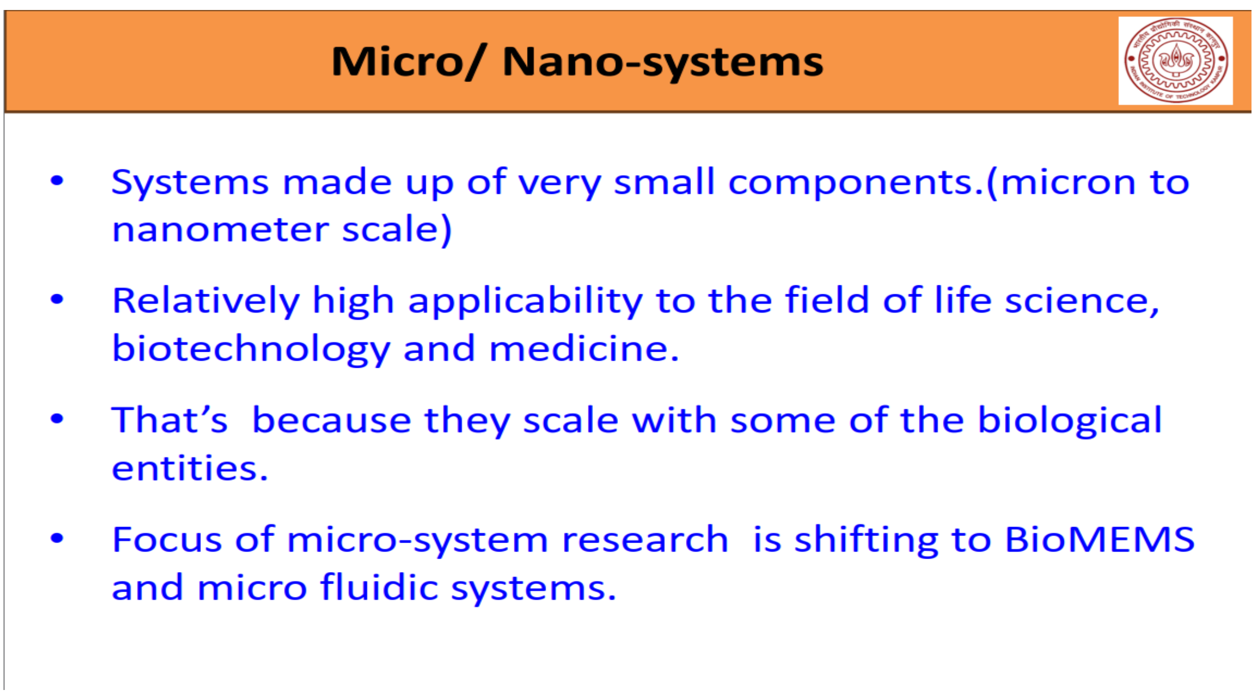
Advanced Machining Processes
Prof. Shantanu Bhattacharya
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur
Week - 01
Lecture - 02
Introduction: Advanced Machining Processes, USM

Hello and welcome to this lecture 2, Advanced Machining Processes. I am now going to take over a little bit of just overall peripheral discussion into the state of you know micro engineering microsystems research. And with just a few slides of where we see this microsystems research going and how you know what kind of modules are available today in the industry. We will try to start with the advanced machining process. The first process, of course, today would be the mechanical processes where we will talk in great detail of USM ultrasonic machining. And we will talk it from a physical model perspective and then later on try to see what can be done you know with such a process to make some of these so-called micro-manufactured systems or microsystems.

So, when we look at the whole micro nano systems they are typically defined as ones which are made up of small components. At least one dimension should be in the micron or the nanometer scale. And these systems they have relatively high applicability to the field of life sciences and biotechnology and medicine. Today you can find out biochips for example, which are abundantly available for carrying out diagnostics in medical industry.



Also, you can find out many examples of surgical interventions within patients where you use microsystems because otherwise, macrosystems may create lot of pain and other effects within human beings. So, one of the reasons why the micro nanosystems are important or they are you know gaining a lot of popularity is that you can think of the dimensions of the systems kind of scale and rhyme a lot with the biological entities. And these entities may have different size domains. For example, if you look at a virus typically it is a few hundred nanometers in size or if you look at a bacteria or bacterial cell it may be few tens of microns. There can be you know red blood cells or even you know leukocytes which will be of several different tens of microns within the human system.

