Fluid Mechanics
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Lecture - 16
Conservation Equations in Fluid Flow Part - IV

Good morning, I welcome you all to the session of fluid mechanics. Well, in the last class we discussed the oilers equation of motion or oilers equation that is the equation of motion for an ideal flow, flow of an ideal fluid with respect to cartesian and cylindrical polar coordinate system.

So, we derived from the principle of conservation of momentum applied to a fluid system, and we considered the fluid to be an ideal fluid that viscous force was not present in the flow of fluid, and then derived the equation for motion that is the conservation of momentum with respect to a cartesian coordinate. Then we converted it to a cylindrical polar coordinate system. Then we recognized the vector form of the equation of motion, and we also discussed this in cylindrical polar coordinate system, and we also discussed that this can be obtained also in different coordinate systems. So, these equations were known as oilers equation that is the equation of motion for the flow of an ideal fluid. Now, we will derive the same equation of motion, that is the oilers equation with respect to streamline coordinates or along a streamline.

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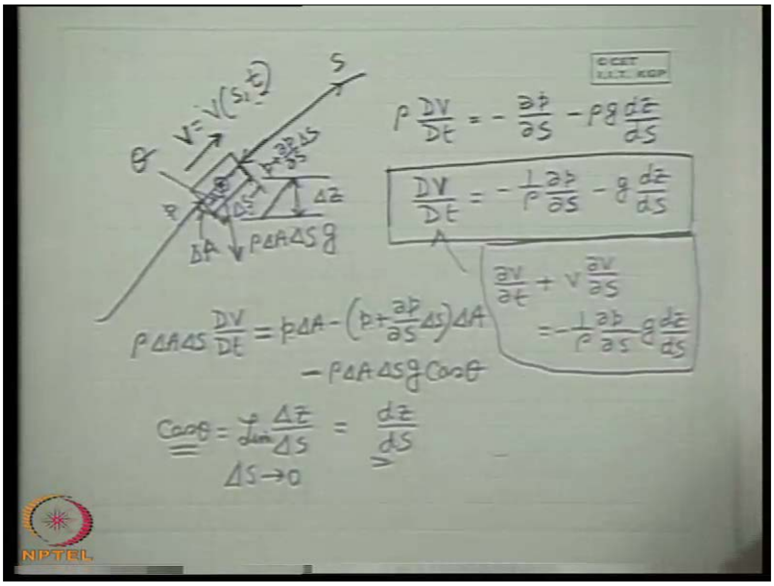

So, let us see how does it look? That means, if we consider the flow along a streamline like this, and we consider a fluid element in this streamline, let us consider this is the direction of the streamline. So, at any point here, we consider a fluid element like this whose length is for example, delta S along the streamline, and let us consider this area is DA or simply A, not DA cross sectional area is A or del A cross sectional area is del A small fluid element that is the cross sectional area perpendicular to this plane.

Now, this is moving let at a velocity at this point. The velocity vector is V. I am not using the vector notation because now V here is identified as a scalar component because this is always along the direction. When I take the streamline in the direction things become one dimensional. That means the velocity at this point is along the direction of this streamline. That means, along the tangent to this. So, this is the situation.

Now, when we have assumed the flow to be ideal; that means, the fluid is an ideal fluid, then the only pressure forces acting on the fluid is only, sorry surface forces acting on the fluids are pressure P at the surface and the downstream surface, it is P plus del P del S into delta S. That means, the change over delta S, that is the fluid apart from what other external forces acting on the fluid body is its weight that is mass times the gravity. That means, because of the gravitational body force field, that is rho into delta A delta S that is the volume times g.

Let us consider the angle made by this vertical line that is the line directing the weight of the body acting the gravitational force, the line of action of the gravitational force. That is the vertical line with the tangent of the direction of the streamline along which the velocity exists. Let these two be theta. That means, this angle, let this to be theta. Then a simple force balance in this direction of motion can be written as from the equation of motion or the conservation of momentum, whatever you tell that mass rho times delta A delta S. This is the mass times the change in velocity.

Let us write this as A DV Dt is equal to the net force acting in this direction, and the net force acting in this direction is contributed by the pressure force and the component of the gravity along this direction. Therefore, at this point if we write the net force will be P into delta A in this small elemental volume minus P plus del P del S into delta S into delta A, well minus the component of the gravity is in this direction minus the rho delta A delta S into g. That is acting vertically downward component is cos theta. That is if this be the theta, so this component will be cos theta. Well, now from a simple geometry, we can write that if we consider this height of this element that is this dimension, the vertical direction these del is delta z. That means, this is the vertical displacement from this point to this point or from this point to this point delta z. Then we can write cos theta is delta z by delta s. This is this delta z. That means, this vertical displacement divided by delta s, the length from a simple geometry this triangle if we consider a triangle like that. That means, this is delta s, this is delta z. That means this vertical displacement between these two points is delta z delta s limit of these because theta is changing from point to point.

