Advanced Machining Processes
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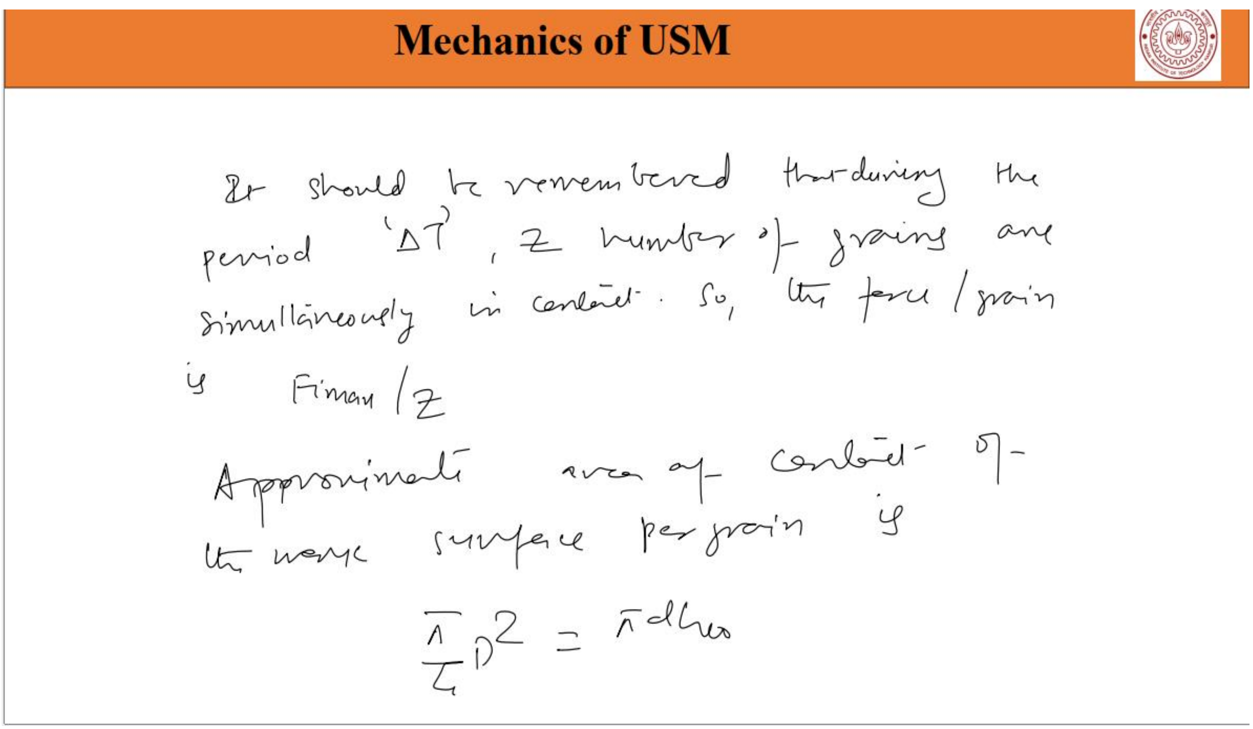
Week - 02
Lecture - 03
Mechanics of USM and process parameters

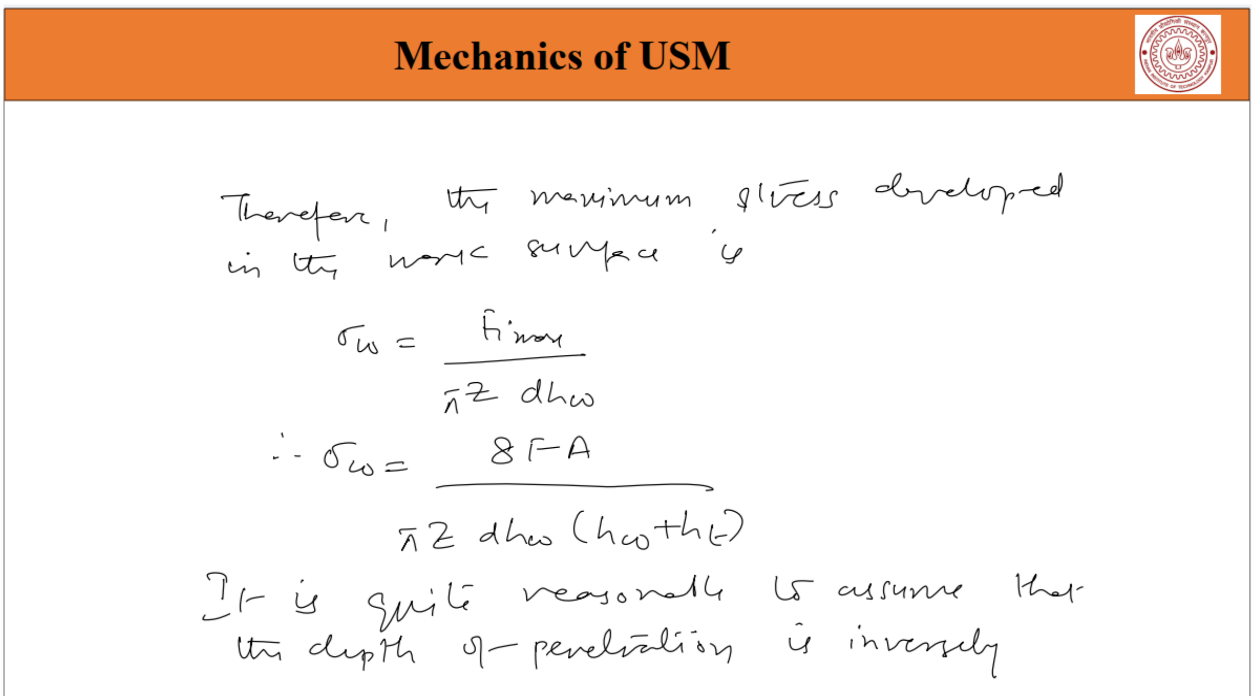
So, let us actually try and do that by looking at what is the force at least per grain which comes between the several grains which are there between let us say the tool and the workpiece surface. So, it should be mentioned that during the period delta small t or delta capital T where the indentation is happening, it is not a single grain which is doing this indentation. So, if we assume that there are Z grains which are there between the tool and the workpiece surface at that small instant of time delta T where the indentation is happening, and they are all placed at equal height with respect to the surface and the tool is a perfectly flat surface. So, therefore, all these grains are together going same distances in terms of its ploughing action onto the workpiece. So, in that case, if supposing Z number of grains are simultaneously in contact. So, force per grain is represented as Fi max by Z for obvious reasons and if we suppose estimate that the approximate area of contact of the work surface per grain is pi by 4 capital D square.



Remember D was actually the indentation diameter. So, this can also be estimated as pi by 4 times of 4 dh where this small d is the grain diameter is the grain diameter and this h here is the indentation of the workpiece surface. Mind you this h is really not equal to ht plus hw because while considering the geometry of the penetration of the grain onto the workpiece only the indentation on the workpiece side was being considered and not the grain. So, this h in the equation for the in the relationship between the indentation diameter the grain diameter and the traverse h there the h actually is the depth of indentation on the workpiece surface.

So, hw. So, that is how we can relate all this and then we can mention this to be equal to pi d hw. Simultaneously, the maximum stress imparted by the grain on the workpiece surface is given by the total force per grain which is available per unit area which is pi d hw from the previous slide. So, this stress should be actually equal to the flow stress of the material sigma w. So, this is the flow stress of the material for the fracture to happen the brittle fracture to happen or for the material to get deformed and the crater would thus start to formulate in that situation.

So, therefore, the sigma w here is represented by the earlier equation as 8 times of force average times of amplitude A divided by pi Z d hw times of hw plus ht from the previous equations. So, we are just simply substituting the value of Fi max in this particular equation for sigma w. So, it is quite reasonable to assume that the depth of penetration is inversely proportional to the flow stress of the material as long as the load in the indenting spheres diameter remain the same. And we can always say that if h be the indentation depth this h can be inversely proportional to the flow stress of the material meaning thereby that if a material has a higher flow stress for a certain force level it would have a lower depth of indentation and vice versa. So, if sigma t and sigma w are the flow stresses of the tool and the work surface then the ratio of ht that is depth of indentation of the tool on the tool side and hw that is the depth of indentation on the workpiece side can be represented by the ratio sigma w by sigma t which this where this is basically the ultimate flow stress of the work material and this here right here is the ultimate flow stress of the tool material.



