Fluid Mechanics
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Lecture - 20
Conservation Equations in Fluid Flow Part – VIII

Good morning. I welcome you all to the session of Fluid Mechanics. Well, in the last class we discussed about the forces that is being exerted by a fluid jet impinging on a flat and curved surfaces. Now, today we will again see that the application of momentum theorem to a control volume. By the application of the momentum theorem to a control volume, we can solve many other practical problems of interest.

Today, first we will be discussing the principles of jet propulsion. What is principle of jet propulsion? This is a principle based on which the aircraft moves. The movement of aircraft in air and sometimes the movement of ship in water are based on this principle of jet propulsion. What is this principle? If you know, probably you know from your popular knowledge, for example an aircraft when it moves it takes air from the ambient. So, atmospheric air comes into the aircraft and where this air is used to burn a fuel, which is being injected into the aircraft, these burning products, the products of combustion at high temperature and pressure.

First of all, air is compressed obviously by a compressor, so that a high pressure air is delivered before it burns with the, sorry, before it is mixed with the fuel to be burned. Then combustion products is being expanded through a nozzle at very high velocity and these products of combustion or burned gases at high velocity is ejected from the nozzle at the rear part of the aircraft, and this gives the propulsive force or the force for the aircraft to move. That means, if the aircraft has to move with a constant velocity, this force should balance the air drag at that velocity.

Otherwise, air will accelerate or decelerate. So, this is the precise the principle of propulsion. In case of ship also, moving ship takes water from the sea, then it is being ultimately pumped to a high pressure and then it is being expanded through a nozzle at the rear end of the ship at high velocity and due to the ejection of high velocity fluid, the ship experiences its propulsive force in the direction of its motion.

So, this is the precise jet propulsion theorem. Assume, just can understand from, this is purely based on the Newton’s second law of motion, second law and the third law. What is that? Because some fluid element, which comes into the system, for example, aircraft or ship gets its momentum change, that is the momentum is increased, which is coming with the low velocity and getting discharged with a high velocity. So, this change in, due to this change in momentum, the system experiences a force exerted by the system. Well, simply the second law of motion. Second law of motion by Newton and then by the third law, the same magnitude force, but in opposite direction is being given by the fluid element to the system. That is, a ship or an aircraft, which is precisely the propulsive force, which allows the vehicle, aircraft or ship to move.

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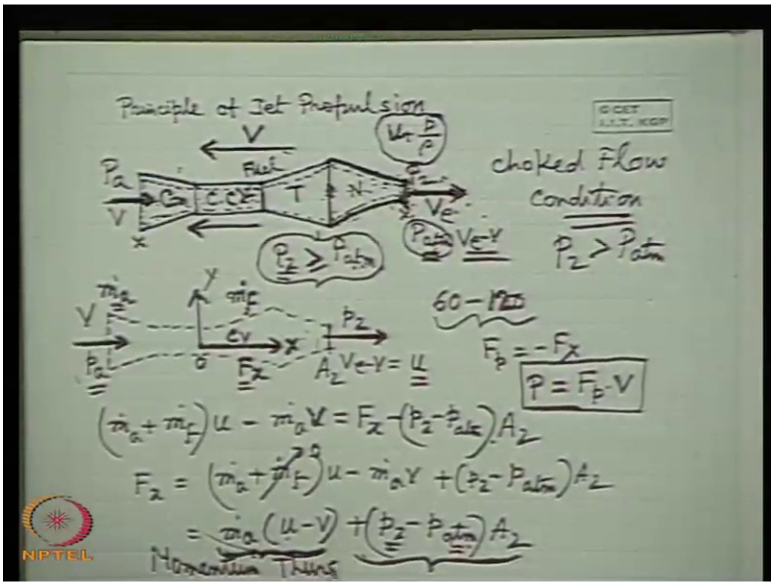

Now, if with based on this principle, if we see, now, let us see these principles of jet propulsion. Principle of jet propulsion. What is this principle of jet propulsion? Well, now, let us see that, this is like this. If we have a jet, it is, first there is a compressor. Let me draw the layout or block diagram of the component like that for a just better understanding.