If you define theta, the angle of implantation at any point with the vertical and the direction tangent to the streamline that will be delta z delta s. Delta s tends to 0. Physically, this is defined like that which gets a mathematical shape as d z d s. That means with the change in A's along the streamline, how the z coordinate is changing? Simple common sense. We can tell that is the implantation cos theta that is the angle theta between the vertical and the direction tangent at any point to the line is d z d s. Obviously, so if we substitute this value of cos theta here and make a simplification u c p del A p, del A cancels del s, del A del s del A del s del A cancels from both the sides. Then simply we get rho DV Dt is equal to minus p del minus del p del s. Sorry, p cancels out minus del p del s minus rho. Rho is there, rho g d z d s or we can write in another form that DV Dt is equal to minus 1 upon rho del p del s minus g d z d s. That means this is the form of oilers equation that is a one dimensional type of thing, where V is in the direction of base along a streamline. So, s is the coordinate along a streamline and V is the velocity along the streamline because velocity will be always along a streamline. V is the resultant velocity of the fluid particle.

This is the form. This can also be splitted as a del v del t here. You see that V is a function of one dimension; V is a function of base of the coordinate along the streamline and the time. So, therefore, with simple split up of these substantial derivatives in terms of its temporal derivative and the convective derivative is this. So, therefore, one can write this is equal to right hand side as a t del p del s minus g d z d s. So, these two equations, either this or this at the oilers equation along a streamline, all right.

Now, I will tell you the most important theorem in this context which is very important in solving different problems, physical problems or applied problems of engineering applications. This important theorem is known as Reynolds transport theorem which is very important, Reynolds transport.

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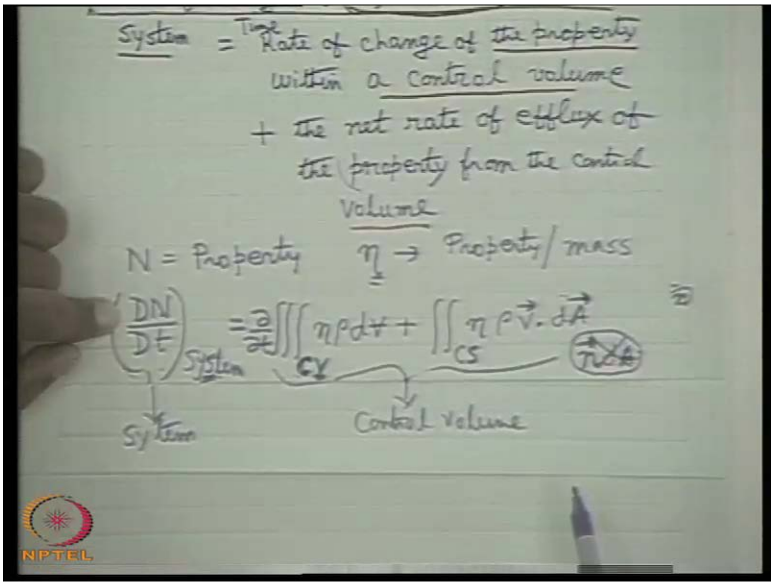

Now, I will come to, probably you can tell that this is the most important part of the section which allows us to utilize all the conservation laws for analyzing fluid flow problems in practice. So, for the time being, I just talk the discussion on conservation of energy. We have discussed conservation of momentum mass and conservation of momentum. Before that we discussed the Reynolds transport theorem. Now, what is Reynolds transport theorem?

Reynolds transport theorem. Now, Reynolds transport theorem is a very important theorem. The proof of which is not that important or I will not do here. If you are interested, you can see in any book, standard book, but what is more important is to know the theorem very correctly. Now, if you see all the physical laws for example, the laws of conservation of mass momentum energy, this type of all physical laws, a thermodynamic law, thermodynamic second law. All physical laws are basically stated with respect to a system or you can tell the birth of all physical laws have taken place with their implication or explanation with respect to a system. For example, one is asked what is the definition of, what is the statement of Newton’s second law of motion. What is the conservation of momentum? That is rate of change of momentum of a system or a particle.

