Advanced Machining Processes
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Week - 9
Lecture – 24
Heat Conduction and Temp. Rise for a circular spot, Numerical problems

Hello, and welcome back to this lecture of Microsystems Fabrication by Advanced Manufacturing Processes. Let us recap quickly what we did last lecture. We talked about laser beam machining, and we probably already know that it is basically the photon-to-matter interaction which converts, which essentially converts into the physics of photon to phonon conversion. And so, there is lattice vibrations induced by the high level of energy pumped in by the photons which causes local temperature to rise, and it is a surface phenomenon. So, the temperature rises to an extent because of an extremely high intensity of energy packed in so that it goes into the vaporization state, and it creates a sort of burst effect and that is why the machining probably is much faster and it is a surface phenomenon which is different than the E-beam machining where E-beam typically percolates to a region or a small skin which is called the beam transparent layer. So, it is more towards the surface that the LBM is geared to.

We talked about introductory concepts of laser, laser being light amplification by stimulated emission of radiation wherein the first stage there is an absorption, there is a frequency which is sent in, frequency of light which results in a very quantized manner an electron to go to a higher orbital from a lower state and an avalanche of these processes are produced together which is known as population inversion. Once this state is reached all the high energy atoms are when incident or when having the same frequency or same energy coming out of a photon which is fresh and which strikes the inverted population system leads to essentially a higher energy, but a more stimulated energy which is independent of the intensity, and this is called stimulated emission. So, there is a huge coherent amount of energy which is lost suddenly, and this energy is tried to retain until there is a threshold which happens after which the energy starts escaping which is the laser beam. So, we also talked about various lasing media like solid state and gas phase lasers where the medium changes from solid to gaseous in nature.

Solid phase includes of course, ruby crystals other different kind of such media where the population inversion is possible with a certain frequency and gas phase of course, can vary from helium to argon, neon all these different kinds of media for lasing action take place. So, some facts of laser-mediated machining was discussed the amount of the magnitude of intensity that is amount of power per unit area clubbed into a surface machining surface is very-very high in laser machining. Also, the time that that makes the time of machining extremely small duration of machining is very small in laser and then of course, because you can super focus through expensive optics the beam to a small spot there is a tremendous increase in the resolution of the system. These days lasers are used for laser beam-based lithography where up to size of about 2 to 1 micron range is possible using laser beams. Also discussed about the mechanics of material removal where there is a photon-to-phonon conversion and started working on the thermal model where we would assume a constant heat flux on a small area or region on the surface where the beam on the beam incident side of the surface and with that we would try to make a model of the heat transfer based on material properties of the work material like thermal diffusivity, density specific, volume-specific heat, conductivity so on so forth.

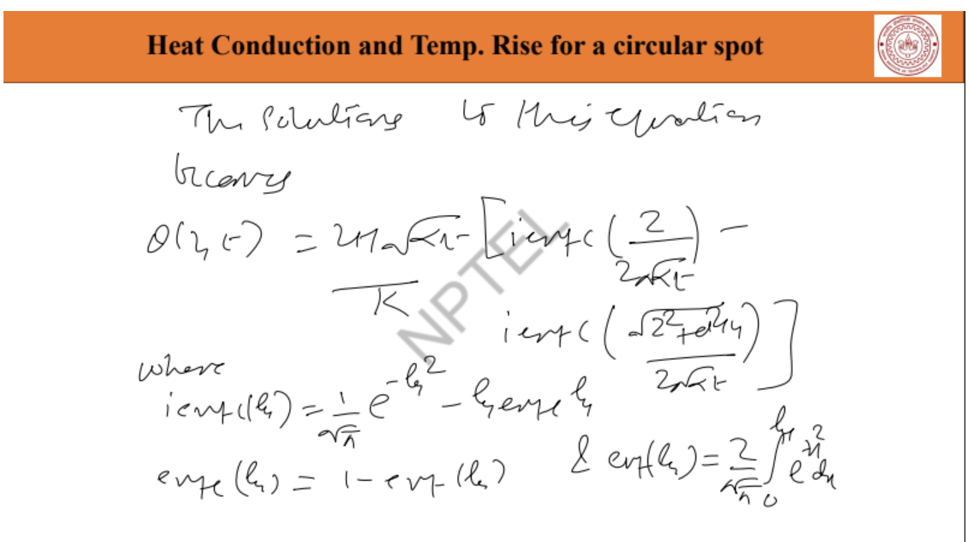
So, let us go back on that model and try to extend this forward a little further. So, we already have seen how the beam can be modeled in a one-dimensional heat using the principles of one-dimensional heat conduction where the depth from the surface is considered to be z and t being the time. So, temperature with respect to different depths from the surfaces z as a function of time was given by an equation represented here as del square theta Z t by del Z square minus 1 by alpha del theta Z t by del t equals 0. We assumed boundary conditions that at the surface of the workpiece corresponding to there would be a constant temperature gradient with respect to the z-direction the depth and it was given by minus 1 by K H t and we further assumed by that it was a semi-infinitely long surface and that the thermal properties of the workpiece remain unchanged and assumed theta equal to 0 at time t equal to 0. Now, this is actually the baseline temperature which is actually the room temperature about 24 degrees or so, but then we assume that to be the baseline and so theta can be equated to 0 at time t equal to 0.