So, this is the compressor C; compressor com. This is the combustion chamber CC. So, this is the turbine. One turbine is there and this is the nozzle. This compressor and turbine is actually attached to a same shack. But I am doing like this. Now, basically from fluid mechanical point of view, what happens? If we consider this aircraft is moving with a velocity V in this direction, for which the air comes into the aircraft. That means, relative to the aircraft the air comes in with the same velocity. That, I will write afterward.

So, because of its motion, let the aircraft is moving with uniform velocity V. Air comes into the aircraft. So, first it comes to a compressor. The first component of the aircraft engine is the compressor. You will learn this thing afterwards in your Applied Thermodynamics class. That air comes to the compressor, which compresses the air at high pressure. So, we are not much interested about the mechanism and processes of each and every equipment. But you should know that in general. After compression, the compressed air is being allowed to flow through a combustion chamber, where fuel is injected.

Fuel is injected fuel is injected fuel is injected and the air reacts with the fuel and fuel is burned and the high pressure and temperature combustion product, gas is being discharge from the combustion chamber. Then it comes to a turbine. As you know, it will be also be taught in the next year in the Fluid Mechanics class. The turbine is a machine, which develops work while the fluid flows through it and the pressure of the fluid is decreased.

That means, we can tell that it is an expansion process. The fuel from high pressure expands to a low pressure, high pressure and temperature expands to a low pressure and temperature and some amount of work is being developed and this work is utilized to drive the compressor, because compressor requires work to compress a fluid from a low pressure to high pressure. After that, the fluid comes out of the turbines. Still, it has got a high pressure and high temperature. So, this energy of the fluid; now, what are the forms of the energy of the fluid? So, as you know, we have recognized the mechanical energy or pressure energy, kinetic energy and potential energy.

But here, kinetic energy is very small. The velocity is very small at this plane. That is, after the discharge to the turbine, that is, there is a nozzle at the end. So, at the inlet to the nozzle. So, only energy is the pressure energy and another energy is the intermolecular energy, because of the high temperature. The sum of this u plus u plus p by rho, you will know afterwards in your thermodynamics class, it is known as enthalpy.

So therefore, these two terms of energy comprises, these two terms of energy constitutes the total energy of the fluid, which is being utilized in terms of the kinetic energy. That means, and expansion through a nozzle takes place where the pressure and temperature is decreased to a low value and ultimately, fluid comes out to a very high velocity. Let, this is the velocity of ejection of the fluid, V e. This is the principle of jet propulsion. This is coming out with a velocity V e. Now you see that, what happens is, a high velocity jet is being ejected. Now, again we should know that, under certain circumstances, the pressure at the exit plane may not be equal to the atmospheric pressure.

If this be the atmospheric pressure and the pressure at the exit plane, let is P 2. Sometimes, P 2 may be greater than the P atmospheric. It may be equal to P atmospheric and greater than. I am not going into detail. But I just tell you that, if it depends, now, question comes that whether the velocity is above the sonic speed or not? That is supersonic or subsonic or sonic. The answer to this is, this depends upon the relative magnitude of the pressure at the inlet to the nozzle and of the atmosphere.

That means, if for a corresponding ambient pressure, the inlet pressure above a certain critical value, then the expansion is possible after supersonic level. Otherwise not. So, if the inlet pressure corresponding to a given ambient pressure is below a certain value, certain maximum value, so, supersonic speed is not possible. Supersonic velocity is not possible by the expansion of this fluid to the ambient pressure. In that case, only a converging nozzle will serve the purpose. But while this pressure is higher than a maximum value corresponding to a atmospheric pressure, then the expansion of the fluid after a supersonic level is possible. In that case, only a converging duct will not be sufficient. Then a converging and diverging duct is used as a nozzle, if we want the expansion up to supersonic level.