Particle is a differential concept of a system or a system is an assemblage of particles. So, it is the rate of change of momentum of a system or a particle which is equal to the force in the same direction. For example, the conservation of mass is defined at mass of a system remains unchanged. Nobody can tell that mass of a control volume remains unchanged. So, whenever we tell that is conservation of mass principle, that mass remains unchanged. It is neither created nor destroyed. Total mass remains constant. So, the definition of system is inherent. For example, the conservation of energy, this energy of a system is equal to the energy of a system within. It is equal to the total energy interacted by the net energy interacted by the system with the surrounding.

So, therefore, the basic laws are first stated in terms of systems, but it has been found while analyzing the practical problems or even the physical problems. For the sake of academics, it is much easier to go for an analysis with respect to a control volume. So, therefore, to apply the principle of conservation or any physical laws to control volume, one has to know the statement for that. For example, if I tell what is the law of conservation of momentum for a controlled volume, can you tell the rate of change of momentum for a controlled volume because control volume is a fixed control volume memo that I will come afterwards. Control volume may move even in, it can move with a change in velocity or with uniform velocity, but there is another provision that control volume is fixed in most of the engineering applications will take a fixed control volume well.

So, when control volume is fixed, the question of its movement does not come over, change of momentum does not come. So, therefore, the question comes that how would you explain the Newton’s second law with respect to a control volume. Similarly, the conservation of mass for the definition of a control volume mass of a control volume may not remain fixed. So, what will be the definition of the conservation of mass for a control volume? For the conservation of mass without knowing any theorem, it becomes very simple. From our simple (( )) that mass conservation is like that the net mass, the mass coming in minus mass going out is the mass which is being accumulated within the control volume. That comes from one simple intuition, but it may not be so for other physical law. So, Reynolds transport theorem is a theorem which helps us to relate or simply we can tell which relates the statement of any physical law for a system to that for a control volume. That means, it relates this again. I am telling the statement of a physical law for a system to the statement of the same physical law for a control volume. How these two are related is given by the theorem known as Reynolds transport theorem. So, without giving a proof here, you can see proof is not that important, but more important is to know the theorem very correctly.

So, I write this way. The Reynolds transport theorem, it tells like that in statement that rate of change, I write this thing rate of change of any property within a system or for a system, better you write rate of change of property for a system. This rate means time rate of change. You can write time rate of change of any property for a system is equal to the rate of change. The ultimate outcome of this theorem, rate of change of the property, the same property rate of change of the property. Here you write within a control volume plus the net rate of a flux of the property from the control volume. It is not a simple intuition. It has to be proved for mass. It comes from simple intuition, otherwise it is not.

So, now, I see that what is the definition that rate of change that is transport theorem gives us the time rate of change of any property for a system is equal to time rate of change. Here also, rate means with respect to time usually in physical system. Rate means with respect to time is equal to time rate of change of the same property within a control volume plus the net rate of a flux of the property from the control volume. So, therefore, this equation relates the statement for a system to that for a control volume and this is operative for any property which is for any extensive property.

So, now if you look, this can be written for any extensive property. Well, you can see, so this can be written in a mathematical form like that if we consider N as the extensive property and we define it as any property per unit mass, then this time rate of change of property for a system, we can write this definition as like this. Now, DN Dt for in a mathematical shape of this theorem is like that system is equal to time rate of change of the property within the control volume. If theta is the property per unit mass, this first term on the right hand side can be written eta rho d v is small elemental volume. It is over the entire control volume here. I write specifically c v, not the v because this implies the control volume, where this implies the system plus the net rate of a flux of the property from the control volume. You know how it can be written in mathematical symbol? If theta is the property per unit mass, it will be what eta into rho v dot.

Now, here I am not bringing n as the unit vector for a area because N sometimes is confusing. So, simply I used dA as the vector. That means, without writing N dA, sorry we are right without writing N dA to show this as a vector that dA is the scalar magnitude of this of a small elemental area, and N is the unit vector along the normal to the area taken positive directed outwards. Without taking that I simply use sometimes, it is used dA as a vector whose magnitude is the area scalar magnitude, and direction is along the normal positive directed outwards. So, this is simply over the entire control surface c s. Its control volume is the net rate of a fluid. So, this for the second term in the right hand side, this is for the first term in the right hand side. So, therefore, right hand side is the statement with respect to control volume, whereas the left hand side is the statement with respect to system and to control volume.

