Fluid Mechanics

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Lecture - 14
Conservation Equations in Fluid Flow Part-II

Good morning. I welcome you all to this session. In last class, we were discussing, we have just started the discussion on continuity equation, and we derive the continuity equation in a Cartesian frame of reference. What is a continuity equation? If we recall, it is an equation relating to the velocity and density field in a flow of fluid, which is deduced from the principle of conservation of mass applied to a control volume that means, continuity equation basically signifies the principle of conservation of mass.

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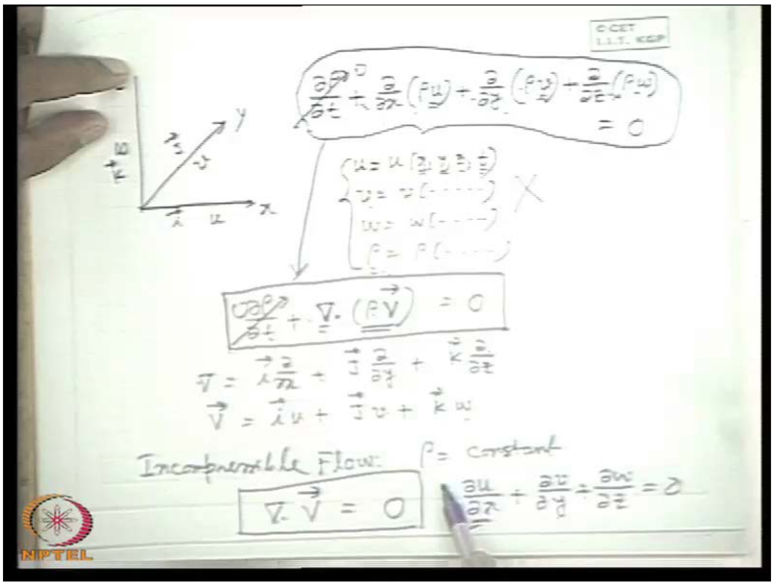

Now, if we recall the equation, you please see here that in a Cartesian frame of reference, if you recall the equation, let this is the Cartesian frame that we discussed x, y, z. The equation of continuity was like this del rho del t plus del del x of rho u, u is the x component velocity, v is the y component and w is the z component. So, typically this equation was derived from the principle of conservation of mass applied to a control volume with respect to a Cartesian coordinate system, rectangular Cartesian coordinate system. This is precisely the equations where u rho is the density and u v w are the velocity components.

Now, one thing we can tell from this equation. That in a flow field, if we get a description of u v w as a function of x y z and t as a function of x y z and t, v is also such functions of x y z and t, w is also such function of x y z and t and rho is also such functions. That means, these dependent variables are hydrodynamic parameters. For example, three velocity components and density are expressed as a function of independent variables like the space coordinates and time. You know these are the independent variables.

Then, this function must satisfy this equation. Otherwise, the flow is impossible. Sometimes we check whether a velocity field and density field describe a physical flow or possible flow or not. If they do not satisfy this equation, means they do not satisfy the conservation of mass. That means, this is an impossible situation. This sets of functions can never represent the velocity field and density field of the fluid flow. It is most important for the use of continuity equation. You should know that.

Now, you see that this continuity equation can be defined or described with respect to different coordinate systems depending upon the geometry of the flow. Now, before doing that, we can just see that this equation expressed in Cartesian coordinate system can be written in a vector form like that, del rho del t plus this term can be written as, if you recall your preliminary knowledge in vector, so, this term is divergence of rho v. rho is a scalar and v is a vector. So, rho v is a vector.

You know the divergence operator is a vector operator, which is I del del x. For a Cartesian coordinate system, this tends like that. I j k are the unit vector in x y and z direction. So, we can write the operator del, which is a vector operator k, just if you brush up your preliminary knowledge in vector. So, this operator being used with a dot product, that is a scalar product with this vector, where v bar that is the velocity field. You know this is a vector. So, this has got distinct three scalar components v and w along x y and z directions. So, if you multiply these two vectors, that is del dot rho v dot scalar multiplication, we simply get this. So, that you know del u del x plus del v del y plus del w del z.

So therefore, one way of converting any equation from one coordinate system to other coordinate system is to see the equation in one coordinate system and to find out its vector form, general vector form. So, if you can do that, then we can tell that this is the vector form and we can expand this vector form in different coordinate system. So that, we can get the equation in different coordinate system.

So, before coming to that, I should describe one thing, which probably we described earlier also. If the flow is incompressible, in the last class also, if the flow is incompressible means incompressible flow. If the flow is incompressible, rho is constant. That means, density does not change in the flow field neither with time nor with space coordinates. What does it mean that del rho del t 0? That means, in this case, del in this form del rho del t 0.

Moreover, rho will come out from the differentials and in this operations, in this vector form rho will come out. So, ultimately the equations will be divergence of the velocity vector in vector form is 0. This is the equation of continuity for incompressible flow. In case of Cartesian coordinate, the expansion of this will be del u del x plus del v del y plus del w. That means, we can write this in vector form and then, expand in case of a Cartesian coordinate del dot v will be del u del x plus del v del y plus del w del z. Because v is a vector defined by like that and del is the or we can straight forward get it from the equation with respect to Cartesian coordinate. That is will be 0 and rho will come out and the common factor which cannot b 0, so, del u del x plus del v del y plus del w del z is 0.