But, in that case, if we restrict the expansion only to a converging nozzle, then it will give expansion to a sonic level. So, sonic to supersonic expansion is possible through a diverging. It is just the reverse in case of subsonic flow, when the area in decreases, the velocity increases. But in supersonic flow, when the area increases, velocity increases. So there, if we do not provide the divergent duct, say expansion will be stopped up to the sonic level. In that case, it will not be expanded up to the atmospheric pressure. In that case, the pressure at the exit plane will be higher than the atmospheric pressure. This as information you know. Sometimes, what we do, that even if the atmospheric pressure and this inlet pressure is such that supersonic expansion of the gases up to supersonic velocity is possible, even if it is so, sometimes it is not allowed like that. It is allowed to expand only up to sonic velocity through a convergent nozzle, so that, P 2 becomes greater than p atmospheric and in some cases, it is not so. A convergent duct is also used, so that, the expansion up to atmospheric pressure is allowed, where the discharge plane pressure becomes equal to atmospheric pressure and the velocity of discharge is supersonic.

This is just because for your information. Otherwise, you will not be able to understand when we will be analyzing the control volume and a control volume theorem, momentum theorem for the control volume, that why P 2 is greater than P atmospheric. So, certain cases P 2 may be greater than the P atmospheric. This is the P atmospheric. Now here, the problem, this is, with this information for this jet propulsion or jet engine performance, now we can think this problem from the basic fluid mechanics point of view, that fluid element comes with some momentum and is going with a different momentum, which is more than this, so that, a fluid element suffers the change in momentum in this direction. For which, the fluid element experiences a force exerted by the system, that is, the entire aircraft engine in this direction.

As Newton's second law, according to Newton's second, the reaction force opposite in direction, but equal in magnitude is exerted by the fluid to this nozzle, propellant nozzle or the entire system. This nozzle is known as propellant nozzle and that is the propulsive force, which is being experienced by the aircraft and which is responsible for its movement in the forward direction.

So therefore, what we do now, our next part, simply from the fluid mechanical point of view, we recognize then what we recognize? We recognize a control volume of fluid. Just we do inside the aircraft like this. Let us draw this control volume. Here, let us draw; where? Let me draw here the control volume. Let is the control volume. So, this is the control volume. So, this is the control volume. So, this is the control volume c v. So now, here also again you can recognize, since the aircraft is moving with a velocity V in this direction, so, control volume is also moving and will have to analyze with respect to this moving control volume. So therefore, the analysis will be made with respect to moving control volume. Only with respect to moving control volume, the flow appears to be steady, because here at fixed point. The flow velocity is 0. But for example here, at fixed point, the of flow velocity is v, but when this is going out, the flow velocity becomes 0 at a long distance; that means, we have to move with the aircraft to see always a flow velocity v e here and always a flow velocity v here in this direction. That means, a person sitting on the aircraft will always see at the inlet air is coming with a velocity v, if it moves with uniform velocity. He will always see a liquid fluid going out with a velocity, not v e, v e minus v. That is the relative velocity. This is fixed. So therefore, to make the problem steady, we have to fix and we have to analyze with respect to the control volume.

Again and again I am telling, that is the reason for which the analysis is made with respect to the moving control volume. Let us consider this as the axis x and y. Let 2 axis is attached to the control volume. This is the control volume. So similar analysis. So now, with respect to the control volume; that means, if we make the control volume fixed; that means, aircraft fixed; that means, in other words, we give a velocity v in the opposite direction. So, that means, with respect to the control volume, the air is coming with a velocity v and it is going out with a velocity v e minus v, which we call as u. That is the relative velocity of the jet, exit jet, fluid jet with respect to the aircraft.

Now, the pressure here is P a and this plane pressure is P 2. Now, it is very simple. What is that? Let, if I consider f x is the force acting in this direction and F y, we are not very much interested because this will be balanced by the weight, because the aircraft is moving in this direction. So, we are interested in the force exerted or force exerted between the control volume and the aircraft mutually between control volume and the aircraft only in this x direction.