So, therefore, we can now say mathematical shape is like that. This is the rate of time, rate of change of any property N in a system is this time rate. Sorry, I am very sorry, del del t. So, time rate of change of the property within the control volume plus the net rate of a flux of this property from the control volume. So, now in steady state, obviously this term will be 0. So, for a steady state, DN Dt is equal to eta rho without D, where eta is the property per unit mass and N is that property. So, therefore, you see for a steady state, automatically this will become 0. So, this is the mathematical form of the equation. Now, let us apply this for different cases. Now, first of all we want to apply this for conservation of mass.

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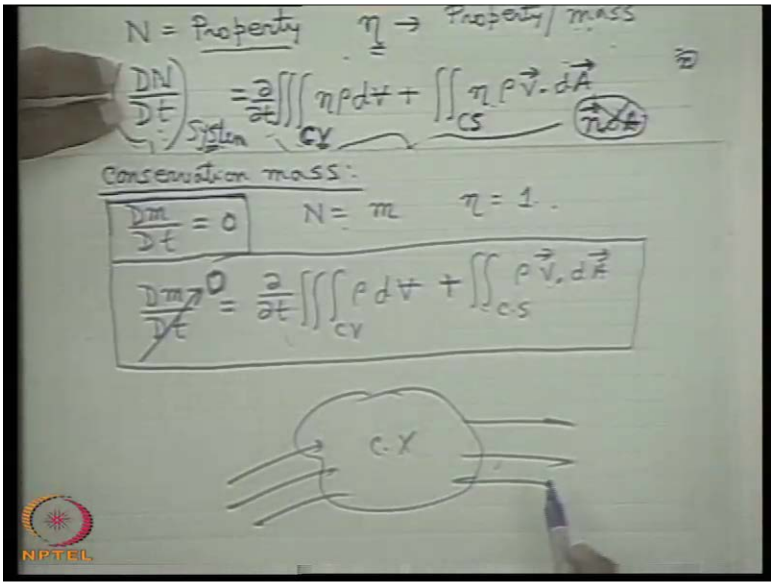

Let us consider the conservation of mass, and let us think in this way that we have learned the conservation of mass first with respect to a system. As we learnt it first at school level that what is that for a system Dm Dt is equal to 0. We have learned it. Now, you want to apply it in influx system. The mass of a system remains unchanged. That means, this is simply the equation of continuity if we use it for a system. That means the conservation of mass applied to a system gives this equation as simply the equation of continuity. Now, if we apply the conservation of mass for a control volume, we take the help of Reynolds transport theorem. How here n is the property of the mass. So, therefore, the value of eta is 1. So, with this concept because eta is the property per unit mass, we write this, we will say that rate of change of mass for a system Dm Dt is equal to del del t.

What is this rho d v over the entire control volume plus rho v dot. Sorry, dA over the entire control centre and since by the law of conservation of mass, this has to be 0. So, therefore, this plus this is equal to 0 which is well known continuity equation in integral, that is the time rate of change of mass within the control volume plus the net rate of mass. A flux from the control volume is 0.

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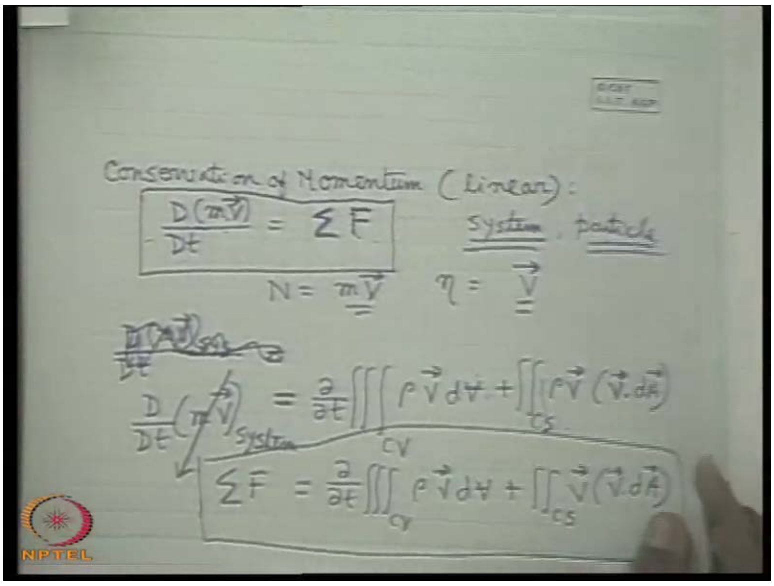

Yes, a boy at school level can tell what is use of all these theorems. Everybody knows that come from the common sense that if there is the control volume fixed region is space where the mass is coming in and going out, the balance mass will be accumulated by the control volume, but this may not be so from simple intuition for other laws, so that we have to invoke or we have to recall this theorem what is known as Reynolds transport theorem that relates the change in property of a system to that for a control volume, all right. Now, let us see what happens for the conservation of momentum which is not so simple conservation of momentum.