So, assuming that we arrived at a solution using a conventional PDE methods where theta Z t was represented by twice H by K root over alpha t by pi exponential minus z square by 4 alpha tau minus tau by 2 error function the first kind z by twice root alpha t. If we tweak this solution to the boundary conditions which have been represented here, we have at z equal to 0 theta 0 t simply represented as twice H by K root over alpha t by pi this goes off and so does this and so therefore, we are actually left with sorry I just need to rewrite this. So, this is z we are actually left with very simplistic expression theta equal to 2 H by K where K is the thermal conductivity to square root of alpha t by pi t being the time alpha is the thermal diffusivity it is the ratio of the conductivity and the volume-specific heat of a material. And if you want really machining to happen you have to assume that this theta 0 t of the surface hits the melting point of the surface almost immediately. So, if the melting temperature of the surface is known we can tentatively estimate the time of machining tm to be pi by alpha theta m K divided by twice H square.

So, that is how time of machining is. We try to calculate in some cases what typically this time would be, and we obtain for normal system where we were trying to machine tungsten surface with only a 10 percent coupled power of the beam, we obtain the time of almost about close to 53 microseconds or 5.3 microseconds which is actually very small number in terms of time of machining. So, this process is really very fast for in comparison to some of the other processes that have been illustrated before. So, let us now work on slightly different problem.

We already have from before the equation delta theta z t minus d z 2 minus 1 by alpha delta theta z t by del t equal to 0. We know that the boundary conditions on the surface become theta z at point of time 0 is 0 and del theta by del z that means the gradient of temperature on the surface in the z direction on the surface corresponding to theta equal to let us say 0 at some point of time t is given by minus 1 by K H t, H t being the heat flux and which is actually constant and continuous on the surface and that is how semi-infinite region being exposed to a laser beam can be modeled. We slightly change the connotation of the problem by converting this semi-infinite region into a circular region meaning thereby that the beam actually now has a diameter d. So, you have a laser beam circular of diameter d, and you want to actually try and see how you model this equation for a circular laser beam which is more realistic and closer to the real-world situation. So, here the boundary conditions have to be slightly tweaked because of that and the new boundary conditions become theta z equal to at any point of time is at a point of time t equal to 0 is 0 because the laser beam is supposed to just get start irradiating the surface at time t equal to 0.