Let us suppose that this ratio is equal to some constant lambda particularly because we are not changing the tool material or the workpiece material. So, it is really a it is a material property and that ratio is represented by this factor lambda here. So, therefore, the maximum the sigma w the ultimate yield stress which is developed can slightly be modified as 8 FA divided by pi Z d hw square times of 1 plus ht by hw which actually is signifying this constant lambda. So, we can write this down as 8 FA divided by pi Z d hw square by 1 plus lambda. So, as of now, we will continue this in the next lecture, but as of now, we have come to know that there is a way that you can actually relate the ultimate flow stress of the work material with respect to the force average force on the tool the amplitude of motion of the tool the number of grains per impact or impacting the surface all at one go between the tool and the workpiece.

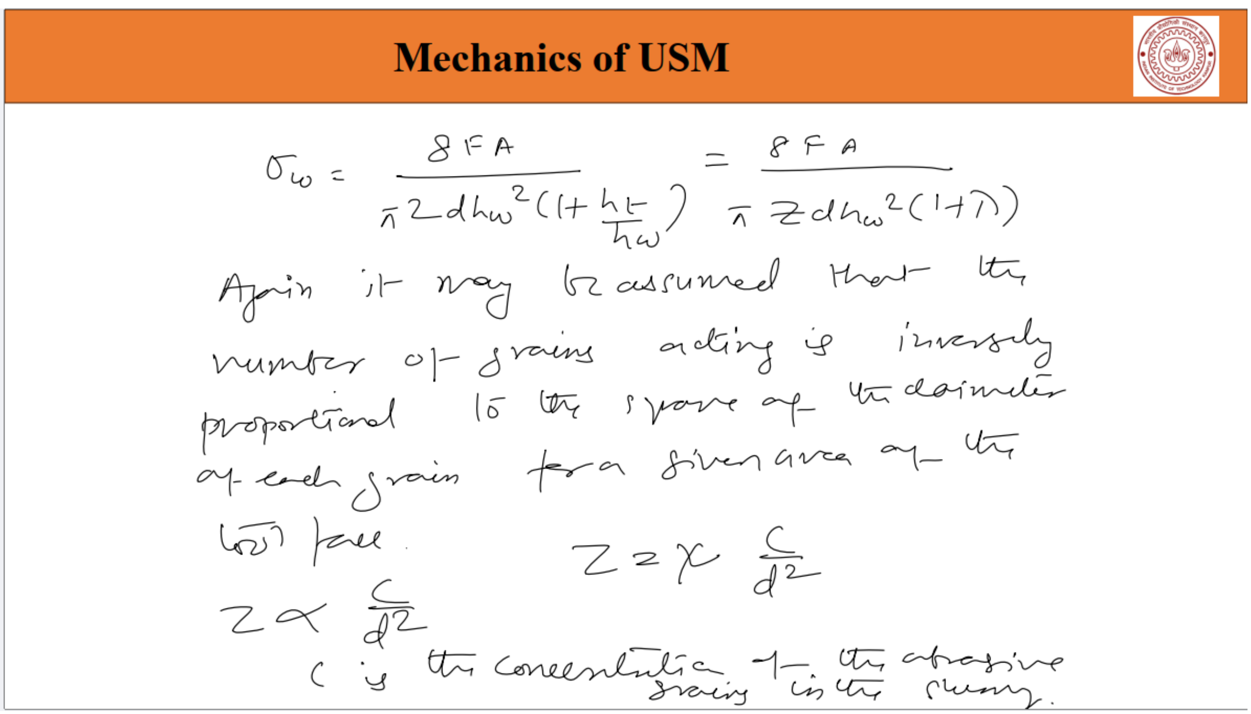
The grain diameter and the penetration depth on the workpiece square times of this material property which is the ratio between the ultimate flow stress of the work and the tool with respect to each other. So, with this, we come to the end of today's lecture and we will continue this in the next class and try to find out how we can put a value for Z in terms of numbers per unit volume of the particle that would be a composition of the that would be indicative of the composition of the slurry the abrasive slurry and then try to find out what is the MRR based on all these different parameters on at least an order of magnitude basis. Then we can also estimate certain plots and trends from the actual experimental methods to this theoretical model and try to ascertain whether they are in unison or they are in consonance with each other. Basically, let us just do a quick recap of what we had finished last time. So, we had looked into the USM or ultrasonic machining process.

We had also tried to find out some estimation about the material removal rate. We also did investigate this M.C. Shaw's model for predictions of material removal and did some assumptions in this model where we talked about that you know the grain, the abrasive grain should be treated as spherical and also it should be treated as if there are many number of grains between the tool head and the workpiece and also that the indentation created by this grain would produce a crater on the surface and for all practical purposes we should consider the amplitude of motion of the tool head to be constant so on so forth. So, there were a set of assumptions that we had made for predicting the MRR or material removal rate and then we started modelling to somehow to estimate what this MRR value would be and in the process of doing that we arrived at a formulation given here in this slide right here where we were talking about the ultimate yield is ultimate yield is the stress of a material of the workpiece sigma w and we correlated this to the average force that the tool the vibrating tool head would give on the grains the amplitude of motion of the tool A and the number of grains or particles making impact per cycle d right here was the grain diameter of the abrasive grains grain diameter.

hw was the indentation depth of the grain on the workpiece and this parameter here known as lambda was basically the ratio of the ultimate flow stresses of the workpiece in the tool as we already have seen before in details that you know the ultimate yield stress is really inversely proportional to the indentation depth. So, we can assume that sigma w by sigma t the ultimate yield stress of workpiece to the tool is nothing, but the inverse ratio of the depth of indentation of the tool to the workpiece and this we considered as lambda here which comes into this equation here. And so therefore, we have ways and means to predict the ultimate yield stress of the work material here let us call it ultimate yield or let us say flow stress yield stress of the workpiece. So, that is how we have very well-defined relation between the various parameters associated with the force given by the vibrating tool head to the area of the grain which is really interfacing with the surface and the ultimate flow stress of the work material sigma w which is in question. Now, if we really try to see what the Z value is the grains per impact number of grains in contact between the workpiece and the tool in one impact would be.

So, let us say that if we assume that the number of grains acting is inversely proportional to the square of diameter of each grain which is obvious because supposing there is an area like this on which you have. So, these many grains right these are the grains on that area and the average diameter of the grain is given by d as we have predicted before. So, this diameter here right here is d. So, the amount of occupation of the grain area would definitely be a function of the overall area that is coming between the tool and the workpiece. So, this is the tool this is the vibrating tool head and the tool is coming down like this and the grains are coming in between the tool and the workpiece this is the workpiece.