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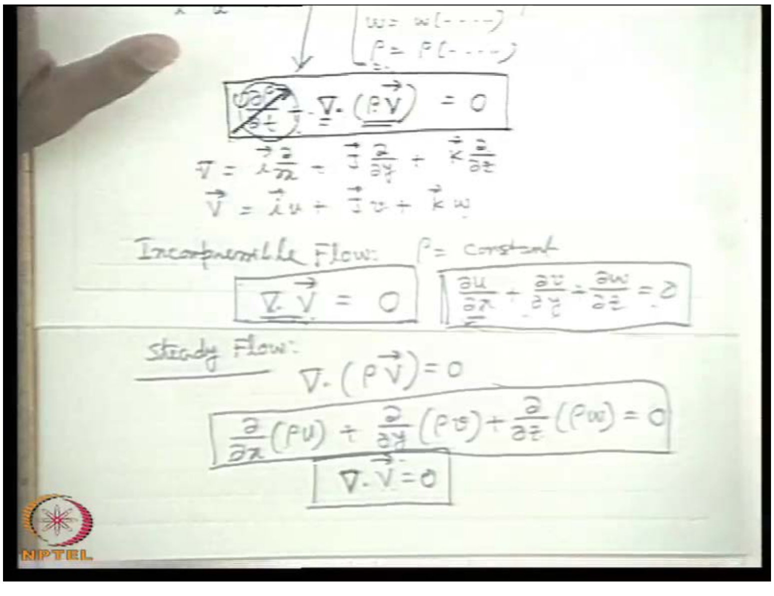

This is the equation of continuity for incompressible flow. What will be the continuity equation for a steady flow? For a steady flow, please tell what is the continuity equation for a steady flow? Which term will be omitted for a steady flow? Steady flow means that parameters will not change with time. So, which term of these two terms del rho del t will be 0? That means, for a steady flow, this is the equation of continuity. That means, in Cartesian coordinate, del del x of rho u plus del del y of rho; v is a special case for a Cartesian coordinate system del del z of rho.

Now, one interesting fact you must remember that this is the continuity equation for a steady flow of both compressible and incompressible. So, if we take an additional constant of incompressible flow along with the steadiness, then rho comes constant. Then, we get from either of the two equation divergence v is 0. So, for a steady flow, this is the continuity equation. Now, for an incompressible flow you see the continuity equation is divergence v 0. So therefore, from an incompressible flow, continuity equation it is very difficult to infer whether the flow is steady or unsteady, because even if the flow is unsteady, but incompressible, the equation remains same which is the fact, what is the, why, what is the fact because the derivative of rho with time only appears.

So, whenever the flow becomes incompressible, so, it automatically goes because rho has to be constant for an incompressible flow. Even if it is unsteady, rho cannot depend on time. Other parameters will depend on time. Since the time derivative of no other parameters appear in the continuity equation, therefore it is very difficult to judge from the continuity equation of an incompressible flow, whether the flow is steady or not. For example, whenever the flow is incompressible state becomes divergence v 0; and that means, this is 0.

So, even if the flow is steady, there is no scope of any further simplification or modification of this equation because time derivative of no variable appears in this equation. So therefore, the continuity equation for both steady and unsteady incompressible flow is given by divergence v 0. But for a compressible flow, there is a change between steady or unsteady equations in the form of continuity equation. This is why? Because this is the compressible unsteady flow and this is the compressible part. That means, the unsteady flow, compressible unsteady flow, a steady flow and this is the compressible unsteady flow.

So, a compressible flow, if it is unsteady, term will be there. If a compressible flow, if it is steady, then this term will not be there. So therefore, we can distinguish the steady flow or unsteady flow, if the flow is not incompressible from the continuity equation. But if the flow is incompressible, whether the flow is steady or unsteady, the continuity equation will always be divergence v is 0. This part is cleared?

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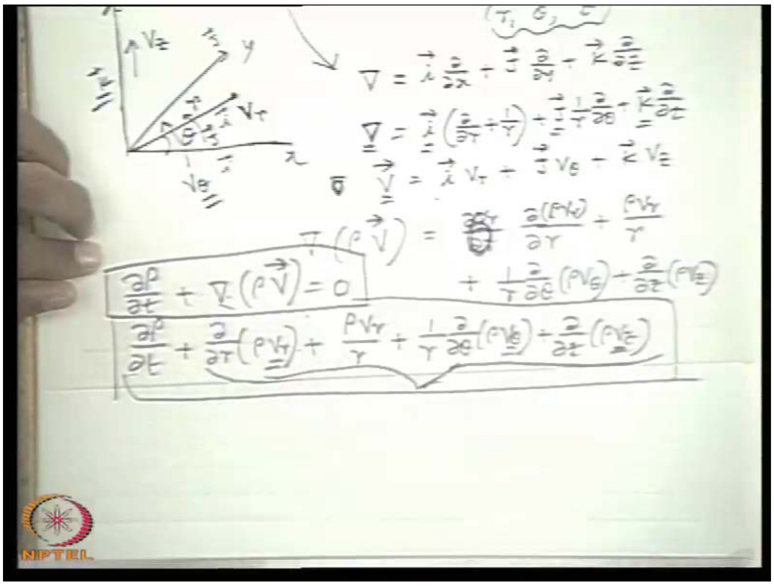

