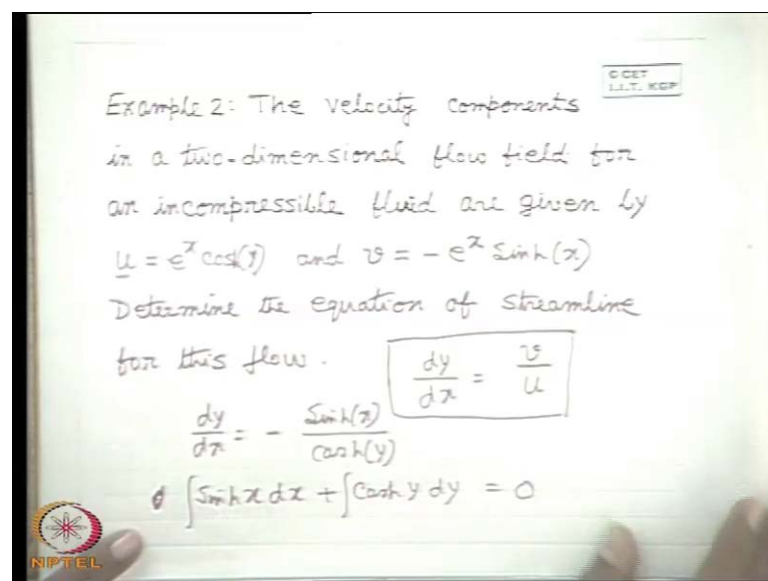


Fluid Mechanics
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Lecture - 13
Conservation Equations in Fluid Flow Part – I

Good morning. I welcome you all to this session of fluid mechanics. So, this class I will start a new section conservation equations in fluid flow, but before that as usual I like to have a closure of the earlier section kinematics of fluid. And before an even prior to that I like to continue with two more problems. Because last class, because of due to the lack of time we solved only one problem, two very simple problems we go through horribly. And then with a quick closure of the earlier section kinematics of fluid, we will start the new chapter or new section that is conservation equations in fluid flow.

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So, let us go to a problem related to earlier section that is a kinematics of fluid. So, just a simple problem the straight for one applications of our theory, the velocity components in a two dimensional flow fluid for in compressible fluid or given by u is this, v is this u is e to the power x \cosh hyperbolic of y . And the y component of velocity minus e to the power of x \sinh hyperbolic of x , well determine the equation of streamline for this flow.

So, in a very straight forward application for the equation of streamline as you know the equation of streamline is that $dy dx$ if you recall for a two dimensional flow field. The

equation of the line, which is the streamline represents the velocity vector as the tangent at every point to this line is given by v by u . So, this is the equation of streamline. So, fluid mechanics ends here, what remains is a simple school level mathematics that is dy by dx or dy dx as you tell is v by u , that is minus what is that? Sin hyperbolic x by cos hyperbolic y . So, if you make it like this $d \sin$, sin hyperbolic x dx plus cos hyperbolic y dy is 0.

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Determine the equation of streamline for this flow.

$$\frac{dy}{dx} = \frac{v}{u}$$

$$\frac{dy}{dx} = -\frac{\sinh(x)}{\cosh(y)}$$

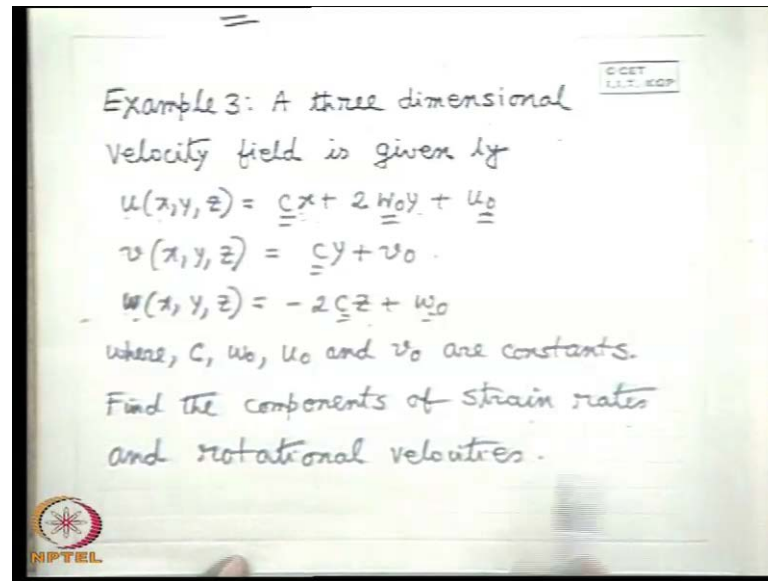
$$\int \sinh(x) dx + \int \cosh(y) dy = 0$$

