Fluid Mechanics
Prof. S. K. Som
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 10
Kinematics of Fluid Part -I

Good morning, I welcome you all to the session of Fluid Mechanics, today we will be starting a new chapter Kinematics of Fluid. But, before that, I like to deliver a closer talk of the earlier chapter, which we discussed in 6 lectures that is, fluid statics that, I will just highlight the main points, so that we can have a quick go through, through all the main points in the chapter fluid statics.

So, these are closer closing the chapter, first we recognise that, in that chapter the in a fluid at rest, there are two types of forces, that act in a fluid body at rest. If we take a fluid body in isolation from it is surrounding fluid, from a vast expanse of fluid, we recognise two types of forces acting. One is the body force, which is due to the external agency, just like the gravitational force, there may be different body forces.

Another is surface force, which act as the surface of the fluid body in isolation, because of the reaction or mutual interactions between these fluid body, with it is neighbouring fluid particles, from which it has been kept separate. Then, we recognise that, fluid at rest can neither developed tangential stress nor tensile stress, only the stresses are compressive in nature. That means, at a point, the stresses are directed from all direction towards the point and they are of equally magnitude that is known as Pascal's law.

Then, what we found that, in a fluid expanse at rest, expanse of fluid at rest, if we consider only gravity force is the external body force field, we see the pressure in any horizontal plane remains same. That pressure does not vary with any coordinate axis in the horizontal plane, it only varies in the vertical direction. And we have recognised the basic equation of fluid statics in that respect is d p d z, the differential of p with respect to z is equal to minus rho g where, rho is the density of the fluid.

The minus term comes because, if you take the z axis positive vertically upwards alright. Then, we can solve the equation explicitly for p, as a function of z provided, we know the variation of rho with z or p; for an incompressible fluid, it gives a simple solution p is equal to rho g z. Where, z is the vertical high plus some constant, which is found out from an arbitrary datum value.

And from this, we will lead to a conclusion that, the pressure at any point in an static expanse of fluid, is over the atmospheric pressure, where there is a free surface. And pressure is the atmospheric pressure by an amount, which is equal to rho g h where, h is the depth of the point from the free surface. Then, we recognise pressures on submerged surfaces; now, if there is a plane surface submerged in an expanse of fluid.

So, the net hydro static pressure force is equal to, this is the formula we derived, equal to the pressure intensity at the centroid of area or the centre of area, of this surface times the total area of the surface. In general, area may be inclined, area may be vertical, area may be horizontal, the formula remains the same. And we also appreciated from common sense that obviously, since the pressure increases with the depth from the free surface, centre of pressure is below the centre of area.

That means, the depth of the centre of pressure from the free surface, is more than the centre of area and the distance between the centre of area and the centre of pressure, along the axis of symmetry, that pass to the centre of area is given by a term. That is moment of inertia of this plane area through a centroidial axis, which is parallel to the line of intersection between the plane surface extended to the free surface.

The moment of inertia, about a axis parallel to this axis through centroid, divided by the area itself, times the coordinate or the distance of centre of area from that reference line that is, the line of intersection between the plane surface and free surface. For a curved surface, we recognize that, each and every force on elemental surface are varying, because they are normal to the surface because of, the curvature of the surface they are varying in directions.

So, in that case, we found out the components in some reference directions and we had the conclusion that, the component of hydrostatic pressure force or a on any curved surface. In any horizontal direction equals to the hydrostatic force on a plane surface, which is the projection of the curved surface on a plane, perpendicular to that direction, in which the component of the hydrostatic pressure force is short.

And the vertical force acting on a curved surface equals in magnitude, by the weight of the fluid vertically above the curved surface up to the free surface. That means, the weight of the bulk of the fluid, which is contained between in the region, if we vertically extend the surface up to the free surface by vertical projection. Then, the bulk of the fluid contained within that volume, is the vertical force acting on the curved surface, vertical component of hydrostatic force acting on the curved surface.

So, this way we recognize, the magnitudes of the three components or different component forces, with respect to a frame of coordinate differences and finally, we find out the resultant force. Then, we come to the concept of buoyancy, as you know, when a body is partially or totally immersed in a fluid, the net effect of the pressure forces around it surface gives no resultant force in a horizontal direction, they balance each other.

But, these gives rise to a net force in the vertical direction, which is acting vertically upwards and that force is known as buoyant force and this phenomenon is known as buoyancy. Now, for a floating or a submerge body to be in equilibrium, the primary condition is that, weight of the body, which is acting downwards must be equal to the buoyant force and these two forces must be collinear, then only it will be in equilibrium.

Then, we recognize three types of equilibrium, we have read in the preliminary mechanics, one is stable equilibrium, neutral equilibrium and unstable equilibrium. For stable equilibrium, for submerge bodies, we have found that the gravity, centre of gravity should be below the centre of buoyancy whenever, the centre of gravity is below the centre of buoyancy, always it is in stable equilibrium. But, if centre of gravity is above the centre of buoyancy, it will not be in stable equilibrium, it will be unstable equilibrium and if it coincides then, the body will be neutral equilibrium.