Now, next I come to the different coordinate system. Now let us, first thing that is equation in general for a incompressible unsteady flow, this is the divergence of rho v. If the question comes, what is the cylindrical coordinate? What is the continuity equation in cylindrical coordinate system? Continuity equation in cylindrical coordinate system. So, one way, for example what is cylindrical coordinate system? Let us concentrate the cylindrical coordinate system. Let this x and this y and this z.

In a cylindrical coordinate system, instead of x y, we define in x y plane, the point by a radial location r and Azimuthal coordinate theta and the z will be same. That is, in the z direction, this has come from the concept of a cylinder, geometry of a cylinder. So that, at any point will be described by a radius radial vector or radial coordinate r azimuthal theta and this z. You know these things. That means, r theta z instead of x y. These are the coordinates in a cylindrical polar coordinate system and not simply cylindrical coordinate system.

In that case, one way of, did you see it? Mathematically, without going for any other physical complications, simply expand this term. But before expanding this term, one has to know the mathematical expression for this del in different coordinate system. Probably, you know, if we again brush up your preliminary knowledge in vector, this del in Cartesian coordinate as I have written earlier , that represents del del y plus k del del z, where i j k are the unit vector along x y and z direction.

Similarly, in cylindrical coordinate system, this will be del del r plus 1 by r. Probably you know this thing, j 1 upon r del del theta plus k del 0, where i j k at the unit vectors along r, along this theta direction j and along the z direction remains as it is. So, if you define i j k, the unit vectors along the coordinate directions r theta and j, this is the operator. One knows this thing. For him, it is easy to find out del dot rho v because v in Cartesian coordinate system, if you represent v r, let this is v r as the radial component of velocity and v theta; that means, this direction, the azimuthal direction is the v theta tangential component of velocity or azimuthal component of velocity. v z is the z component, which we are using w in case of Cartesian coordinate system.

Now, we are using v z; that means, the velocity field can be expressed as its components v theta plus k v z. So, one can find out now, after knowing this and these two velocity fields, rho is a scalar function that divergence of rho v. Well, any questions please? So, this will be this multiplied with this, that means, del v r del r. I am sorry. This will be multiplied with rho. So, del rho v r del r plus rho v r by r plus 1 upon r del del theta of rho v theta plus del del j of rho v. Just simple mathematics. That means, what I do? I expand this term, this second term in a cylindrical polar coordinate or simply cylindrical coordinate system. If I do so, then I can write the continuity equation, another additional term; that means, is del rho del t plus, again I repeat the same thing, del del r of rho v r plus rho v r by r plus 1 by r del del theta rho v theta plus del del z rho v z. That means, what I do? del rho del t, the equation plus divergence of rho v 0. This is the general vector form.

That means, I have expanded this in cylindrical coordinate system. That means, therefore, I can tell precisely this is the continuity equation in cylindrical coordinate system, where v r v theta v z are the respective velocity components corresponding to that coordinate system. So, this is one way of reducing the continuity equation in different coordinate system. We can do it for spherical coordinate system also. That I left as an exercise to you. By expanding this term, this is a purely mathematical exercise. But one can again find out these from physical concept or from fundamentals.

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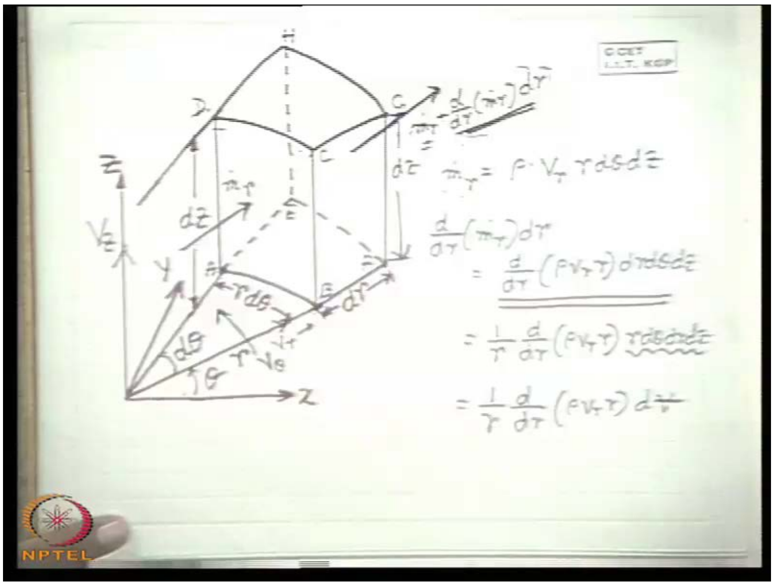

What I told in the last class is that, if we have to derive the continuity equation from the fundamentals, means application of conservation equation to a control volume, then the first term is that, you will have to take the control volume appropriate to a coordinate system. Now, look into this figure. Now, if we want to derive the continuity equation in a cylindrical polar coordinate system, which is defined by a radial location R azimuthal, this is the azimuthal theta and Z, then we will have to consider as volume, which is parallel in case of a Cartesian coordinate system. Now, the control volume will be like this, where the different planes of the control volume will be parallel to the coordinate planes.