$$\cosh(x) + \sinh(y) = \underline{\underline{\text{constant} - C}}$$

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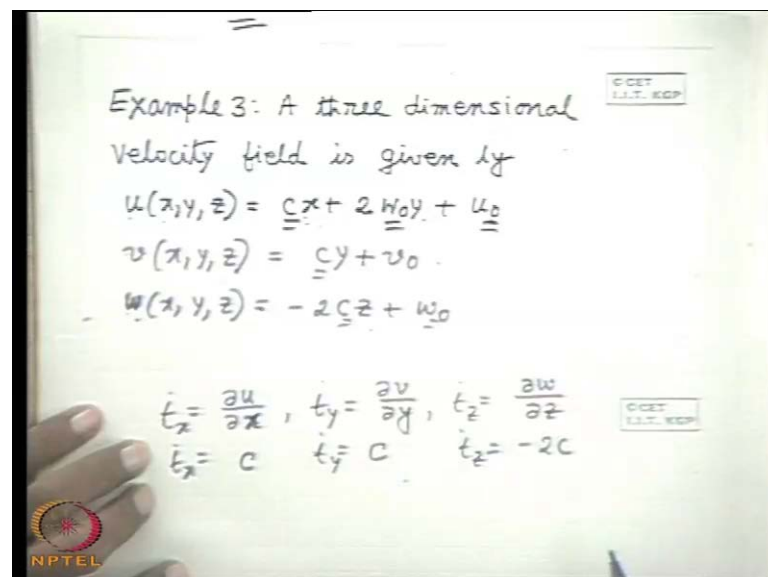
So, we integrate this, this is the differential equation if you integrate this you get the form like this. What is the integration of sin hyperbolic x dx ? Cos hyperbolic x ; cos hyperbolic x , and this will be sin with the same sin for the hyperbolic function and that will be a constant integration of 0. So, this is precisely the equation in the x y plane for streamline; this is precisely the equation in x y plane for this streamline. So, this constant the values of these constants can be found out. If we define these streamline that this constant value for a giving value of x and y , this you will come afterward I newly introduce the concept of steam function. So, constantly represent the parameter; that means a series of cos representing this streamlines with differ in values of this constants.

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See for example, as the parameter is a very simple straight forward application. Another straight forward application for well in kinematics of fluid regarding the fluid motions you see. A three dimensional velocity field as you know the I told you yesterday the three dimensional flow means were the 3 components of velocity exist and all the 3 components are functions of x y z all the 3 space co ordinates in general.

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So, this is known as a three dimensional flow field which is given by the u component of velocity of could s get is not a function of j x y and u_0 . This u_0 is a

constant this is define in the problem, y component of velocity is given by this and z component of velocity is given by this. In this equations $C \neq 0$ and u_0 these are the constants $C \neq 0$ u_0 ; these are the constants. So, variables are x y z as the independent variable and dependent variable at the velocity components u v w which are function of x y z. These are study velocity field as you see because there is no dependence with time.

So, what we have to find out? Find the components of strain rates and rotational velocities. It is again a straight forward applications, of the fluid kinematics that how do you define the strain rates? Let x the rate x co ordinates strain rate is a sin on x which is given by $\frac{\partial u}{\partial x}$ epsilon on dot y; that means, the rate of strain in y direction. That means, these are the gradient of the velocity components in that direction with respect to the space co ordinates in that direction. So, it very simple to remember this strain rate in z direction is the differential of z direct components velocity with j. So, it is simple now class school level thing that $\frac{\partial u}{\partial x}$ is C. So, it is C well, what is $\frac{\partial v}{\partial y}$? Again C. So, is a constant strain rate in y direction $\frac{\partial w}{\partial z}$ is minus 2 C. So, these are the values of epsilon on dot x straight forward application epsilon on dot z.

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Example 3: A three dimensional Velocity field is given by

$$u(x, y, z) = Cx + 2W_0y + u_0$$

$$v(x, y, z) = Cy + v_0$$

$$w(x, y, z) = -2Cz + w_0$$

$$\underline{\underline{\dot{\gamma}_{xy}}} = \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) = (0 + 2W_0)$$

$$\dot{\gamma}_{xz} = \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) = 0 + 0 = 0$$

$$\dot{\gamma}_{yz} = \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right) = 0 + 0 = 0$$

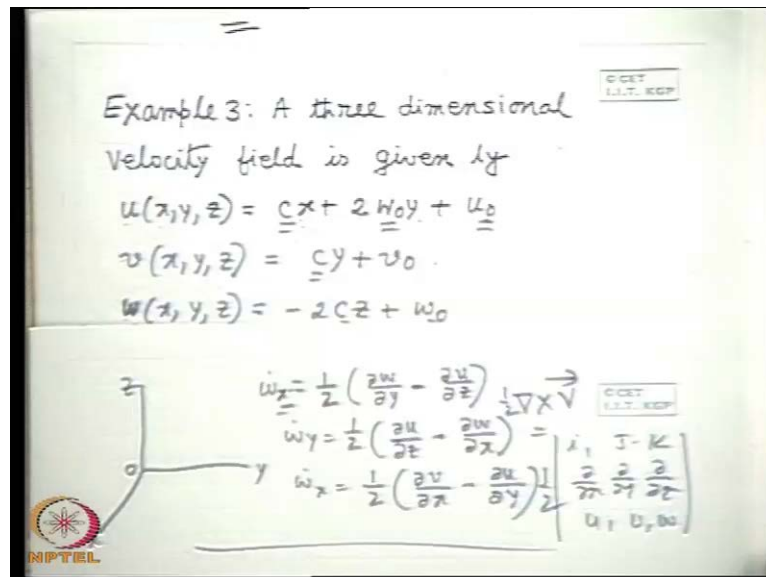
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Next is the, next is the share rate or angular deformation, rate of angular deformation, angular strain rates whatever you called. We know gamma dot x y in the x y plain it is $\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$ similarly, gamma dot x z x z plain what will be the value $\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$. And well gamma dot y z is equal to what gamma dot y z is $\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}$

del y plus del v del z. Now, what is del v? Del is now straight forward of substitution it is 0 v is not a function of this. But del u del y is not 0 it is twice w 0; that means, it has got a fixed angular strain rates in x y plane which is twice w 0.