Well, but for floating bodies the beauty is that, when the body is giving angular tilt, to check it is stability so that, it can come back or not to it is original position, the centre of buoyancy also changes. Because, the some part of the body gets up from the free surface, some part gets down, is immersed. So, therefore, even in this case, the g is above b, there is a chance for the body to be stable equilibrium, which is decided by another point known as metacentre.

So, if the metacentre is above the centre of gravity, the definition of metacentre is, the point of intersection between the vertical lines drawn through the new centre of buoyancy to the old line old vertical line, containing the old centre of buoyancy and the centre of gravity. So, this is the definition of metacentre and when the metacentre is above the centre of gravity, the floating body is stable equilibrium. If metacentre is below the centre of gravity, the floating body is unstable equilibrium and metacentre coincides the centre of gravity, the floating body is in neutral equilibrium alright.

Then, we appreciated that, for small angular tilt, this metacentre is a geometrical parameter of the floating bodies. It is a function of the geometrical shape and it is dimensions and we found out, the distance between the centre of buoyancy to the metacentre, along the line of symmetry or the vertical line, containing the centre of gravity and old centre of buoyancy.

Under equilibrium conditions, centre of buoyancy, under original equilibrium condition can be found out, if we designate this as b and m, and the distance as b m is equal to, I by V. Where what are these nomenclatures, I is the second moment of area of the plane, of floatation of the floating body. Second moment of area of the plane, of floatation that means, the plane of floatation, if I see the section of the floating body and the plane of floatation.

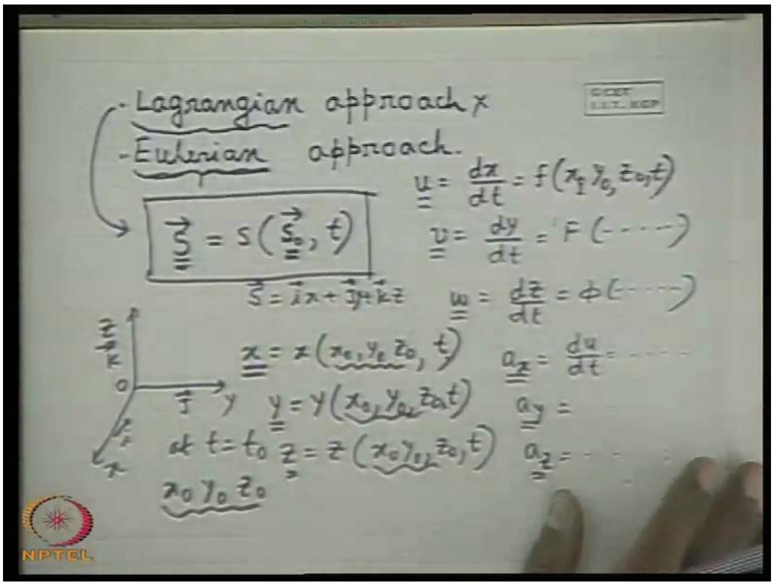
This second moment of area of this sectional area, at the plane of floatation, about the axis of rotation that means, about an axis perpendicular to the plane of floatation floatation. So, this is the nomenclature of I and V is the immersed body so, this we recognize in the last class and this is all so, now, I will start the fluid kinematics. Now, what is kinematics of fluid or fluid kinematic now, kinematics of fluid describes, the fluid motions basically, the geometry of motions and the different aspects of fluid motions.

What are the different aspects of fluid motions but, without finding out the cause, the motion will take place only, if there is a force but, kinematics does not describe the force part, only the geometry of motion. What are the different aspects of motion and consequences of motion, for example, a solid body just a very simple thing, we know. There is solid body, if it moves, all particle moves with the same velocity or same acceleration, if there is a linear motion of its solid body.

So, particle to particle, the velocity or acceleration does not change similarly, if a solid body is given a rotation so, all particle in the solid body have the same angular rotation or angular acceleration. But, in case of a fluid, the different particles move with different velocity, as we have already appreciated at the beginning of this course, when we discuss the viscosity. So, these are the aspects, which gives certain different characteristics of fluid motion like translation, deformation, rotation.

This we will be recognizing in this chapter of fluid kinematics, without going into detail, without going into any of the causes, that force or anything else, which will come in the fluid dynamics part.

(Refer Slide Time: 10:36)



So, in kinematics, let us first then, concentrate that there are two approaches, which describe fluid kinematics. There are two approaches one is the Lagrangian approach approach one is the Lagrangian approach try to understand, one is the Lagrangian approach, another is the Eulerian approach. There are two approaches Eulerian approach, Eulerian approach this will be very important, even when you will read thermodynamics, this you should know, one is due to the scientist Lagrange, another is due to this scientist Eulerian, Euler.

So, if we see the, what is Lagrangian approach, now, in Lagrangian approach let us understand first without, before writing the mathematics. Because, we will have to understand physics first, mathematical will automatically follow, mathematics is a tool, it will automatically follow. So, Lagrangian approach what is done, each and every fluid particles are traced that means, in a flow fluid, flow fluid composed of several fluid particles.