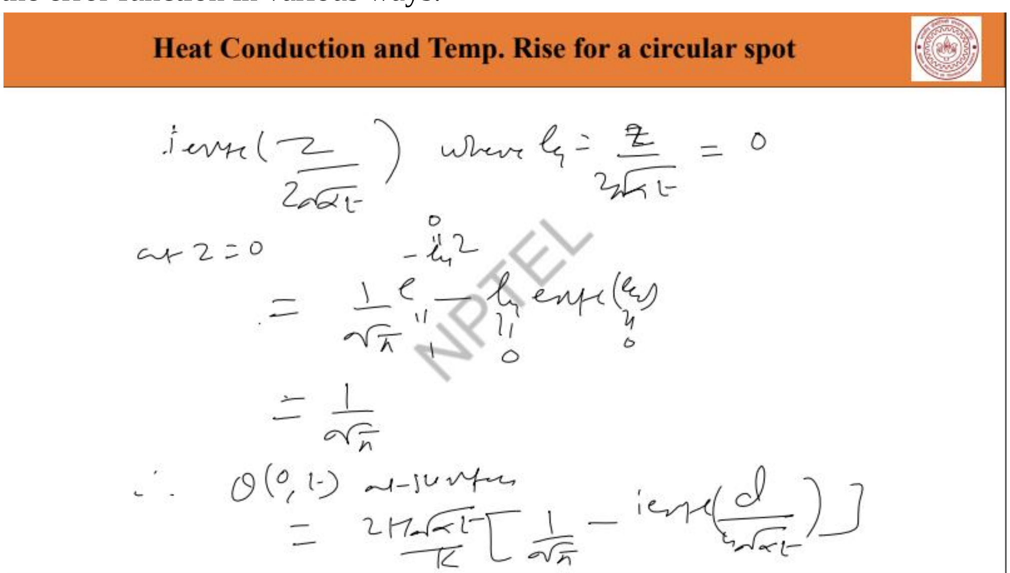
Therefore, the temperature is still the room temperature the baseline temperature. The only other difference which would have in this particular case is that the gradient of temperature at the surface for corresponding to all different points of time t equal to 0. Now, really becomes a function of beam diameter and we can consider this to be minus h t by K pi d square by 4.



So, if we assume this to be the new boundary conditions the solution that would emerge to this equation for a circular beam become equal to theta z t equals twice h root of alpha t by K times error function of the third kind z by twice root of alpha t minus same again times root of square of z plus d square by 4 divided by twice root of alpha t.

Just worth mentioning that these three different error functions of different kinds would be represented as the basic error function of variable zeta is the numerical integration 2 by root pi integral varying between 0 and zeta e to the power of minus square of x d x.

The second kind is basically a variation of this error function and just write it algebraically as 1 minus error function zeta. This is only for simplicity sake that we are assuming this and there is another representation of the same error function, and we call it error function third kind or third type it is 1 by root pi e to the power of minus zeta square minus zeta times of error function of zeta that of second kind. So, we call this third kind. So, that is how we have defined these different values if you may recall in the earlier slide as well, we had if the heat flux were a step function meaning thereby that we assume that the heat starts at point of 0 at point of time t 0 equal to a constant heat flux H and continues there in for all points of time. So, it is like a step function in that case the solution that came out involved this second kind and it is basically nothing but 1 minus error function and just for simplicity sake for algebraic representation, we are trying to represent the error function in various ways.



So, that you can shorten the notational representation of the whole formulation that has been arrived at. So, here the same thing is done with a third kind which again is slightly complex form of what we had the error function 2 or second type. So, in a nutshell if we were to really find out the value of theta 0 t. corresponding to z equal to 0 and for all points of time t that means the temperature variation on the surface with the exposure to the beam starting from point of time t equal to 0 onwards. So, it just amount to putting the value of z to be 0 in this particular expression here and trying to find out how it would behave with respect to this new value.

So, the value of the error function of the third kind ierfc zeta for zeta which is equal to again z by twice root of alpha t as we have assumed in earlier in this coefficient here, right here that is what the zeta value is. So, this becomes equal to very simply just 1 by root of pi e to the power of 0 minus 0 error function of zeta equal to 0. So, this is simply 1 by root pi. So, if we represent this value or we substitute this value in the equation for the temperature on the surface theta 0 for all point of time theta 0 t we get this essentially boils down to let us just first write the whole expression twice H alpha t by K the error function of third kind z by twice root of alpha t minus the error function of the third kind again root of z square plus d square by 4 by twice root of alpha t corresponding to z equal to 0. So, this further becomes equal to 1 by root pi we just derived it in the last step minus ierfc the error function of the third kind ierfc and the z goes away here.

So, it is basically d by 4 root of alpha t. So, that is essentially how the temperature variation on the surface of the machined workpiece the function of time can be represented as. So, if you know the beam diameter in this particular case d is the beam diameter and you are a pair of the different material properties of the material like K alpha so on so forth, and you also are aware of the coupled heat flux which in this case also is assumed to be like a step function. So, starting at time t equal to 0 you have a finite heat flux H which translates over all point of time in space. So, all point of time and so therefore, the theta the surface temperature as a function of time can really be equated to the melting point or melting temperature of the workpiece material, and you can have a good estimate of the time of machining based on looking at the various values obtained in this formulation here let us call it equation 1.

