Advanced Machining Processes
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Week - 9
Lecture – 22
Derivation of Functional characteristics of EBM, Power requirement in EBM, Mechanics of EBM process

Hello, and welcome back to this microsystem's fabrication by advanced manufacturing processes. Quick recap of what we did in the last lecture, talked about surface roughness of EDM operations, electro-discharge machining operations. We also talked about the various EDM defects like overcuts, electrode wear and taper due to the unequal exposure of the workpiece to the sparks coming from the tool electrode. We also talked about tool and electrode material and dielectric fluid, particularly the tool material should be chosen in a manner so that the wear is minimum. And dielectric fluid which actually is circulated in the space between the tool electrode and the workpiece should have typically a high breakdown constant, there can be water-based or oil-based fluids which are used. And then we started talking about electron beam machining, and the way that the resolution of a system can be improved or enhanced by using super focused high-energy electron beam.

So, just go back to that and try to recap some of the things regarding electron beam machining. So, in this EBM process, there is an electron beam which is created through a thermo-ionization effect using a grid cup-shaped electrode charged at a negative voltage. And then subsequently there is a perforated anode which is used to pull off the electrons and focus them subsequently with a magnetic field so that it can be focused into a very small spot size. There are typically two magnetic fields which are created the first lens system which is creating this electromagnetic field is used to focus and make the beam narrower, and the second is used for rastering the beam over the surface.

And basically the relative change of the beam with respect to the surface according to guided by the different shapes or sizes that the beam has to incorporate onto the workpiece surface is controlled by the second magnetic lens. So, there are certain disadvantages that we discussed about EBM, one of the major shortcomings of the process is that it is a high vac, high vacuum process meaning thereby that substrate sizes are limited because typically these vacuums are established in columns. And the other issue about E-beam machining is that it is really a high-resolution process. So, that is an advantage for the E-beam machining. So, you know you can do a lot of writing at very small resolution, nowadays the E-beam machining is done on the nanoscale, on to the nanoscale by making a feature size of as small as about 10 nanometers separated by equal spacing, and this process is known as E-beam lithography where you can write it on a resist surface.

So, let us actually look at some of the mechanics associated with the E-beam process. So, let us say the temperature rise of a surface on the beam incident side can be approximated by solving the following one-dimensional heat conduction equation for the heat source placed inside the metal. So, if theta z t be the local temperature at a certain depth z from the surface at a certain point of time t then the dou theta z t by dou t becomes equal to alpha this second space derivative temperature with respect to the depth z square plus 1 by specific heat capacity times of density of the material times of the heat flux H z t. This equation alpha is the thermal diffusivity of the material and H z t is the heat source intensity that is heat generated per unit time per unit volume assuming the heat source to be a steady one steady heat source. The intensity then would depend only on distance from the surface z.

Derivation of Functional characteristics of EBM

The temperature rise can be approximately estimated by solving the following one dimensional heat conduction equation for the heat source placed inside the metal.

$$\frac{\partial \theta(z,t)}{\partial t} = \alpha \frac{\partial^2 \theta(z,t)}{\partial z^2} + \frac{1}{\rho c} H(z,t)$$

Where θ is the temperature, α is the thermal diffusivity of the metal, z is the distance from the surface, t is the time & $H(z,t)$ is the heat source intensity, i.e., heat generated per unit time per unit volume

So, we can actually represent this H z equals A e to the power of minus let us say some constant b times of depth from the surface where A and b depict the energy absorption characteristics of the material.

Derivation of Functional characteristics of EBM

Assuming the heat source to be steady, the heat source intensity only depends on z according to the relation

$$H(z) = Ae^{-bz}$$

Where A & b depict the energy absorption characteristics of the material.

Using this expression in the PDE we have

$$\frac{\partial \theta(z,t)}{\partial t} = \alpha \frac{\partial^2 \theta(z,t)}{\partial z^2} + \frac{A}{\rho c} e^{-bz}$$

So, if we use this heat equation for describing the steady state heat source as if the beam has hit on a surface and it is a cylindrical beam and the heat conduction across the surface is time-invariant that means it is steady heat flux into the surface. So, the equation that has been earlier obtained here equation 1 can be really written down in terms of it can be slightly modified and written down in terms of this steady state heat source. And so, H typically would now depend at time t equal to 0 only on z and also corresponding to all other times after 0. So, that is how the PDE can be expressed if we solve this PDE assuming that the metal body semi-infinite in nature of the surface of the metal is insulated except for the hot spot and the rate of heat input remains uniform with time during the pulse duration.