So, each and every fluid particle is traced and is kinematic behaviour is found out, which means, that, if we identify a fluid particle. How a fluid particle will be identified, by giving it’s space coordinates at given interval of time for example, at time t is equal to 0 from when, we start our observation. Then, we try to find out it’s location in the flow field so, it’s path, whether it’s path in the flow fluid is traced with time and this way we cover all the fluid particles, that compose the flow fluid.

This is the basic approach of the Lagrangian, that is the Lagrangian method or Lagrangian approach. So, let us now, write this mathematically that, if we have a flow field, the this placement s of any particle, of any fixed identity particle is a function of the identity of the particle, which is s 0. The identity is fixed by it is position vector at time t is equal to t 0 and time t that means, this is in simple one line, the definition of analytical expression of Lagrangian approach.

That means, the position vector of any identified particle, particle is identified by it’s initial displacement or initial position, rather you can tell initial position from a frame of reference. And it is a function of the identity and the time, and this way all the particles have different identities are traced. For example, if we consider, x y z coordinate, let us consider an x y z coordinate where, the position vector can be written as i x, i j k are the unit vectors in i j x, y and z direction.

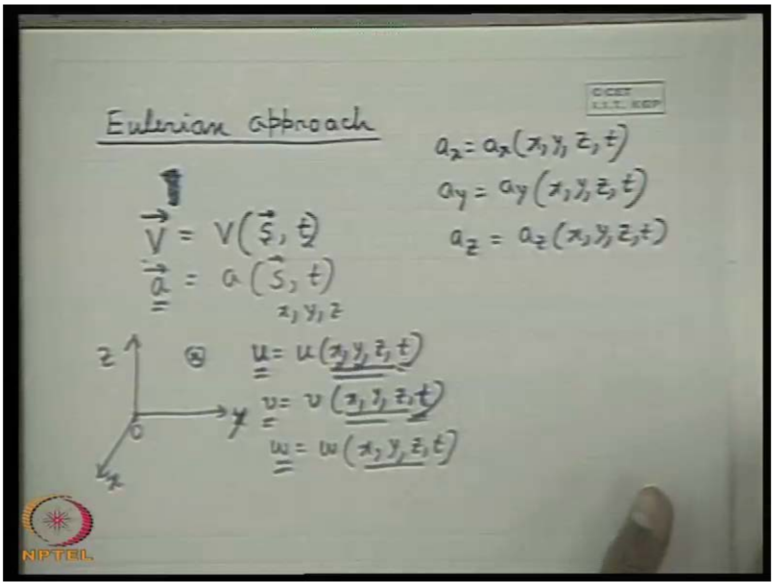
Then, this we can write, that x in scalar components k z, i j y very good plus k z so, x is a function of the identity of the particle x 0, y 0, z 0. That means, we consider at t is equal to t 0, set t 0, x 0, y 0, z 0 is the position of a particular particle that means so, this is the identity of the particle and time t. Similarly, we can write y as a function of x 0, y 0, z 0 and t similarly, we can write z as a function of, in terms of scalar component, is a vector representation x 0, y 0 that means, the displacement of a particle is a function of it’s identity and the time.

Similarly, if we want to find out the velocity of a particular particle of fixed identity, it will be d x d t and this will be some function of x 0, y 0, z 0 these are the variables and time t. Similar is the case for d y d t, this will be some other function of the same thing, same variables similarly, the y component of velocity d z d t is the sub function of let sub function of this. Similarly, if we want to find out a x, a x is d u d t or double derivate so, this will be a function of x, y, z 0, t 0 similarly, a y, a z.

So, therefore, we see all the kinematics parameters that means, this displacement, velocity and acceleration of a particle, is a function of their identity and the time. So, this way, we can find out the description of kinematics behaviour of all particle. So, this approach is very fundamental in nature this approach is very fundamental in nature because, it traces all the fluid elements that composes the flow.

And one very important point of this approach is that, the conservation of mass is inherent in the Lagrangian approach, in this approach. Because, when a fluid flows, the mass has to be conserved so, the kinematics should be such, the kinematics of the flow should be such, it must obey the law of conservation of mass. So, conservation of mass is automatically change because, each and every individual particles are traced.

(Refer Slide Time: 16:52)



So, this is one advantage but, this method is not followed or it is not convenient, it is not in use because of, the most disadvantageous point that, the integration of equations become very tough. Just see this, that, if we are prescribed to with u v w, for a particular particle a x, a y, a z so, to integrate this equations, which I will show you afterwards problem. Sometimes the integration of this, to find out the path of the particles become very difficult so, because of this mathematical complications, this method is usually disregarded. So, therefore, the method which is in common use is, the Eulerian approach and it is the most convenient method.

Now, let us see, what is Eulerian approach then, Eulerian approach or Eulerian method, Eulerian method or Eulerian approach, whatever you call. In Eulerian approach, what we do, we do not trace the particles with identity, in a flow field, we just concentrate at a point that means, a field is described by the space coordinates. That means, we define a frame of preference and the entire flow field is described, with respect to the coordinates preference, with respect to frame of reference.

