

Advanced Machining Processes
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Week - 03
Lecture – 07
Introduction to Electrochemical Machining

Today, we will be looking into a slightly different aspect into the electrochemical machining that is another very important non-conventional machining process or fabrication process very widely used again in the microsystem's fabrication. And as the name suggests electrochemical has a component of electricity and is also related to the chemical aspect which means essentially it is a ion transport. So, as any other electrochemical process would be this machining also involves the transport the material removal getting removed by means of displacement through ions into an electrolyte solution you need conducting surfaces for such machining operations. And then the idea is that instead of depositing on the tool side if supposing the ions that emanate out of the workpiece material, they can be somehow precipitated into a state which is undissolvable. So, they can be like debris and by circulating the electrolyte you can carry away those materials away from the main work zone and that way you can do bulk micromachining by using electrochemical operations. So, let us look this process at this process from a fundamental standpoint as we always normally do.

So, it is one of the again electrochemical machining is one of the most sort of wanted unconventional processes that are existing. The process is a reverse of electroplating some minor modifications and again it is based on the principle of electrolysis that can be seen from the way that the process is described. So, as you already know that in a metal in a piece of metal electricity is typically converted by free electrons, but in the solution, the electricity actually gets converted or the electricity gets carried by ions. So, in metal on one side the electrons are kind of bound within the clutches of the nucleus formulating sometimes part of a large cloud, electron cloud which moves around the various orbitals, but in solution the major current you know comes out of the ions and the mobility of those ions in the solutions.

So, that is the major charge carrier positive ions, negative ions, cations and anions in a solution. So, the flow of a current in an electrolyte always is accompanied by movement of matter as opposed to what typically would happen in metallic conductors where it is really the electron of the hole vacancy which is moving around and that results in a current. And somehow this movement of current on an external circuit should be somehow balanced to the movement of charge carriers or ions in the solution for creating a continuous process or continuous state of operation. And so, electrochemical machining is to establish that kind of a continuous operation where charge carriers on the solutions are kind of neutralized by the electrons and holes carried from the in the external circuit of such a process. So, as you can see here back in the slide in the ECM process the workpiece is connected to the positive electrode.

So, it is made the anode and the tool to the negative terminal for the metal removal. So, that is conventionally followed in some of these processes. So, tool is the cathode, and the workpiece is the anode as you can see in this figure here. And one good point that I would like to mention here

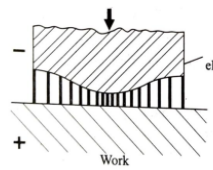
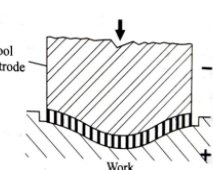
is that it is a die-sinking process meaning thereby that whatever is the profile of the tool, however complicated the profile of the tool may be is kind of impinged or impregnated into the workpiece surface eventually. Because of the you know improper distribution of the electric field to begin with.

So, electric field as you all are aware is basically the gradient of potential and the distances as you can see between this, and this is quite large. So, the sides here of the tool are at a much higher distance thereby meaning that the electric field here is lower and the electric field here is very high as can also be illustrated by these field lines. You can see the density of the lines closer to the interface which is closer of the tool surface in comparison to the sides. So, therefore, it is obvious to assume that if by Faraday's principle, we assume the material removal rate MRR to be proportional to the electric field somehow. The field vectors are very high at the centre and sort of trace amounts at the sides which means thereby that the MRR from the central portion of the workpiece would be the highest.

So, MRR is highest where the tool is closest and the MRR subsequently is lower as you move to the sides. And therefore, it is obvious to assume that the workpiece would kind of achieve a shape after a while where the electron electric field kind of gets homogenized and beyond which the whole shape the negative of the shape of the tool electrode is impregnated and the ECM process gets carried out after that in the same shape as once this full impregnation in shape of the tool is embedded on to the workpiece surface. So, that is what the beauty of this ECM process is that it is a die-sinking process. So, whatever shape you want to generate you create a negative of that shape of the tool and automatically impregnates that shape finally, on the workpiece. So, this can be also sort of more mathematically represented by the Ohm's law.