And, then if we plot the nature of the theta z t the temperature with respect to the depth from the surface z and the time t we obtain the plot of theta as indicated here with respect to z. Here one thing that we can observe very well is that the variation of the temperature from the surface or as a result of the distance from the surface that really is a function of the various pulse durations of the E-beam. So, if the pulse time is greater in this particular case for example, tau 3 is greater than tau 2 is greater than tau 1 there is a gradual shift observed of the maximum temperature point towards the surface. So, as if the beam transparent layer that the electrons seek through while going into the metal is decreasing because of an increase in the pulse duration. In other words, you may think of it physically as there is some kind of a homogeneity of the temperature if the pulse duration is large and it really achieves a steady state.

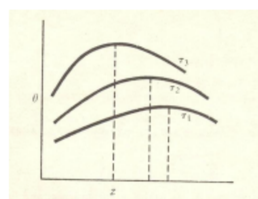
Let us assume the following

(a) The metal body is semi-infinite

(b) The surface of the metal is insulated except for the hot spot

(c) The rate of heat-input remains uniform with time during the pulse duration.

The nature of variation of the temperature found by solving PDE equation is shown below.



As the pulse duration increases, the peak temperature shifts towards the surface.

Variation of temperature with distance from the surface for various pulse durations.

And so, therefore, already the temperature is over a certain critical point and the beam when it comes new on to the surface does not see that much transparent layer that it was supposed to see before because already it is very heated up and already there are lot of lattice vibrations which are happening. So, in reality, the physics of the problem also kind of gets replicated by the variation of theta with respect to z as can be seen here. So, therefore, as the pulse duration increases the peak temperature shifts towards the surface. So, we would now like to perform a sort of dimensional analysis for also checking the consistency of the various parameters of cutting with respect of the EBM process with respect to the material removal rate. So, using the Buckingham's pi theorem.

So, the first thing of importance is to be able to look at what are the independent and dependent parameters in the whole EBM process the EBM cleaning process. And so, let us look at the various quantities of importance here. They are beam power and we already know that this beam power can be written down as the beam current times accelerating voltage. They are beam diameter, velocity of the beam, let us call it V thermal conductivity of the metal call it K here, the volume-specific heat rho c as has been used in the earlier term as well. Melting temperature theta m and depth of penetration of the melting temperature z.

So, we have z is equal to a function of so many different things the beam power, the rastering velocity, the beam diameter, the thermal conductivity of the material, the volume-specific heat and finally, the melting temperature of the material. So, the idea behind this analysis, this dimensional analysis is to be able to in step by step first predict all the independent parameters like in this case, you have the beam power, beam diameter, rastering velocity of the beam etcetera into the basic dimensions. So, basic dimensions in this particular case because we are using terms related to either work energy or velocity or even temperature. So, there will be four basic dimensions mass, length, time and temperature. And so, we express all these different independent parameters in terms of these basic dimensions.

Derivation of Functional characteristics of EBM

we have

$$Z = Z(P, v, d, K, \rho c, \theta_m)$$

Let us express all quantities in terms of basic units M L & T, θ

$$\therefore P = ML^2T^{-3}, \quad v = LT^{-1}, \quad d = L$$

$$K = MLT^{-3}\theta^{-1}, \quad \rho c = ML^{-1}T^{-2}\theta^{-1}$$

$$\theta_m = \theta, \quad Z = L$$

According to Buckingham's π theorem we can form $(n-m)$ numbers

So, let us start with power. Power, for example, is force into distance per unit time. So, the basic dimensions would be that of force that is M L T to the power minus 2 times of l divided by T. So, this is M L square T to the power minus 3. Similarly, you have for velocity L T to the power minus 1, d of course, is the diameter.

