

Advanced Machining Processes
Prof. Shantanu Bhattacharya
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur
Week - 9
Lecture – 23

Laser Beam Machining, Mechanics of Material Removal, Heat Conduction and Temp. Rise

Hello, and welcome back to this lecture of Microsystems Fabrication by Advanced Manufacturing Processes. Quick recap of the last day's lecture, we were talking exclusively about E-beam processes, E-beam machining, and as we have already discussed in detail before it is about beam of electrons which are accelerated at a very high kilo electron volt of kilo, I mean thousands of electron volts of potential, and which results in a stream of electrons at a very high velocity interfering with the workpiece. And there is a conversion of energy from the striking electrons to phononic vibrations on the surface of the workpiece which results in a local temperature increase thereby sublimating the material. As usual, there is a beam transparent layer while interfering with a sample surface, the beam does not see the layer for some small delta value before it goes into the depth and hits a certain region at the bottom of this layer, and in that region, it creates a small vapor pocket which can come out, and somehow this is responsible for all the machining operation. We exclusively discussed about how the beam can be rastered on a surface, and how it is a high-resolution process of writing because of the extremely small wavelength, wavelength and limited diffraction which would happen to the beam. And then we talked about certain modalities about the machining parameters, and how the beam process can be modelled in terms of the requirement of power, and the other parameters of a beam like the diameter, the velocity of raster of the beam, the workpiece material properties like thermal conductivity so on so forth.

We found out that the actual velocity of rastering is much faster in comparison to the velocities that is being taken over I mean is being offered if you just use a simplistic model of power equal to CQ where C is a coefficient which is determined experimentally from the material removal rates and the power of the beam. Today, we will be slightly changing gears and go into a different process of the laser beam machining system. Here the system of machining is driven by high-intensity laser which is focused on to a surface creating some absorption of the material of the surface, and this absorption is again governed by Beer's law, and there is a conversion from photon to phonon in this particular case. So, there is a photon-to-phononic vibration conversion, and that is the physics of this process.

The phononic vibrations result in a temperature increase, local temperature increase, and there is a big issue of reflectance in the laser machining process, because extremely reflective surfaces,

smooth surfaces have a lot of power loss factors induced by the reflectivity which results in effectively utilizing a very small percentage of the laser beam. So, in order to investigate the laser beam machining process, one needs to really look into the fundamentals and the basics of laser. Laser as we all know is light amplification by stimulated emission of radiation and let us look at some of the properties of what laser beam is capable of just like a high energy beam of high velocity electrons. The laser beam was also capable of producing a very high-power density locally on a surface, it is highly coherent beam as most of us know laser to be, it is electromagnetic radiation, and it has a wavelength which varies about 0.1 to 70 microns.

So, this is really the range of how much wavelengths can be achieved as high as 70 microns wavelength can also be achieved by a lasing source, and as low as about 0.1 micron about 100 nanometers wavelength can also be achieved. So, lasers happen in almost all parts of the spectrum including the UV laser to the visible laser to even the infrared region. And because of this unique property laser beam machining can be used for a variety of applications of this photon-to-phononic transduction process which really results in machining a range of materials. In one instance you can machine human bones for example, using laser, in other instance you can just machine tissues, and other example can be machine hard metals or sometimes plastics.

And so different wavelengths, different energies of the laser beam can if suitably tailored by creating a pulsation effect of the beam could actually give you a lot of different surface finishes, and there is a vast range of materials with a vast range of finishes that you can cover using laser beam machining. The power requirement for a laser beam machining operation restricts the effective usable wavelength range 0.4 to 0.6 microns although these days there are quite a bit of UV lasers which actually operate in less than 400 nanometers region of the wavelength as well. And because of the fact that rays of laser beam are perfectly parallel and monochromatic in nature, and we focus to a very small diameter and can produce a power density as high as about 10 to the power of 7 watts per millimeter square.