Electrochemical Machining

- The dissolution rate is more where the gap is less and vice versa.
- This is because the current density is inversely proportional to the gap.

By ohms law $V = IR$

$$V = \frac{i \rho d}{A}$$

$$\frac{V}{d} = \frac{i \rho}{A} \Rightarrow \text{Electric field} = \text{Current density}$$

if ρ is constant (depending on material properties)

\therefore highest electric field is the current density

So, which is actually followed here V equal to IR potential is current into resistance and here we just sort of modify this as ρd over A where d is the distance between the 2 electrodes and A is the interfacial area between the 2 electrodes. And so therefore, we can easily find out that V by d which is nothing but the electric field the magnitude of the electric field is actually equal to ρ times of the current density. So, this actually is the magnitude of the current density and so

therefore, electric field is proportional to the current density meaning thereby that if current density is more the electric field would be higher and vice versa. So, the current density definitely is more here in comparison to here because you have more amount of current travelling for. So, because of the increased electric field as can be seen here.

So, you have more amount of current density at the centre because the E field is high and less amount of current density at the sides because the E field or the electric field is lower and MRR of course, as I told you before is proportional to the current density. Therefore, if E field is high meaning thereby the current density is high the MRR also is high and vice versa. So, that is how electrochemical machining is really performed and that is how you know the die-sinking nature of the ECM process comes. So, there are certain basic fundamentals about this ECM electrochemical machining process. One of them is that the tool is provided with a constant feed motion and as I already mentioned before that the whole idea is to somehow be able to precipitate whatever comes out of the workpiece of the anode.

And that precipitate can be circulated outside the work zone by means of the electrolyte which is a flowing or a moving electrolyte and that way the debris removal would mean that machining would happen. So, the electrolyte has to be, therefore, circulated. So, it is pumped at a high pressure through the tool and the small gap between the tool and the workpiece and the electrolyte chosen is such that the anode is dissolved, but there is no deposition on the cathode meaning thereby that whatever properties the electrolyte has would let the anode ion out, but also precipitate the ions released. So, a fundamental choice in any ECM system is to first be able to identify what electrolyte would do that with what pair, with what paired electrode which would be the tool in turn. So, the order of current and voltage that are normally found in such processes are a few 1000 amps current and about 8 to 20 volts voltage and the gap of the ECM where the ECM is conducted between the tool and the workpiece is typically about 100 to 200 microns 0.1 to 0.2 millimetres. Typical material removal rates can be about 1600 millimetre cube per second for about 1-kilo amp current. So, this is how highly energy you know. So, it is high energy process typically energy densities are very high specific energy of the material is very high and just some other factors figures approximately 3 kilowatt hour of power are needed to remove about 16000-millimetre cube of metal and this is about 30 times the energy required in any conventional process. So, in general we what we are seeing when we did this mechanical processes or even now in electrochemical processes is that these non-conventional advanced machining processes are very highly energy intensive.

So, the question is why do you use them? So, the first reason although the processes are low yield is that you can more accurately do surface finishing and also if you can somehow constrain the operations to a very small zone, you can do micro machining very wonderfully. Particularly let us say for example, the modality involved in the ECM process would make it amenable to crevices or corners which may not be otherwise get machined or be able to get machined by a conventional process as such which involves a metal-to-metal cutting. So, if supposing the shape complexity of the final you know product that you want to make is huge some of these non-conventional processes are really better choices over the conventional processes. Even though the fact that the yield is low, even though the fact that the energy density which is needed for removing a certain volume of metal may be very high energy required may be very high and energy density may be higher. So, still do people do use because of shape complexities, because of surface finishes

particularly in microsystems design sometimes very complex shapes need to be fabricated and there is no other choice left apart from some of these non-conventional ways and means of fabricating.