Similarly, for x z plane del w del x del w del x is 0 since w is not a function of x del u del z it is also 0; that means, it does not deal any angular strain, rate of deformation rate or shear rate these are the terminologies used in the x z plane. Similarly, if we inspect these strain angular strain rates in y z plane del w del y del w del y is 0 del v del z is 0; that means, it has got only strain rates in x y plane. Now, again if we think of rotation it is very simple rotation the same application.

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But here is sign convention comes that means, if you take this as x y z probably you recall I discussed in the class that the omega x omega dot x; that means the rotation about the x axis. That means, in the y z plane will be half the positive sign is this; that means, del w del y minus del v del z minus del v del z. Then omega dot y will be similarly, half del u del z minus del w del x. Well similarly, omega dot x will be half this was derive in the last class del v del x minus del v del y. That means, about any axis it will contain the velocities in the components not in that then x above rotation about x axis will contain the z and y components velocities. And they are cross differential z component with y component with z with a minus sign.

So, it can come from the determinant concept as you know $i j k$ that is $\nabla \times$ again I am repeating this things $\nabla \cdot \mathbf{u}$. That means, this is the curl of the velocity vector this is the definition of half of course,, they are half of course,. So, from an analytically you want can find out now the list. But is to substitute the value accordingly and you can find out the values that this left to you a simply substitution of the value as I did for this strain rate. So, this is the formulae and you can find out. So, this is all for the examples which highlight to show you in this class. Now, I like to give you close up, close up of the chapter 3 that what we actually observed that read in chapter 3.

First is the, it is the fluid kinematics first there are two approaches to describe the fluid flow. One is the Lagrangian approach which considers each and every fluid element of given identity and to trace they are path in the fluid flow. The identity is fixed by fixing the position vector of a fluid element at a given interval of time. So, therefore, at a give in at particular interval at, they particular use tend of time particular use tend of time whereas, Eulerian approach solves the fluid flow problem by concentrating at the particular point, and describing the flow velocities, at that point as a function of time. So, they are putting an entire flow field the velocity field is described the acceleration field is describes that the function of space co ordinates and time.

So, Eulerian approach describes the velocity, acceleration an all hydrodynamic parameters is a continuous function of $x y z$ that t ; that means, the space co ordinates and time. And this is the most convenient defining of fluid flow then we recognize what is the study flow and what is the non study flow. When all hydrodynamic parameter become independent of time in a flow fluid the flow is said to be study, and if he does not do. So, the flow is a non study flow similarly, a flow is said to be uniform move in the velocity and accelerations in a flow fluid are independent of the space co ordinates they are same at each and every point in the flow fluid. So, this is known as the uniform flow the flowing general may be on study and non uniform.

But any combinations of these four can occur next we appreciated very important thing that is the acceleration. The basic thing is like that when any parameter changes because of the time and also it due to convection, let me is a parameter is changing because of the convection the parameter is convective. So, the time and the convection both are coupled to make the change of the parameter with time because the parameter moves with the flow fluid. So, therefore, with time it has got a movement. So, they are put the change

composed of change with respect to time at a given point. If you observe the movement plus the change along with the convection a simple example is that if a fluid particle. If you trace its change in velocity with time then as we fluid particle moves with time its change of velocity will depend.

Because, of this movement from 1 position to other position along with the change in the velocity field even at the particular position with time. So, in general therefore, this is the rule that total derivative with respect to time contains a temporal derivative which is the change with respect to time at a fix point plus the convective derivative. So, this gives the concept of temporal and convective acceleration. So, total acceleration or substantial acceleration consists of two parts; one is the temporal acceleration which is the change of velocity at a point with respect to time.

Because of an steady in, in the flow and another is the convection that is the change of velocity even for a steady flow for a fluid particle moving from 1 point to other. Because of the non uniform nature of the flow. So, they are both temporal acceleration plus convective acceleration is total or substantial acceleration. Then we recognize stream lines path lines and straight lines stream lines and imaginary lines drawn in a fluid flow. So that the tangent at any point on this line represent the velocity direction of the velocity vector at that point. Now, the series of stream line changes from time to time a steady study the path line are the locus of the, different fluid particles with different identity and straight lines.

We are defined as the locus of the feet of the end points of several points several fluid particle that across a particular points fix points. So, straight line specified by that fix point through which a number of particles have crossed and it give in steady what are the end points of those particles along they are locus this is a straight line. In steady flow stream line, path line, straight lines are coincident. Since the Lagrangian version Eulerian version become identical then the most important part and that is the last part of that section which we recognize that we fluid particle in general had 3 distinct features.