Numerical Problem

A laser beam with a power intensity of 10^5 W/ mm^2 falls on a tungsten sheet. The focussed diameter of the incident beam is 200 microns. How much time will it take for the center of the circular spot to reach the melting temperature (3400 deg. C). thermal conductivity = 2.15 W/cm. deg. C, Volume specific heat = 2.71 J/ cm^3. deg. C. Assume that 10% beam is absorbed.

Substituting the appropriate values we get $H = 10^7 \, W/cm^2$ $\alpha = 0.79 \, cm^2/sec.$

$$3400 = \frac{2 \times 0.1 \times 10 \sqrt{0.79 t_m}}{2.15}\left[\frac{1}{\sqrt{\pi}} - ierfc\left(\frac{0.2}{4\sqrt{0.79 t_m}}\right)\right]$$

$$ierfc\, l_1 = \frac{1}{\pi} e^{-l_1^2} - l_1\left[1 - erf(l_1)\right]$$

Let $l_1 = \frac{1}{200\beta}$ & $\beta = \sqrt{0.79 t_m}$

So, let us just now slightly change this problem we had an earlier problem here where we found out assuming the surface or the workpiece to be a semi-infinite region we found out that the time of machining in this case was a very small value about 53 microseconds. Now, the same problem if we just change the beam from interacting with the semi-infinite region of the workpiece to a circular beam of diameter d how the whole time would get modified let us have a clear look at it. Now, in this case we have tweaked the problem slightly. So, you have a focused beam of diameter or 200 microns and remaining conditions being same the workpiece is tungsten sheet power intensity is about the same 10 to the power 5 watts per millimeter square assume about 10 percent absorption remaining 90 percent is reflected of the surface, and then all other properties like thermal conductivity volume-specific heat are given for tungsten sheet. So, it is merely the same problem with assuming in this particular case the beam is not a semi-infinite beam, but it is actually a circular beam of diameter 200 microns.

Numerical Problem

$$\therefore 3400 = 9.30 \times 10^5 \times \beta\left[\frac{1}{\sqrt{\pi}} - \frac{1}{\sqrt{\pi}} e^{-\xi^2} + \xi\{1 - \mathrm{erf}(\xi)\}\right]$$

$$\mathrm{erf}(\xi) = \frac{2}{\sqrt{\pi}} \int_0^{\xi} e^{-\xi^2} d\xi$$

$\therefore$ erf(ξ) has been computed for different ξ as per the following table

Let us see how the difference would come in terms of machining time for both the cases. So, let us write down on the complete equation for the temperature theta 0 T equals theta m and this particular case as you know it is 3400 degree Celsius, and this can be equated to twice H root of alpha t by K times 1 by root of pi minus ierfc error function of third kind d by 4 root of alpha t. We plug in the various values here for example, H in this particular case is 10 percent of 10 to the power of 5 watts per millimeter square amount of power which is coupled comes out to be 10 to the power of 4 watts per millimeter square, and we have alpha in this particular case as 2.15 watt per centimeter degrees Celsius volume-specific heat rho c 2.71 joule per centimeter cube making I am sorry this is K thermal conductivity making the volume making the thermal diffusivity alpha the ratio of K by rho c 2.15 by 2.71 this comes out to be about 0.79-centimetre square per second. So, with all these parameters from the question we try to plug in these into this equation here and try to obtain the calculate the value of the time of machining t which is involved at several places here in this equation as you can see. So, we get 3400 equals twice times of if we just prefer converting this into watts per centimeter square this comes out to be equal to 10 to the power of 6 watts per centimeter square, because you know everything else has to be consistent the units have to be all in CGS, and so to the reasonable extent and what we are left with is 2 times of 10 to the power of 6 times of root of 0.79-centimetre square per second times Tm the value of the machining time divided by K which is 2.15 watts per centimeter degree Celsius times of 1 by root of pi minus the error function of the third kind of this whole term. So, let us convert this 200 microns into centimeters. So, this comes out to be 210 to the power of minus 6 that is about 210 to the power of minus 4 meter or about 0.02 centimeters.