So, how we will choose it? This one direction will be r d theta. If this is r and this radius vector is at an angle d theta, so, this angle is d theta. Another length will be d r and this will be definitely d z. That means, a control volume of dimensions d r d theta r d theta d z, this type of a fluid element or a slide of a fluid element have to be considered. Now, let us give some name, otherwise it is difficult to A B C D and this one is E F G H. Now, A B C D E F G H represents the control volume.

Now, you see we have to represent the mass flux in different directions. So, there is velocity in r direction. So therefore, there is a mass influx. Now, let us consider, due to the r direction velocity v r, so, velocities are in r direction, theta direction v theta , this is the v theta, theta direction, this is the v r in r direction. This is as usual v z. Here, we represent v z not as w, v z direction velocity.

Now, if we concentrate in the similar fashion, the mass flux in the radial direction which is across the surface A B C D. This is A B C D. That means, the r surface which is perpendicular to the radial direction. So mass influx, if we consider as m dot r and the mass efflux from the r plane; that means, this plane E F G H, there are two r planes; that means, planes perpendicular to radial direction. One is A B C D and another is E F G H.

So, because of the existence of the velocity vector, we are in its usual positive direction. Mass will come into the control volume across A B C D surface and mass will leave the control volume across E F G H surface. So, this will be m dot r plus d d r of m dot r d r because this mass flux has changed by this amount because of a change in d r. In similar fashion, we write what is the expression of m dot r, that is the mass flux across this surface. It is the volume flux times the area. So, volume flux will be density. Sorry, density will come for the mass. Let us write the volume flux will be the velocity times the area. What is the area of this surface r d theta and d z? That means, already I get this d z. Again it is duplication. Does not matter r d theta d z and rho is multiplied to keep m dot r. So therefore, what is this m dot r d r m dot r d r? That means, let us write it only d r m dot r d r. This will be equal to d d r of rho v r r. So, d r d theta d z, I take out.

Now, you see the net mass efflux. Because of the mass flux across the r surfaces or r planes; that means, the plane perpendicular to r, because of these two planes A B C D and E F G H is this minus this. So, net mass efflux, I am not writing. I just tell you net mass efflux from the control volume. Because of the mass flux is across the r planes, that is planes perpendicular to r direction; that means, these two planes A B C D E F G H is equal to this minus this. That means, m d d r of m dot d r. That means this quantity. This is the net mass efflux, because of the flux across the r planes.

This we can write by taking 1 by r multiply and then v r, little rearrangement d theta d r d z. What is r d theta d r d z? Volume. Very good. So, 1 by r d d r of rho v r r into d v. In the similar way, we can find out for mass fluxes across the theta.

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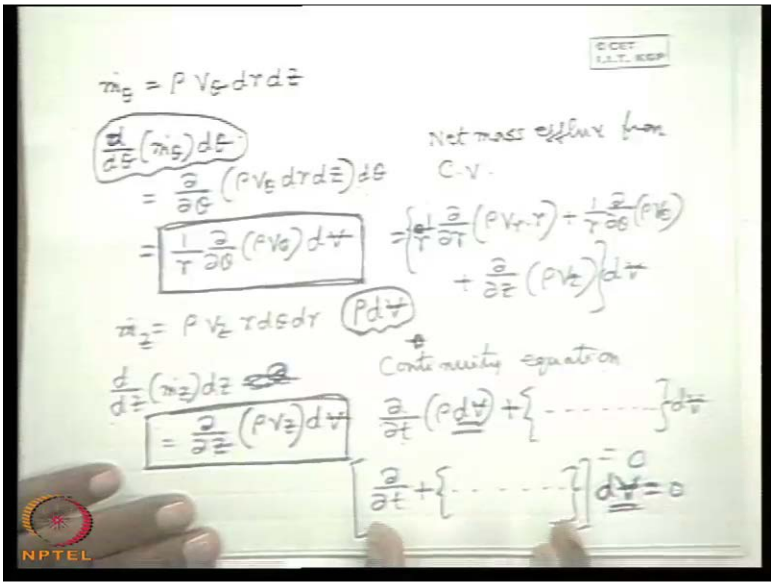

That means, m dot theta. That means, theta direction. That means, the mass flux due to the planes perpendicular to theta directions; that means, now we are interested for mass flux across the planes B F G H and A E H D. Two parallel theta planes perpendicular to theta direction.

Now, because of the existence of the which is the component of the velocity in its usual positive direction, the mass flux will enter the control volume across B F G H. Let us define that m dot theta. Because of the same reason, the mass flux will go out of the control volume across this plane or phase A E H D, which we should write m dot theta. Why because this is change at this location, because of a change in r d theta.