One is translation simple translation it translated without changing its shape without changing any dimensions linear dimensions of the body or angular dimensions plus rotation and deformation. Deformation is the most distinct which able feature almost important feature that distinct which is a fluid element from solid element that fluid in

continues motion continuously defunds is getting defund is shape the shape get defund continuously. And the dimensions linear dimensions and angular dimension goes on changing continuously.

So, fluid elements have got 3 distinct, part 1; translation rotation and deformation where are the solid body has translation and rotation without deformation. That means, for solid body if it is translate if it is translated without any change in the linear dimension and the angular dimension of the body dimension remain as it is. Similarly, when fluid body get a rotation is dimension remains same; that means, all the particles in the solid, solid body all the particles are the solid body moves really same angular velocity. The solid body does not have the deformation which is the distinguishable feature for a fluid body in its motion. Now with this event the close the lecture and kinematics of fluid today we will be discussing the conservation equations in fluid flow. Now, you know that heavy physical system on that any process transferring energy exchanging energy or any process which is performed by any physical system mass obey 3 laws of conservation. One is the conservation of mass; that means, mass is neither, created not destroyed mass of a system remains unchanged.

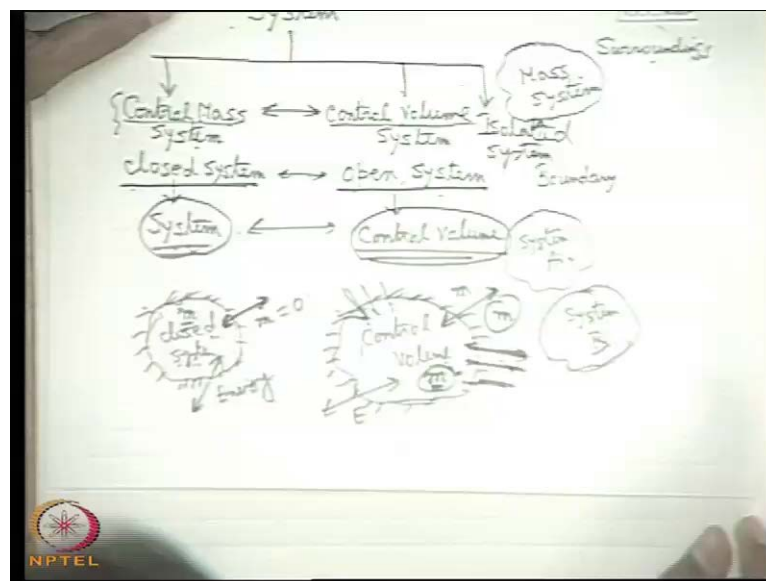
Another is the conservation of moment that you have already come to know from your school level that conservation of momentum is giving by the Newton. The Newton second law of momentum should be concern with respect to the force applied on a system according to the laws motion. Another is conservation of energy; energy is neither, created not destroyed. So, there is any conservation of energy transformation of energy total energy remain same of course, one thing as to be told in precaution that they are all physical process were mass energy is been mutually converted with to each other..

So, if you take care of those process then the there are 3; there are this 3 conservation equations are not independent. For example, conservation of mass and conservation of energy then jointly make is conservation statement that conservation on mass and energy; that means, mass plus energy remains constant. However, if we discuss that particular part of the physics with which at present we are not interested in our fluid flow problem where the mass energy mutual exchange is there we can till the 3 independent laws of conservation. Conservation laws; that conservation of mass conservation of momentum and conservation of energy have to be follow the obey by flow of fluid also.

As it has to be obeyed by all physical system under any condition or executed or executing any process.

Now, before examining this, all three conservation equations our main objective of this chapter will be to apply this conservation equation to a fluid flow problem. And finally, derive an equation in the thing is hydrodynamic parameters like velocity pressure and all this things. And you till that this is an equation will the thing to velocity, pressure which comes from the conservation principle either conservation of mass. And conservation of momentum and conservation of energy generally give a name to that particular equation, but to derive those equations following the conservation principle, we first need certain terminologies, which will be very helpful in studying solid mechanics. Of course, of course, you are learned in solid mechanics, fluid mechanics, thermodynamics any physical system to learn we must at least have to know some terminology.

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Let us concentrate on those terminologies, now what are those things how do you define is system, how do you define a system. Now, a system is usually defined as some amount of mass, some amount of mass at some amount of mass of the working fluid or working system some amount of mass within a giving boundary. So, system definition includes 2 things some amount of mass and some boundary, boundary is very important for defining a system. So, a system distinguishes a giving mass, with a giving boundary. So, boundary is a very important for this system. So, system has 2 important characteristic;

one is the mass and another is within the boundary and everything external to the boundary is known as surroundings, surroundings.

So, this is the definition between this is a system and this is a surrounding. So, surrounding everything external to this system we are may be another system. For example, we can think of system a with a boundary and system b interacting between each other. In that case if we tell system a, and system, system b will be, be surrounding to the system a similarly, system a will be surrounding to system b. So, surrounding means it is external to a particular system on which we are paying our attention. But now in this characteristic feature we can still there are two types of system which is very important; one is control mass system control mass system you will get in my book..