So, it has the dimensions of length K here is the thermal conductivity it would have the dimensions of M L, let us just write this down here, do not have space. So, K can be expressed in terms of M L T to the power minus 3 theta minus 1 rho c which can be expressed in terms of M L to the power minus 1 T to the power minus 2 theta to the power minus 1 so on so forth. Of course, theta m is nothing but having the basic dimensions of temperature, z is L. So, according to the Buckingham's pi theorem, the methodology that is followed is to be able to see how many dependent or independent parameters are there. In this case the total number of parameters that are there are 7.

You can see this z is 1, p is 2, V is 3, d is 4, k is 5, rho c is 6 and theta m is 7. So, basically, there are 7 such parameters which are either dependent or independent and they can be expressed in terms of only 4 basic dimensions that is mass, length, time and temperature. And so according to the Buckingham's pi theorem, this n value happens to be 7, m the number of basic dimensions happen to be only 4 in this particular case. Meaning thereby that there exists at least n minus m subgroups which are dimensionless. And so, we have to somehow be able to correlate by raising these different quantities to different powers to arrive at this condition that at least 3 subgroups formulated by the various combinations of these 7 parameters would be having no dimensions or they would be completely dimensionless.
So, let us assume this 3 groups m minus n equal to 3 groups to be equal to let us say pi 1 and pi 2 and pi 3.

Derivation of Functional characteristics of EBM

$$\therefore \pi_1 = z \cdot P^{\alpha_1} V^{\beta_1} K^{\gamma_1} (\rho c)^{\delta_1}$$
$$\pi_2 = d \cdot P^{\alpha_2} V^{\beta_2} K^{\gamma_2} (\rho c)^{\delta_2}$$
$$\pi_3 = \theta_m \cdot P^{\alpha_3} V^{\beta_3} K^{\gamma_3} (\rho c)^{\delta_3}$$

Substituting the dimensions of each quantity, we equate to zero the ultimate exponent of each basic dimension since, the π_i's ($i=1,2,3$) are dimensionless groups

So, we can combine these or formulate these 3 independent subgroups pi 1, pi 2 and pi 3 by combining some one or all of these parameters together. So, that these are completely dimensionless in nature. So, how many parameters we have earlier illustrated are the depth of melting temperature, the beam power, the velocity of rastering of the beam, the thermal conductivity, the volume, specific heat of the material, the temperature of melting and finally, the beam diameter. So, there about 7 such parameters which are dependent or independent.

And the first estimate shows that the only things which are independent of time here are the dimensions, the length dimensions that is z and d and the temperature theta m. The remaining all dimensions are dependent on time. And so, if we were to raise the time-dependent parameters to different powers we would arrive at an easier solution of this equation. And so, therefore, the idea is that let us actually formulate a subgroup pi 1 with the length dimension z to the power of 1 times of the other which are dependent, or which are time-based like power to the power of alpha 1, rastering velocity to the power of beta 1, thermal conductivity to the power of gamma 1 times of volume-specific heat to the power of delta 1. Similarly, we have some other dimensionless parameters like pi 2 which can be represented in terms of diameter of the beam, power to the power of alpha 2, velocity of rastering to the power of beta 2, thermal conductivity to the power of gamma 2, and rho c volume-specific heat to the power of delta 2.

Similarly, the other dimension which is the temperature dimension is in terms of theta m, power to the power of alpha 3, V to the power of beta 3, K to the power of gamma 3, rho c to the power of delta 3. So, substituting the dimensions of each quantity we equate to zero, the ultimate exponent of each of the basic dimensions. We can call these set of pi's, pi i with i varying from 1 to 3. And since the dimensions of both z and d are the same, alpha 1 is equal to alpha 2, beta 1 equals beta 2, gamma 1 becomes gamma 2, delta 1 becomes delta 2. As you can see here if supposing all the basic dimensions are equated to zero, this particular pi 1 would have a zero dimension, and so the remaining alpha 1, beta 1, gamma 1, and delta 1, these would all be sort of equal to length inverse for making this dimensionless, which means thereby that, because this also has the same dimension length L, and alpha 2, beta 2, gamma 2, and delta 2 would combine together to have again length inverse dimension.