That means, we concentrate on fixed points, which are specified by it is coordinates and then, we describe velocity or acceleration at that fixed point, at all points. Fixed point means, all these fixed point, as a function of the space coordinate of the points and the time. In general, we can tell in a flow field, Eulerian approach writes the equation of velocity or acceleration, as a function of space coordinates and the time.

That means, velocity will vary from point to point, even at a given interval of time, at same interval of time, you will see the velocity and acceleration varies from point to point. Similarly, the velocity and acceleration may vary this kinematic parameters at a point with time so, therefore, velocity and accelerations are expressed as function of space coordinates and time.

So, if you clock the Lagrangian and Eulerian approach, it will describe the Eulerian approach, a flow field is described by Eulerian approach then, the kinematic Lagrangian approach a fluid particle is traced. So, the velocity and acceleration of a fluid, particular at any instant, will be the velocity and acceleration of the point, at which the fluid particle exist at that instant.

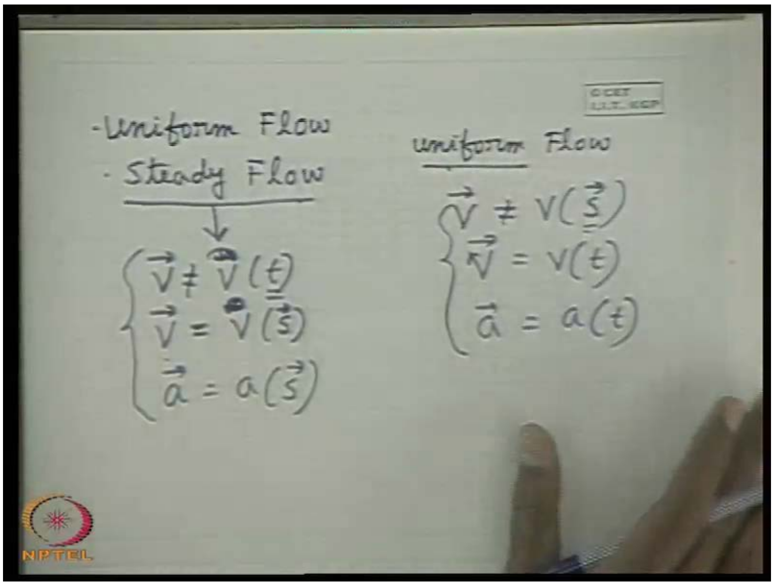
So, a fluid particle passes through a different points at a different instant so, the instantaneous values of the velocities and acceleration at the point where, it crosses is the velocity and acceleration that, the particle assumes, there is the physical in between Eulerian and Lagrangian approach. Let us write the Eulerian approach in mathematics so, Eulerian approach in mathematics writes the position sorry not the position the velocity vector, is a function of velocity vector is a function of space coordinates and time.

Similar is the acceleration vector, is a function of space coordinates and time that means, if you consider a frame of reference simple rectangular Cartesian coordination, sorry this is y, this is x and proper sense of rotation. Then, with x, y, Z, if a flow fluid is described with the coordinate x, y, z at any point then, we can write the x component of velocity is a function of x, y, z.

So, we can break this vector form into scalar components, is a function of x, y, z that means, this x, y, z is not the coordinate of a particle, it is the coordinate of a point so, w is a function of x, y, z and t. Similar is the case for acceleration, a x is a function of a x is a function of x, y, z and t similarly, a y is a function of x, y, z and t. And similarly, a z is a function of that means, here the velocity and accelerations are described as a continuous function of space coordinates and time.

And this is permissible in the continuum mechanics, with the continuum approach, assumption for a continuum as you know, the any property can be described as a continuous function of the spaces variables and time. And this is done for Eulerian approach, to represent the velocity components or the velocity vector in one vector equation and the acceleration vector, as a function of space coordinates and time so, this is the Eulerian approach.

(Refer Slide Time: 21:05)



So, therefore, Lagrangian approach tells that, when a particle comes at a particular point at a particular time so, the velocity vector at that point and that time is the velocity of the particle, which comes at that point.

Next, we will come to the concept of uniform flow and steady flow, uniform flow and what is uniform flow ,what is steady flow. Now, what is steady flow, first we start what is steady flow, this one, A steady flow is defined where, all hydrodynamic parameters all, not the kinematics parameters only, they are variant with time, there is no change with the time. The parameter may have a distribution over the space coordinates that means, there may be a variation of the parameters over the space.