So, that is the reason why non-conventional processes go hand in hand with manufacture of microsystems or micro-machined systems so on so forth. So, let us look at some of the other aspects of ECM. As we have already illustrated here, the electrolyte has to be flowed somehow between the tool and the workpiece and the best way to do that is to sort of coaxially move the electrolyte as you can see here. This electrolyte is being moved through this coaxial tube which is actually present at the centre of the tool and there also needs to be provided some insulation at the sides of the tool to ensure that the machining is not done sideways. The machining is only done in this vertical manner and then, of course, this tool should have constant feed motion.

So, there has to be a precise control on the feed motion of the tool and then of course, the electrolyte which is splashed needs to be collected and circulated back into the pumping system. So, that the debris which moves along with it can also go and does not clog the gap, the very small gap between the workpiece and the tool. So, with the ECM the advantages are that the rate of metal removal is independent of the workpiece hardness. So, the workpiece may be of any hardness value there is no scribing as such which is involved or there is no ploughing action which is involved. So, it is not mechanical and therefore, the dissolution is really an electrically dependent property, and you need not have a higher force typically to remove the material which may happen in a conventional process.

So, hardness does not matter here, any kind of hardness can be machined amenable because the basic principle is dissolution, dissolution because of ions coming out of the surface. So, it becomes advantageous when either the work material possesses very low machinability, or the shape machined to be machined is complex. So, this I have emphasized greatly in the last slide and unlike most other conventional and unconventional processes here there is practically no tool wear. Although that is not true really because sometimes great pressures get generated into the small gap here which may be a reason for tool deformation even if not wear. But the idea is that because there is no deposition on the tool as such or there is no change in the characteristic of the tool as such whatever comes out of the workpiece goes into the solution and gets precipitated and this precipitate is getting carried away.

So, the tool should be left alone by and large by this process and the wear of the tool is not is minuscule actually. So, though it appears that since machining is done electrochemically the tool experiences no force the fact is that the tool and work are subjected to large forces. And we will do some design problems later where we will try to find out whether these forces are in consonance with the ultimate flow stress σ of the tool side which would give you a feasibility or non-feasibility of the process in question. So, we will do some of these design examples when we discuss this in some more details later in electrochemical machining. So, let us now look at a little bit of modelling about how you know material removal in such a ECM process electrochemical machining process can be predicted.

And so, the basic theory is which come to our mind or the basic formulations which come to our mind are the Faraday's laws of electrolysis. So, let us see what those 2 laws are. So, fundamentally

which governed this whole machining process as this is as could as any other electrochemical business. So, the Faraday's laws are pretty much valid in the machining also. So, one the first Faraday's law says that the amount of chemical change produced by an electric current that is the amount of any material dissolved or deposited.

So, that is essentially what the current is going to do either carry out ions or deposit ions to the electrodes that is proportional to the quantity of electricity that is passed. So, if current is higher, current that is passed is higher the amount of chemical change produced by the electric current will also be higher which typically means the material dissolved or the material deposited would also be higher. So, that is the first law. The second Faraday's principle or law of electrolysis says that the amount of amounts of different substances dissolved or deposited by the same quantity of electricity are proportional to their chemical equivalent weights. And this chemical equivalent weight as we will just see little bit later is basically the is a function of the total atomic weight and also the function of the valency state in which it has to be moved.

So, if it is moving by plus 2 the chemical equivalent weight would be the atomic weight by 2 so on so forth. So, it is a function of the valency at which the ion is coming out and also a function of the atomic weight. So, the amount of different substances dissolved or deposited by the same quantity of electricity are proportional to their chemical equivalent weights. So, in quantitative form the Faraday's 2 laws can be stated here that the mass, the weight in grams of a material dissolved or deposited is proportional to the current in amperes. So, I is from here at the time for which this current flows in seconds and this gram equivalent epsilon weight of the material which is actually equal to the atomic weight per unit the valency of the material.