You can compare this with a E-beam process which had been discussed before, and see that it is actually quite comparable to the power densities that are achieved by the electron beam. So, for developing a high-power laser normally UV laser is used, these days there is also high-energy laser, UV laser, which is very often used, and then there are continuous CO₂, carbon dioxide, nitrogen lasers which have been very successfully used for all kind of machining operations. Let us talk a little bit about the principle of laser machine, laser or as I already defined laser to be light amplification by stimulated emission of radiation. This is really what is important phenomenally, important in this laser emission is the stimulated part of the emission, and we have to understand the physics little bit about what these stimulated emissions are, and basically it comes from Einstein's quantum theory which talks about that under appropriate conditions light energy of a particular frequency can be used to stimulate the electrons within an atom to emit additional light

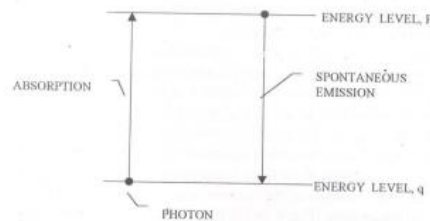
Principle of LASER



- The term LASER stands for Light amplification of stimulated emission of radiation.
- Einstein hypothesized that under appropriate conditions, light energy of a particular frequency can be used to stimulate the electrons in an atom to emit additional light with exactly the same characteristics as the original stimulating light source.
- An atom initially in any excited state does not remain forever in that state
- Einstein proposed that when an atom at 'q' energy level has the light of right frequency acting on it, it absorbs the photon of that energy and the transition

takes place from lower energy level 'q' to higher level 'p'.

- This phenomenon of movement of an atom to a higher energy level is called absorption.
- On the other hand the transition from 'p' to 'q' is called emission.



with exactly the same characteristic as the original stimulating light source. So, let us say, let us consider various states here that an atom is initially in an excited state, and we know that all these excited states typically are very short-lasting in nature, they are not really in a very long-lasting state may be a couple of femtoseconds over which the transition happens, the electron changes its orbital, and there is some kind of a $h\nu$ or radiation which is emitted because of that.

So, let us suppose in this particular case you have two different energy levels of an atom, energy level q, and energy level p, there is spontaneous emission of an electron because of the absorption of a photon from energy level q to a higher energy level P, where the total amount of $h\nu$ or the energy which is added here to the electron is actually only the difference of the two energies P and q, and so there is a phenomena of movement as all of us know of an atom to a higher energy level. Now, this principle is actually the absorption of light. So, the physically, the physics behalf of absorption is nothing but that there is an incident radiation onto an atom, and there is a change in the electronic state from a lower level to a higher level thus absorbing the energy, and after a small amount of time, a brief amount of time the electron decides to come back, emit the same amount of energy back, and this is called the emission. So, the absorption part is when the light energy comes in to make this transition happen from a lower level to a higher level, emission part is when the electron decides to return back to the ground state or another lower level and emits an energy which is equal to the difference in the two states of the electron. So, what is important is the process following that step, that once this emission has happened there would be another process called stimulation which would happen at joint, and we are going to learn about the physics of the stimulation.

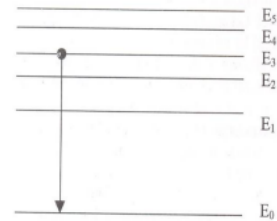
After the electron transits back to its ground state, and the emission is made of the particular

energy, what next, what happens which causes stimulation of several different atoms within a lasing media, so that all of them emit the radiation together. So, let us talk about this here, let us say this is a complex model with different energy levels of an atom, E_0 being the ground state of the ground energy level, and these are different higher energy levels varying from E_1 to E_5 . So, therefore, there are two different kinds of emissions which can happen, emission as you know is the return back of an electron from a higher to a lower level. So, one emission is actually called a spontaneous emission, it means that it is independent of the intensity of the light, it happens spontaneously, and the other is depending on the light intensity, intensity is a stimulated emission. So, there are different, two different categories, one in which it is dependent on the ambient light, and it is independent, so it is called stimulated if it is dependent on the ambient light, and if it is independent of the light intensity, it is called spontaneous in nature.

Principle of LASER



- The emission can be of two kinds, viz. spontaneous emission (independent of light intensity) and stimulated emission (depending on light intensity).
- Suppose each horizontal line indicates the allowed value of energy level.
- Let an atom or molecule be brought to high energy level ' E_3 ' by an outside energy source from its ground state ' E_0 '.