So, you can write in terms of d theta, d d theta because in the theta direction, angular direction we are doing it m dot theta d theta. So, what is m dot theta? Please tell me. m dot theta now, we will be mass into the volume flow, that is v theta times the d r into d z. Very good. d r into d z. All right. Now, what will be this m dot d d theta of m dot theta d theta? This will be equal to del del theta is rho v theta d r d z d theta. So, this can be written as 1 by r del del theta of rho v theta into 1 by r r d theta d r d z. That means, this is d v, where d v is the elemental volume of this control volume. Again, this is the quantity which represents the net mass efflux from the control volume due to the mass fluxes across these two planes. That is theta planes B F G C and A E H D.

So, this represents simply the net mass flux, from the mass efflux, from the control volume due to the mass flux across these two parallel planes. That is, the planes perpendicular to theta direction. Similar way, if we see or if we investigate the mass fluxes through planes perpendicular to z direction. What are the planes? A B F E and D C G H. That means, bottom and top plane in this drawing.

So, due to the existence of the positive direction velocity v z, in usual positive direction of the coordinate axis, the mass will come into the control volume across the phase A B F E. Similarly, the mass will leave the control volume across the phase D C G H. Simply we can write now rho, this will be v z times r d theta d r. Simply this will be m dot z plus d d z of m dot z d z as usual. That means, there is a change in the mass flow because of a vertical displacement of d z. So, d d z of m dot z d z can be written as, rather I can write d d z that is del del z. I am writing in this fashion though I am writing d d, but ultimately this is a partial differential. That is why I am changing from d to del. Does not matter.

So, this will be equal to del del z. You can write all these in terms of del, because in the conception there is no problem. But in a mathematical notation, this is not a total differential concept because these are all partial differential. Because all the quantities vary with both r theta z and time also. So, this is an instantaneous picture. So that, when we make differentiation with j this should be del only.

However, for the conception there is no problem. So, del del z of what we can (( )) rho v z and r d theta d r d z will automatically make d v. This is nothing but the amount, which corresponds to the net efflux to the control volume due to the fluxes is across the z planes. That means, across two planes A B A V and D C G H. So therefore, we get this is the net efflux from the control volume due to the mass fluxes across r planes. Similarly, this is the net efflux from the control volume due to the mass fluxes across theta planes and this is the mass efflux from the control volume due to mass fluxes across z plane.

So, net mass efflux from the control volume will be sum of this. That means, del del r, if you want to have a look, 1 upon r del del r rho v r. So, I now write with this 1 by r del del r rho v r into r. You can have a look; plus d v, we will take common, plus this 1 by r del del theta of rho v theta plus del del z of rho v z into d v plus. If you recall this is the net mass efflux. So, continuity equation will be what? Continuity equation therefore, if you recall the continuity equation in its statement form that the net rate of mass efflux from the control volume plus the rate of change of mass within the control volume.

So control volume, volume is d v and rho is its density. So, it is the instantaneous mass within the control volume. So, rate of change of mass within the control volume will be del del t of rho d v plus this quantity. That means, I am not writing it again d v is equal to 0. That means this quantity. So, d v will come out of this del del t because control volume by definition and d v is fixed. So therefore, we can write del del t plus, this in bracket; that means, this term, the entire thing d v is equal to 0. d v cannot be 0. It is valid for any volume, any finite volume of the control volume. So, this part will be 0.

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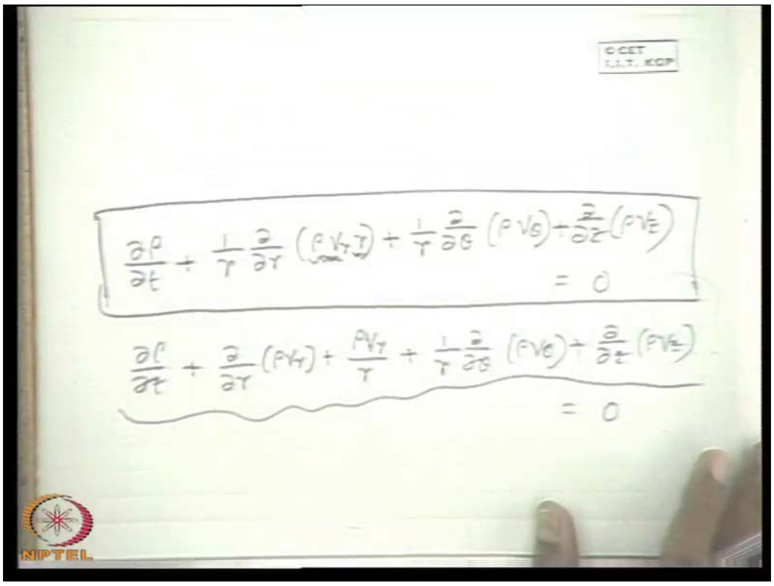

So therefore, in the similar fashion, we can write that finally, del rho del t plus, now I write 1 by r del del r of rho v r into r plus this probably. You can see that this I am writing 1 by r del del theta 1 by r del del theta of rho v theta plus del del z of rho v z is equal to 0 . So, this is the precisely the continuity equation. We can write it in a different form, del rho del t. If we just expand this, the differentiation v r r, so we can get we can write del del r of rho v r plus rho v r by r, by taking rho v r the first function, and r is the second function. If we differentiate it, this cooled level thing, del del theta rho v theta plus.