So, this can divided by the term 4 root of again 0.79 t m. Let us call this value beta root of 0.79 t m, and we try to calculate the value for beta from this particular equation here right here. So, the first thing we need to do is to put the beta value in write in this equation here, and we can rewrite this equation as 3400 equals this whole thing can be calculated and written down as 9.30 times 10 to the power of 5 times of beta, this is the beta value times 1 by root of pi minus the error function of the third kind of this whole term here which we call zeta. So, therefore, as you can rightly see here that zeta is basically nothing but 0.02, I am sorry let us just write this thing here. So, 0.02 by 4 times of beta which makes it equal to 1 by 200 beta. So, that is how zeta can be classified. So, in summary, beta has been estimated as 0.79 time of machine t m, and zeta new parameter here is 1 by 200 beta.

And we can write down this equation if as you already know the error function of the third kind of a parameter zeta can be represented as 1 by root of pi e to the power of minus zeta square minus zeta error function of zeta. So, here the zeta value of course, is 1 by 200 beta. So, we can have this represented as 3400 equals 9.3 into 10 to the power of 5 times of beta times of 1 by root of pi minus 1 by root of pi exponential to the power of minus zeta square plus zeta times of 1 minus error function of zeta.

That is how you can represent 3400, and you already know that the error function of zeta is actually twice by root pi integral 0 to zeta e to the power of minus x square dx. This is a numerical integral, and so you have standard tables calculating the error function as an area under the curve of e to the power of x square with respect to x, and for different zeta values zeta ranges varying between 0, and different zeta values real values of zeta you have different error function values. So, this is represented here in this particular table as you can see for certain coefficient x here this represents the error function of x. And so, this essentially the zeta over equal to 2 0.2 the area under the curve twice root of pi of the area under the curve which is actually the error function is 0.2227. And so, you can take this all the way to about 2 for the curve to have complete area of about 1.

So, actually 2 4, so essentially the variation from 2 to 4 is very less as you can see almost going to 0.9953 here, and then goes to about 0.9999 at 2.8 x value, and then after that following this whole region it is about unity.

So, it converges the value converges at about 4 the error function. So, in this way we can actually try to estimate by using a software the various values of zeta equal to 1 by 200 beta by plugging the beta value and trying to see what the error function comes out to be equal to plug this back here, and numerically try to determine what beta can be. So, the equation can really be solved by numerical methods in an iterative manner.

Error function tables

Error Function, Sine and Cosine Integrals [see (35), (40), (42) in Appendix A3.1]

x	erf x	Si(x)	ci(x)	x	erf x	Si(x)	ci(x)
0.0	0.0000	0.0000	∞	2.0	0.9953	1.6054	−0.4230
0.2	0.2227	0.1996	1.0422	2.2	0.9981	1.6876	−0.3751
0.4	0.4284	0.3965	0.3788	2.4	0.9993	1.7525	−0.3173
0.6	0.6039	0.5881	0.0223	2.6	0.9998	1.8004	−0.2533
0.8	0.7421	0.7721	−0.1983	2.8	0.9999	1.8321	−0.1865
1.0	0.8427	0.9461	−0.3374	3.0	1.0000	1.8487	−0.1196
1.2	0.9103	1.1080	−0.4205	3.2	1.0000	1.8514	−0.0553
1.4	0.9523	1.2562	−0.4620	3.4	1.0000	1.8419	0.0045
1.6	0.9763	1.3892	−0.4717	3.6	1.0000	1.8219	0.0580
1.8	0.9891	1.5058	−0.4568	3.8	1.0000	1.7934	0.1038
2.0	0.9953	1.6054	−0.4230	4.0	1.0000	1.7582	0.1410

Of course, we can start with the value of beta corresponding to the semi-infinite region time which was obtained 53 microseconds is 0.000053 seconds, and putting this t m value get the corresponding beta value, plug this beta to find out the zeta value, and the first approximation first iteration that can come out from the zeta is by plugging, and playing with the zeta here this is the error function you can calculate what is the e to the power of minus eta square by root of pi, put it back here, and try to find out how close this equation comes to 3400.