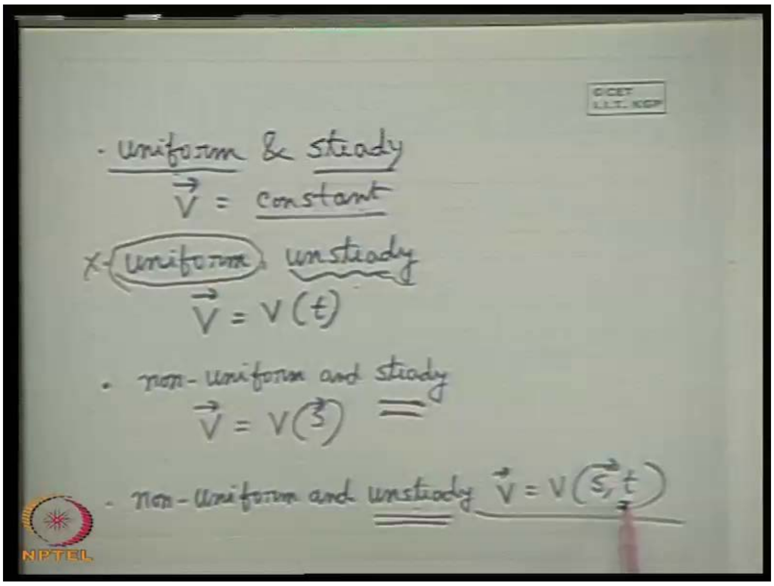
Let us discuss in terms of velocities only because, we are discussing kinematics only, velocities and acceleration that changing with space coordination. So, we have a variation in a three dimensional plane, we can consider the plane of variation in a two dimensional plane, we can consider a variation by a single curved. That means, there is a variation, there is a map of the very map of the velocities and acceleration but, these map or these variations, over this space coordination remains invariant with time.

That means, in a simple we say that, if we fix a particular point, fixed point then, the para-hydrodynamic parameter all hydrodynamic parameters remains same in variant with time that means, this is known as steady flow. So, therefore, in a steady flow, the Eulerian approach say that, V bar is not a function of time that means, the only consequences is that, V bar is a function of s, space coordinate only sorry this is a functional notation I am sorry.

Similarly, the acceleration is a function of space coordinates that means, the hydrodynamic parameters and all other parameters sees to be a function of time, this is known as steady flow. On the other hand, what is a uniform flow, please tell me what is a uniform flow? You know that thing, when the velocity and acceleration is not a function of space coordinates very good that means, V is a function of time.

Similarly, the acceleration is a function of time, is a function of time only, this sees to be a function of space coordinates, uniform flow that means, the velocity and accelerations are same throughout this space. Now, in general, a fluid will be both uniform I sorry non-uniform and unsteady so, unsteady flow is a flow where, the hydrodynamic parameters along with the kinematic parameters are functions of time. Similarly, a non-uniform flow is a flow where, the hydro kinematics parameters is a, or a function of space coordinates.

(Refer Slide Time: 23:55)



So, different possibilities may appear, most simple case is uniform and steady flow, flow is uniform and still so, nothing to be done, no solution is required in this case. So, that means, the velocity vector for example, or acceleration vector does vary, neither with space nor with time that means, in this case velocity vector if we is throughout constant, is a constant.

Now, in case of another now uniform unsteady flow another of another case is, it is these are very simple uniform unsteady flow uniform unsteady flow. So, in terms of V, we can tell uniform flow so, it is not a function of space coordinates because of, the unsteadiness it is a function of time only. Another thing may come that is, uniform unsteady so, non-uniform steady, non-uniform and steady, in this case what will happen, it will be a function of s only.

Because, it is the steady flow, it is the function of s, and most general is non-uniform non-uniform non-uniform and unsteady and unsteady unsteady where, V is a function of both s and t general nomenclature, it is a function of s and t. In engineering applications, flows are never uniform never uniform, uniform it is never, there always because, uniform flow is available only in solid body motion. Usually, in a fluid this is by, virtue of the property of the fluid because of, it’s viscosity flow is always ununiform or non-uniform.

But in some cases, we assume the flow to be non-uniform but, regarding with steady and unsteady situations, many engineering applications part into steady flows but, many engineering situations, part into unsteady flow. It is very clear that, steady flow pose more simplifications, rather unsteady flow pose more complication because, the function time comes into it. So, therefore, if a flow is steady, it becomes more simple, if a flow is uniform, it is the simplest one, which is not the case in practice.

So, therefore, unsteady flow problems becomes complicated in practice whereas, steady flow problems are simple. This is the reason for which, sometimes we choose axis properly to make many unsteady flow problem to become steady. So, a problem whether it is steady or unsteady, problem may be in absolute scale unsteady but, sometime if you choose the frame of the reference, against which the analysis is made, some flow may become steady.

I can give you one very simple example, which I have given in my book, if you have read that, that if you just go with a boat or a ship, or with a launch through a river. You will see that, while you are sitting on the boat, any point on the river, boat is moving and you fix any point in the neighbourhood of the boat or anywhere in the river, you will see the flow condition is steady. That means, the total flow fluid, which is being generated, that we can see.

