That means at the point two that means sum the of the three energy three components of energy are same in all radial location because the constant mechanical mechanical energy is constant. So, from that one also we can get the pressure distribution equation.

Now, after this we must know that there are certain practical situations where the free vortex flow takes place. But one very important consequence of free vortex flow you see that free vortex flow is that there is no energy added or no energy taken out. That means energy remains constant. Therefore, free vortex flow exist, because of some rotation which was previously important on the fluid due to some energy interactions and just it is maintaining its motion due to some internal actions. There is no dissipation of mechanical energy no addition of mechanical energy no withdrawal of mechanical energy, it remains constant.

So, this type of flow takes place usually it is resembles to certain practical situations for example, if the water is taken out from a shallow vessel with a from, from a shallow vessel, from a (( )) at the bottom then the circulatory flow which is induced is more or less similar to a free vortex flow which is often found in, in our house when the water comes out from a bath tub or from a washing basin. Another case is that if in a whirlpool caused in a river at a far distance from the eye of the whirlpool the flow field resembles more like a free vortex flow. In a tornado in an airflow in a tornado also the flow of air resembles in some part, in some region to a free vortex flow. In certain cases like fluid machines, in a pump the flow at the after the impeller casing probably we have not read that centrifugal pump in a centrifugal pump flow coming out from the impeller casing as it enters into the volute chamber the rotary flow or the rotational flow is similar to a free vortex motion, free vortex flow.

Another example is that if you stir a liquid mass of liquid in a vessel by a stirrer or a paddle wheel the flow field far away from the stirrer not very close to the stirrer resembles to that of a free vortex flow. So, one very important and interesting thing in a free vortex flow is that if you just recall the equation V theta is C by r where r is measured from the origin, one thing you see mathematically when r tends to 0, V theta tends to infinity, which means mathematically or physically that there is a singularity at the centre that means at the centre there is a singularity this is the mathematical definition means velocity is not defined. Velocity becomes infinity. What does it means? That means free vortex flow can never be extended physically up to the origin from where the r the radius that is the radial co-ordinate is measured, which means in practice free vortex motion is never defined at the very near to the origin or at the origin mathematically it is undefined.

So, what happens in practice whenever there is a free vortex flow near the centre or at the central core either there is another type of vortex flow or there is a hollow core? You will see there will be always a hollow core, so that free vortex flow is never extended up to the origin, so that is always a singularity as it mathematically found that V theta is C by r, so when r tends to 0, V theta tends to infinity, these are all details about the free vortex flow.

Now, we come to another type of vortex flow that means the circular motion with only V theta component of velocity, which is known as force vortex flow. This type of vortex flow the difference is that V theta is directly proportional to the radius not inversely proportional to the radius. But V theta directly proportional to the radius for example, V theta is some constant into radius is very familiar we are very familiar to it since our school days this is because we know this is this motion which is exhibited by a solid body. If the solid body rotates then at each and every point at different radial locations the tangential velocity is the angular velocity into r. Being omega is the angular velocity, so V theta is r into omega, so we know that solid body rotates with same angular speed. That means all points at different radial location rotates with the same angular speed, and as we know the tangential velocity bears the relationship with the angular velocity as tangential velocities r times the angular velocity, omega is the angular velocity r omega.

So, which means the tangential velocity is directly proportional to radial location means the angular velocity remains constant in the entire motion, in the entire field of motion. So, in a liquid or fluid flow of fluid where V theta is directly proportional to the radius means that fluid particles moves with constant or equal angular velocities which means that fluid body moves like a solid body, that is why this type of motion described by V theta is some constant into radial location is known as solid body motion, solid body rotation or solid body vortex or forced vortex.

So, this forced vortex generates is generated in practical situations when a liquid is taken in a vessel contained in a vessel if vessel is given a rotation if a vessel containing liquid is rotated about its axis then the solid then the fluid, fluid body which is being turned with this motion of the solid body executes the rotary motion a rotational motion. That means exhibits tangential velocities which follows a solid body rotation. That means a force vortex motion where V theta is constant into the radial location varies directly with the radial location. Sometimes if certain practical applications the rotation is important in the fluid flowing through a pipe enhancing the heat transfer coefficient. So, when fluid flowing through the pipe pipe itself is rotated about its axis then the fluid following a rotary motion is very close to that of a force vortex motion. So, force vortex motion is a solid body motion where the tangential velocity is directly proportional to radial location or radius.