So, atomic weight can be obtained from the periodic table and this valency states can be multifarious there can be plus 2 plus 3 plus 1 states or maybe you know minus 1 minus 2 states. And so, basically, 1 has to be aware of at what stage this is electrochemical dissolution is happening is it a divalent dissociation is it a trivalent or a those kind of things have to be taken care of while doing electrochemical machining. So, let us actually look at a little slightly different aspect of this theory and what I would like to now go ahead and work out is how ions or how solutions would behave at the interface of or interfacial boundary of a solid and a solution. So, whenever there is a solid which is immersed in a solution there is almost always the formulation of a certain layer of charges which is also known as the double layer of charges. So, automatically by tendency you know by the natural tendencies the metal which you are immersing in a solution would try to bleed out ions and it would retain electrons and there would be an ion-electron pair which is thus generated with the ion floating in the solution and the electron on the electrode.

And then there is going to be a very thin layer of dielectric high dielectric like water which is separating both these ions and electrons and not allowing them to intermingle again once they have been liberated. So, each ion starts formulated as a bunch of water molecules around it and it does not let the ion go back and take the electron from the metal surface. So, it is very important to somehow see the characteristics of such ions may be positive and negative in a solution and how they are transported. And such a transport theory was for the first time proposed by you know which came as Debye Huckel's theory of ion transport. And so, basically, we want to look into that aspect before starting with electrochemical machining because a corollary of that which is the zeta potential of any surface is very important for the process of electrochemical machining.

So, let us see what happens when strong electrolytes are completely ionized, and their ions are not entirely free to move independently of one another through the body of a solution except when this is infinitely dilute. So, the following situations may happen. So, let us say there are positive ions in this and there are bunch of different negative ions and each of these positive ions are formulating a cloud of negative ions around it something like this. So, these are cloud of negative ions around it. So, the first effect that one can imagine intuitively is that the ions will move randomly with respect to each other due to fairly violent thermal motion.

So, this is also the Brownian movement that the ions would move due to their ambient thermal energy which is a function of the temperature state at which they are in. They would actually move randomly with respect to each other. And then there are going to be coulombic forces between opposite kinds of ions like for example, in this case let us say the positive charge centres and the negative cloud around it of negative ions around it. There are going to be coulombic forces of attraction which would lead essentially to the assembly of these charge clouds around the positive central charge systems or vice versa. So, this kind of leads to time-averaged ion atmosphere of one kind with respect to a central ion of the opposite kind.

In this case, it is a positive central ion, and the time-averaged atmosphere is a negative ion. And this is the distribution which will almost always happen when you pour you know such an ion pair or an ionic solid into, governed by coulombic forces into a solution like NaCl in water. So, Na plus and Cl minus and then water comes in between. So, water molecules kind of because they are high dielectric they get into and engulf the Na plus and the Cl minus and they do not they seldom allow them to come together. But then what can happen is because there is an overall positive charge centre in the Na plus and overall negative charge centre in the Cl minus there can be a assembly in a manner. So, that there is an average atmosphere of Cl minus around Na plus or vice versa.

So, that is what we are meaning by this coulombic forces. And then supposing you make an external electric field applied to such a system where this let us say is the positive side this is the negative side. So, all the negative ions will start moving in the positive end and the positive ions, of course, will start to move in the negative side. And therefore, movement of such ions under an external electric field first of all will be very slow.

And the atmosphere starts moving in the opposite direction as in this case you can see the negative ion atmosphere moves to the positive side and the negative the positive central ions start moving to the opposite that is the negative side of this field or potential which is created by 2 electrodes. Therefore, there would be a continuous disruption and reformulation in the ion atmospheres with respect to the central ions. So, for example, when they start moving these negative ions are going to get reformulated in the cloud of the second positive ion from first positive ion it moves to the second positive ion so on so forth. So, therefore, there is an assemblage of charges which kind of disrupted from the ionic environment of ion 1 into the ionic environment of ion 2 and vice versa and so that happens all way throughout the solution. So, therefore, there may be many effects like electrophoretic effect which is essentially the movement of the charges across a medium over a potential externally applied to the system.