So therefore, write the equation. What is this? That net efflux of the momentum in the x direction. If M dot is the mass flow rate of air; now here, one thing, if I neglect the fuel mass flow rate, usually fuel ratio is very high. Sometimes we can take or sometimes we can neglect. Let us consider M f dot. M f dot. M f dot is the rate of burning of fuel, rate of fuel burn we can consider. So therefore, this is usually very small. This is the total mass flow rate at the discharge times the velocity u minus m dot a u. That is the, sorry, M dot a V, that is the momentum influx; efflux positive direction and influx positive direction b with the minus sign is equal to F x. Now, we know that we have to add the pressure forces with their gauge values. If we finally we want to arrive at the net force acting on the aircraft taking care of the atmospheric pressure forces exerted on it, we have seen that earlier, that if we do not take the gauge pressure, absolute pressure, find out the pressure exerted on the control volume of fluid. Then minus of it is the pressure exerted on the system. That is here aircraft for example. Then if we add on it the effect of atmospheric pressure on its side, then we will see it is same as if we find out the pressure force exerted on the control volume with the pressure as the gauge pressure at this two sections, where the fluid coming in and going out. This already we have established earlier.

So therefore, which take with the gauge pressure. So, this is 0. So, only pressure is this. P 2 minus P atmospheric. That is, the gauge pressure P 2 is the absolute pressure, if we consider. P 2 minus P atmosphere into let A 2 is the area of the propellant nozzle at its discharge plane. So, simply this is the equation. Alright? So, we can write F x is equal to M dot a plus M dot f into u minus M dot a v plus P 2 minus P atmospheric into P 2. Usually, this is very small. In usual aircraft engine, the fuel air, air fuel ratio is even sometimes 120. 60 to 120 is the ratio.

So, sometimes we can neglect this and if we neglect this, then we can write a simple expression like this. So, this is simply the P atmospheric. When in case of a subsonic expansion; that means, when the fluid is coming out with a subsonic velocity, velocity less than the aquatic speed, when mack number M less than 1, then the expansion takes place up to completely the discharge atmospheric pressure, because when it is discharged, the atmospheric pressure, we consider the pressure at the discharge plane is always atmospheric pressure. So, within this scope of our syllabus for this fluid mechanics class, we are considering only the subsonic flows. So, always whenever there is a flow situation, which is coming out or discharging at the atmospheric pressure, so, discharge plane pressure is always atmospheric. So, this part will be 0. But there is a phenomenon which you will learn afterwards; when the fluid flows both in subsonic and supersonic region.

That means, in that case, what happens? If the outlet pressure corresponding to inlet pressure is such that the fluid may be expanded after a supersonic velocity, then if it is to be done, we have to add a convergent part and then a divergent part and a typical design length is there, up to which it will be convergent and then it will be divergent. So, if you do not do and if you only provide a convergent duct, then what will happen? The fluid will be expanded only up to sonic velocity. It will not be expanded up to supersonic velocity and in that case, it will be unable to utilize the entire dropping pressure. That means, the pressure will not be decrease up to the ambient pressure and this will be coming to a pressure at the discharge plane, which is higher than the ambient pressure. Immediately, there will be a pressure drop after the fluid is being discharged into atmospheric pressure. So, only under those situations, so, P 2 is more than P atmospheric. Otherwise, you may feel difficult, that why at the discharge plane, when it is discharged to atmospheric pressure is taken more than atmospheric. This is precise with the information. This condition is known as choked flow condition. This will be told afterwards in the class of Compressible flow, choked flow condition. When this choked flow condition. So, p 2 will be greater than P atmospheric.

So, in that case only we will get this. So, this is known as, now, before that I tell that, what is that F p? That is the propulsive force on the aircraft. Now, you see the F x is positive, U is greater than V, P 2 is greater than P atmospheric. That means, the force on the control volume is acting along the positive direction of the x axis as I have taken. So therefore, the force acting on this aircraft will be in this direction. That is, minus f x ; that means, this is the direction of the motion. So, the propulsive power P can be written as a P into the velocity, uniform velocity. So, this analysis is valid for this aircraft moving with a uniform velocity because the control volume is an inertial control volume, moving with uniform velocity. So, this is the propulsive power expression.

So, this part of this thrust, propulsive thrust or propulsive force is known as momentum thrust. This is known as momentum thrust. This part of the thrust is known as pressure thrust, which is generated due to the pressure thrust. Now, you see that this is the expression for the propulsive power. Now, consider a situation, what happens in rocket? In rocket, the difference between a jet and a rocket is, in a jet engine, probably you know from your popular knowledge, what is the difference between a jet engine and a rocket engine? Can you tell what is the difference between the operation of a jet engine and a rocket engine?