So, they are in terms equal to each other, and they can be equated to each other. So, that is why alpha 1 equal to alpha 2, and so on so forth. So, let us now pick up one of them, let us say pi 1, and try to represent this in terms of basic dimensions. So, this is l dimension for z, times of the dimension for power here, which is M L square T to the power minus 3 to the power of alpha 1, times of the dimensions for velocity L T to the power minus 1 to the power of alpha 2, times of I am sorry beta 2, times of k, which is actually again represented as M L T to the power minus 3 theta minus 1 to the power of gamma 1, this is beta 1, times of M L minus 1 T to the power minus 2 theta minus 1 times of delta 1. So, alpha 1 plus gamma 1 plus delta 1 is equal to 0, twice alpha 1 plus beta 1 plus gamma 1 minus delta 1 equal to minus 1, thrice alpha 1 plus beta 1 plus gamma 1 thrice gamma 1 plus twice delta 1 equal to 0, and gamma 1 plus delta 1 is 0.

And so solving all these equations we get alpha 1 equal to 0, beta 1 is 1, gamma 1 is minus 1, and delta 1 equal to 1. Thus pi 1 the first dimensionless group comes out as z v rho c by K pi 2, the second dimensionless group comes out to be d v rho c by K. In a similar manner alpha 3, beta 3, gamma 3, and delta 3 are found, and pi 3 that way emerges out to be K square theta m by power p times of rho c v. So, if we get a functional relationship pi 1 is f pi 2 pi 3, in this particular case pi 1 is z v rho c by K, and this can have a functional relationship with respect to the other two non-dimensional numbers pi 2 and pi 3. So, d v rho c by K, and K square theta m by power p rho c v.

Z has been found out to be experimentally proportional to P, thus Z rho c v by K comes out to be equal to the power P times of rho c v by square of K theta m function f 1 of d v c by K. Thus, that is the only way to have the proportionality to the power as linear, the other term does not have the power term in it, which is inside the, which is actually the function f 1. So, it has been therefore, so therefore, we arrive at a term that if you just rearrange this a little bit this goes away, this also goes away. So, you have z theta m by K times of power P is equal to a function of d v rho c by K. Now, if you do an experiment of the E-beam where you observe the various relationships which happen between Z theta m by, z theta m K by power P on one hand, and this d v rho c by K on another hand, you do have such an experimental relationship emerging from the observed data, and this can be written down this is more empirical by just doing a curve fit.

Derivation of Functional characteristics of ECM

$$\therefore \frac{ZK\theta_m}{P} = f_1\left(\frac{dv\rho c}{K}\right)$$

It has been experimentally observed that

$$\frac{ZK\theta_m}{P} = 0.1\left(\frac{dv\rho c}{K}\right)^{-0.5}$$

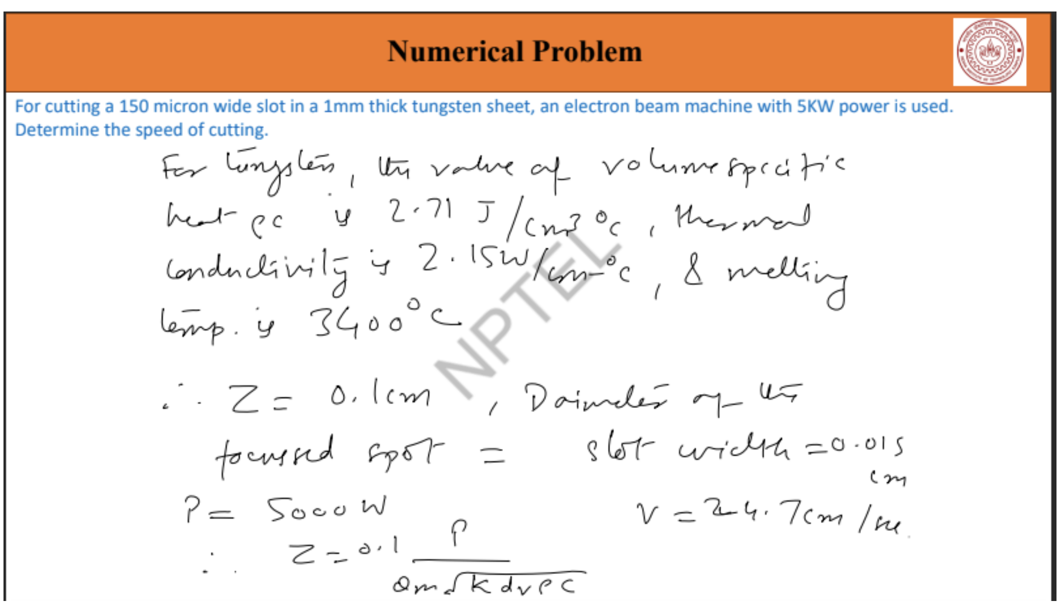
$$\text{or } Z = 0.1\,\frac{P}{\theta_m\sqrt{Kdv\rho c}}$$