- Now if it is allowed to decay back to its ground state energy level (E_0), a photon is released.
- If this photon comes in contact with another molecule or atom at higher energy level ' E_3 ' then this atom will also decay back to the ground state releasing another photon.
- This chain of events would produce photons having same characteristics (wavelength, phase, direction and energy).
- This sequence of triggering clone photons from stimulated atoms is known as stimulated emissions.
- Therefore, to produce a working laser, the energy source should be so powerful that most of the atoms or molecules of the lasing material are at higher states. This state is also known as population inversion.

Supposing each horizontal line here indicates an allowed value of the energy level, and an atom or molecule be brought to the high-level E_3 by an outside energy source from its ground state E_0 . So, if it is allowed to decay back to its ground state energy level E_0 , photon is released, and this photon now comes into contact with another molecule or atom which is already existing at a higher level E_3 , and this is what the spontaneity of stimulated emissions would mean that supposing there is already an electron transiting back from a level E_3 which creates a photon, and this photon instead of being fully emitted by that atom goes and strikes another atom which is already in the energy level E_3 . And this phenomena of all the atoms going to this inversion level at higher energy level by the absorption of the ambient radiation is also called the population inversion. So, all of them are now high-energy states. So, there is another atom which this photon which comes out faces at a higher energy level E_3 .

So, what would happen essentially is that the electron from that E_3 higher excited state atom would as well transit back to the lower level, thus generating a photon additional to the photon which has

already been sent. So, therefore, this increases the amount of photons, the amount of light, which is emitted, and they are all coherently done. So, therefore, they are exactly at the same phase angle, same wavelength, same characteristics everything together, they all emit together, and it creates a situation which is called stimulated emissions. So, stimulation of higher energy state of the atom by an already emitted photon from an atom which is already transitioned back to a lower energy state. So, a chain of events would produce photons having same characteristics that is wavelength, phase, direction, and energy.

This is what we mean by really coherence, and the sequence of triggering clone photons from stimulated atoms is actually known as stimulated emission. Therefore, to produce a working laser this energy source should be so powerful that all most of the atoms or molecules of the lasing medium are at the higher states. And therefore, all the lasing systems essentially would have an ambient light source which would be responsible for making the population inversion happen, go to a higher level E3 as is indicated here in this particular diagram. And this population inversion is all done by a source which typically has a very high power or a high energy, and this is the power of the laser. So, how much, how many atoms or what is the density of the atoms which would all achieve a higher energy state, and an inverted state, a population, the whole population of atoms getting inverted is what the goal of a lasing medium or a laser source or a laser torches.

So, there are certain aspects of a laser which are very important. Now, once it has been stimulated, the stimulated emissions happen, and there is an amplification, and there is an amplification, and the stimulation amplification because of, because of stimulated emission which is happening from the medium that is why laser. Feedback mechanism of lasers are important property of lasers to understand once we talk about laser machining. So, feedback mechanism is essentially the element of the laser producing system, and it will not let the, or it will minimize the losses of the lasing system essentially. So, one of the roles that a feedback mechanism has is that it captures and redirects a part of the coherent photons back into the active medium.

As if these photonic emissions if they are captured and put back into the medium would be responsible for all the stimulation. So, they can stimulate further higher energy state atoms, and all of them can stimulate together the emissions. So, as many you can save from getting lost, the more energy-intensive the stimulated state would be in terms of number of photons. So, the mechanisms also permit a small percentage of coherent photons to exit the system, and this is what is responsible for the laser light. So, you have to develop a may be synchronous asynchronous I am sorry time-based at different points of time system where there is a gating of the light as opposed to a non-gating.

And so, there are different switches associated with the laser light output some of them are called Q switches which pulsates at a very high frequency thus gating the light back and forth and losing

some of the photons which have been coming out of the stimulated emission back and forth into the light source, the laser source, and into the medium. So, therefore, there is a retention on one end of those photons into the medium, and on other end there is a slow draining out of those photons which actually are the laser light. So, most of the photons which are generated by the stimulated emissions are captured or retained in the system. So, that you can have more amplification, more stimulated emissions, and a part of it does the active machining or the active processes, and it comes out of the laser torch which we can normally see, or it can be impacted onto a workpiece to have machining process. So, let us look at some of the varieties or types of laser which are available there are solid-state or gas-based lasers each of them have their own limitations and shortcomings.