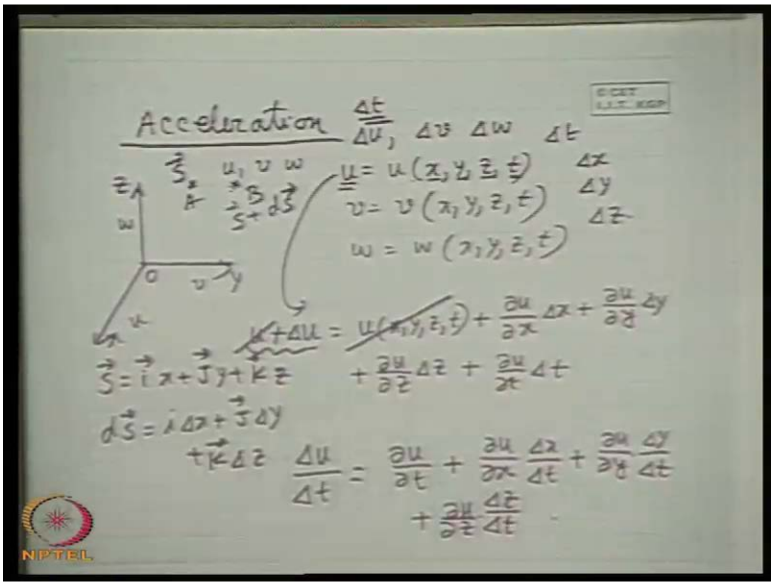
Always you see similar type of flow of water surrounding the boat or if you fix your eyes to a large distance, that far away stagment water for example, there also you can see always. As you move with the boat, the far away point is always at rest so, this far away point with respect to you. Now, what happens, you are moving with the boat that means, you are observing it, the coordinate axis are the fixed to the boat so, with respect to the boat, the flow fluid is steady.

But, if you consider an observer, who is at absolute rest that means, an observer standing on the bridge, or an observer standing on the shore of the river. He concentrates at the particular point, with an Eulerian approach and tries to find out the flow fluid, he will see the flow fluid is changing with time. For example, a person standing on the bridge but, if he just looks at the bottom at the point at the bottom of bridge in the river, he will see that, when the boat is far away, this point is at rest, V is equal to 0.

And the boat is approaching, some flow fluid is been generated and it is getting change, it acquire some flow velocity, which is very high, when the boat is very close and when it goes away, the flow velocity is again dying out. So, this way, if we see at any point, he will find out an unsteady flow fluid, when as a person moving with the boat, if he concentrates at any point in the river, he will find out the flow fluid is same with time that means, a steady flow.

This is a very good common example where, you can appreciate afterwards, when will be studying the control volumes,. The reference to a particular frame of axis, unsteady flow may be steady for simplification in analysis alright.

(Refer Slide Time: 28:51)



Now, let us come to the definition of acceleration now, I will ask you one simple question. Since, we are acquainted with this term acceleration, velocity since our childhood you can say, at from class 8 or class 9 level, whenever we have come across physics, physical science. Now, what is definition of a acceleration, please tell me what is the definition of acceleration? It is the rate of change of velocity with time.

Now can you tell, when the flow is steady acceleration is 0, yes yes let us now see, though it is wrong but, I tell you because, I know that, it is difficult for you to tell yes.

For example, if a fluid flows through a convergent duct nozzle, as you know that, if a fluid flows through a convergent duct at the inlet where, the diameter is more, fluid velocity is less.

This will come afterwards afterwards from the continuity but, as atleast I know, at this stage you can appreciate it. When it comes flows through the converging nozzle and comes out from the small discharge area then, the fluid acquires a very high velocity so, therefore, fluid enters with a very low velocity and acquires a high velocity. Now, under steady conditions, as per a definition of steady flow, I can tell that, the fluid velocity varies from point to point.

That means, if you consider at the time being the flow is not varying in this direction and cross section, only varying in the direction of flow one dimensional then, I can tell that at the inlet the velocity is low, at the outlet velocity is high. But, whatever may be this velocity variation, it remains invariant with the time that means, the velocity of flow at the inlet is constant with time.

Velocity at any sections in the nozzle is invariant with time and the velocity of discharge is also invariant with time that, the liquid is coming with the same velocity out of the nozzle. So, in this case, the flow is steady or not steady or not, steady but, is there an acceleration of the liquid, liquid is accelerated from a lower velocity to a higher velocity so, why do you tell then, if the flow is steady, the liquid will not be accelerated.

Now, you see a liquid particle accelerated not because, the velocity will changes with time but, because of it is convection. Why in this case, we call the fluid particle is accelerated, if you consider a fluid particle now, you come to the Lagrangian approach, as it flows from inlet to outlet. It suffers a change in velocity that is why, it has an acceleration, it is positive or negative, depending upon the shape.

As you know that, if there is a diverging passage, the fluid will be decelerated, with the change in velocity it suffers because of, it is spatial variation of the velocity. So, when things are flowing, it’s acceleration is because of, it’s convection and these acceleration is due to a gradient in it is space. That means, because of, change in the velocity in this space coordinates and at the same time the change in the velocity with respect to time.

I have given you an example, a steady flow but, non-uniform now, if the flow becomes unsteady that means, if in this case practically, if I vary the flow rate at the inlet that means, the flow at the outlet will be varying. And as a whole in the convergent duct, the variation of velocity will vary from instant to instant. So, in that case, what will happen, a liquid or fluid particle at inlet has got some velocity, which is the velocity at the inlet at that instant.