Student: Rocket engine takes all fuels

Rocket engine takes So, all fuel and Fuel is also there. Oxidizer. Very good. That means, fuel is also there within the fuel tank in the aircraft, but oxidizer, that is oxygen required for burning of the fuel is taken from the atmosphere. For a rocket, both oxidizer and fuel is contained within the rocket. That means, for a rocket, from fluid mechanical point of view, it does not take ambient air. So, in that respect; that means, general propulsive power, its performance is independent of the atmosphere, atmospheric condition. That means, there is no influx; that means, this is 0. That means, though rocket is moving with a velocity V, but this influx momentum is 0, because no air is coming. There is no provision made, where the air will be shacked in or air will be taken into the vehicle.

So therefore, which part will be 0? This part will be 0. So, for rocket engine, so, the analysis is that, that this is the rate of the burned gasses coming out of the propellant nozzle of the rocket plus if there is a pressure thrust is equal to the force exerted on the control volume or opposite of that is the propulsive force. That is the only difference for the operation of the rocket engine. Clear?

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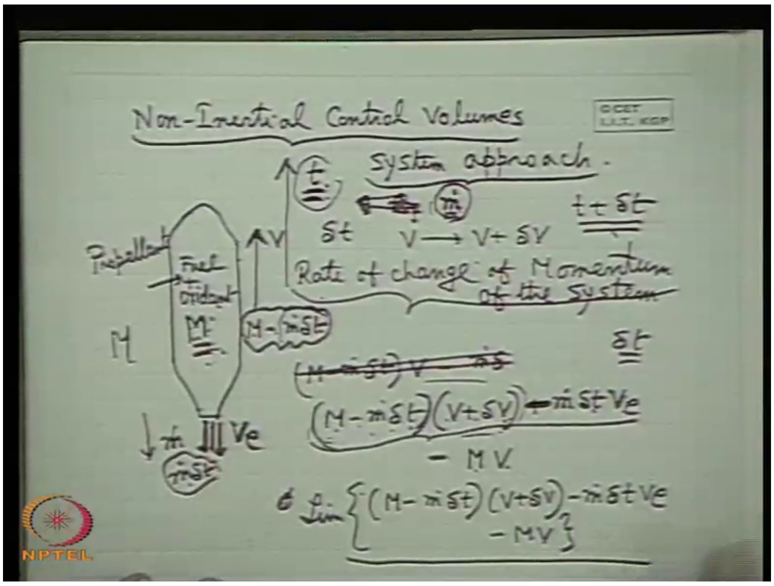

Now, I will tell you what happens in case of accelerating non-inertial control volumes. Now, I will come to the application of non-inertial control volume. Propulsion of ships with uniform velocity in sea water is also based on the same principle. So, that you can see in the book. So, I am not discussing that. Now, I come to the non-inertial control volume. What is non-inertial control volume? The control volume which is accelerating. Now, consider the situation. The jet engine is accelerating. Not moving with the uniform velocity; that means, propulsive force is always more than the dragracetance of the air at that particular speed. So, speed is going to increase. Similarly, rocket engine is also accelerating. It is not only fixed per jet engine, but rocket engine also. So, in that case, we will have to analyze the situation by a momentum theorem applied to an accelerating control volume.

But, usually the momentum theorem is very difficult to apply in a accelerating control volume, because the usual transport theorem, which we write, that for principle of momentum, that is the net momentum efflux plus the rate of change of momentum within the control volume, that is valid for a control volume, which is either fixed or moving with the uniform velocity. So, this has to be modified by taking into consideration the inertia force of the control volume. But usually when the accelerating system are analyzed, then it is analyzed on the basis of system approach. That means, your usual approach, conventional approach, the rate of change of a particle or rate of change of a system, rate of change of, sorry, of momentum of a particle or a system is equal to the force acting on the system in that particular direction.

Now, before that, I, after that, I like to tell that, why we are not doing this. This is valid; now, what I will be doing for any accelerating system, but usually jet engines moves with uniform velocity. So therefore, uniform velocity analyses, that analysis for, with inertial control volumes are alright. But accelerating systems are only the rocket. Why? Because rocket at least, initial, that is during its initial flight, it is accelerated to a very high velocity. The reason is that, rocket contains its own propellant. What is propellant? Propellant is the fuel and the oxidant. If rocket has to travel a long distance, see, it has to carry a very huge propellant for its travel.