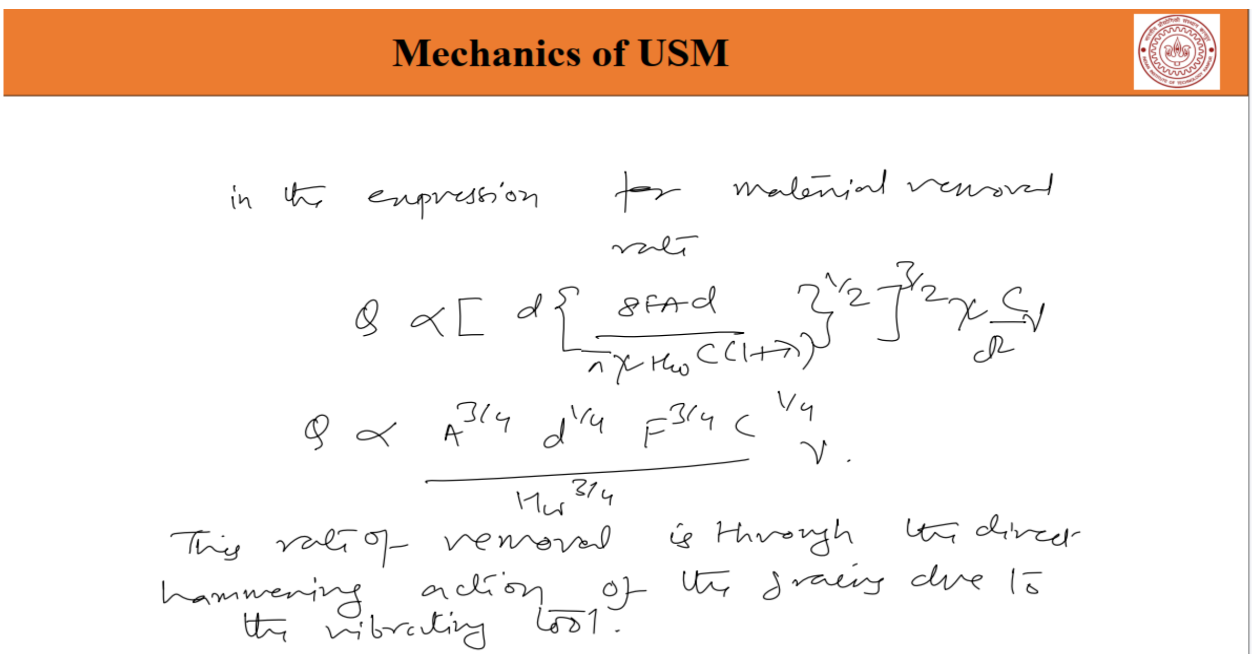
And so, the influence of the diameter of the grain on the effective area of the workpiece that can be machined which is showed by the shaded region is obvious. So, therefore, if we assume that the number of grains acting let us say these are Z numbers in one impact between the tool and the workpiece. So, if that is inversely proportional to the square of the diameter of these grains which also is signified or signifying the sort of area of projection of 1 grain. So, it will not really be improper to assume in this kind of a relationship. So, for a given area of the tool face Z is actually proportional to inversely proportional to the square of the grain diameter and also Z would be proportional to the concentration of the grains in the slurry.



So, if c is the concentration of the grains in the slurry or a concentration term. So, more is the concentration more would be the number of Z's more would be the value of Z the number of particles between the tool and the shaded area here workpiece. So, therefore, we can say that Z is equal to some constant psi times of c by d square and we can actually substitute the value of Z from here to here in this particular equation. So, the final form of the equation can come out to be square of hw is actually equal to 8 times of average force times of amplitude A times of grain diameter divided by pi times of psi and the flow stress of the material is nothing, but the hardness of the material. So, sigma w and hw are kind of interconvertible.

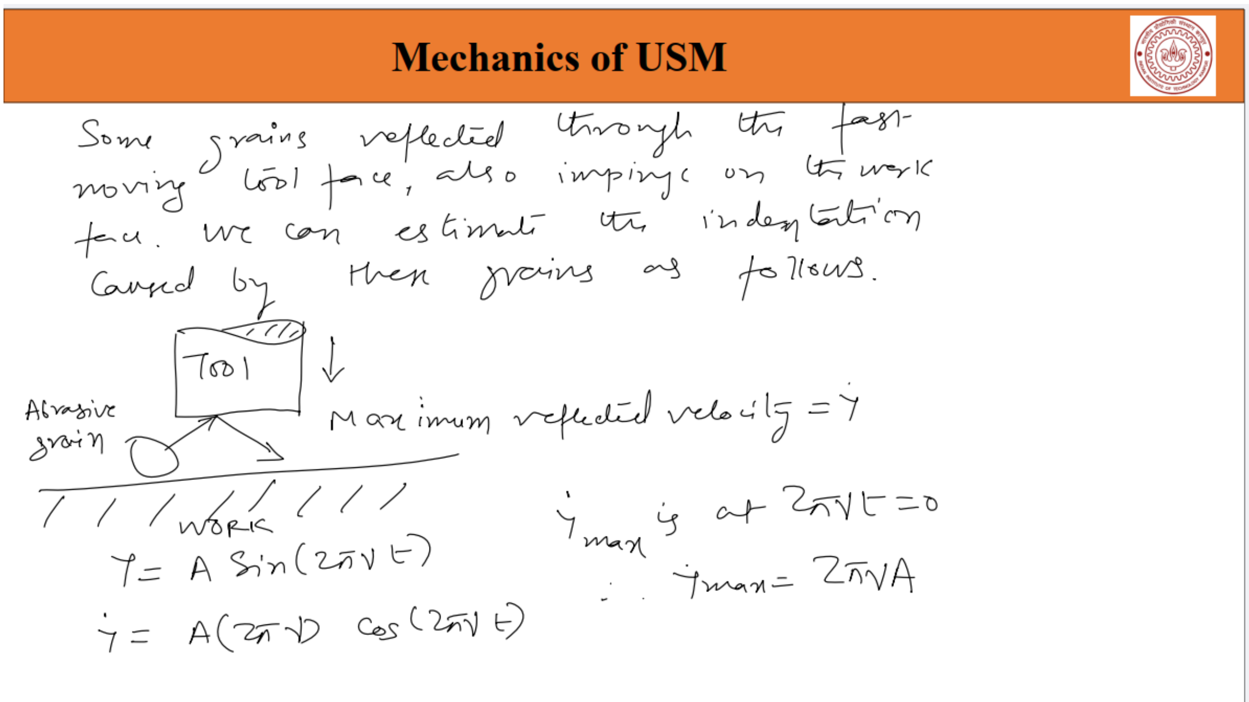
So, hw times of c times of 1 plus lambda and where hw now this is small hw as you know is the depth of indentation of the grain on the workpiece surface. So, therefore, hw always becomes equal to the square root of 8 F average A d divided by pi psi hw c 1 plus lambda. And if we substitute this value of hw in the equation for Q, you remember Q earlier was actually determined to be proportional to d hw to the power of 3 by 2 times of the value of Z times of the frequency nu. So, Q here can become of course, we can substitute all these values here. So, the Q finally, after substitution of hw and the value of Z which is actually square of inverse square of d times c times of psi and nu, of course, is the frequency we get.

Q is proportional to amplitude to the power of 3 by 4 diameter d to the power of 1 by 4 the average force of the vibrating tool head to the power of 3 by 4 times of concentration to the power of 1 by 4 divided by the flow stress of the material or hardness of the material hw to the power 3 by 4 times of nu. And so, the rate of removal is through the direct hammering action of the grains due to the vibrating tool. So, this actually we can say as Q direct or in other words Q direct is nothing, but the direct hammering action of the vibrating tool head on the grains thus creating a ploughing action. So, as I told you there are 2 modalities of this material you know removal 1 is of course, the direct hammering action of the vibrating tool head and the other that is more that is not very important or not significant although it is to be considered in model is the impact that a free grain would have on the surface. Meaning thereby if the gap between the tool and the workpiece is very high and there is a possibility of the abrasive grain to freely flow between the tool head and the workpiece.



So, impact that the tool would give on the abrasive grain would be converted as a sort of kinetic energy of the grain and this kinetic energy would come and impinge on the surface thereby removing the material from the workpiece. So, that is separate mechanism. So, this Q that we have determined now is really the direct hammering action where you are squeezing the grain between the tool head and the workpiece and you are giving a force average, average force F average between the grain and on the grain by the tool which is creating a direct ploughing action. So, that is what the first part is. Let us look at now the second part of the problem and the second part is related to the kinetic energy of the grains.

So, let us actually try to model that part. So, some grains get reflected through the fast-moving tool interface also impinge on the workpiece. So, we can estimate the depth of indentation in that case by looking at the following. So, let us say this is the tool and this right here is the workpiece and there is a grain, there is an abrasive grain which because of the motion imparted by the slurry goes and strikes the tool in this particular direction coming out of it in this direction and we will have to somehow predict what is the maximum reflected velocity. So, this is the direction of reflection of the grain.