When it is convicted to the outlet, it will have an acceleration why, this is because, it is velocity that it will assume that is, the velocity existing at the outlet at that time. Because, this takes care of both, the change in the velocity due to space coordinates and the change in the velocity due to time. So, therefore, this two things are responsible to make a change in velocity of any particle, which is convicted in a fluid.

So, therefore, acceleration comes from this two causes, both the cases that non-uniformity of flow field, there is the change of velocity with the space coordinate. And also, with the unsteadiness of the flow fluid or unsteady part of the flow fluid that is, the change of velocity with the time both together, determine the acceleration. So, we can tell conclusively, the acceleration is 0, when velocity is neither a function of space coordinates, nor a function of time.

That means, velocity all points are same, invariant with the space coordinates and also with time. So, if there is a convectional fluid particles, it moves from one point to other point, it will have the same velocity so far, same velocity that means, the flow is during the flow is uniform and steady, acceleration is 0. You have understood, even another case I am telling that, if the flow is uniform but, unsteady that means, there is no change in the velocity along the direction of the flow. But, at any instant, this constant flow at all points goes on changing, there is also an acceleration.

So, space wise, when the fluid particle flows from one space one point to other point, it changes it is velocity because, it takes time to change cross it takes time to come to a point from one point so, during that time the velocity changes. So, therefore, we see that, the acceleration of a fluid particle is because of, the variation of the velocity with time and space coordinates both, you have understood this. So, this is most important point in the fluid flow, that is the acceleration now, let us derive an expression for this acceleration component.

Now, well let us consider in a cartesian coordinate system, the velocity of a point is given by u, v, w, three components u, v, w. And let a particle is at a point, whose displacement and the position vector is fixed, with respect to this frame of preference. Now, let us find out after a small time interval delta t, the change in the velocity component, the particle has gone here for example, from a to b, the change in the velocity components is delta u, in v delta v and in w component delta w.

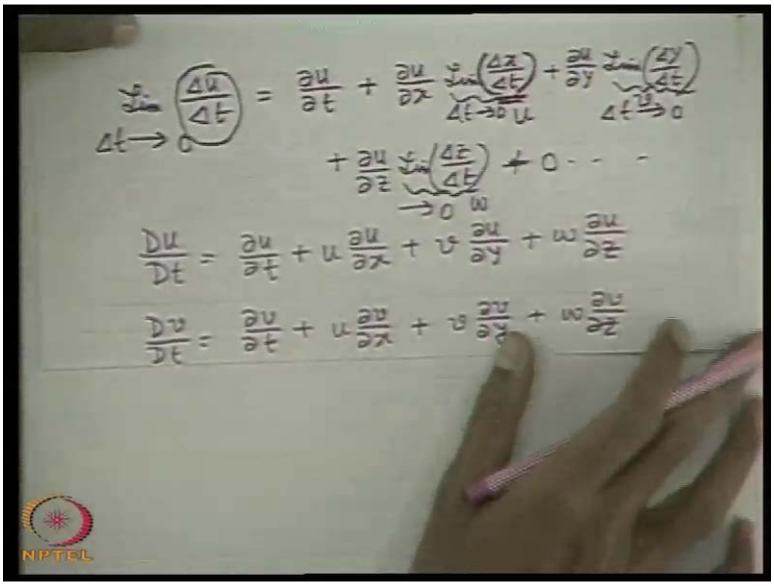
Then, we can write, that now u, v, w we can write from Lagrangian approaches, u is function sorry Eulerian approaches sorry x, y, z and t. With the Eulerian approaches, we can write x, y, z and t, we can write this is simple mathematics that, at any point the velocities can be written like this. Now, the increment in velocities, in any one of the components with delta u with that, we can write the u plus delta u. That means, the change in a final u component velocity after a time delta t, when the particle is displaced from a to b, can be written as u, as a function of x, y, z, t.

And with the Taylor series expansion, we can write del u because, u is a function of x, y, z so, the change in u is caused by a change in x, y and z and t simultaneously. These are the independent variables defining this function del u del x, del x plus this is the simple Taylor series expansion. That means, a function a dependent function is a function of it is independent variable and a change of this function, due to the change of the independent variables simultaneously, plus del u del z, this u are functions of x, y, z, times del z, plus well del u del t del t.

Now, here we assume, that during this time interval delta t, when the particle has moved from a to b, it has got a x coordinate change by del x, y coordinate change by del y, z coordinate change by del z del z. That means, the position vector is s, here the position vector is s plus del s, understand so, s is i x plus j, i j k are the unit vectors along the x, y and z axis, y plus k z. Similarly, the displacement vector d s is i del x plus j del y plus k del z that means, del x ,del y, del z are the displacement during delta theta.

Now, this u can sense so, y one can write del u and dividing it by delta t, del u delta t and taking this term first, del u del t plus del u del x, del x del t plus del u del y, del y del t plus del u del z, you can see del z del t. So, I can write this del u del t is del u del t plus del u del x just dividing by delta t, del x del t plus del u del y, del y del t plus del u del z del z del t.