Let us write the conservation of momentum. Well, conservation of momentum, you know momentum may be linear or angular. Let us consider the conservation of linear momentum as we have done for oilers equation. Now, let us first see that we have learned this conservation of momentum as we have really learned even from our e school days with respect to a system because all the statement explanation for conservation of momentum or law of motion is given for a system. What is that? That is rate of change of momentum for a system that mass into velocity is equal to the net force, the total force acting in the same direction. This is precise. The conservation of momentum or equation of motion that is being learnt at the first step because the basic laws are always expressed in terms of the system. That means rate of change of momentum is equal to the force in the same direction of what of a system or a particle.

So, this is always implied. Sometimes, a very tough school teacher may deduct marks if you do not write that. So, rate of change of momentum of a system or a particle is equal to the net force total sum of the forces acting in that direction. This is precisely for a system. Let us apply for a control volume. Now, then routine work is like that what is N in this case is the momentum. Let us denote the momentum with a vector. So, v is a vector momentum. So, what is eta? It will be simply the velocity vector v because by definition, eta is the property per unit mass. In this equation you have to know all the nomenclature. So, therefore, we can simply write this Dt of m D or we can write this way Dm b Dt for a system. This is for a system is equal to Dm b Dt for a system, rather this looks very odd D Dt of m v this way you write for a system.

All right, it is equal to what del del t. Now, it is routine mathematical proceed here control volume eta is v rho. So, we can write rho g bar d v plus, please any problem you ask me, control surface eta is v. So, rho v v dot, this should be little like that this is a vector and this is also a vector. These becomes a scalar v dot dA. Simply I am writing these equations with the value of eta v. So, this is precisely the Reynolds transport theorem. That means, precisely the form of the Reynolds transport theorem for conservation of momentum. That means, we are using the conservation of momentum statement for system and control volume with a equality sign with the help of the Reynolds transport theorem, which states physically that the rate of change of momentum of a system is equal to rate of change of momentum within the control volume plus flux within the control volume net rate of momentum. So, well this is the precise definition of the Reynolds transport theorem for the equation of motion.

Now, what is this is nothing, but the sigma F. So, therefore, sigma F becomes equal to I am writing again this thing because this is so important. Control volume rho v d v plus control surface v b dot dA. So, this is what is known as momentum theorem. That means, this is what is very important known as, please rho is missing. Is rho correct? This is what is known as momentum theorem.

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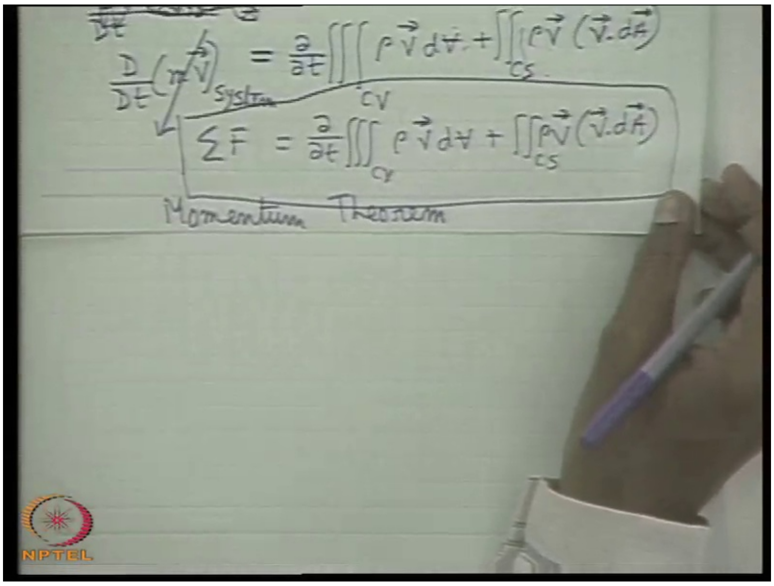

This is what is known as momentum theorem. That means, this is the equation of motion or conservation of momentum applied to a control volume. So, now onwards, we will recall this formula that the rate of change of momentum within the control volume plus the net rate of momentum a flux across the control surface, that is from the control volume across the control surface is equal to the net force acting in the same direction for which momentum you will take is equal to the net force acting on the control volume in that direction.