Let us talk about solid state first. So, solid-state lasers because of poor thermal properties cannot be used for very heavy-duty work, and also, they are not very high frequency. So, they can operate typically lesser than or with the speed lesser than 1 to 2 Hertz. So, once or twice in a second that is about how much solid-state laser can deliver. So, how they are used? So, in some situations in machining, you do need a continuous type of process where like for example, operations like drilling or operations like spot welding where you need to deliver continuously the beam radiation.


So, that absorption can be created locally, and it results in continuous removal of the material, and in case of spot welding it creates a tag by melting both the materials and fusing them together. So, in such applications, the solid-state lasers can be very well used because of their poor thermal properties, and lower frequency of operation. So, you can increase the pulse to very high value where sometimes even up to the level of about nanoseconds or picosecond pulses can be generated by any lasing system. And the most important examples of such solid-state lasers which are slightly more powerful than what other I mean systems are these Nd-YAG kind of lasers. So, these are used in cutting applications other like for example, where completely large size sheets have to be cut into small pieces that is a rastering of the laser which would take place, there would be continuous addition of power.

So, heavy-duty lasers are used there and Nd-YAG forms one of those varieties in the solid state where you can deliver enough power for such complete cutting action to happen. Some materials which are developed for lasing action are calcium fluoride crystals doped with neodymium, aluminum oxide with chromium ion impurities also called crystalline ruby so on so forth. And these materials are actually the typically the lasing medium which would do all this population inversion stimulated emission so on so forth. Let us talk about gas lasers now. So, solid-state lasers were kind of outdated because of their slow speeds.

The next generation lasers were actually where the medium was instead of a solid like a ruby crystal or something is a gas. Several types of lasers exist in this category there are carbon dioxide

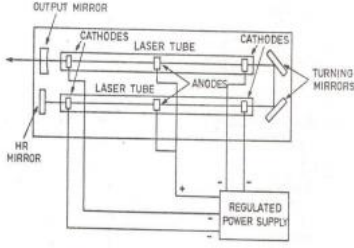
lasers, helium, nitrogen lasers, and all these gases typically act as a lasing medium meaning thereby that all that stimulated emission fundamentals that we discussed in the last two slides are instead of a solid medium done in a gaseous medium. These gases of course, are re-circulated and replenished and this reduces the operating cost of a laser. In some of the cases, direct electrical energy is used to provide the energy for stimulating the laser medium. For example, this right here shows CO₂ laser where you can see that there is a lasing tube and there are these electrodes the cathode and anode which actually produces a discharge, and it is the discharge which actually produces the further population inversion as well as the stimulated emission.

Types of Lasers



Gas Lasers:

- In this type of laser, CO₂, He and Nitrogen act as the lasing medium. These gases are re-circulated and replenished to reduce the operating cost.
- Direct electrical energy is used to provide the energy for stimulating lasing medium.
- Axial flow CO₂ laser is shown in the figure on the right. Its power delivering capacity is usually 100 W each meter length of the tube.
- Some of the details of the gas laser systems are the following:
 - Large amount of gas volume is used.
 - The resonant mirrors are positioned to reflect the beams several times before it escapes through the output mirrors.



Most of the lasing systems are computer controlled for maximum and optimum output.


So, the power delivering capacity of the CO₂ laser is usually about 100 watts, and this is about per meter of the length of the tube. So, that is how high it is about 100 watts per meter of the tube length of the laser. So, there are some details which are needed to be understood for gas lasers particularly. One is that large amount of gas volume is used, the resonant mirrors are positioned to reflect the beam several times, and sort of amplification process. So, the more you amplify you basically put the same light which is coming out of the medium back into the medium again to have more and more stimulated emission till it grows over a certain intensity, and it escapes through the output mirrors which are these mirrors right here.