(Refer Slide Time: 38:02)



Now, if I take a limit of this as delta t tends to 0 delta t tends to 0 then, we can write this is simply del u del t plus because, at delta t tends to 0, delta u will tends to 0, this will be a finite one and a delta u delta t, this in variant with that. So, this limit we can write, del u del x into limit of del x del t this quantity, as del t tends to 0, plus del u del y limit of this quantity, del y del t, as del t tends to 0 plus del u del z limit of del z del t.

Now, you see this quantities, limit of del x del t, del y del t, del z del t, at del t tends to 0, both del x tends to 0, del t tends to 0, del y but, this retains a finite value and by definition, this is the x component of velocity at a point. That means, the x displacement divided by the time and limiting value, as time tends to 0 that means, it quizzes to a particular point, when del x del t tends to 0, del x tends to 0.

So, by definition from pulmonary mechanism, this is the x component velocity, this is the y component velocity that means, this is u, this is the y component velocity v and this is the z component of velocity w. So, therefore, we can write now, this as now, this del u del t at t tends to 0 can be written as D u D t, big d this value is written as, it becomes is equal to del u del t plus, if I write u, u del u del x plus v del u del y plus w del u del z.

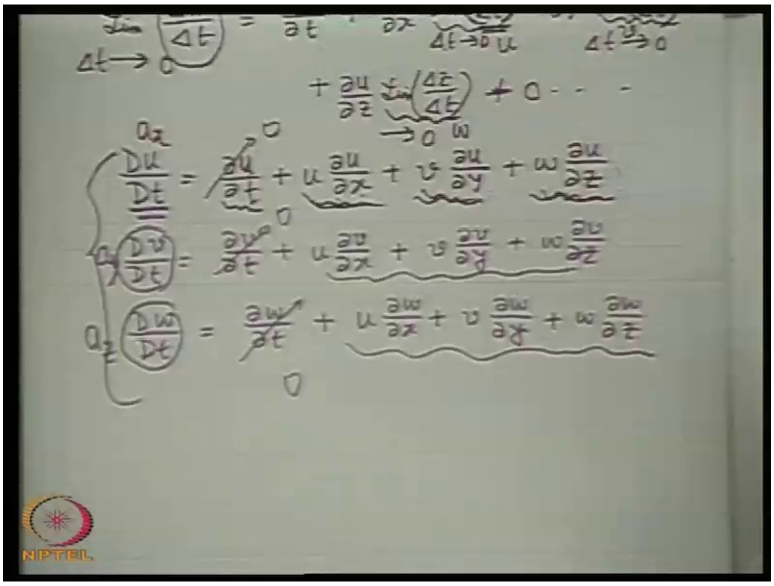
In a similar fashion in a similar fashion, if we expand the V component, we can write d v d t is equal to del v del t plus u del v del x you do it, you will see in similar fashion, we can find the find out v del v del y. Now here, I have forgotten to tell you one thing, I am sorry that when I expressed it, please go back to the earlier slide. Here, there were terms this is higher order terms because, you can ask me, sir why you have not written this in Taylor series.

Higher order terms in del x, del y, del z, del t that means, this is the first order term, as you know then, the term will come of the higher order of these where, delta x square, delta y square, delta t square will be there. So, therefore, when you divided it by delta t, this terms terms of order delta t and more so, therefore, when you divided it by delta t, so you will see the higher ordered terms will be there. So, automatically, when you will take the limit, this term will be cancelled because, this term will automatically becomes 0.

When we take the limit of these as del t tends to 0, the higher ordered terms in delta x, delta y, delta z, delta t will automatically go down to 0 obviously, it will go down to 0. Here, this first ordered term will not go down to 0, any term delta x whole square by delta t limit will be 0, as delta t tends to 0. Similarly, any term containing delta t in the numerator as a net product or delta t square, the limit will be 0. This simple mathematics you can see that, the higher ordered terms in delta x, delta y, delta z where, it comes here in the terms square then, q.

So, higher ordered terms if we take then, after dividing it by delta t the terms, which is left, which will be yielding 0 values, when you take the limit of these, as delta t tends to 0 so, therefore, we have take this as 0 so, all the terms are 0.

(Refer Slide Time: 42:23)



Now, we come here u and v, and similarly, we can write that, d w d t if we explain if we expand w component, we will get del w del t plus u del w del x plus v del w del y plus w del w del z so, these are the three. So, this is the acceleration in x direction that is, a x this is the acceleration in y direction that is, a y this is the acceleration in the z direction. So, therefore, we see for example, any one component the x direction acceleration is composed of one term, which is responsible for the change in x component of velocity with time.

And other three terms are connected with this space derivate that is, the term responsible for this space wise variation of x, y, z. Now, if the flow is steady that means, u, v, w is are not the functions of t then, the first term is always 0 for the acceleration. But, still acceleration is there because of, this component that is, it's space wise derivative. If the flow is uniform then, this three terms in the right hand side, in the expression of accelerations are all 0. So, only the first term exist so, therefore, we see this is the situation.

Thank you.