So, the maximum reflected velocity needs to be somehow determined in this particular case. So, that is actually let us say y dot maximum. So, as we know that you know the grain velocity here of the abrasive grain the initial velocity by which it is striking the tool is of hardly any significance in comparison to the overall inertial component of the tool because the tool first of all is very heavy and number 2 is it is also vibrating at a certain velocity or ultrasonic frequency meaning thereby that its velocity is also very high. So, therefore, the velocity the initial velocity of the abrasive grain as it strikes the tool surface here for example, in this position A here is not really of great significance and we can say that whatever is the velocity of the tool head is at that particular point of collision at the time of collision would be equal to the velocity of reflection of the particle. So, it is simply imparted there is no specific inertial component associated with the abrasive grain because of its small nature it is few microns as I told you abrasive grains could be in the range of 20 to 25 microns.

So, let us find out first the operating velocity of the tool head as a function of time. So, yt as you know because it is a sort of simple harmonic motion imparted by the tool. So, yt can be written in form of an equation as the amplitude of motion A times of sine 2 pi nu t, nu is the operating frequency of the tool and A is the amplitude. And so, therefore, that is what the equation of motion of the tool head would be. So, the operating velocity of the tool head would be dependent on this equation of motion here.

And so, operating velocity can be written down as y dot t the first differential of y with respect to time which is equal to A times of twice pi nu times of cosine twice pi nu t. And as you know here that at time t equal to let us say 0 which signifies probably the mean position of the particular tool where the velocity is supposed to be the maximum. So, this value y dot t would be y dot maximum which is actually equal to A times of twice pi nu. A is the amplitude of motion nu is the operating frequency and so, A twice pi nu is basically the y dot max or the velocity of motion. So, now we look into the aspect of the kinetic energy of the particular tool once the maximum velocity of the grain is there.

So, the corresponding KE or kinetic energy actually will be equal to the maximum kinetic energy because it is half mv square, v is the velocity of motion and v is equal to v max corresponding to the maximum velocity the time when the tool is at the mean position. So, therefore, the maximum kinetic energy of the abrasive grain I already explained to you before that it really is nothing but the maximum velocity of the tool. The inertial component of its own self of the grain is so small that we do not really treat that in this equation. And so, therefore, the maximum kinetic energy of the abrasive grain is given by the term half mv square and m here because it is a spherical grain that we are assuming with diameter d we can assume it equal to be the volume of the grain which is 4 by 3 pi r cube d by 2 whole cube times of the grain average grain density rho A. So, this is actually the density of the abrasive material.

Mechanics of USM

During vibration, the maximum velocity of the tool face $= 2\pi \nu A$

So, the corresponding K. Energy (max) of the abrasive grain is given by:

$$(KE)_{max} = \frac{1}{2}\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \rho_a\right)(2\pi\nu A)^2$$

assuming full penetration of the grain.

$$(KE)_{max} = \frac{1}{2}\left[\frac{\pi}{6} d^3 \rho_a\right]\left[4\pi^2 \nu^2 A^2\right]$$

$$= \frac{1}{3}\pi^3 \rho_a d^3 \nu^2 A^2$$

$\rho_a =$ density of the abrasive material

This is not really the number of grains per unit volume, but it is the density of 1 grain per unit volume of that material. So, that basically is the mass component in the motion. So, it is half mv square and v as you know is 2 pi nu A square, where nu is basically the frequency, A is the amplitude of motion of the particular tool head and then this is a characteristic property of the grain itself. So, if we really try to solve this round, we get a term 1 by 3 cube of pi rho a d cube nu square A square, where rho A is the density of the abrasive grain. So, that is what the maximum kinetic energy in this particular case would be.

Cut. What is the problem here? So, basically, now we want to really find out the amount of energy which is needed for indentation caused on the surface by a flying, free-flying abrasive grain that comes and strikes onto the tool surface and impinges on to the workpiece surface as a result of the reflected velocity. So, assuming that during the indentation caused by such an impinging grain, the contact force increases linearly with the indentation. So, the depth, the KE max whatever has been imparted onto the grain surface or the free-flowing abrasive grain by the tool surface should really be equal to half Fi dash max hw dash. If you remember the plot here, cut. So, in this graph here let us say we assume that Fi dash is linearly varying with respect to the depth of indentation hw dash.

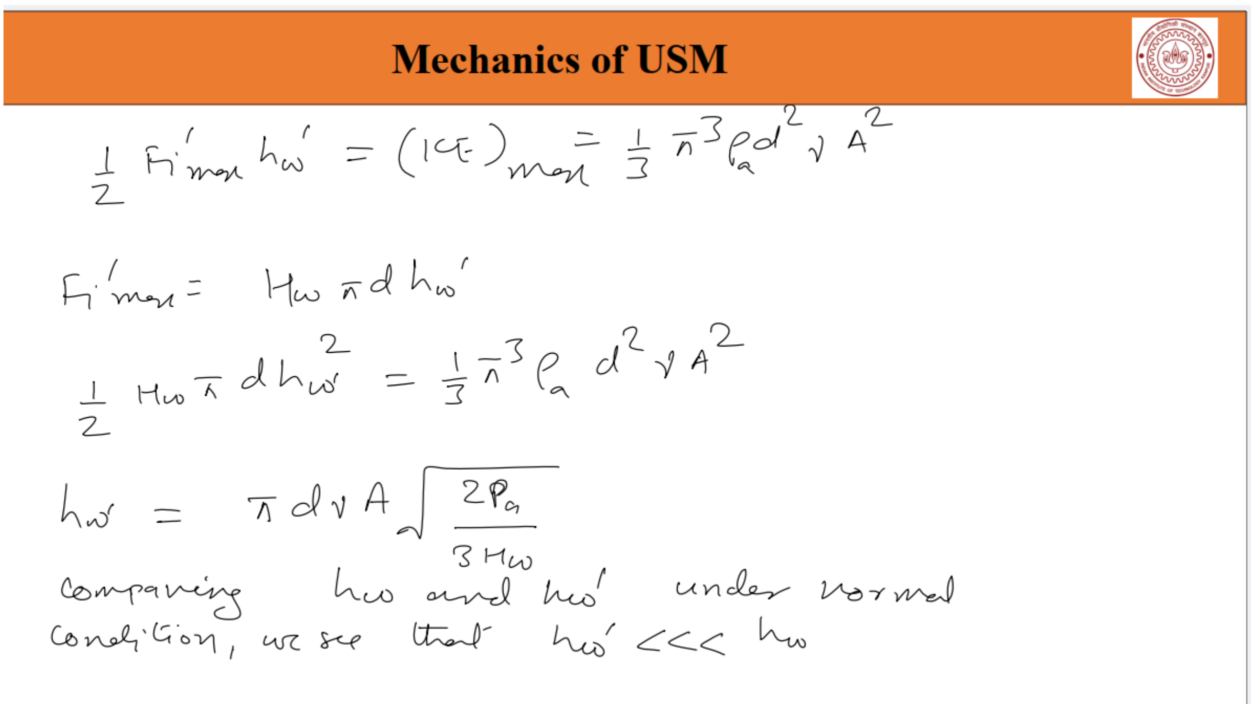
Mind you we are using different subscripts here because you know just to differentiate it from the case of direct impact where Fi average and hw were the 2 subscripts which were used there. So, this is the linearly varying model meaning thereby that when the force is 0 at the beginning and when the grain has not yet stuck on the surface and then the force slowly increases because the grain gives you know all its momentum all its energy to the surface and also faces the reverse force from the surface. And then after a while after the full indentation has been realized the amount of force at that point can be treated as Fi dash maximum. And then you can assume that the grain slowly releases contact meaning thereby that it flies off the workpiece surface and it goes all the way to force equal to 0. So, the area under this curve here showed by the shaded area is really the work done the amount of work done because of which the indentation has happened.

So, during indentation, an area is actually given here by half Fi dash maximum hw dash and so we equate that to the maximum kinetic energy of the grain that has been obtained before. So, therefore, you know we can easily find out. So, sigma w which is actually equal to the also the hardness of the workpiece. So, these are all flow stresses is related to the maximum force at that instant of point when the indentation had gone maximum.