So, most of the lasing systems are kind of computer-controlled these days, and they are designed for their optimum output. So, that is what fundamentally a lasing system would do whether it is a solid state or a gas laser. So, having understood the fundamentals of how lasing happens, let us actually look into some of the facts based on the machining processes of these laser systems. So, the very important fact which needs to be mentioned here is that the efficiency of the LBM is very-very low in the range of about 0.3 to 0.5 percent. Meaning thereby that most of the energy which is coming out of the lasing process is actually either reflected or it does not do anything useful. Other important aspect of such lasers is that the output, typical output energy of laser like this

about 20 joules, and that is with the pulse duration of almost 1 millisecond. Now, this has improved a lot over the past so many decades of research in laser technology, and these days we can get femtosecond lasers based on molecular switches, and which has an extremely high frequency and low duty cycle. The peak power in lasers reach typically to a value of about 20000 watts particularly when machining is involved, material removal is involved. And in such a situation the divergence of the laser beam is only about 0.002 radians. So, there are very less diffraction effects that the laser would have, and you can easily focus a laser to a spot of almost 50 microns.

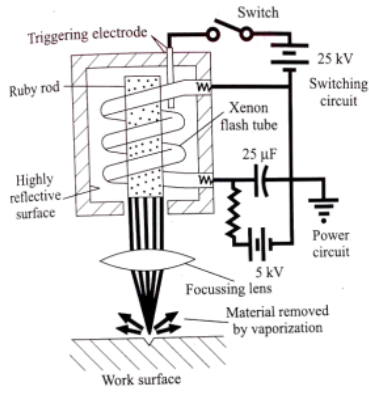
These days the limit has been pushed in case of lithography systems, laser lithography systems to about close to 2 to 3 microns, and that is how low you can get this spot size to be with the technology in the optical technology getting parallelly developed. So, like an E-beam, the laser beam is also used to drill micro holes and cut very narrow slots. Resolution of these systems are having same amount of fineness as E-beam system, fin holes typically up to about 250 microns diameter can be easily drilled by a laser with a dimensional accuracy of around 25 microns 0.025 millimeters plus-minus. So, when a workpiece thickness is more than about 0.25 millimeter, there is a taper which is produced which is almost about 50 microns per millimeter thickness as noticed. So, that is how laser machining processes parameters related to that can be understood. Let us talk about the mechanics of material removal in a laser. So, this figure right here shows typically a pulsed ruby laser, and as you can see that there is a ruby rod which is the lasing medium in this particular setup here.

Mechanics of Material Removal



- The figure below shows a typical pulsed Ruby laser.
- A coiled Xenon flash tube is placed around the Lasing material and the internal surface of the container walls containing the Lasing system is well polished and is made highly reflecting so that maximum light falls on the ruby rod for pumping operation.
- The capacitor is charged, and a very high voltage is applied to the triggering electrode for the initiation of the flash.
- The emitted laser beam is focused by a lens system and the focused beam meets the work surface, removing a small portion of the material by vaporization and high-speed ablation.

- A very small fraction of molten material is vaporized so quickly that a substantial mechanical impulse is generated, throwing out a large portion of the liquid material.
- Since, the energy released by the flash tube is much more than the energy generated at the lasing head therefore the system needs to be continuously cooled.



The diagram illustrates the mechanical setup of a pulsed ruby laser for material removal. It shows a central ruby rod surrounded by a coiled xenon flash tube. A triggering electrode is positioned at the top, connected to a 25 kV switching circuit. A capacitor with a 25 μF value is part of the power circuit, which also includes a 5 kV component. A focussing lens is used to direct the laser beam onto the work surface, where material is removed through vaporization. The entire lasing system is enclosed in a highly reflective surface to maximize light efficiency.

Fig. Schematic diagram of laser beam machining.

There is a coiled xenon flash tube which is this region coiling the ruby medium. The xenon flash tube is responsible for all the primary emission which results in the population inversion in the ruby medium, and the whole setup is housed within a highly reflective surface meaning thereby that this surface here inside which this whole lasing system is being placed is highly reflective. So, there are hardly any photon losses either from the xenon flash tube or the ruby laser, and the capacitor right here is charged to a very high voltage, and that is applied to the triggering electrode

which actually leads to create this primary excitation in the xenon flash tube. And once this xenon flash tube flashes and sends photons out it creates a population inversion in the ruby medium. So, the emitted laser beam is focused now, and so there is of course, this whole principle of amplification and emission which is happening, and there is occasionally loss of photons most of the photons are retained within the medium for the amplification purpose, but occasionally loss of photons which formulate the cutting beam, and it comes out of the optical system hits this lens right about here, and then gets focused very narrowly on to a small spot near the workpiece.