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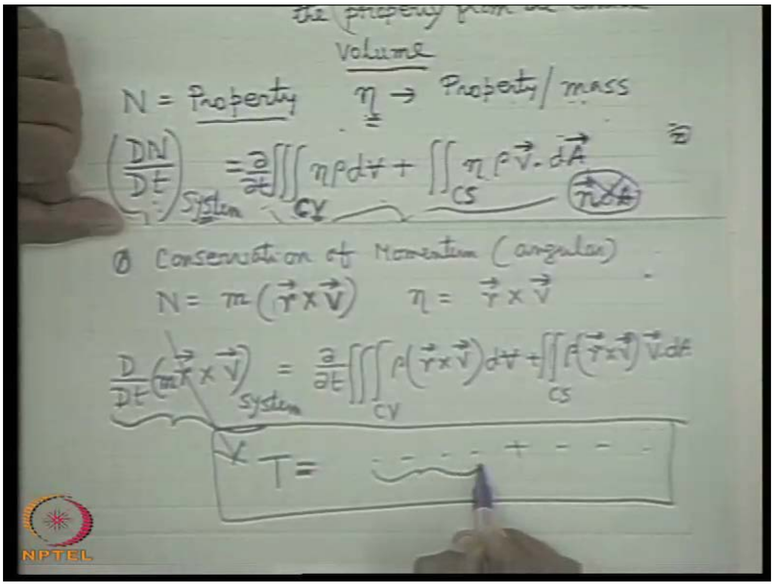

Similarly, we can derive this for conservation of momentum angular. Similarly, conservation of momentum we can do it for angular momentum, simple. In case of angular momentum, if we exploit this equation what is our N is m times mass times, you know the angular momentum is defined in this fashion that this is the angular momentum of any particle or any fluid element about any point or perpendicular axis at a point is the cross product of the radius vector. That means, if you join these two points, then we get a radius vector and the velocity vector. That means, physically it is the distance of that particular point from the point about which the angular momentum is considered times the velocity component perpendicular to this radius vector which is denoted as r cross v. It may be v cross r. V sign changed. That depends upon the sign convention r cos v. This you know how the angular momentum is specified. So, eta will be r cross v.

So, this is the only thing. Then again we write this thing in terms of the D Dt of r cross v for a system is equal to del del t eta is 1. That means, first I write rho r cross v, this d v well plus vertices over the control volume. So, what the control surface rho r cross v into v dot dA. So, therefore, we see that the rate of change of angular momentum of a system is equal to the b is missing. Where is b missing? Del t of Del del t D Dt of m is missing.

M is missing del m is missing, naught v is missing, m is missing, yes. So, correct. Very good. Now, try to understand. So, therefore, this is equal to simply the rate of change of angular momentum within the control volume plus net rate of angular momentum from the control flux control surface, all right. Now, again in the similar fashion we have learned the angle conservation of angular momentum for a system first and what is this quantity. Torque. Very good. The rate of change of angular momentum to a system is torque. So, therefore, first plus second. So, this is the angular momentum theorem. That means, the torque applied to a control volume must be equal to the time rate of change of angular momentum within the control volume plus the net rate of a flux of the angular momentum from the control volume. Obviously, for the steady case, this part will be 0. Similarly, for the steady case for angular conservation of linear momentum, this part will be 0. The first part will always be 0 for the steady state.

So, these are very important theorem, angular momentum theorem and the linear momentum theorem which will be discussed in analyzing the problem. Now, I will come to the conservation of energy.

Now, conservation of energy again if you recall, the conservation of energy if we first start with the a system with the definition with respect to a system, we know the conservation of energy for a system is first given by the first law of thermodynamics. As we have already read at school at first year class that conservation of energy and the first law of thermodynamic is synonymous. What is that for A system? Now, conservation of energy tells that the total energy in universe or in an isolated system which does not interact with the surrounding means anything external to the system with the surrounding. If we consider universe as an isolated system, then what universe is used in the physical science in a lose sense of an isolated system that it does not interact with anything outside. That means in universe or in an isolated system, total energy remains constant. So, there are many systems within the universe or an isolated system, they are interacting with the surroundings.