So, this comes out to be Z K theta m by P equals 0.1 times of d v rho c by K to the power of minus 0.5, or in this case Z becomes equal to 0.1 power P divided by theta m root of K v d rho c. So, that is how you can equate Z with respect to the various dependent parameters, the beam power, the depth of melting temperature, the K value, thermal conductivity of the material, the beam diameter, velocity, density, specific heat so on so forth.

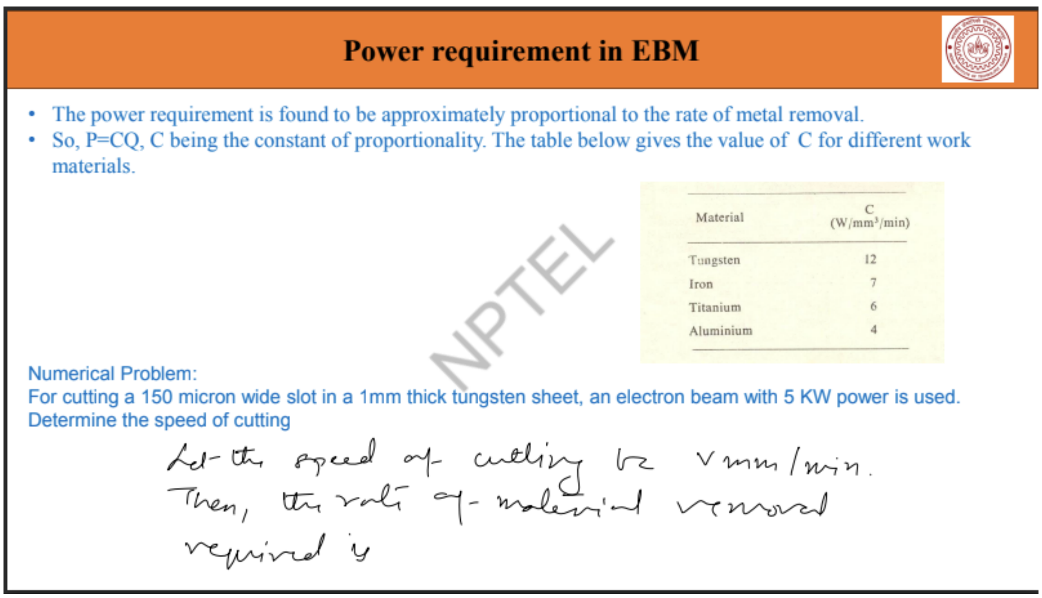
So, in a nutshell, we do have now a comparison based on dimensional analysis and experimental data of this E-beam machining, and we have already arrived at a relationship of how the temperature varies with respect to the depth, where the plot suggest that with the control or in the pulse duration, and the variation in the pulse duration there is a gradual shifting of the depth of melting temperature towards the surface. So, having said these two things I think we are pretty much ready for doing micromachining using E-beam, which we will probably cover in the last few lectures, where we will talk various aspects of resolution, beam power so on so forth, using this fundamental knowledge about the E-beam process. Let us now do some numerical examples to strengthen our understanding in this particular area. Let us look at this numerical problem that for you want to cut a 150-micron wide slot in a 1 mm thick tungsten sheet and use an electron beam machining process with the 5-kilo watt power, and we have to obtain the speed of cutting in this particular numerical model. So, we already know that there is a formulation which has been obtained with dimensional analysis and experiments as Z equal to 0.1 times of power P divided by theta m root of K d v rho c. We already know for tungsten the value of volume-specific heat rho c is 2.71 joule per centimeter cube degree Celsius. The thermal conductivity is 2.15 watts per centimeter degree Celsius, and these are some material properties which can be obtained from any standard book, and the melting temperature for tungsten is around 3400 degree Celsius.