So, you have to really focus a laser in system with respect to the workpiece very closely before you start all the melting and vaporization process. So, immediately with this laser light hitting onto the workpiece a very small fraction of the molten material is quickly vaporized, and there is a substantial mechanical impulse which is generated because of this sudden vaporization. So, it is sort of a sublimation process the absorption of that local region, because of the high intensity that is power per unit area, area of the spot being very small goes up to a level where the phononic vibrations or the phononic energy which is delivered is so high that the material altogether absorbs the photons, and gets heated up kinetic energy increases, and the molecules are almost let loose as gases or vapors. And so, moment this vapor formation is completed there is a sudden escape of that vapor out of the workpiece which creates a secondary mechanical impact. So, this is the energy released by the flash tube is much more than the energy generated at the lasing head.

You have to continuously cool the system otherwise it may as well hit the melting point of some of the materials involved in developing this lasing torch. So, all the lasing media typically needs to be continuously cooled for proper operation. So, machining by laser beam is typically achieved by the following phases. Phase 1 is the interaction of the laser beam with the work material.

Mechanics of LBM

Machining by laser beam is achieved through the following phases:

- Interaction of laser beam with the work material.
- Heat conduction and temperature rise.
- Melting, vaporization and ablation.

Interaction of laser beam with work material:

- The interaction is a thermo-optic interaction between the beam and the workpiece.
- It is obvious that the work surface should not reflect back too much of incident energy.
- The figure on the left shows a laser beam falling on a solid surface.
- The absorbed light propagates into a medium and its energy is gradually transferred to the lattice atoms in form of heat.
- The absorption is described by Lambert's law as:

$I(z) = I(0) e^{-\mu z}$ where $I(z)$ denotes the light intensity at a depth z and μ is the absorption coefficient.

Fig. Laser beam falling on work surface and variation of intensity below surface


There is a photon-to-phonon conversion which raises the local temperature, and local energy, and

there is of course, heat conduction because of a sudden temperature gradient which is produced from the center of the beam to the side of the beam, and sudden rise of the center of the beam's temperature.

And then of course, there is melting, vaporization, and ablation which takes place because of such an interaction of photon to the phonon or the vibration, molecular vibration of the particular lattice with which the photon is interacting. The interaction is thermo-optic in nature between the beam and the workpiece. It is obvious that the work surface should not reflect too much of incident energy because that is a loss. And the idea is that how the absorption happens is reflected in this figure here, where the incident intensity on one side falls onto the surface in question, and there is a gradual absorption and shortening of the intensity as one goes into a certain depth which is given by this Beer's law. I_z is the intensity at a certain distance z from the surface, depth of z from the surface, I_0 is the intensity towards the surface that is the intensity of the incident beam.

And so, I_z can be determined by looking at $I_0 e^{-\mu z}$, where this μ is actually the molar absorptivity of the material, and so this is also known as the absorption coefficient. So, the absorbed energy propagates into the medium, and energy is gradually transferred to the lattice in form of heat and vibrations, and that is what the Beer-Lambert's law says about absorption, and how the energy is dissipated within the medium. Let us look at heat conduction equation, and effectively try to predict the effective temperature rise based on this theory. So, we will perform a one-dimensional analysis. By assuming that the beam spot diameter is larger than the depth of penetration.

If we assume the thermal properties like conductivity and specific heat remain unaffected by this temperature change and also assume a sort of uniform heat flux at the surface of a semi-infinite body.

Heat Conduction and Temp. Rise


$$\frac{\partial^2 \theta(z,t)}{\partial z^2} - \frac{1}{\alpha} \frac{\partial \theta(z,t)}{\partial t} = 0$$

Where α , is the thermal diffusivity
 θ is the temperature.