But I tell you that really in any book it is define like this control mass system or sometimes we calling that close system or in a more liberal way convention we call these as the system. Then you can ask me sir other type of system you are told to that is not called system usually not. So, this I am coming cosmologically it is basically control mass system, another is control volume system. One is control mass system another is control volume system. So, control volume system we sometimes define as open system or we sometimes define as control volume we do not refer to system. So, when system and control volume the 2 what that define system refers to control mass system and sometime close system. And control volume system sometime depends as open control mass system goes with control volume system these are this comparison is very clear. Close system at it is open system and one is system this we call system this we call simply control volume what is the difference?

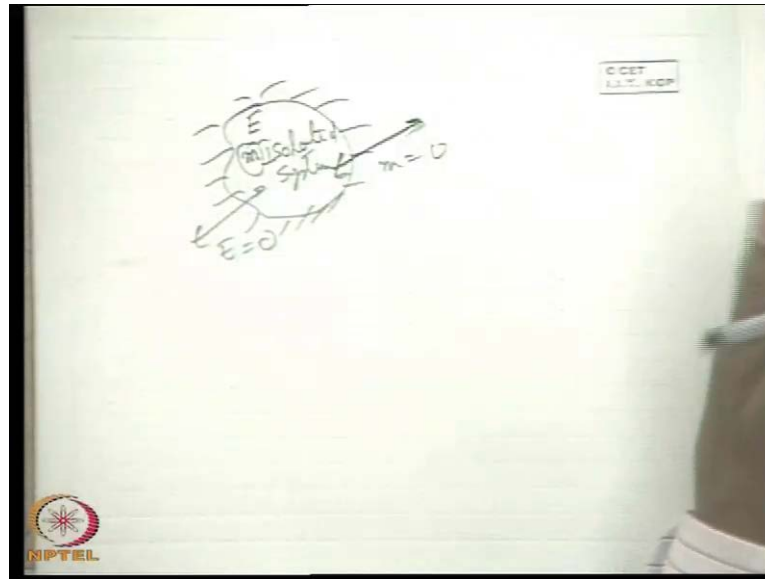
In a control mass system or close system, close system this is the boundary of the system the restriction is that they are is no mass flow m is 0. There is no mass in flow or mass out flow to the system only energy in flow and energy out flow is their energy may either come in or energy may either go out. So, system boundary does not allow any mass to come in or go out which means that in a close system it is not only the amount of mass m , but its identity remain same. That means, the same mass with the same identity remain same remain within the close system close system does not allow the mass transfer. So, mass does not came in go out to boundary of a close system may expand may collapse because of the mechanical work done there is no restriction for the volume of the close system. But boundary may expand or collapse without allowing any mass to

come in boundary may be fixed it may not expand then collapse it may receive only heat or it may reject heat.

So, any form of energy transfer across the boundary is possible were as in a control volume system. So, they are what you see this is known as close system it is closed it is not allowing any mass and it is refer to as system sometime only system. Whereas in a control volume systems, simply we tell is that sometimes control volume the mass transfer is relax. That means, they are may be mass coming in and they are may be mass going out. Some portion the mass may come some portion mass may coming in sorry mass may go out along with the energy. So, both mass and energy in flow and out flow takes place, but here the restriction is that this boundary is reached. That means, this boundary does not move; that means, in a control volume the volume is fixed. Whereas in a control mass or close system the mass is fixed that is why this is known as control mass system. And that is why it is known as control volume system, but in a control volume mass may come in mass may go out.

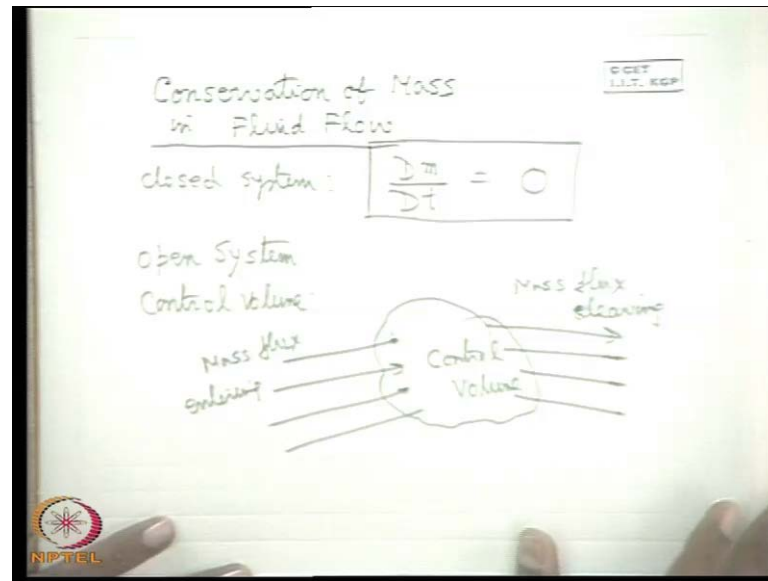
So, therefore, as per as the mass of the system the control volume characteristic this that the identity of the mass changes; that means, at some condition the mass may remain same in the control volume that amount of mass coming in exactly equals to that out. But still if that case control volume system or open system or control volume differs from the close system is that it is not only the mass, but the total identity of the mass remains you understand. So, here the identity is loss because the mass is the going out also coming in mass may or may not remain same depending upon the balance between in flow and out flow, but identity is always loss. So, this is the close system and the control volume or open system another type of system also we define, which is isolated system please write isolated system. So, this system is very simple. So, this is control mass system or close system or system. So, sometimes we will refer only by system and this is control volume system or simply you will refer by control volume system or open system.