So therefore, at the initial part of the rocket flight, the extra work that is being done to raise this huge amount of propellant is usually wasted. So therefore, what is done is, to have the effective use of the material as much as propellant has to be burned to attain a very high velocity during a small interval of time. That means, during the initial flight of the rocket, for any rocket flight, the rocket is accelerated very high. So, this high acceleration of the rocket at the initial part of the flight is very important and has to be analyzed to find out the acceleration which is required. What will be the amount of fuel to be burned and all these things, which we will not go into detail at this present moment, but you must know.

So, during this initial part of the rocket flight, rocket is being accelerated. So therefore, any analysis during this part cannot be done with respect to a inertial control volume. So therefore, what usually is done is, this type of system is being analyzed with respect to a system approach and this is not only about a rocket. So, in books, you will see, always they describe the non-inertial control volume and analysis for a rocket engine. That does not mean it is only for rocket engine; for any accelerating system. So, how it is done for a rocket engine? Let us see. Now, let us consider a rocket like this. Let us consider the propellant nozzle and it is moving vertically up as it usually does.

So, let us just symbolize the rocket like that, which is moving. At any instant, its velocity is V. At any instant t, let velocity is V. Gas is ejected. Gas is ejected at a velocity V. It carries its fuel plus oxidant. There is no doubt in it. So, that means, fuel plus oxidant, which is known as propellant is the terminology. Probably you know. Propellant. Now, how it is analyzed? Because if you make a control volume, it is accelerating; that means, the velocity is changing. This is the instantaneous velocity V at any time t. So, it is analyzed with respect to system approach. What is the system approach? It is very simple. It is school level thing; system approach. Let, at this instant t, M is the mass of the rocket along with its propellant, because the total mass of the rocket is mass of the rocket plus its propellant.

So, M is that mass. Let us consider, after a time delta t, let at the time delta t, the burning rate of the fuel is constant. Let that is M dot f, that is the burning rate of the fuel or rather, I will tell that burning rate of the propellant, M dot, not fuel, burning rate of the propellant. So, this is the burning rate of the propellant. That means, this is the rate at which this plane gasses. Burning rate of the fuel and then combined with air; that means, the burning, this is the rate of reactants participating in the reaction or you can tell, this is the rate at which the burned gasses being ejected. So now, after a time delta t, we consider that velocity V is changed to v plus delta v. That means, the velocity is increased to delta t.

Well, now, then we can write the system approach equation of motion, that is the rate of change of momentum. Rate of change of momentum. Rate of change of momentum of the system. Now, you ask me that, what is the system. Now, I take the system, that is the system approach. You must understand, that t, the rocket and its propellant at time t comprises the system. So, therefore, at time t plus delta t, this system comprises a mass within the rocket, which is M minus M dot delta t plus M mass M dot delta t, which is coming and having a velocity V e. That is the exit velocity. All right? So therefore, the momentum of the system at time t plus delta t minus momentum of the system at t divided by delta t will be the rate of change of momentum of the system. So, rate of change of momentum of the system is equal to the momentum of the system at time t plus delta t minus the momentum of the system at time t divided by delta t.

Now, what is the system? Let us recognize at time t plus, that the system approach if you recall, it contains the same amount of mass with the same identity. That means, identity has to be traced out. So, what was our system? Our system was the rocket and the fuel and oxidant whose mass was again at t. But now the system after t plus delta t will be this mass. Now, after time delta t, the mass of the rocket and the fuel is M minus M dot delta t. So, this M dot delta t amount of mass, which was originally there within a, is now appearing. If we trace its identity in the exhaust gas, this M dot delta t with a velocity V. So therefore, the momentum of the system at time t plus delta t is M minus M dot delta t into v minus M dot v plus, I am sorry, v plus d v, I am sorry, M minus, I have not talked about this mass and this is having now a velocity v plus delta v, I am sorry, v plus delta v. Then plus M dot delta t. This is with velocity V e. So, this is the momentum.