And there can be also viscous drag of the atmosphere because mind you these all moieties charge

moieties are present in solution. So, therefore, if the positive centre is moving ahead and the negative centre is trying to move in the opposite direction it is obvious that there would be viscous effects or there would be shear between the layers of the solution which is containing these ions in turn. So, there would be viscous drag which is associated with the forward motion of the ion through this whole so-called atmosphere. So, all these parameters need to be included in a model where we can really estimate what would happen to the layer of moiety which is formulated across a metal electrode. So, let us look at how we will be able to in a simplified manner model such a situation in a solution.

So, let us look into the formulation of this sort of ion transport equation which is also known as the Debye-Huckel equations. So, let us suppose there is a positive charge and single positive charge, and it has an environment made up of both positive and negative charges and at a distance of at an infinite distance from this charge. The charge density of the positive the number density of the positive or the negative charges of the environment of this positive charge are N_+ or N_- per unit volume. So, this is the positive or negative charge number density meaning thereby number per unit volume of this ion of interest, central ion of interest which is a positive charge. So, supposing if we were to find out a point A here in the near vicinity of this positive charge, this central positive charge of interest here.

Ion-Ion and ion-solvent interaction

Debye-Huckel

Ion Transport - kinetics is determined from the famous Debye Huckel equations

Point in the vicinity of positive ion where potential = ψ

ψ work done for +ve charge $\rightarrow +z_+ E \psi$

\leftarrow work done for -ve charge $= -z_- E \psi$

$\psi_{\infty} = 0$

N_+
 N_-

}

positive &
-ve ions
per unit
volume

The amount of work that and let us say at particular A there is existing potential ψ defined by a function. So, this ψ is the potential at a point near the central charge. So, the work done for a positive by the system for let us say moving a positive charge is actually given by the charge itself. Let us say the charge has a valence Z plus. So, Z plus times of electronic charge e this e as you all know is 1.6×10^{-19} coulomb electronic charge. So, the total charge on the ion times of the potential. Similarly, the work done for moving a negative charge from distance infinity where ψ is 0 actually all the way to this point A. So, from for moving a positive charge from infinity to A negative charge moving a negative charge from infinity to A would then be given by minus Z minus $e \psi$. ψ is the potential at A Z minus is the valence E on the negative ion and E of course, as you know is the electronic charge.

So, that is the amount of work which is done in case of a positive charge or negative charge to come from infinity to the point A close to the central ion of interest. Now if we look at, so therefore, there would be a possibility of a charge distribution at the point A which can now be given as N_+ and N_- . And these minds you are numbers per unit volume, number of positive charges per unit volume, number of negative charges per unit volume. So, we are concluding here that the ion which is the central ion of interest is assembling its own charge atmosphere around it and at a point A the numbers are N_+ and N_- of positive and negative charges per unit volume. Assuming that at point in space which is infinite distance from the central ion the distribution of the positive and negative ions are N_+^0 and N_-^0 in terms of numbers of charges per unit volume.

So, and we already know what is the work done for moving 1 charge of a certain value of a certain charge from infinity to A. So, there is a correlation which exists, and I am not going into the details of that correlation between you know the numbers which are at the point A to the numbers which are far away or at infinite distance from the point A. So, that correlation actually emanates out of the Boltzmann distribution. So, from Boltzmann distribution the N_+^0 or number density near the point A is actually equal to N_+^0 at infinity times of exponential to the power of minus the total amount of work which is done. So, $Z_+ e \psi$ by kT and this is also a function of the absolute temperature in which the system is stalled.