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Isolated system by definition is isolated in all respect; that means, isolated system is a Pasic material; that means they are isolated. That means, there is neither mass interaction nor energy interaction; that means, then isolated system does not interact to in this surrounding. While a cos system interacts with the surrounding in terms of the energy transfer. A control volume system interacts with the surrounding in terms of both mass and energy transfer this is the surrounding external to the system. While and isolated system is isolated from the surrounding in all respect. That means neither mass transfer nor energy; that means, total energy and mass, mass even with the identity remain same in an isolated system does not change with time. So, these are the 3 terminologies that close system open system or control volume isolated system, which will be required after words in describing other many hydrodynamic parameters and analysis of many engineering problems.

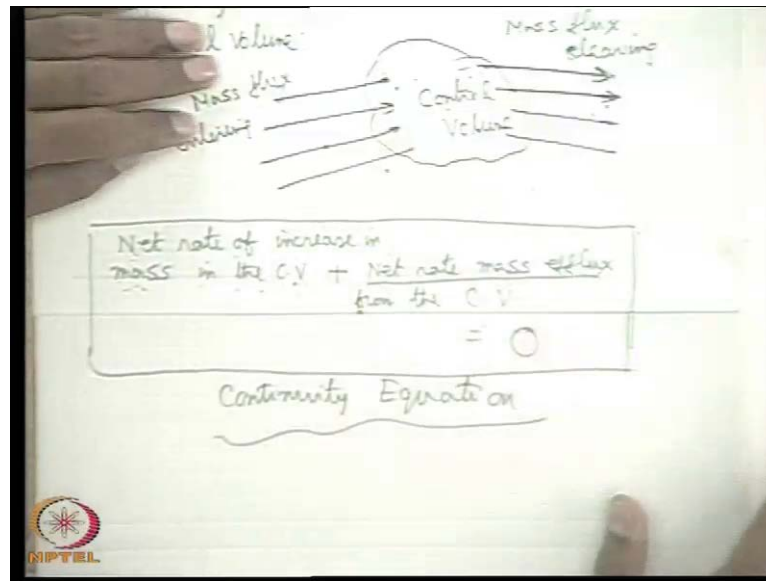
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Now let us see what is continuity equation? Continuity equation is now first of all we start conservation of mass, conservation of mass in fluid flow, conservation of mass in fluid flow fluid flow. If we apply the conservation of mass in fluid flow as we know the close system. If we apply it to a close system, if we apply it to a close system you can tell me sir what is great in it? If the mass of a close system is 0, the conservation of mass is giving by these expression $D n D t$ is 0. But this is not true for an open system or control volume. So, what is then with respect to an open system, what is this with respect to an open system or control volume please tell me. What is this with respect to an open system or control volume the mass of a close system is 0 if we have an open system or control volume, control volume. So, we can simply tell the conservation of mass like that if the mass enters to control volume is the mass, flux entering and the mass, flux leaving.

What is this with respect to an open system or control volume leaving? What should be the generous statement for conservation of mass for a control volume where the mass, mass flux coming in continuously and mass, flux is leaving continuously? Can you always tell that the mass flux leaving is equal to mass flux entering we cannot always tell. These we can tell are the particular condition when the control volume mass will not change. So, this is the particular condition control volume may as observe mass that. That means, may remain with the in the control volume or control volume may lose mass some mass may go from the control volume.

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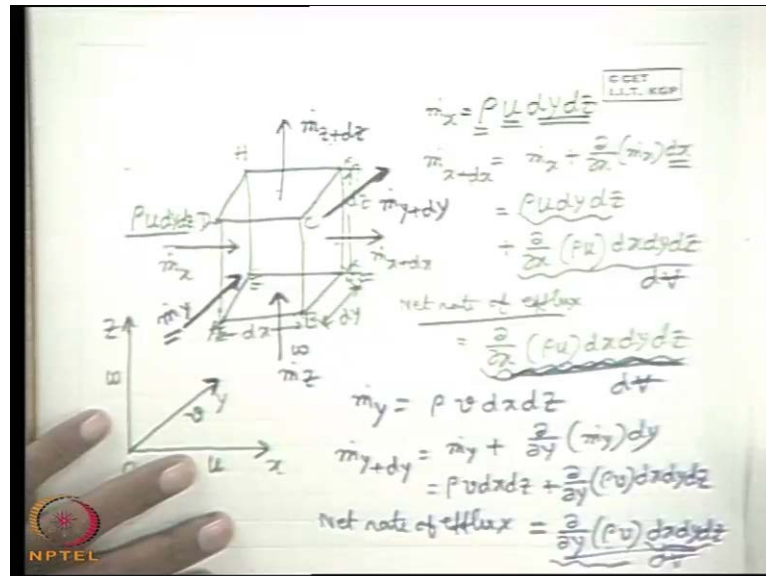


So, the most generalized statement relating to that is the mass rate of increase. Let us consider increase in mass in the control volume plus net rate of mass a flux mass a flux from the control volume is equal to 0 it comes from basic intuition. So, that net rate of mass a flux plus net rate of increasing mass in the control volume is 0; that means, net rate of increasing mass in the control volume is the negative of net rate of mass a flux. That means, the net rate of mass in flux; that means, mass in flux minus net rate of mass a flux means mass a flux minus mass in flux. So, one can see from very common sense that rate of mass in flux minus rate of mass out flux is the net rate of increasing mass in the control volume.