So, Fi dash max per unit area. So, at that time if we assume that you know the total grain dia which has been projected onto the workpiece surface is capital D. And capital D is as you already know twice root of d hw where hw is this depth of indentation. So, if you remember the first exercise on USM that we had done this modelling that how about a grain with a diameter d impacting on a surface producing a depth hw. So, there was a relationship between this capital D here the projected diameter of the grain on the surface and the grain dia. So, therefore, force per unit area that you get out of this equation where F goes to Fi dash max the maximum force of the grain on the surface per unit the area at that time which we assume to be pi capital D square by 4.

So, we assume this area to be pi capital D square by 4 or in other words you can have this as Fi dash max divided by pi d hw. So, that is what has to be equated to the hardness of the surface or the flow stress of the surface for the condition that the grain would actually produce some deformation on the surface. And we already know from the previous equation that this KE max can be related to this Fi dash max and we would like to now formulate an equation for that. So, half Fi dash max times of hw dash where Fi dash max is the maximum force at maximum indentation hw dash this can be equated equal to this kinetic energy maximum which had come from the last derivation 1 by 3 pi cube rho A square of d nu A square. And therefore, also from the equation that you have derived earlier here in this particular instance let us call it equation A here.

From this equation, A you already know that Fi dash max can be equated equal to Hw pi d small hw dash where this is the maximum indentation depth of a freely flowing abrasive grain on the surface. So, thus if you substitute this in this particular equation for Fi dash max we get a formulation half Hw pi small d small hw dash square half F_i max dash times of hw dash is actually equal to 1 by 3 cube of pi rho A square of d nu A square. And that way you can actually have hw dash as the under root of twice rho A abrasive grain density by 3 Hw times of pi small d nu A. So, comparing this hw dash that you have obtained with the earlier hw that was for case of a you know hammered grain or a direct impacted grain we find out that hw is very high in comparison to hw dash.



You can compare both parallelly. So, if you may just recall in the earlier case the hw dash came out to be this whole 8 force average Ad by pi psi hw c 1 by 1 plus lambda. So, it really included a lot of terms and magnitude-wise this hw dash coming from the direct hammering action is always very high in comparison to the hw dash that you obtain by the free-flowing action of the grain. So, therefore, really the maximum material removal rate we can conclude here. So, the maximum material removal rate is highly dependent on the free flowing sorry the direct hammering action of the grain. So, it is dependent on the direct hammering action of the grain.

So, it can be concluded that most of the material is really removed by the direct hammering and very less amount of material comes out because of the free-flowing impact which is really not relevant to mention here also. And from the earlier relationship, we already have seen that the MRR Q is proportional to A 3 by 4 grain diameter cap small d to the power 1 by 4 average force to the power of average tool force to the power of 3 by 4 times of concentration to the power of 1 by 4 divided by hardness to the power of hardness of the workpiece to the power of 3 by 4 times of nu, where nu is the operating frequency A is the amplitude small d is the average small d is the grain diameter this is the average force and Hw is the hardness of the workpiece. And as I already discussed that the Q because of free-flowing grains the MRR because of free-flowing grains is negligible. Therefore, this really is the MRR value and therefore, it is safe to say that MRR is proportional to the d power 1 by 4 where d is the grain dia unfortunately that is not so, but because in experiments it has been observed that the material removal rate is proportional to the first power of d and not d to the power of 1 by 4 here. So, this was a discrepancy that you know arose from the Shaw's model because of which some explanation needed to be given.

Mechanics of USM

∴ It can be concluded that most of the material is removed by the directly impacting abrasive grain.

From earlier relation for MRR

$$Q \propto \frac{A^{3/4} d^{1/4} F^{3/4} C^{1/4} \nu}{H_w^{3/4}}$$

$MRR \propto d^{1/4}$ but in reality

$MRR \propto d$

So, that somehow this experimental data which comes out to be proportional to d could be easily fit inside the you know the data which has been theoretically derived by the Shaw's model. And so, therefore, Shaw actually tried to find out in reality what goes on or what happens. So, this discrepancy was addressed by Shaw finally, by looking at the overall shape of the grain. So, Shaw actually looked at the grain shape under a microscope and found out that the grain actually is not a spherical grain, but a sort of flowery structure on the surface something like this. And what really was impacting the workpiece surface was not this overall average spherical grain diameter small d as has been illustrated in many times in the model, but this small diameter here of 1 such you can say this can be a spherulite d1.

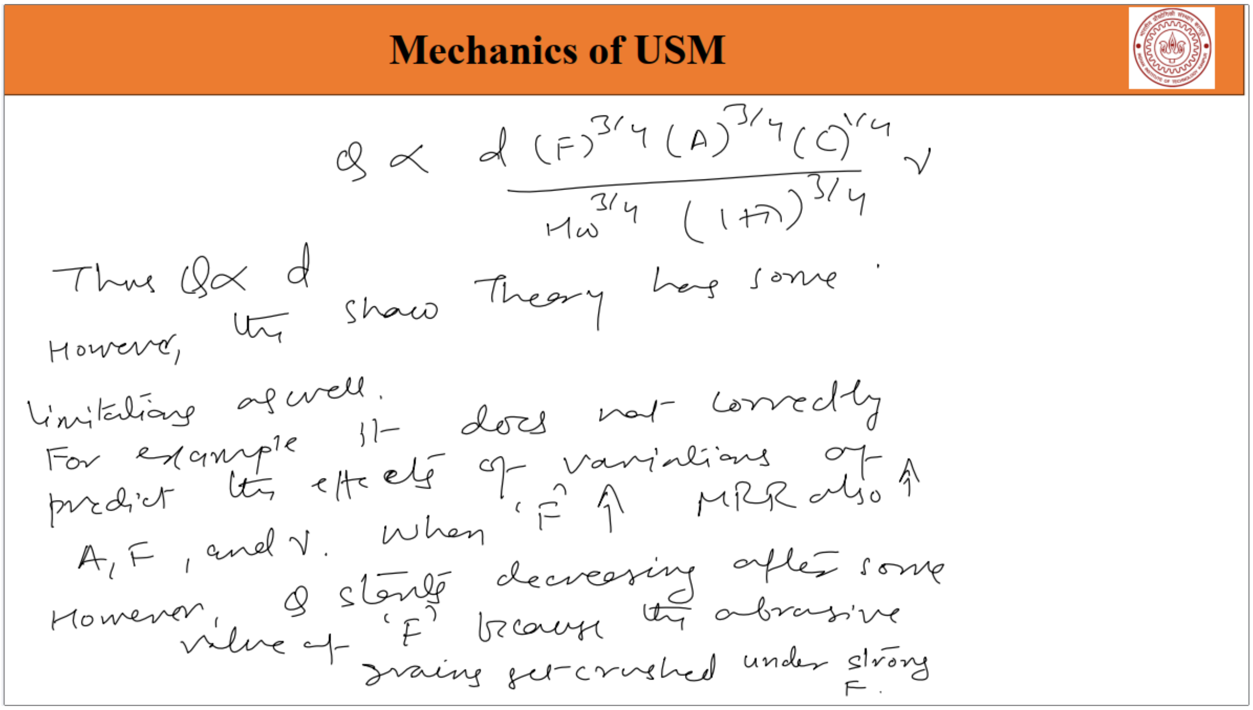
So, essentially this is the diameter which would affect the material removal process and it would in turn indent on the surface. The surface area also would be determined by d1 and not d. So, he very closely monitored if there exists a correlation between the grain diameter d and this small we can call it the projection diameter of the grain d1 dash d1. And what interestingly he found out is that yes there exists a correlation where you know these 2 things can be very well you know correlated as d1 the projection diameter being proportional to the square of the average grain diameter capital small d meaning thereby that if this diameter increases d1 almost increases as a square of this diameter. And therefore, it is safe to assume that d1 is actually equal to a constant mu times of square of d and this mu can vary between close to 1 or somewhere less than 1.