Numerical Problem

$$\therefore\ 3400 = 9.30\times 10^5 \times \beta\left[\frac{1}{\sqrt{\pi}} - \frac{1}{\sqrt{\pi}}e^{-\zeta^2} + \zeta\left\{1 - \frac{2}{\sqrt{\pi}}\int_0^{\zeta} e^{-\zeta^2}\,d\zeta\right\}\right]$$

Using the table & numerical integration-

$\zeta = 0.5$

$\therefore\ \beta = 0.01$ & $t_m = 0.00078\,ms$

Then you can actually vary the t m to a slightly lower value and try to again estimate beta vary the t m to a higher value, and try to estimate the beta, and see what is the trend here is it going closer to 3400 or far away from 2400 this equation. And so based on that you can actually figure out a good beta value for which this whole expression right hand side would be equal to the left-hand side. In this particular case using the table, and the numerical integration value the solution of this equation 3400 equals 9.3 10 to the power of 5 times beta by 1 by root of pi minus 1 by root of pi e to the power of minus zeta square plus zeta times of 1 minus twice by root of pi integral 0 to zeta e to the power of minus zeta square d zeta. This comes out to be corresponding to a zeta value of 0.5, beta of 0.01, and time of machining 0.00073 seconds sorry it is 13 second. So, the time of machining in this particular case, as you can see 0.00013, is very small in comparison to the very large in comparison to the time of machining which was earlier for a semi-infinite region for obvious reasons that you are trying to reduce the beam area from a semi-infinite interaction with the workpiece to almost a small value of the diameter d equal to 200 microns. So, therefore, more time would be needed for this machining to happen because of heat losses across the beam boundary to the remaining part of the solid. So, we will just see the effect of power intensity.

Heat Conduction and Temp. Rise for a circular spot

by the general approximation considering a flat-beam

$t_m = 0.0000537$ secs

This is different than the more accurate exp. with circular boundary condition.

If power intensity high and

$\frac{1}{\sqrt{\pi}} >>> \text{ierfc}\left(\frac{d}{4\sqrt{\alpha t}}\right)$ $\theta_m = \frac{2H}{K}\left(\frac{\alpha t_m}{\pi}\right)^{1/2}$

So, if let us suppose the power intensity in this particular case is H is very high of course, because of a higher H the t m should reduce, and if the t m reduces then you have this value of zeta here which has been estimated to be 100 1 by 200 beta root of 0.79 t m. So, as the t m reduces zeta value goes up. So, some changes should happen in this particular equation based on if the zeta is either reducing or increasing. So, supposing there is a case when the power intensity of the beam is low or time of machining is high, and subsequently the zeta is falling down here, zeta goes down.

So, obviously, if we look at this part of the equation here, this equation or this part of the equation with the smaller value of zeta should typically go down. For example, if zeta were approaching 0 then the let us just write down the equation once more 3400 equals 9.3 10 to the power of 5 times of beta times of 1 by root of pi minus 1 by root of pi to the power of minus zeta square plus zeta times of 1 minus error function of zeta that is what this term is corresponding to in this particular case. So, if zeta is going to 0 then this term goes to 1, and effectively we are having a zeta value here which is d by 4 root of alpha t as we have already cited before, and of course, it is small, but then we just want to find out because there is a smaller term which is here which is also beta. Let us find the overall effect on this equation because of that.

Heat Conduction and Temp. Rise for a circular spot

If power of beam decreases t_m increases

Then

$$\theta(0,t) = \frac{2H\sqrt{\alpha t}}{K}\left[\frac{1}{\sqrt{\pi}} - \frac{1}{\sqrt{\pi}}e^{-\zeta^2} + \zeta(1-\text{erf}\,\zeta)\right]$$

where $\zeta = \frac{d}{4\sqrt{\alpha t}}$ as $t_m \uparrow$ $\zeta \downarrow$

$$\therefore \theta(0,t) = \frac{2H\sqrt{\alpha t}}{K}\left[\frac{d}{4\sqrt{\alpha t}}(1-\text{erf}\,\zeta)\right]$$