Student: Sir v e minus

V e

Student: Minus V e it is (( ))

Why V e minus v?

No, not related. With the system approach I am doing. Why you are doing relative to the rocket? You do not get confused. This is M dot V e. So, V e in this direction. So, you are correct. So, if you write the momentum, momentum here, velocity is in this direction. So, because of that, you can make a minus sign.

So, this is the momentum of the system at time. You must follow. This is a system approach. We are finding out the momentum, net absolute momentum of the system. Why you are making a relative velocity? Earlier, we used the relative velocity, because we analyzed with respect to a control volume, which is moving to make the system steady in a control volume analysis. But here we are making a system approach for an accelerating system. So, it is simply the rate of change of momentum of the system, absolute momentum. Now, what is the initial momentum? minus M v. It is clear? There is no doubt in it. This is the change of momentum. This is the change of momentum. Is there any doubt regarding this change of momentum? Please tell me. There is no question of relative velocity. It is only the change of momentum of the system, which is accelerating. That is why I have taken for a time delta t the change in velocity. This is precisely the momentum of the system. It is a typical system approach.

So, it is system, which was, what is the system? That is the M mass of fuel and oxidant plus the rocket at time t. So, this system comprises this mass with v plus. Initially, the system was M mass. Again I am telling of the rocket plus fuel and oxidant moving with velocity V in this direction. This direction I take as the positive direction. So, after a time t plus delta t, what is this system and what is the associative velocity? System comprises M minus M dot delta t mass with V plus del V velocity and M dot delta t mass with V e velocity. It is formal thermodynamic steps are not important. It is mass and the associative velocity. Why minus? Because V e is in the opposite direction. We consider this V e is the exit velocity in this direction.

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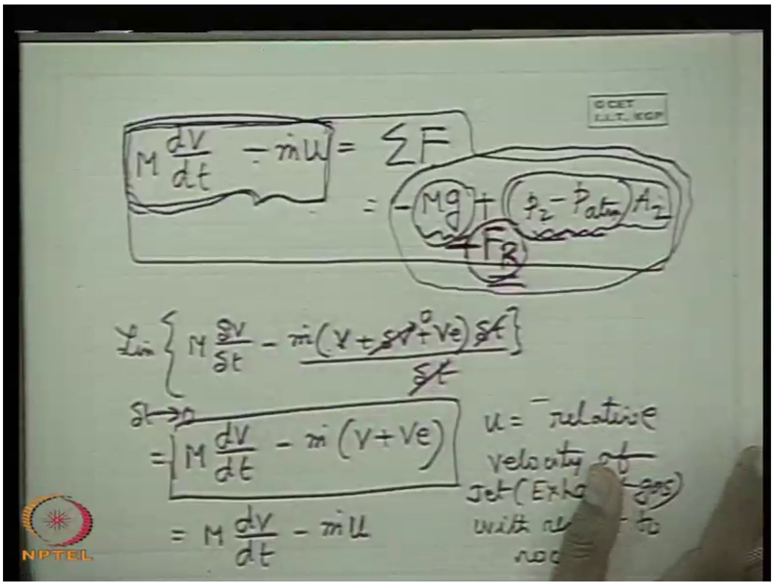

So, minus the initial momentum M V. So therefore, rate of change of momentum will be limit of this value, limit of M minus M dot delta t into V e plus del V minus M dot delta t V e minus M V. All right? Divided by delta t. Delta t tends to 0. Is there any difficulty in this? So, I can write this as the, sorry, this as the rate of change of momentum. So, rate of change of momentum of the system is this minus this divided by delta t, for a delta t time. Delta t tends to 0. Now, if we make this, you will get M V and M V cancels.

So, you get M into delta V by delta t. So therefore, we can write this expression. Simple mathematics. Limit M into delta V by delta t and minus M dot into, what we can write? Delta V M dot into V plus delta V plus V e times delta t divided by delta t. Delta t tends to 0. So, this delta t. Delta t cancels. So, only trick is that, when delta t tends to 0, this also tends to 0. V e is constant. So, this is alright. So, the limiting value will be V plus V, where delta V also tends to 0, when delta t tends to 0. So, delta t delta t cancels, but here delta V by delta t. So, this tends to 0 and this tends to 0. So, according to our calculus, so, this represents the rate of change of velocity with respect to minus M dot V plus V e. So, this is the rate of change of momentum. So, this is the rate of change of momentum.