And that way you can have a very nice formulation between d1 and square of d. So, if you actually use this let us call this equation B in the theory of Q and Q you already know is proportional to this now it is d1 hw to the power of 3 by 2 times of z times of mu. Remember this is the volume of material removal and this d1 hw to the power of 3 by 2 is now indicative of the new projection grain diameter which is actually the diameter causing the indent or the impact on the surface times of the depth of indentation hw which does not remain we which remains almost same to the power of 3 by 2. And this is correlated by that formulation square of D is actually equal to 4 times of d1 hw. So, we are taking the modified diameter of this projection which is actually causing the indentation and trying to find the relationship between this diameter and the overall diameter here the grain diameter d.

So, if you put this expression d1 into you know this particular expression here you get that of course, hw as I already told you for a hammering case hammering grain case can be correlated by this relationship 8 F average times of amplitude A divided by pi Z d1 Hw 1 plus lambda. And we already know that Z is actually proportional to the concentration and inversely proportional to the square of the grain diameter. So, if we put all these together on the equation for Q the Q equation becomes equal to cube of d times of hw to the power of 3 by 2 times of psi c by d square nu. One thing which is interesting to observe here is that the Z value still is dependent on the average grain diameter value for obvious reasons that the number of particles which are making impact on the surface between let us say a fixed tool area and a fixed working surface is really determined by the area of an area of projection of an average overall grain. And the area of projection of an average overall grain is nothing, but proportional to the square of the diameter the average diameter of the grain not the diameter of the projection.

Projections can be many on a grain surface. So, that is why the Z value does not alter the Z value is still inversely proportional to the square of the diameter because that is ultimately the determinant of what would be the grain-to-grain spacing. The average diameter of the grain is the determinant of the grain-to-grain spacing between the tool surface and the workpiece surface assuming a fixed tool area. So, therefore, this expression here becomes conveniently changed and Q becomes conveniently proportional to the grain diameter the average grain diameter d which is in consonance with, of course, the experimental observation. And therefore, this gives you the total prediction of Shaw's theory towards the different parameters involved in the material removal rate of a USM process. So, what I am now trying to what I am now I will try to do is basically try to evaluate some of the characteristics typical characteristics of how Q will vary with what parameter.

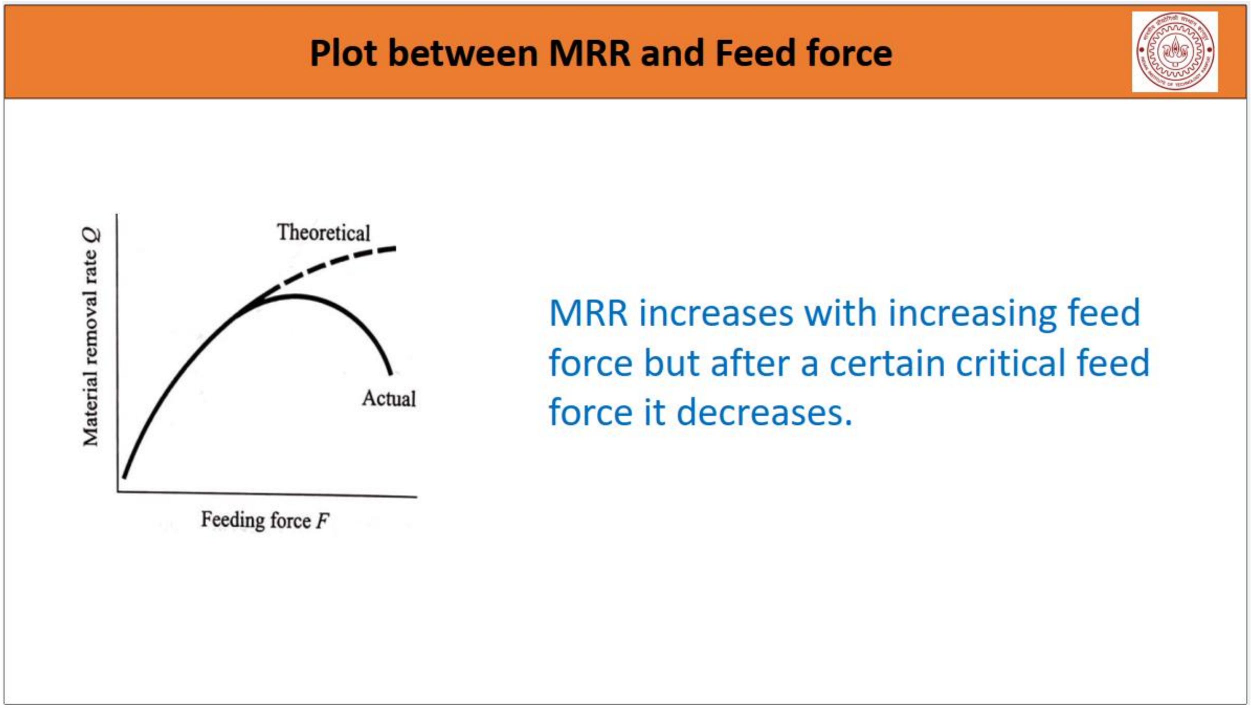
So, let us actually write this whole thing down here. So, Q as you know now is proportional to d times of the average force F average to the power of 3 by 4 times of amplitude of motion of the tool to the power of 3 by 4 times of c concentration to the power of 1 by 4 divided by the hardness to the power of 3 by 4, 1 plus lambda to the power of 3 by 4 times of nu, where nu is the average frequency. And thus, as you know that Q would increase if the grain diameter would increase. Obviously, because there is a direct proportionality between the two. And if supposing all these other parameters like the force average, the amplitude of motion, the concentration of the grain and the average frequency if they have increases, they would significantly impact the Q.



So, the Q increases because of them. And if the hardness of the workpiece is more then of course, the Q falls down. So, Q is inversely proportional to it. And also, so is true about the hardness ratio, hardness ratio and hardness ratio as you have earlier defined is very well defined as the relationship between what the workpiece hardness of flow stresses with respect to the tool hardness of stress. So, if the workpiece hardness is more, it is obvious that the Q or the material removal rate would fall down. So, that is in a nutshell what the predictions of Shaw's theory actually show.

And experimentally they have been many times verified by various people that these trends are actually true. So, we would now like to go ahead and look at some of the experimental trends of different you know aspects of the Shaw theory. And how actually and theoretically predicted values would differ above a certain limit of 1 parameter may be. So, one case is the MRR plot of with respect to the feed force or the average force. So, this actually is a plot between the average feeding force F average as you saw as obvious to assume that if F average is more than Q is more.

So, theoretically predicted trends would look something like this which is represented by this dotted line here. As if the F average keeps on increasing and the Q should increase. But then what is interesting here is that above a certain limit of the feed force let us say above a certain limit of the F average force the there is a depreciation of the material removal rate. And the material removal rate comes down up to a certain critical feed force. And that happens because of a very important effect which practically you know almost always happens into in these USM systems which is also known as the grain crushing effect.



So, if the feed force is higher than higher to a value that this F average per unit area of the grain actually equals to the you know the ultimate flow stress of the grain itself abrasive grain itself. So, therefore, there is a possibility that the grain itself would get broken into pieces and there is a crushing effect. So, the number of active grains which are now valuable at that critical feed force would simply you know go down. So, that because they are themselves getting crushed. And therefore, the material is almost always reasonable to assume that because of this crushing effect of the grains etc.