So, therefore, this is written like that when the net rate of increase in mass in the control volume is 0 that control volume does not increase its mass or decrease its mass by this mass transfer process. Then net rate of mass a flux from the control volume is 0 or net rate of mass in flux which you tell with a positive or negative sign is 0. That means the rate of mass in flux than rate of mass a flux is balance. So, this statement of conservation of mass apply to a control volume or open system which is the very basic thing which comes straight from the common sense in initialize in deriving an equation. That means, these constrain of conservation of mass is utilize with respect to a control volume is deriving an equation relating the velocity field in the flow and known as continuity equation continuity equation. So, continuity equation is an equation which is derives by

using the conservation of mass with respect to control volume. Let us derive the continuity equation all of you have understood?

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Now let us consider a x y z coordinate axis x I have given x in this direction let us consider this as y and this as j. Now, with respect to any frame of reference if you like to define the continuity equation that conservation of mass apply to a control volume. First type is that you define a control volume whose planes are parallel to the co ordinate planes. That means, here I define the control volume like this I define a control volume like this I am not drawing in the spirit of the drawing actually this, this will be doted. So, this is a parallely piped. So, if the first step is to derive or example deriving the continuity equation and again I am telling with respect to any frame of reference. First step is to draw the control volume whose planes are perpendicular to the co ordinate planes. So, therefore, in a Cartesian co ordinates system it will be a parallelepiped. Let us give this name A let is B C D this is E F G H now we define a control volume with imaginary boundaries.

Because the boundary of a system or control volume is not necessary into be a real and r easy it is an imaginary boundary which one can choose depending upon the need of the problem. And let the dimensions of this v d x in the direct this length along with the co ordinate direction con convention d y and let this is d z; that means, this one is d z well. Now we should find out how the mass flux is coming in, let us consider the u component

of velocity that is a positive x direction. V component of velocity in the positive y direction and z component of velocity in the positive z direction existing the field.

So, therefore, with this positive x direction velocity there is a mass flux coming across this plane A E A H D this plane we call as x plane. Why because the normal to this plane is x direction; that means, x plane there are 2 x planes a e a h d and b what is this A B C D E F? This is B F G C. So, through this x plane A E A H D let us consider a mass flux $n \cdot x$ is coming. So, due to this typical velocity field the mass flux $n \cdot x$ plus $d x$ we consider these mass flux changes over a distance $d x$ as $n \cdot x$ flux $d x$ which is nothing, but the mass a flux across this x plane.

That means b f g C which is $d x$ distance apart from this x plane in the positive direction. So, what is $n \cdot x$ dot that is mass flux coming in to the control volume through the phase A E A H D, you know the volume flux in a fluid flow is giving by the velocity times the area. So, if the velocity is u what is the area, area is $d y d z$. So, $d y d z$ and volume flow times the density is the mass flux. So, mass flux is simply $\rho u d y d z$. That means, u is the velocity, velocity times the area is the rate of volume flow times the density is the rate of mass flow that is coming in...

So, the mass flow going out from the x plane; that means, from the planes parallel to this x plane that is B F G C it is parallel to A E A H D which is going out that will be $m \cdot x$ plus $d x$. That means, it will be $m \cdot x$ plus $\text{del del } x$ of $m \cdot x d x$ and since $d x$ is the small we can neglect the higher order term in the delaxary expansion. So, simply then we can write this is equal to $\rho u d y d z$ plus $\text{del del } x$ y $\rho u d y d z d x$; that means, $\text{del } d x$ of $\rho u d x d y d z$. So, this is the term extra over this. Now, therefore, the net rate of a flux net rate of a flux because of this flow through x planes there are 2 x planes x planes means the plane parallel to y z plane.

That means, is normally is x is equal to $\text{del del } x$ net rate of a flux means a flux minus in flux $\text{del del } x$ of $\rho u dx dy dz$. So, this is the net rate of a flux into the control volume due to the flux is crossing the x plane. Similarly, if we consider the flux is crossing the y plane let this is the flux is crossing the y plane. So, the flux which is coming into the control volume thorough be y plane; that means A B C d. So, A B C d is the y plane through which the flux is coming in because of the velocity components v existing in this

fashion this is the positive direction of y. So, let us consider this $m \cdot y$ similarly, which is going out through another y plane.

So, this parallel to this y plane A B C D that is what A F G H; that means this is these y plane perpendicular to the y axis that we can designate $m \cdot y + d y$; that means, it is the change of the mass flux over a distance $d y$. Then we can write with the similar concept that $m \cdot y$ dot is ρ times the volume flux that is the flow velocity in these direction v times this area $dx dz$. And with this similar rotation $m \cdot y + d y$ is $m \cdot y + \frac{\partial}{\partial y} (m \cdot y) d y$ what we can write? We can write this is equal to $\rho v dx dz + \frac{\partial}{\partial y} (\rho v dx dz)$.