So, this is the equation we also derive straight from expanding the vector form of the continuity equation. So, there is no need always of deriving it from the fundamental. Just for your conception, I show you. Because earlier, what we did was we know this vector form of the continuity equation. We simply expand this with the idea or with the knowledge that the del operator is defined in a cylindrical coordinate system like this, so that, we can straight away this in different coordinate systems.

For example, the expression in Cartesian coordinate system will be del del x of rho u plus del del y of rho v plus del del z of rho w. Similarly, for a cylindrical coordinate system, this will be the expression. But this can also be derived again from the fundamental. That means, taking a control volume appropriate to a coordinate system. For example, they are cylindrical coordinate system and applying the law of conservation of mass, considering all the mass fluxes coming in and coming out from the control volume across different plane surfaces, so that, I can or we can derive the continuity equation.

Similar way, the continuity equation can be derived in a spherical coordinate system. Again we will make more complications, because the control volume will be little complicated by geometry either by using the fundamental concept or by expanding these forms, which is left as an exercise to you.

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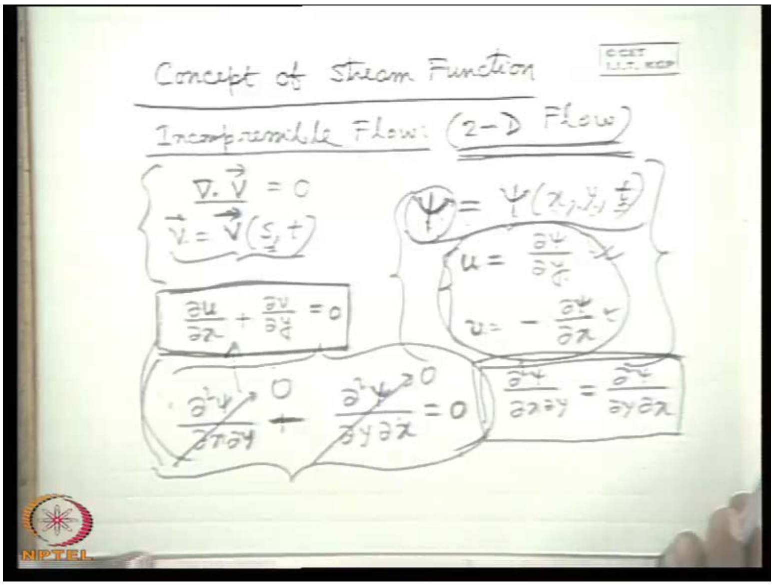

Now after this, I will go to a very important concept in a fluid flow, which is the concept of stream function. What is a stream function, concept of stream function? Now, we know that for an incompressible steady flow or we should not always tell that is incompressible steady even for an incompressible flow. That means, even if it is steady or unsteady, the continuity equation is such that, means, if (( )) I have an incompressible flow. My velocity fields are given as a function of space coordinates and t, with scalar components u as a function of x y z t, v as a function of, if it is a Cartesian coordinate system. Or, if it is a cylindrical coordinate, it will be a function of r theta z. So, I am just describing in generally one test and claims, that is flow is incompressible one can tell him. Please wait. I will check whether your velocity functions explicitly given in this form satisfies this particular equation or not. Divergence of v 0, if it is satisfied, we will tell your flow field is possible. If it does not satisfied, then we can tell it is an erroneous flow field. This velocity function can never define an incompressible flow.

This is the concept. So therefore, if we come again to mathematics, the divergence of the velocity vector is 0 for an incompressible flow. Let us consider a Cartesian system. First, simply it is nothing but del u del x plus del v del y is 0. Always we consider a two dimensional flow stream. Concept of stream function is associated only for a two dimensional flow. You must know, this is not for a three dimensional flow. In three dimensional asymmetric flow only, which again reduces the three dimensional flow in a two dimensional flow. I will explain it afterwards. Then, we can define this stream function. Usually the stream function is defined only for two dimensional flow.

So, for a two dimensional flow, the expansion of this term for a Cartesian coordinate system is like that. Or in other words, in a two dimensional flow, defining u and v as a function of x y and t, the continuity equation for an incompressible flow at any instant is this. Now, if I define a function psi, which is a function of x y and t, at any instant t. If it is an unsteady flow, the variable t will come. Otherwise, it will be a function of x and y only. That means, if in a two dimensional flow field, I defined a function. Think mathematically first. I defined a function, so that these functions satisfies this condition u is equal to del psi del y and v is equal to minus del psi del x. That means this function is such whose partial derivative with respect to y, defines the x component of velocity at a particular point. If it is a function of time, this will be a function of time also. Similarly, its x derivative with respect to x with a negative sign defines the velocity, y component velocity at that instant and then, this function is defined as the stream function.