The boundary conditions applied are
 At $t=0$, when the heat flux has just started the temp. of the body $\theta = 0$
 At $z=0$ $\frac{d\theta}{dz} = -\frac{1}{k} H(t)$

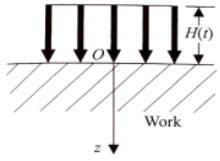


Fig. Idealized heat flux for solving heat transfer problem.

Then the heat conduction equation becomes $\frac{\partial^2 \theta}{\partial z^2} = \frac{1}{\alpha} \frac{\partial \theta}{\partial t}$ function of depth and time by $\frac{\partial^2 \theta}{\partial z^2} = \frac{1}{\alpha} \frac{\partial \theta}{\partial t}$

1 by thermal diffusivity alpha del theta time derivative by del t equals 0. Alpha as we all know is thermal diffusivity, theta is temperature as a function of depth z from the surface, and time t.

We apply some boundary conditions for estimating this equation on this beam workpiece interface, assuming that there is a constant heat flux which is coming through the laser beam into the surface into the work material. So, the boundary conditions that are applied are, that at time t equal to 0, especially when the heat flux has just started.

Heat Conduction and Temp. Rise for a circular spot



Here the equation is

$$\frac{\partial^2 \theta(z,t)}{\partial z^2} - \frac{1}{\alpha} \frac{\partial \theta(z,t)}{\partial t} = 0$$

and B.C.'s become

$$\theta(z,0) = 0$$

$$\left. \frac{\partial \theta(0,t)}{\partial z} \right|_{z=0} = \frac{-H(t)}{K \pi \frac{d^2}{4} t}$$

where $H(t)$ heat flux.

We assume the theta to be equal to 0, this is actually room temperature. Now, we assume room temperature with the baseline. So, we can consider theta to be 0 here. So, everything gets developed over the baseline. So, at the surface with depth z is equal to 0, the d theta by d z is actually minus 1 by K H t, where H t is basically the temporal flux, heat flux which is sent within the workpiece. K is the thermal conductivity of the material. So, if we really apply these two boundary conditions in order to solve this partial differential equation, the solution to the above PDE, and we can actually use any method like variable separation may be or parametric method for solution of the PDE. So, the PDE above solution is theta z t equals twice H by K, H is a constant heat flux times of root of alpha t by pi exponential to the power of minus square of the depth by 4 alpha tau minus z by 2 the first kind of the coefficient z by twice root of alpha t. Let us evaluate what happens at z equal to 0, we have already mentioned about this before, that at z equal to 0 that is the surface the theta 0 at any point of time t is actually given by this twice H by alpha root of alpha t pi, put the z value 0 here, I am sorry this is K. So, the total amount of theta at distance z equal to 0 on the surface at any point of time is actually twice H by K root of alpha t by pi, where obviously K is the thermal conductivity of the material, material which is to be machined, alpha is the thermal diffusivity, t is the time of the whole machining process, and H is the total amount of heat flux which comes in the system. You can talk about the heat flux as the amount of power per unit area that is coupled to the surface, the work surface, the units are watts per millimeter square.

Heat Conduction and Temp. Rise



The solution to the PDE above

$$\theta(z,t) = \frac{2H}{K} \left[\sqrt{\frac{\alpha t}{\pi}} \exp\left(\frac{-z^2}{4\alpha t}\right) - \frac{z}{2\sqrt{\alpha t}} \operatorname{erfc}\left(\frac{z}{2\sqrt{\alpha t}}\right) \right]$$

At $z=0$, surface

$$\theta(0,t) = \frac{2H}{K} \sqrt{\frac{\alpha t}{\pi}}$$

If θ_m is the melting temp. of the material, the time required to reach

So, why does melting happen in a laser machining or how does melting happen? That the surface of the machine, surface of the workpiece which is being hit upon by the incident laser is reaching the melting point of the material. So, this theta essentially here θ_m is nothing but the melting point or the melting temperature of the workpiece or work material. So, we can have a very good formulation here on first principles that theta M is related to the incident power, twice of the incident power by thermal conductivity of the workpiece times of twice root of alpha t by pi, alpha being the thermal diffusivity, t being the time of machining. So, in other words we do know what is theta m from properties of the material, we do know what is the beam power that we are using. In fact, it is very critical to estimate the beam power considering all the reflective losses which are there from the surface, because most of the power gets off from this coupling mode by the essential evil called reflection, particularly for smooth and shiny surfaces.