So, the idea is that we build up some structures or some features which kind of sink well with these biological world. When we talk of a little more downstream you know entities you get DNA, or you get proteins again which are few nanometers. One helical turn of a DNA for example, as I already mentioned in some earlier lecture is close to about 3 to 4 nanometers 30 40 angstrom. So, if you had features and structures with the processes that you can make which can rhyme or sink with these biological entities it is useful because the sensitivity levels of their detection increase. And perhaps one of the reasons why this is small world you know the micro nano world becomes suddenly very important you know for humanity.

Focus of such systems are gradually shifting to the microfluidic systems because obviously biological entities and fluids they hold you know sort of similar kind of coexistence. Human body is made out of fluids most of the biological systems that we talk of entities are within the human body or interacting with the human body. So, obviously, some fluidic part is kind of necessary in such microsystems research. Now, when we talk of it let us look at some example problems where microsystems are really helpful in today's even industry. So, one of the finest examples of microsystems that comes to my mind is this bulk micromachined accelerometer.

This has been developed by silicon microstructures and basically, the idea here is that you have a proof mass which kind of changes directions, and you know there is some kind of a change in the orientation based on where it is the characteristics of the frame where it is mounted. For example, if such an accelerometer is in a car and there is a sudden need of de-acceleration in terms of braking etc. It should be able to create a signal which should actuate a safety mechanism like a airbag coming up. So, these accelerometers are used everywhere you know for many different applications. Now, the idea of a microsystem here again is that your sensitivity should be very high.

The moment there is a start of such a huge de-acceleration process which may be brought in by a collision for example, it should be able to immediately generate a level of signal which actuates the airbag or any other safety mechanism that an automotive may have. So, that is why going small you know becomes an advantage sometimes. This right here is an example again I keep illustrating about it is you know a probe for a atomic force microscope. It is a atomically sharp tip made out of silicon as one can see and this tip is basically you know when you scan this tip over a surface it obviously creates it exposes itself to a lot of forces electrical forces sometime Van der Waals forces and that leads to the frequent oscillation of these tips. And you could actually look at the way that the tip changes you know its orientations and try to get a inverse scan of the surface on which this tip is proceeding through.

So, that is how AFM's typically work and again the element here is of course, of micron size and made sharp you know to a sharpness of almost the atomic level. So, the idea is to be able to develop something which does not have a lot of its own inertial impact because of the extremely small mass that it may have, and it may generally carry forward even small forces which it faces while scanning over surface. This again is a very interesting example you know from the digital micromirror device chips these are basically manufactured by Texas Instruments. And the idea here is what goes into your projector. There is an image projection which happens from a digital file you have a powerpoint or any other you know digital stored set of information in a formatted manner.