Now, what is V plus V e? Just I see in physical system, I described V is the velocity in that direction and V e is the velocity of the jet engine. These are absolute velocity system approach. So therefore, the V plus V e is the relative velocity of the jet with respect to the aircraft. Sorry, with respect to the rocket. So therefore, this is usually written in the term of this, in the terms of this, is equal to M d v d t minus M dot U, where U, you can write the relative velocity of jet with respect to the rocket. Relative velocity of jet or exhaust gas, whatever you call. Exhaust gas with respect to rocket. So, this is the rate of change of momentum. So, this rate of change of momentum now will be equal to M d v d t according to system approach. It will equate with what? It will equate with the net external force acting in the same direction.

So, momentum is accorded into the vertically upward direction. So, momentum is accounted for the vertical. So, what is the net upward force acting in the vertical? So, what are the forces? That is, the weight at any instant, weight of the rocket at any instant, that is M, so, that will be, and that may be minus m g plus the pressure forces. That means, if the pressure here is p 2, which is higher than the atmospheric pressure, if the atmospheric pressure is p atmosphere. So, p 2 minus p atmosphere into the area of the propellant atmosphere plus p 2 minus p atmosphere into area of the propellant. Now, if the gravity effect is negligible, so, it is only the pressure difference which gives the net force for two account for this change in momentum. So, this is precisely the net force acting on the rocket, which is causing its change in momentum.

So, this is done with the help of system approach. Of course, there is a resistance force. You can ask, sir, why not? Air resistance. If there is any resistance force, air resistance is there. So, it is the difference in pressure. So, if the rocket goes to space, where there is gravitational effect is negligible, so, there is no resistance. It is only the pressure difference times propellant nuzzle area, which gives the external force to the rocket and responsible for its acceleration. So, there will be plus F r. F r is usually in the opposite direction to its motion. That means, F r will be there with a negative sign. So, there will be a minus f r. That is the air track resistance. That means, a rocket is moving in the ambient. So, this is the atmospheric track resistance. So, in general, this part will have to recognize from the physical system, that what are the different forces acting on the rocket. That is equal to the rate of change of momentum of the rocket at that instant. So, this is the system approach.

Now, after that, I will come to a most important section of this chapter is that motion with uniform acceleration. Of course, time is limited for today's class. This is most important topic that I will be describing in the next class. Just I tell you in brief. Now, we will discuss certain prosecutions, where fluid is accelerated or moved with uniform acceleration or velocity. If fluid is in motion, it is with uniform motion. That means, all the fluid particles will be moving with the same velocity. As I told earlier, this is the case of pure translation or if the fluid is accelerated, it will be accelerated with uniform velocity.

For example, a tank containing fluid is accelerated horizontally or vertically or in any arbitrary direction. Then what happens? We can consider the fluid within the tank is accelerating uniformly. That means, each and every particle on the fluid is accelerating with the same value, same acceleration. Another case is, the pure rotation or solid body rotation; that means, if a fluid body is rotating about a axis in such a way that, all fluid particles are moving with uniform angular velocities, uniform angular. That means, a solid way the solid body rotates. This happens in practice if we take a fluid into a cylinder and if cylinder is giving a spin, is rotated about its axis, then the fluid within the cylinder will be charged; will be rotated in a way that it will give a solid body rotation. That means, the fluid will be rotating each and every particle of the fluid at different radius, will be rotating with the constant angular velocity, so that, the tangential velocity at any point will be defined as the angular velocity times the radius. That it will be a linear relationship with a radius. That is the basic equation of the tangential velocity for a solid body rotation. You understand?

So, these situations will be analyzing in the next class. How the pressure varies in these cases of fluid flow. That, if fluid is uniformly accelerated or uniformly moved or a fluid is given a uniform rotation. That means, a solid body rotation or rigid body rotation. What is the pressure field within the fluid? This will be our discussion in the next class, because time is up. I cannot start this, because I cannot finish it.

Thank you.