So, therefore, if we define its conservation of energy for an interactive system, system interacting with the surrounding means everything external to the system. There may be number of systems which are external to a particular system on which we are concentrating our attention. So, therefore, we can tell that for a system interacting with surrounding, the conservation of energy is obvious and it comes again from the physical intuition from our school days that the total energy remains constant. That means, the net energy interaction by the system with the surrounding must balance with the energy contained in the system with the energy contained in the system, and as you know or you will read afterwards in more detail that thermodynamic, first law gives the two distinct status of energy. One is the energy transit which always occurs in transit. For example, heat and work and another is, the energy which is contained or stored in a system. It is known as internal energy.

So, therefore, the total energy interaction by a system in the form of heat and work with the surrounding must be balanced by a corresponding amount of internal energy stored in the system. That means, in a process if system exchanges energy and it gives energy to the surrounding, so its internal energy will be decreased or it takes energy from the surrounding in a net energy interaction process. Its internal energy will be increased.

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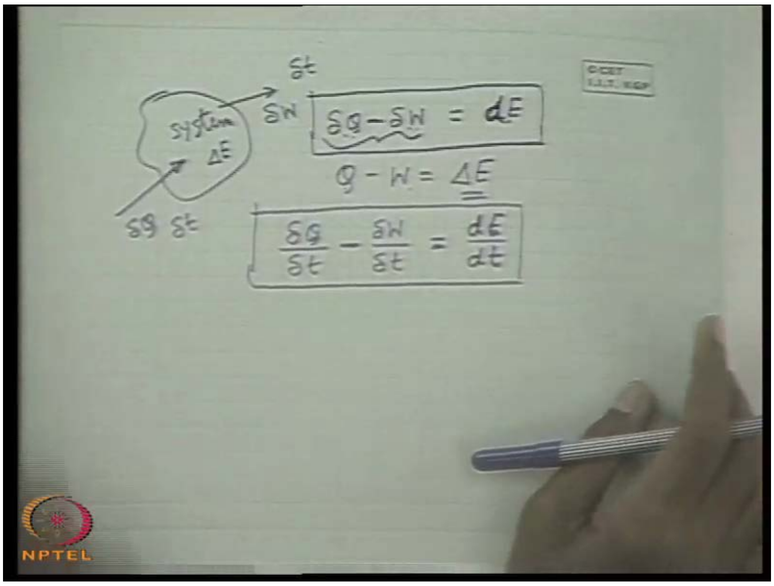

So, therefore, the first law of thermodynamics for a system which is the law of conservation of energy applied to a system which you learnt at the basic level can be written like that. Please see that if there is a system and if we consider in conventional positive sense that amount of heat delta Q is added to a system and the system torque and work delta W and by doing. So, if the system goes for a change in energy delta E, internal energy E, then from the first law of thermodynamics or the law of conservation of energy whatever you call delta Q minus delta W, this is the net energy that system has gain mass b equal to the increasing delta E.

So, this is the conservation of energy in thermodynamics. You will see that all kinds of energy interactions is divided into heat and work. This work takes care of all kind of energy interaction apart from heat. That means, this will be mechanical work, this will be magnetic work, this will be electrical work, all kind of work. So, heat and work, this difference is equal to the energy change in internal energy. That means, heat added to a system minus the work done by a system is equal to delta for a finite process. You can express this as differential expression as well this is dE, rather we call it as dE, because differential is dE. This is the finite delta E finite change it call it dE Q minus W is dE.

Now, on a rate basis if you write, we are more interested on a rate that is the rate of change of Q for a change in. That means, if delta Q is added for an interval of delta t and delta W for an interval of delta t, so it is the rate of heat addition and this is the rate of work done. As you know that heat and work are the path function. They are not the point function. We cannot write in a strict differential sense. So, del Q del t minus delta w, the rate of heat added and the rate of work added work done. That means, delta Q for a time delta t delta W for a time. So, this is equal to d E d t which simply can be written in a differential manner with the real spirit of differential from mathematics. That means the rate of change of internal energy because internal energy is a point function.

So, this is the time rate basis the first law applied to a system. Now, if I use the Reynolds transport theorem again, if we recall the Reynolds transport theorem, what is this Reynolds transport theorem that D and Dt again if we write now Reynolds transport theorem.