Therefore, the Z value can be expressed as 0.1-centimeter 1 millimeter, diameter d of the beam can be equated to the slot width that you want to machine here. In this particular case, the slot width is 150 micron, and this is in the best interest of the quickest machining step.

So, it is 0.015 centimeters. The beam power that is used is basically 5000 watts, and velocity has to be determined, the rastering velocity can be easily determined from this relationship here, and the velocity comes out to be equal to 24.7 centimeter per second. So, in order to cut a small slot of 150 microns in a 1 mm tungsten sheet, the amount of speed that is used for cutting the slot is about 24.7 centimeter per second.



So, cutting speed is not that fast. So, there is a lot of dwell time, and this helps in melting, and removal of the material like any other process would do, and so that is how the E-beam process works. So, if you may recall there was another way of estimating the beam power which was done before, and there it was mentioned that the beam velocity actually, the rastering velocity of the beam actually obtained on a surface may be much much more in comparison to that predicted by that method. Let us just do a quick comparison to see how that is true. So, if you may remember the power equation in the earlier slides were given out by an expression P equal to CQ, where C was the constant of proportionality, and the value for example, for in this particular case it is a tungsten sheet, the C value experimentally observed in case of a tungsten sheet came out to be about 12 watts per millimeter cube per minute. Q of course, is the MRR material removal rate, P is the total amount of power which is needed.



Now, we also talked about the similar kind of setup where we were cutting a 150-micron slot in a tungsten sheet using 5 kilo watts beam power. So, let the speed of cutting be V mm per minute, then the rate of material removal required is Q equals 150 by 1000 times of 1 times of V mm cube per minute. The corresponding beam power is given by P equals C tungsten times of the material removal rate Q being estimated above here, and if we assume this power to be 5000 watts as is the case given in the question, and the C tungsten to be about close to 12 times of this 150 by 1000 V, we obtain a velocity V of 4.6 centimeter per second. So, this is much-much small as you can see in comparison to what we have obtained using dimension analysis and other criteria.

So, in a general, the actual E-beam velocity of rastering is much-much more in comparison to the velocity which is predicted by a simplistic equation P equal to C cube. The other important points about E-beam before we stop this lecture is that since the machining by E-beam is achieved without raising the temperature of the surrounding material, there is no effect as such on the work material. So, it is a very high-resolution process as has been illustrated before, the surrounding material

$$Q = \frac{150}{1000} \times 1 \times v \ \text{mm}^3/\text{min.}$$

The corresponding beam power is given by

$$P = C_{tungsten}\, Q = 12 \times \frac{150}{1000} v$$

Since, P is given to be 5000W.

$$v = \frac{5000}{12 \times 0.15} \ \text{mm/min.} = 2778 \ \text{mm/min.}$$

$$= 4.6 \ \text{cm/sec.}$$

This speed is much less than the actual speed as will be derived later.

really remains unaffected, because of the extremely high energy density the work material even up to the extent of only 25 to 50 microns away from the machining spot that still remains at room temperature. So, whatever deliverance of heat energy is associated with the E-beam process is really limited to the work area for which it is intended or targeted. So, distances as small as about 50 microns from that work area by and large are unaffected.

So, E-beam is a very good process as far as machining accuracy is concerned. And also one more factor is that the chances of contamination are very less, because the process is mostly carried out in high vacuum, and therefore, material getting formulated into its oxide state you know or some other state by combination with the reactants which are present or the free radicals which are present in the atmosphere that in this case gets limited by the fact that the beam is within a column which has a high vacuum. So, with this we would like to end this lecture on E-beam machining. In the next lecture I would talk about a little more details of laser machining process, and how that is suitable for doing micromachining or micro-manufacturing, and following which all these processes how they can be used in actual MEMS technology would be illustrated in great details. Thank you.