And similarly, the N_- at the point A mind you these are number densities at point A these are number densities are infinity point infinity. So, $N_-^0 e$ to the power of $Z_- \psi$ by kT . So, k is the Boltzmann constant and N_+ or N_- are the numbers per unit volume near or at A N_+^0 N_-^0 numbers per unit volume at infinity or at infinite distance. So, the total electrical density, the total electrical charge density near A or at A can be given as the numbers per unit volume at A times of the amount of charges. Let us assume the charge on one ion is $Z_+ e$ minus the number of negative charges per unit volume times the value of the charge $Z_- e$ on the negative ion.

Now from Boltzmann distribution

$$N_+ = N_+^0 e^{-(Z_+ e \psi / kT)}$$

\Downarrow

$\psi = 0$

$$N_- = N_-^0 e^{+(Z_- e \psi / kT)}$$

k is the Boltzmann constant & N_i^0 is the no. of ions of either kind per unit vol.

Electrical density = $\rho = N_+ Z_+ e - N_- Z_- e$

$$= N_+^0 Z_+ e e^{-(Z_+ e \psi / kT)} - N_-^0 Z_- e e^{+(Z_- e \psi / kT)}$$

Applying Taylor series

If we substitute the values of N plus and N minus from this set of equation 1 earlier, we can get this as N plus 0 Z plus epsilon times of e to the power of minus Z plus epsilon psi by K T minus of N minus 0 Z minus epsilon times of e to the power of Z minus epsilon psi by K T. So, that is the electrical charge density at the point A. And if we just apply Taylor series here and just assume that the higher orders are negligible x square by factorial 2 plus so on so forth. And we assume that the higher orders because of the low value of x are negligible.

we know $e^x = 1 + \frac{x}{1} + \frac{x^2}{2} + \dots$
 neglecting higher (2nd & more order) terms
 $e^x \approx 1 + x$

$$N_+^0 z_+ e^{\frac{-z_+ \epsilon \psi}{kT}} - N_-^0 z_- e^{\frac{z_- \epsilon \psi}{kT}}$$

$$= (N_+^0 z_+ - N_-^0 z_-) \epsilon - \frac{N_+^0 z_+^2 \epsilon^2 \psi}{kT} - \frac{N_-^0 z_-^2 \epsilon^2 \psi}{kT}$$
 ||
 Principle of electro neutrality
 Assuming $N_+^0 = N_-^0 = N_i$ (Average charge density at ∞)

$$= -\frac{\sum N_i z_i^2 \epsilon^2 \psi}{kT} = \rho$$

We can write this equation number 2 in the following manner. So, you have rho at A the overall charge density at A written as N plus 0 Z plus epsilon times of 1 minus Z plus epsilon psi by K T minus of N minus 0 Z minus epsilon 1 minus Z minus epsilon psi by KT. And therefore, this can further be written down as N plus 0 Z plus epsilon minus N minus 0 Z minus epsilon minus of N plus 0 Z plus epsilon square psi by K T minus of N minus 0 Z minus epsilon square psi by K T. This of course, is 0 because at infinite distance we can assume the principle of electro neutrality to exist and this becomes 0. So, the other term which can be taken care of is basically minus N plus 0 Z plus epsilon square by KT minus N minus 0 Z minus epsilon square by K T times of psi. So, if we assume that the charge densities of the positive and negative kinds are same at infinity which is actually true because of again the principle of electro negativity, electro neutrality coming into existence.

And this we assume to be equal to let us say Ni. So, we formulate a system where I is a set of all positive and negative charges. Let us say I varying between 1 and N meaning thereby if it is exactly half 1 2 N by 2 can be the positive charge and by 2 to N can be the negative charges so on so forth. So, therefore, we can write down this charge density equation rho A as in a very simplistic manner as sigma Ni Zi square. Now, I am actually getting rid of all these pluses and minuses in the subscripts and representing them by i's. So, Ni Zi square, square of epsilon times of potential function psi by K T.