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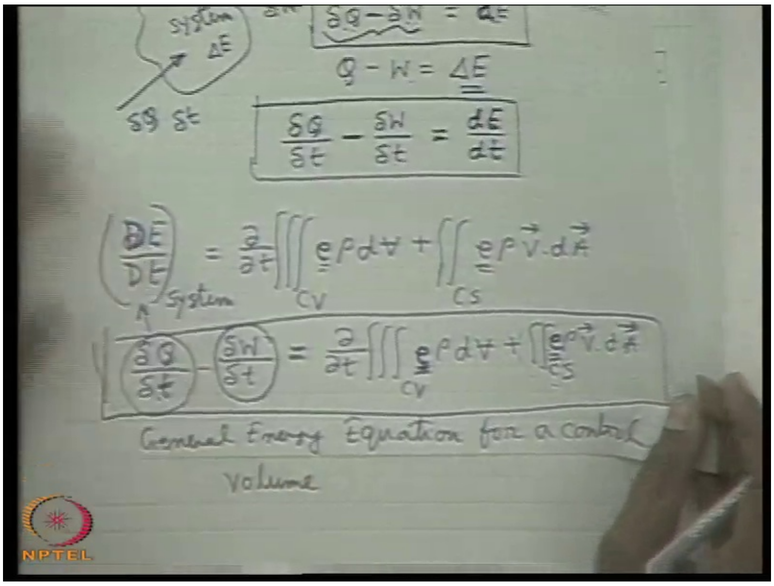

So, DE Dt, sorry big D. Usually D followed DE Dt for a system. What is this del del t of control? What will be eta? Let e where e is the at internal energy per unit mass without knowing express it form of the energy in terms of other parameters. You cannot write anything either for e or e. So, we define small e as the counter part of big E per unit mass. It is a total energy, total internal energy per unit mass. So, therefore, this simply becomes equal to d v plus what is this please. Tell me any problem. Eta is e rho, absolutely simple. So, this is that means the right hand side represents that the rate of change of internal energy within a control volume plus net rate of a flux of energy from the control volume across its control surface is equal to d E d t for a system. So, what is DE Dt for a system? Open it because what we have learnt first, so we see my way of approach first. We see what we have learnt for any conservation law with respect to a system then using the Reynolds transport theorem. So, this is replaced by del Q del t minus del w del t is equal to del del t of c v e rho d v plus c s rho vita dA.

So, this is precisely then the conservation of energy applied to a control volume which tells that the heat added to a control volume at the rate basis, the rate of heat added to a control volume minus the rate of work done from a control volume is equal to the rate of change of energy within the control volume plus the net rate of a flux of energy from the control volume across the control surface. This is known as a general energy equation for a control volume general energy equation for a control volume.

Now, next task is that this energy equation is well known for a control volume. Next tas task is to recognize what is the internal energy E here because here also I have forgotten to write E E has been. So, what is this E? That means, what is the internal energy which is associated with a flowing particle because this is going out when it is going across the control surface. So, our main task is now to recognize the internal energy. So, after this mathematics, I think we should now physically try to recognize one of the different forms that a fluid element or a fluid particle possess when it is in flowing codes of flow because if you consider a control volume, so definitely when the fluid particle crosses, its control surface comes into control volume. It is some energy within the control volume.

Similarly, when it is going out of a fluid particle or lamp of fluid system going out for the control volume, it takes away energy from the control volume along with its flow. So, therefore, now as it is essential to recognize what are the different forms of energy that a fluid particle possess or a fluid element possess by virtue of its flow or in codes of its flow. What are those energy's? Can you tell what are those energies? One is the first for any system under any conditions the energy possessed by it is the intermolecular energy that is the energy by virtue of the kinetic and potential energies of molecules comprising that sustain. So, one is the intermolecular energy, another energy is the kinetic energy this is because of motion. So, when any particle is in motion whether it is solid particle or fluid particle by virtue of its motion, it has energy which is the kinetic energy whose magnitude is given by mass time, the square of velocity by 2 m b square by 2, that is the potential energy and another one is the, sorry which is kinetic energy and another one is the potential energy.

What is the potential energy? Please define what is potential energy. So much of potential energy you have. Read at school level. What is potential energy? Yes, very correct. It is the energy which a particle or a system or any element possesses by virtue of its position in a force. What type of force by virtue of a conservative force field? Yes, by virtue of its position in a conservative force because work is done to place the body in at a particular position in a conservative body force speed if it is the only gravity as the conservative body force speed will...

Now, the value on the potential energy is m g h. If we consider h is the height from any arbitrary datum level, where we take the energy to be 0. That means, from that level to bring the particle at a level height h, the work in g h is being done. So, this is the potential energy. Apart from that, there is another energy known as flow work or pressure energy which I will discuss in the next class. Time is up. So, we just finish it here for today. So, next we will be discussing another form of energy along with these three forms which appears for a flowing particle of fluid or a flowing element of fluid, that is pressure energy or flow one. This I will discuss in the next class.

Thank you.