Now, question comes, why so arbitrarily we are defining a function such that u becomes del shy del y and v becomes del shy del (( )). Mathematically it is understandable. We will define a function shy, which is a function of x y and t in such a way, that u and v are defined in terms of this function in this manner. Then, this function is called this stream function. But what is the significance of it?

Let us now see the mathematical significance. If you defined this way, then if we put this stream function in continuity equation; that means, if the continuity equation is now substituted in terms of the stream function, what we will get first term? del square shy del x del y. What we will get in the next term? It is minus del square shy del y del x. Now, if shy is a continuous function of x and y, you know that this order change does not have any difference. That means, del square shy del x del y is del square shy del y del x. That means, if you differentiate shy first, we takes and then with y or first with y and then with x, they will be equal if the function is continuous.

Again brushing up your school level mathematics. So therefore, this is equal to 0. That means, it is automatically satisfied. That means, we do not get any extra equation as a continuity equation, if the flow field is defined in terms of stream function. Try to understand. This is a real tough concept. This level, only by reading books you may not understand this. That means, instead of defining flow field in terms of u v, if we define a flow field in terms of stream function, it automatically satisfies the continuity equation. Because if we substitute the stream function, because stream function is defined this way.

So that, if we substitute this continuity equation get 0; that means, if we define this flow field in terms of a stream function, so, continuity equation is automatically satisfied. That means, we do not have an extra equation to satisfy the conservation of mass; that means, the equation is automatically satisfied. This is the mathematical implication of stream function.

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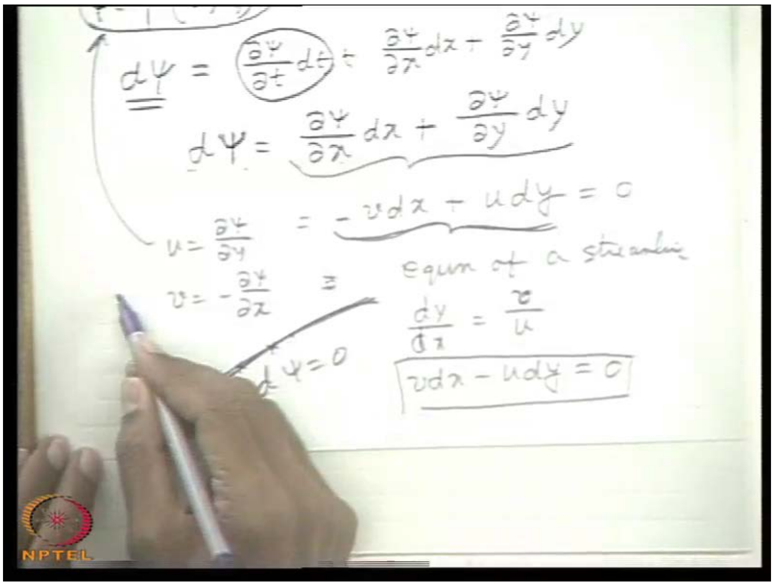

Let us see the physical implications of stream function. Now, before recognizing the physical implications of stream function, let us consider one thing that stream function is a function describing shy as a function of x y. Now, a change in stream function. One interesting thing now we like to express. Now, we write a total change in stream function x y t. It can be written in mathematical form del shy del x del shy del t d t plus del shy del x. This you know d x plus del shy del y d y in a two dimensional flow.

Now, at any instant if you are interested that any instant value or for a steady flow, this term is not coming into picture. So therefore, d shy is equal to; that means, the change in the stream function at any instant or in a simplified manner, we can think of a steady flow, where the change in the stream function can be defined like this. It is very simple. It is no way connected to mechanic fluid. If there is a function of x y, the change in the functions shy d shy is the change due to x and change due to y, del shy del x d x plus del shy del y d y.

Now, what is the definition of stream function? This function is not a very arbitrary function. This is such a function that u is equal to del shy del y or simply del shy del y is equal to u and v is equal to minus del shy del x. So therefore, here I can write minus v d x plus u d y. What is the value? Can you tell me along a streamline? Along a stream line, what is the value of right-hand side? For a streamline, the equation of streamline we know. What is the equation of a streamline? If you recollect, it is d y d x is v by u because the tangent at that line, at any point is the direction of the velocity vector. For a two dimensional case, streamline is d y d x v by u. So therefore, v d x minus u d y or u d x minus v or u d y minus v d x is 0 along a streamline.