So, therefore, we can write the net rate flux, net rate flux, net rate of a flux from the control volume net rate of s flux from the control volume due to the fluxes parallel to the y planes is equal to $\frac{\partial}{\partial y} (\rho v dx dy dz)$. In the similar fashion we can do for the z plane that means the planes through which flux is crossing through z planes; that means, there are 2 z planes.

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$m_z = \rho w dx dy$
 $m_{z+dz} = \frac{\partial}{\partial z} (m_z) dz + m_z$
 $= \rho w dx dy + \frac{\partial}{\partial z} (\rho w) dx dy dz$
 Net rate of ^{mass} flux in the Control Volume
 $= \left\{ \frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) + \frac{\partial}{\partial z} (\rho w) \right\} dx dy dz$
 Net rate of increase in mass within the Control Volume

Now, 4 planes we have consider rest to planes are they are one is A B A v and D C G i. So, flux is coming in through this bottom plane because of the existing velocity component. So, this is $m \cdot j$ similarly, flux is going out through this z plane D C G H which is at a distance $d z$ above this plane. And we just give it n value $n \cdot z + d z$ and in the similar way we can write $n \cdot z$ is $\rho w dx dy$ the area of this plane.

And similarly, we can write $\rho \dot{z} + d z$ as $\frac{\partial}{\partial z} (\rho \dot{z} + d z) + \rho \dot{z} + d z$. So, $\rho \dot{z}$ will come fast. So, this can be written as $\rho w dx dy dz + \frac{\partial}{\partial z} (\rho w dx dy dz)$. So, $dx dy dz$ I can substitute as $d v$ that is the elemental volume of this control volume $d v$. So, that earlier also I can write it $d v$ and I can write it $d v$. Now what I can write net rate of a flux in the control volume, in the control volume, in the control volume due to the flux is across all the phases. Due to the flux is across all the phases will be some of these net rate of a flux due to fluxes across x plane plus net rate of a flux due to fluxes across y plane plus the net rate of a flux due to flux is across z plane. That means some of this, this I am writing as $d v$ some plus this some of this, this and that means, this is equal to $\frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) + \frac{\partial}{\partial z} (\rho w)$ times $d v$ which is a constant.

Now, what is net rate of increase, because if we again recall our continuity equation which came simple from physical common sense. Net rate of increase in mass in the control volume net rate of mass a flux from so net rate of mass a flux from the control volume we are found out. But what is net rate of increase of mass in the control volume net rate of a mass a flux I am net rate of every time I have writing mass a flux what a flux mass a flux net rate of am mass a flux.

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$$\frac{\partial}{\partial t} (\rho dV) = \frac{\partial \rho}{\partial t} dV$$

$$\left\{ \frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} \right\} dV = 0$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} = 0$$

Continuity equation

So, now, net rate of increasing mass, net rate of increasing mass, net rate of next thing we have to find out of increasing mass net rate of increasing mass within the control

volume. What will be its expression within the control volume? What will be its expression within the control volume? What will be its expression, which expression will be the rate of change of mass within the control volume? That means, if I define the mass at the control volume at any instant what will be that ρdV ? And if it changes with time control volume is fixed.

So, if it changes with time; that means, this is the mass of the control volume this will represent this thing that net rate of increasing mass within the control volume that instant in as mass of the control volume its ρ and its volume dV . So, volume of the control volume as I have told by its definition is in very end with time it will come out. So, this is very important to know that how we are ρ is coming out dV , because of the definition of the control volume. So, according to this statement physical statement now I can write $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u})$. So, according to the conservation of mass apply to a control volume in a Cartesian coordinate system we can write this into dV is equal to 0. True and this is valid for any value of dV it is irrespective of the volume of the control volume which is an arbitrary parameter which means this quantity has to be 0. Because dV is not 0 it is valid for any value of dV any finite volume of the control volume this quantity has to be valid..

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The image shows a whiteboard with handwritten mathematical equations. At the top, a boxed equation is written: $\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$. Below this, the text "Continuity equation for an incompressible flow" is written. At the bottom, another boxed equation is written: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$. A hand is visible on the left side of the whiteboard, pointing towards the equations. In the bottom left corner, there is a logo for NPTEL (National Institute of Technology Education and Learning).

So, finally, therefore, we can write this has been continuity equation $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u})$ so in a Cartesian coordinate system. So, this is the continuity equation;

that means, in a Cartesian coordinate system the consequence of conservation of mass at application of conservation of mass in a control volume gives an equation which is known as continuity equation. This continuity equation form this is the continuity equation in Cartesian coordinate system $\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)$ is equal to 0.

So, when a special case now we consider that when the fluid is incompressible. For incompressible fluid or incompressible flow we are not interested fluid is incompressible or compressible we have already recognized earlier that whether density changes in the flow or not we are interested. For the flow at density does not change it is an incompressible flow where the Mach number is below 0.33. Then density is not worried no matter in the flow fluid density changes density is not a function of x, y, z and similarly, these become 0 and this comes out. So, therefore, the equation becomes $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$. So, it is very important that for incompressible flow at density is neither a function of time or nor a function of x, y, z . In a Cartesian coordinate system with u, v, w are respected velocity components all x, y, z direction this is a special case of continuity equation for an incompressible fluid. So, today after this well, next class, we will discuss again you cannot animation to this.

Thank you.