Heat Conduction and Temp. Rise



to reach temperature θ_m is

$$t_m = \frac{\pi}{\alpha} \left(\frac{\theta_m K}{2H} \right)^2$$

Numerical Problem

A laser beam with a power intensity of 10^5 W/mm^2 falls on a tungsten sheet. Find out the time required for the surface to reach the melting temperature = 3400 deg. C , thermal conductivity = $2.15 \text{ W/cm. deg. C}$, Volume specific heat = $2.71 \text{ J/cm}^3 \cdot \text{deg. C}$. Assume that 10% beam is absorbed.

$$\begin{aligned} \text{Thermal diffusivity } \alpha &= \frac{K}{\rho C} = \frac{2.15}{2.71} \\ &= 0.79 \text{ cm}^2/\text{sec.} \end{aligned}$$

So, once we have estimated that and we are sure that what is the power density, the flux which is coming into the surface in terms of watts per millimeter square. We can find out machining time here t_m by plugging in the various values of these are material properties of the workpiece or this is the power, this right here is the melting temperature. So, t_m is basically equal to π by α times of $\theta_m K$ by twice H whole square. So, that is how the melting temperature t_m . So, let us do a small problem here that this is a laser beam and we have a power intensity being coupled of about 10 to the power of 5 watts per millimeter square falls on a tungsten sheet and find out the time required for the surface to reach the melting temperature which is 3400 degree Celsius.

We have the following parameters here the thermal conductivity is 2.15 watt per centimeter degree Celsius, volume-specific heat, $\rho \times C$ is given to be 2.71 joule per centimeter cube degree Celsius, and you have to assume that about 10 percent beam is absorbed on to the workpiece surface. So, let us say let us look at thermal diffusivity first. So, let us calculate thermal diffusivity α is given by thermal conductivity by volume-specific heat K by ρC , and this happens to be equal to 2.15 by 2.71 about 0.79 the units are centimeter square per second. So, α is calculated now, and we already know that we are coupling beam power H which is about 10 percent of the actual power being coupled, actual power being 10 raise to the power 5 watts per millimeter square. So, this happens to be about 10 to the power 4 watt per millimeter square. So, that is how much the beam power is which is coupled to the surface in question. And if we change this slightly from to a more convenient unit just because everything else is in centimeters this happens to be 10 to the power of 6 watt per centimeter square. So, that is how much power gets coupled, and we can find out machining time t_m very easily here by using the expression earlier derived π by α - $\theta_m K$ by twice H whole square.

So, 3.14 divided by 0.79, and θ_m is basically 3400 times of 2.15 by twice 10 to the power of 6. So, many seconds it comes 0.000053 seconds. So, that is how small a time it takes for the laser to heat the surface up to a melting temperature of almost about 3400 degree Celsius.

Heat Conduction and Temp. Rise



Assuming the beam power is remain uniform with time, the heat flux H is given by

$$H = 0.1 \times \text{Power Intensity}$$

$$10^4 \text{ W/cm}^2$$

$$\therefore H = 10^6 \text{ W/cm}^2$$

$$t_m = \frac{\pi}{\alpha} \left[\frac{\theta_m \times K}{2H} \right]^2 = \frac{\pi}{0.79} \left[\frac{3400 \times 2.15}{2 \times 10^6} \right]^2$$

$$= 0.000053 \text{ sec}$$

So, you can think of the amount of laser power which is being utilized just at 10 percent of the laser power. So, how much amount of power is really important for changing the material properties, taking it to the vapor state is very much a very small percentage of how much power is generated within the ruby medium of the crystal or even a gas laser. So, that shows the difference in the power levels between what is inside a laser torch and what actually gets delivered from the laser torch. So, the next lecture towards the end of this particular lecture we would actually try to make a little more complex model by assuming instead of a steady state problem, a transient problem and try to see or evaluate how the temperature varies in that particular case by making model a little more complex, and by changing also the system from a semi-infinite plate to a circular beam over a small region. Thank you.