So, that is the density of the charges the overall charges at a point A in space close to the central ion of interest. So, we somehow need to be able to derive the potential function which exists at A near that central ion of interest owing to the charge density which is around it. And in the next step what we are going to do is to utilise the Poisson's equation in the space coordinates and try to find out that at least in the CGS system the gram centimetre second system. How this equation can be

used for determining the electrical field at a point A because of the complete amount of positive and negative charges with densities N_i , where i varies between all the positive and negative charges close to the central ion of interest. So, the electrostatic potential and charge density are related by the Poisson's equation as $\text{del}^2 \psi$ by $\text{del}^2 x^2$ plus $\text{del}^2 \psi$ by $\text{del}^2 y^2$ plus $\text{del}^2 \psi$ by $\text{del}^2 z^2$.

This is the special all second derivatives space derivatives and that is actually equal to minus 4π charge density which is actually the charge density at A ρ at A by the dielectric constant. Here ψ is the potential at A ρ is the charge density at A and D is the dielectric constant of the medium. So, we want to somehow be able to solve for this equation to find out the relationship between ψ and all these other terms like charge density, dielectric constant and of course, this space term. So, that you know ψ can come as a function of the special variation and also the variation of the charge density between different shells around the central ion of interest. So, what we will try to do is to just use a simple conversion technique to convert this whole you know equation into polar coordinates.

from Taylor series and only taking first order terms we have

$$\rho \sim - \sum N_i z_i \epsilon \left[z_i \epsilon \psi \right]$$

$$= - \sum N_i z_i^2 \left[\frac{\partial \psi}{\partial z} \right] \quad \text{--- (1)}$$

The electrostatic potential & charge density are also related by the Poisson's equation

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = \frac{-4\pi \rho}{D}$$

Using coordinate transformation to rotational

Assuming that the above plane is spherically symmetric & the ψ is only a function of (r)

So, here what we assume is because there is a central ion, and you know potential function really varies as a function of distance r from the central ion. And so, somehow, we have to convert this whole equation into polar coordinates or spherical coordinates and then assume only variation along of the radius vector r or we assume spherical symmetry. So, not going to go into the derivation of it, but the approach can be represented here let us say you have a Cartesian coordinate system x, y, z and you have a radius vector here r . So, you have the first projection of the radius vector as on the x, y plane by so, this can be projected by let us say an angle γ . So, this component here is $r \cos \gamma$ and this component here is $r \sin \gamma$ and then if you rotate this at an angle let us say θ .

So, the x component can come out to be $r \cos \gamma \sin \theta$ and y component can come out to be $r \cos \gamma \cos \theta$ or $r \sin \theta \cos \gamma$ and then of course, you know that the z component is already $r \sin \gamma$. So, if you were to change this whole form of Poisson's equation into polar coordinates you will have to just represent ψ by differentiating with respect

to r , γ and θ using the chain rule and then assume spherical symmetry. So, the variation that you are assuming is only along r . So, this whole equation now gets converted or changed into the form $\frac{1}{r^2} \frac{d}{dr} (r^2 \frac{d\psi}{dr}) = -\frac{4\pi\rho}{D}$. So, this is the spherical coordinates, or you know this is the equation in spherical coordinates or polar coordinates assuming that there is spherical symmetry and there is no variation along the angle γ or θ the only variation is along the radius vector r .