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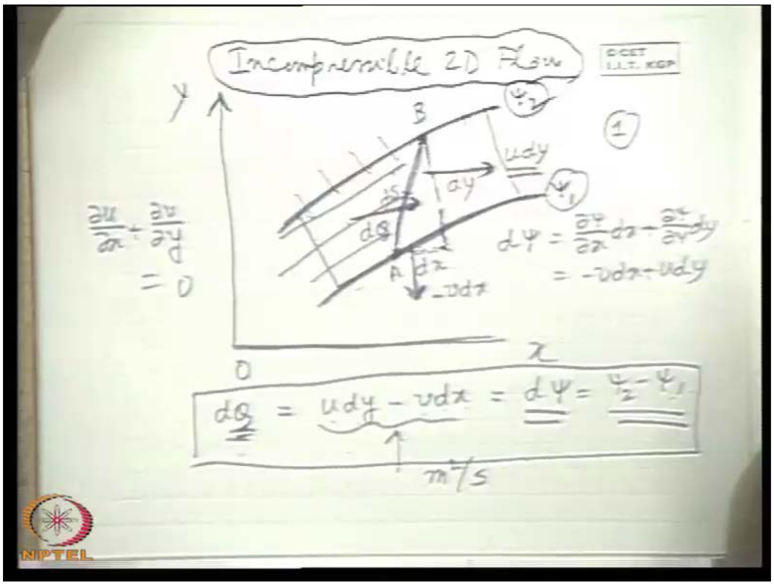

So therefore, along a streamline, if this is a streamline this value is 0, which means, d shy is equal to 0 along a streamline. That means, the shy function along a streamline, let us consider this is the x, this is the y, this is the z and this is a streamline. So, z concept does not come. It is only x y concept. Let it is like this and there is x and y and this is a streamline. So, shy functions, this is a streamline and this is a streamline. So, what we get? Along a streamline, the values of shy are same. That means, shy is equal to constant along a streamline. So, this is proved.

So, this is one of the very important conclusions that along a streamline d shy is 0; that means, shy is equal to constant. So, shy is equal to constant along a streamline. So therefore, in a two dimensional flow field, we can define streamlines by telling different values of shy; shy 1, shy 2, shy 3 and shy 4, which are the constant values because shy is constant along a streamline. It does not change along a streamline.

So, a streamline can be specified by giving a constant value of shy stream function. From this, we can proceed another step further regarding the streamline, physical implication of streamline. Let us consider again x y, the two dimension and let us consider two shy 1 and shy 2, the two streamlines, shy 1 and shy 2. Let us consider one point A on a streamline, any other point A and B on two streamline. Let us join. Now, let us make; Let us consider a control volume, whose dimension in this direction perpendicular to this plane of the figure or plane of the paper is unit. Let it be one unit and this length be d s, that is the length joining two points on two streamlines defined by stream functions shy 1 and shy 2.

Therefore, this length, this is dy and this is dx obviously. Now, let us consider the flow field is such that because of which there is a flow coming into this control volume across this surface of length d s and plane unit dimension in this direction. If there is a flow coming into the control volume and if we consider the flow going out from these two planes, then what is the flow coming out from this? It is u d y volume flow. Now, I have told that this concept of stream function, we are discussing for an incompressible 2D flow. It is always valid; incompressible 2D flow. Because from the very beginning, we have defined that the continuity equation, which is getting satisfied is del u del x plus del v del y 0. That means, basically it is two dimensional and incompressible.

So, in case of incompressible flow, the volume flow rate instead of mass product, we can use because rho becomes a constant scale factor. That means, the volume flow which is coming to the control volume across these planes must go out from these two planes. If we consider through these two planes, the volume the fluid is going out. So, this is the volume fluoride across this phase and this is the volume fluoride across this phase because the area of this phase is d y into unit distance and area of this phase is d x into this distance unit. So therefore, we can write the d q. If I tell the d q amount of volume flow, q is the nomenclature for volume flow rate is crossing these planes, joining a and b must be equal to u d y. Now, this v d x in this frame of reference, where y is positive in this way, then this will be minus v d x because v is always negative sign. So, minus v d x plus of minus v d x; that means, u d y minus v d x.

Now, what is u d y minus v d x? This is the difference d shy between the two points because if I relate shy 1 shy 2 minus shy 1 as d shy, it will be del shy del x as we have seen, d x plus del shy del y dy. That means, u minus v d x plus u d y. That means, this quantity, this is equal to d shy; that means, this gives the difference of stream function between these two streamlines. So, this is the flow rate by unit length in the perpendicular direction. So, this gives the most important physical conclusion, that the difference in stream function between two adjacent streamlines, between two any streamline gives the volume flow rate within the streamlines per unit in the normal direction.

There is no volume flow rate across a streamline. That means, if we consider a control volume like that, the volume is flowing like this. So, this is given. So, if we consider two streamlines and this consists of a stream two; that means, bounded by two streamlines, then we can tell the volume flow through this channels made by two streamlines is given by the difference of this stream functions defining these two streamlines, per unit length or per unit width in the normal direction. Because if you see the unit, you see that shy 2 minus shy 1 u d y minus v d x, this unit is meter square per second. That means, this is the volume flow rate per unit within this direction.

So, it is cleared. Therefore, we can tell that this stream function signifies this physical role that difference between this stream function gives the flow rate within the two streamlines. Well, I think we can conclude here. So, streamlines is like stream function is constant along a streamline. Stream function is a function of x y, such a way that it is derivative with respect to x and y coordinate, defines the velocity component in such a way that it automatically satisfies the equation of continuity and incompressible two dimensional flow. This is number one. Number two is the streamline function remains constant along a streamline. Number three is the difference between the stream functions between two streamlines gives the rate of flow through these two streamlines along the channel found by two streamlines per unit width or length in the perpendicular direction.

Thank you.