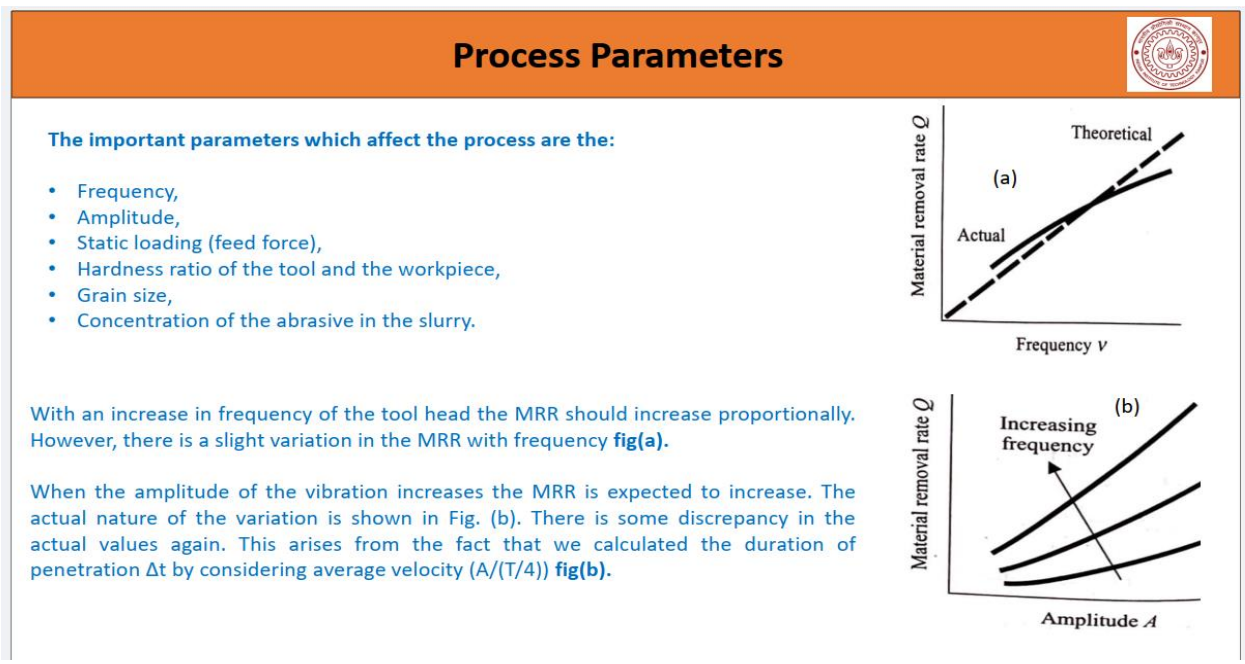
The number of available complete grains which come between let us say the tool head and the workpiece are lessened and so would be the material removal rate. And therefore, the actual trend of the material removal rate is shown by this particular illustration. So, this really is a critical force above which the grain crushing would start to take place. Critical force at which grain crushing effect would be observed. So, that is in a nutshell what would happen to the trend of material removal rate with respect to the feed force.

There are some other interesting factors to be discussed. For example, as I have already pointed out that with frequency, if the frequency goes high and the material removal rate would go high. So, is visible in this particular trend here. Of course, you know the actual varies slightly from the theoretical. Although, the theoretical shows almost a direct relationship, linear relationship with increased frequency. But the actual is slightly different because of reasons associated with the inertia of the slurry and the inertia of the tool head.

So, is the case with amplitude as you may recall that amplitude if it increases here A is proportional to 3 by 4. So, therefore, sorry to the A to the power 3 by 4 is proportional to the Q

MRR material removal rate. So, therefore, any increase in amplitude would also record an increase in the Q value which is true here. As you see in 1 of the cases for a certain frequency let us say nu 1 for you know an increase in the amplitude there is a recorded increase in the material removal rate. And if supposing the nu the operating frequency keeps on varying between let us say nu 1 to nu 3 when nu 3 is greater than nu 1.

You can see that there is a double effect. So, 1 is the effect because of amplitude and another is an overall increase because the frequency domain in which you are operating and mind you frequency is proportional to the MRR here is also increasing. So, as you increase the frequency the overall material removal rate with different you know for different frequencies of the amplitude, they would have a linear increase. So, we have already studied this aspect the feed force where you saw that there is a grain-crushing effect which is there. And some other trends that can be useful are that related to the you know what would happen for example, with increasing amplitude and feed force.



And so, this actually is illustrated by this particular figure here. So, with an increasing amplitude if the feed force is higher for every feed force there is a crushing critical limit. For example, if the feed force is at a lower amplitude meaning thereby that the gap between the overall gap between the pool surface and the workpiece surface is lower. So, at a certain critical feed force value here the grain crushing would happen, and this would keep on increasing. So, the critical limit of the feed force goes on increasing as you can see here at which grain crushing begins. For example, at a lower amplitude, it begins much earlier and at a higher amplitude, it begins later.

And that is probably obvious because the gap in this case between the tool and the workpiece is more. And so, you know it is important to see if the gap is more then the critical feed force which would be needed for having this grain crushing effect would actually be higher. Because the tool has a higher relaxation time for going from the surface all the way towards this other extremity amplitude of motion is more. So, if you have more relaxation time then there is a possibility of crushing to happen at a higher feed force in comparison to if you have less relaxation time in case of a lower amplitude. Also important is that if the lambda value that is the work hardness to the tool hardness as I had illustrated before is increased there is a reduction in the material removal rate which comes obviously, because of this equation here as you know 1 by lambda to the power of 3 by 4 is inversely proportional to the mean material removal rate Q.

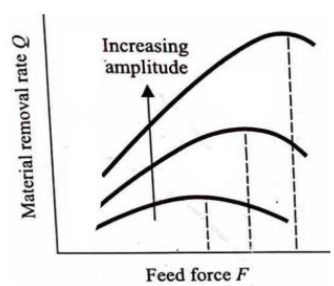
And therefore, it is good to assume that if lambda increases the material removal rate would fall down. And these are some of the relative material removal rates for a frequency of let us say 16.3 kilohertz of the vibrating tool head and amplitude of 12.5 micrometres of the vibrating tool head and a grain size of 100 mesh. So, you can see that for different work materials like more brittle materials glass, the material removal rate is very high which effectively means that the work hardness by tool hardness is lower in this particular case.

Process Parameters

We already said that with an increase in static loading, the mrr tends to increase. However, at higher force values of the tool head due to grain crushing the mrr decreases.

The ratio of workpiece hardness and tool hardness affects the mrr quite significantly, and the characteristics is shown below.

Apart from the hardness the brittleness of the work material plays a very dominant role. The table below shows the relative mrr for different work materials. As can be seen the more brittle material is machined more rapidly.



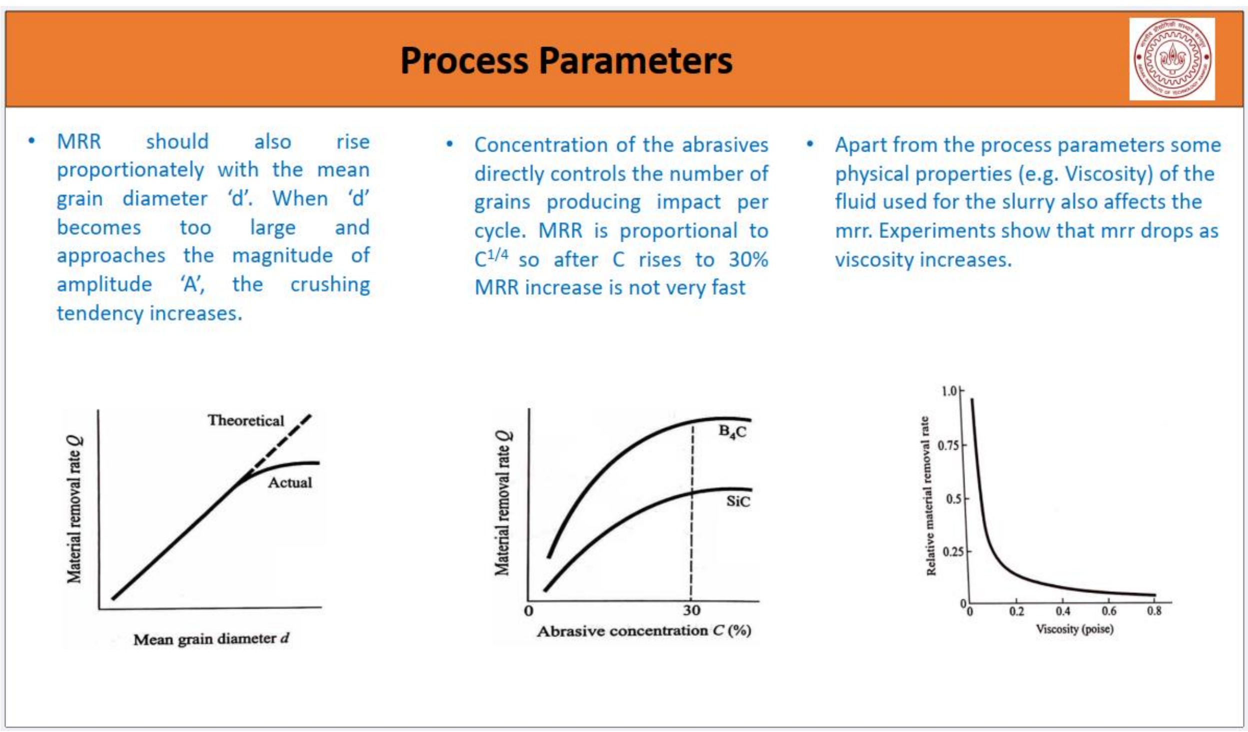
Relative material removal rates
(v = 16.3 kHz, A = 12.5 μm, grain size = 100 mesh)