$$= \frac{Hd}{2K}[1-\text{erf}\,\zeta]$$

So, one thing is that if this goes to 1 corresponding to zeta tending to 0 these two terms 1 by pi cancel with each other, and we are left with 9.3 10 to the power of 5 times of beta times of zeta, and let us just write the value of zeta here which is d by 4 root of alpha t on the surface corresponding to z equal to 0 times of 1 minus the error function of the value zeta. Zeta of course, is 1 by 200 beta as you have seen before. So, therefore, if we just sort of put this whole expression back in place try to see how this equation would change. We have theta on the surface corresponding to z equal to 0 at any function as a function of time t is twice h root of alpha t this is the beta value mind you divided by K times of now we have d by 4 root alpha t times of again 1 minus error function of zeta, and zeta is d by 4 root alpha t as we already know here this root alpha t goes away we are left with h d by twice K 1 minus error function of zeta, and that is equal to the surface temperature as a function of time.

Heat Conduction and Temp. Rise for a circular spot

$$\therefore \theta_m \approx \frac{Hd}{2K}\left[1-\text{erf}\left(\frac{d}{4\sqrt{\alpha t_m}}\right)\right]$$

Max value of $1 - \text{erf}\,\frac{d}{4\sqrt{\alpha t_m}} = 1$

at t_m very very high & $\zeta = 0$

$\text{erf}\,\zeta = 0$

This would correspond to the minimum input power needed

If supposing the again value of this error function d by root of 4 alpha t m this tends to 0, and if you may look into this step reason for that that as x tends to 0 the error function tends to 0. So, we are talking about a typically zeta value between 0.2 and point I mean 0.0. So, in that event, the expression here would change to the simple formulation that total amount of power which is needed.

Heat Conduction and Temp. Rise for a circular spot

$\therefore H_{cr}$ = minimum input power to melt the material

$$= \frac{2K\theta_m}{d}$$

and below this critical value the melting temp. will never be reached

So, that only the minimum possible melting temperature is hit upon let us say for example, this is equal to theta M. So, H d by 2 K is typically theta m, and in other words, the power which is needed which is called the critical power. That means, power enough for the melting temperature to hit upon is represented simply by twice K theta m by d, d is the beam diameter, and these are simplistic assumptions for a machinist where you can assume that you know the d value is in microns, and it goes to an extent that this whole value d by 4 this argument here d by 4 root of t m kind of tends to be between 0.2 and 0 something like that. So, in that event, the critical power which is needed for the temperature to go to a melting point of the workpiece is represented by this H c r equal to twice theta m K by d.

So, having said that we can actually solve a small numerical problem where we define, or we try to find out this value of the critical power for a laser machining system. Here let us say for example if we already know the diameter to be 200 microns of the beam, and we assume the same tungsten workpiece meaning thereby that the other properties thermal conductivity and the thermal diffusivity remains similar to what we have taken earlier, and we assume that 10 percent of the beam is absorbed 90 percent is reflected. So, we need to find out that critical value of the beam power and let us look at how what kind of power values would be hitting upon in this particular case. So, we already know that H c r here critical power is represented as twice K theta m by d theta m as we know is the melting temperature of tungsten about 3400 degree Celsius.

Numerical Problem

If the diameter of the focused laser beam incident on a tungsten work is 200 microns and 10% of the beam energy is absorbed, find out the minimum value of the beam power intensity to achieve the melting.

$$H_{cr} = \frac{2K\theta_m}{d}$$

$d = 0.02\,cm \quad \theta_m = 3400^\circ C \quad K = 2.15$

$$\therefore H_{cr} = \frac{2 \times 2.15 \times 3400}{(0.02)} = 7.31 \times 10^5\ W/cm^2$$

As only 10% of beam energy is absorbed H_{cr} of beam $= 7.37 \times 10^6\ W/cm^2$

We already know the K value as defined earlier to be 2.15 watt per centimeter degree Celsius thermal conductivity of tungsten workpiece. So, in this particular case beam diameter, of course, is 200 microns let us put it in centimeters as 0.02 centimeters. So, we get the critical power H c r to be equal to twice 2.15 times of 3400 by 0.2, and that comes out to be 7.37 10 to the power of 5 watt per centimeter square. We already know that 10 percent of the beam power is the only power which is coupled to the system. So, we are left with 7.37 10 to the power of 6 watt per centimeter square as the incident power for just about reaching the melting point of tungsten on the surface by because of the beam matter interaction.

So, I think we have come to the end of today's lecture. Thank you.