So, we want to now be able to solve this so that ψ comes as a function of r , ρ , dielectric constant D so on so forth and it is very amenable for us to now try to find out what is the work done by a charge at infinity to come very close to the surface in question which is actually A where the potential is ψ . So, here let us try to solve this differential equation which has been obtained $\frac{1}{r^2} \frac{d}{dr} (r^2 \frac{d\psi}{dr}) = -\frac{4\pi\rho}{D}$. And we already know that the ρ has been illustrated before in an equation which you formulated earlier as $-\frac{N_i Z_i^2 \epsilon_0 \psi}{K T}$. So, therefore, this can be represented as $\frac{1}{r^2} \frac{d}{dr} (r^2 \frac{d\psi}{dr}) = \frac{4\pi N_i Z_i^2 \epsilon_0 \psi}{K T}$ and the minus goes away $\frac{1}{r^2} \frac{d}{dr} (r^2 \frac{d\psi}{dr}) = \frac{4\pi N_i Z_i^2 \epsilon_0 \psi}{K T}$. So, if you were to just take this portion stand-alone and consider this to be square of small k the capital K of course, here as you are seeing is the Boltzmann constant.

So, we will not go into that the small k . So, the small k square can be represented as $\frac{4\pi N_i Z_i^2 \epsilon_0}{K T}$. Let this be equation 3 and we can represent this equation $\frac{1}{r^2} \frac{d}{dr} (r^2 \frac{d\psi}{dr}) = k^2 \psi$ as square of small k times of potential function ψ . So, this, of course, is a standard equation over the standard solution and this also a general differential equation it is not partial anymore because only one variable is involved. And the solution to this standard solution to this comes out as $\psi = A e^{-kr} + B e^{kr}$ where A and B are constants.

So, that is how the standard solution of this equation comes out as. So, we will take the solution and try to investigate what these values of A and B are based on the boundary conditions that are imposed to on to us through this problem. So, here the first condition that we can actually think of is that the potential function ψ is actually 0 at r equal to infinity and therefore, so ψ infinity is 0. So, if you consider ψ to be equal to $A e^{-kr} + B e^{kr}$ at r equal to infinity $A e^{-kr}$ automatically becomes 0 otherwise this term will you know be undefined by undefined. Of course, here because it is e to the power minus kr as r increases to infinity this term goes very small and so therefore, overall, the equation can be 0.

So, therefore, B is 0 from the boundary conditions and ψ becomes equal to the constant $A e^{-kr}$ to the power of or e to the power of minus kr by r . So, that is the solution for the potential function at a close to the central line of interest which is $A e^{-kr}$ by r . So, we already know that from the previous equation that charge density had been earlier defined as a function of $-\frac{N_i Z_i^2 \epsilon_0 \psi}{K T}$ the potential function by Boltzmann constant K times of T . And so, we can actually substitute the value of ψ that we have obtained here to obtain the charge density at space close to that point A as $-\frac{A e^{-kr} \sigma N_i Z_i^2 \epsilon_0}{K T}$ just a

substitution. So, that is what the charge density value comes out to be or in other words, if we were to involve small k into picture you know that small k was defined by equation 3 here.

So, therefore, we can say that this whole term can be written down as A small k square by 4π times of $D\epsilon$ to the power of minus small kr by r . So, this way you can actually get rid of this whole complex sigma by $K T r$. So, that is how the charge density would equate to close to the central line of interest at a point a very close to it. We will try to now find out. So, we kind of coming at the end of this lecture, but in the next lecture, we will try to find out what is the value of A which we would like to investigate.

$$\begin{aligned}
 \text{At } r \rightarrow \infty \quad \psi_0 &= 0 \\
 \text{if } \psi &= A e^{-kr} + A' e^{kr} \\
 \text{Then } A' &\text{ should be zero} \\
 \therefore \psi &= A e^{-kr} \quad \text{and} \\
 \rho &= -\frac{A (\epsilon_0 \epsilon \nabla^2) e^{-kr}}{4\pi} \\
 &= A \frac{\epsilon_0 \epsilon}{4\pi} D(e^{-kr})/r
 \end{aligned}$$

And finally, the potential function will come out once the value of A comes from a certain condition that we would do in terms of principle of electro-neutrality of charges around the central line of interest. So, with this, we close today's lecture and we will try to solve this or take it up and solve this further in the next lecture. Thank you.