Work material	Relative removal rate
Glass	100.0
Brass	6.6
Tungsten	4.8
Titanium	4.0
Steel	3.9
Chrome steel	1.4

And if it is more ductile in nature as is going slowly on a higher and higher scale you can see the MRR is reducing because of change of material here. Of course, the hardness and the brittleness both of the work material plays a very dominant role in this process. And therefore, particularly in MEMS applications or microsystems applications when we talk about silicon micromachining or when we talk about glass micromachining, and they are very brittle in nature. So, the paradigm is really very high material removal rate which has to be well controlled. So, that you can actually have a small channel imprinted through a masking technology that will probably show at the end of all this fundamental process analysis of the mechanical kind.

So, that is what how these processes would be applied to fabrication of microsystems technology. And so, basically, there are certain other aspects which I would also like to point out here. For example, let us say if we talk about how the variation of material removal rate would be with the mean grain diameter. It is obvious to assume that as the grain diameter increases the material removal rate theoretically should be proportionally increasing as you already have mentioned earlier that Q is proportional to the mean grain diameter d. But, again the important aspect of grain crushing comes here because if the grain is too high in diameter there is a tendency of the tool to crush or start crushing the gains as you can see here.

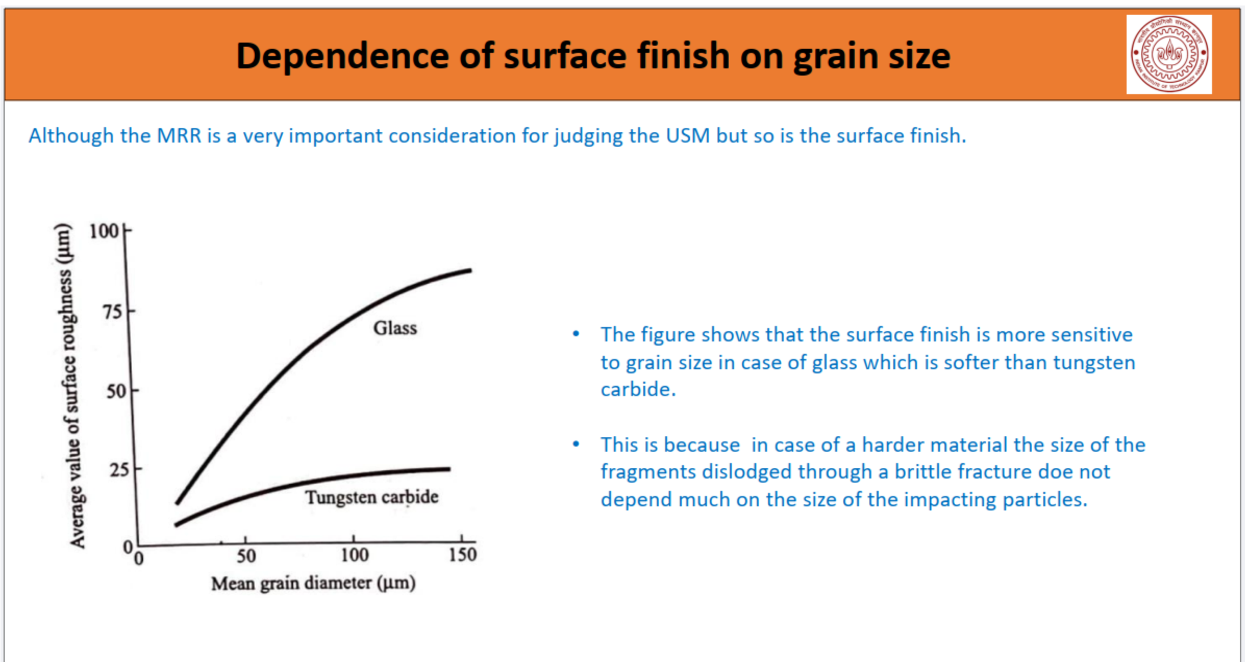
So, crushing of grains and the moment this crushing phenomenon happens as you know the MRR goes down. So, the actual value of MRR for a higher diameter grain greater than let us say a critical diameter d1 here would be more would be not following the theoretical trend, it would actually start coming down. And so, is true with concentration. So, for example, you know if you keep on loading the grains in the slurry at higher and higher concentrations for 2 different materials it has been proposed here. Let us say for boron carbide with the different hardness and grain hardness and silicon carbide with the different relatively lower grain hardness.



You can see that with the increase in the abrasive concentration, the MRR kind of plateaus and that is because you can always between the tool and the workpiece. If supposing this is the tool surface and this other is the workpiece surface and there are lot of grains on it. So, you can only pack this area available of the tool to its fullest capacity. For example, if you load more number of grains this density of the grains per unit area of the tool-workpiece surface would keep on increasing up to only a certain limited value beyond which any further grains cannot be accommodated. So, even if the concentration is increased beyond that any further you do not see much you know material removal because the amount of grains which are at probably the critical concentration here are fully packed into this area.

So, therefore, there is a plateauing action of the MRR with the increase in concentration beyond a certain critical concentration. So, is the case with viscosity a very important term for the slurry particularly when you already know that at the very beginning, I had mentioned that the MRR in a USM is really dependent on how or what the constitution of the slurry would be made up of abrasive particles and a fluid medium. And so, if the viscosity of the slurry is more meaning thereby that the you know interlayer shear between the fluid carrying the particles are more there is a tendency that you know it will have a creepy motion or just like molasses it will move very slow. And because of that all the material which comes out essentially because of indentation etcetera would not be easily dissolvable in such a situation. So, the diffusion gradients that need to be established should be very high for the debris material which is formulated because of the indentation and the brittle fracture do not get carried away very easily in that case.

So, therefore, with an increasing viscosity as you have seen there is a relative reduction in the material removal rate as can be illustrated from this trend here very important to know that if the viscosity is higher the removal of the material debris that would happen would be kind of at a lower rate. So, that is in a nutshell what some of the trends operating trends would be. Another interesting factor is what happens you know for a brittle and a harder material. For example, in this case, you can compare 2 such materials of average surface roughness values in microns between tungsten carbide and glass as you can see here. And with the mean grain diameter increased, of course, there would be a critical grain diameter beyond which there would be grain crushing which takes place.



But what is important here to see that if the brittle if the surface is more brittle then the surface roughness value which would eventually arrive at would be higher in comparison to a more harder material. For obvious reasons that a brittle material would be more amenable to brittle fracture and greater chunks of pieces or materials would come out and they would form in turn larger craters and because of the larger craters, the overall average roughness of the surface would be higher. So, these are some of the dependencies of the various parameters associated with the AJM process. And what I would like to next do today we are of course, at the end of the lecture, but we would try to design some USM problems and predictably ascertain what is the material removal rate which would emanate from such a design.

So, in probably the next class whatever theory we have learnt by the M.C. Shaw's model of material removal where we saw that the prominence of the direct impact or the direct hammering is much more in comparison to the free-flowing grains and the way that that removes material. We would like to now design some problems in a manner so that we can estimate the material removal rate. So, you have a ballpark idea of what are the rates that we are talking about and in terms of the specific energy that is needed through this process as opposed to some of the other comparative processes we will try to compare. And then of course, once we are done with all that designing, and the very important aspect of tool design would be taken into picture. And finally, we would like to apply these two microsystems fabrication technology. Thank you.