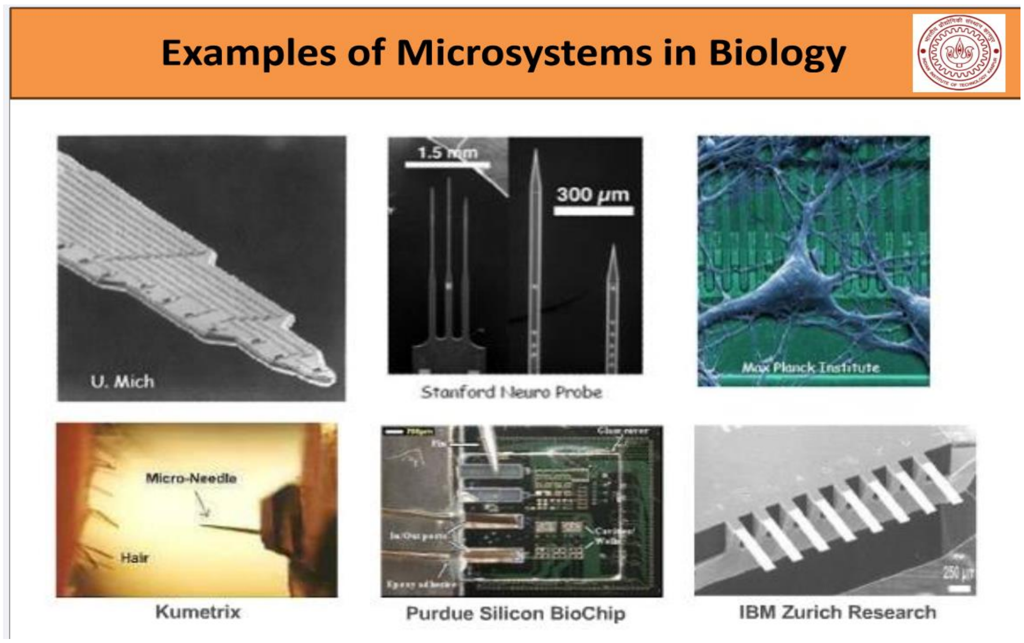
And you are wanting to project that up screen and for that, you need to be able to operate you know a set of mirrors which would align, de-align, make you know the spot bright out of reflection or dark, and then based on this brightness or the darkness you define the features you know in the projection capacity. For example, you can look at such mirrors here which are in the path of this light signal. Of course, there is a set of you know red green blue filters, and basically, the idea is that now if the mirrors twist and turn and each mirror has a three-dimensional view represented here. You have a flat silicon piece which is pivoted on two pedestals and there are some electrical interconnects which are down here based on it. You could actually twist and turn this mirror and make it either out focus or in focus in the beam that goes reflected or past it into this collecting lens which is projected of course, onto a screen right here.

This is the collecting lens. So, this is a very interesting example of what microsystems can do because these mirrors are very light in weight. They can twist and turn and create you know different patterns and that is how you visualize a big image from a digital file which sends you information related to whether the pixel should be bright, or the pixel should be dark. So, that is how the DMD chip works. Few more examples for you one is again the single-chip accelerometer you know from analog devices single single-chip microphone right here.

This is based on again the principle of if you look at the cross-section of how this is being manufactured. You have acoustic holes and basically, there is a piezo material which is in the top of these holes right here and the idea is that as I speak and you listen basically the tongue within the oral cavity it creates a set of compression and rarefaction which is what sound is and those waves would travel through a media a medium which is there between the speaker and the listener and listener would have again a vibrating membrane which otherwise called the so it is called the ear drum. So, the vibrations coming out from this media between the speaker and the listener it changes the vibration pattern of the ear drum and the membrane there in which creates auditory impact on the human brain etcetera. So, these same principles are kind of mimicked here as you can see there are acoustic holes in this particular you know structure which are perforated, and whatever sounds are emanated from the oral cavity are picked up in terms of some vibration vibrating effect, and that kind of gets converted into an electrical signal which can be again amplified through some amplification means you know by increasing the gain, etc. And that goes into a speaker which is a much wider more diameter high diameter membrane.

Which again creates the same compression rarefaction impact on the surrounding air which goes into the listener's ears. So, that is how this the principle of microphone is based upon. Now if I looked at what is the real USP for all this process it is the thickness of that membrane you know this membrane right here which is able to pick up the smallest possible change in pressure of the media. So, you can only do if you can go small and make you know microstructure. So, that you could very accurately pick up a lot of signals that is how microphones are based upon the MEMS microphones.

Now there are a team number of examples of what micro systems do in day-to-day processes particularly live processes. This is a neuro probe developed at University of Michigan which is used to observe you know the electrical activity of brain tissue. This right here is again a neuro probe developed for providing deep electrical stimulation in patients who are suffering from Parkinson's diseases. Again, there is a set of MOSFETs here in this particular figure and there are various nerve cells growing on the top of this MOSFET. This area again is called nanobiology that how do you see how the cells interact with respect to each other when they are closely growing or they are far apart is being carried out through measurements of you know small surface potential changes across this array of MOS transistors.



Now there is a very famous example of micro needle. It is carried out based on the motivation from a mosquito needle and the fabrication is done because such a needle which maybe around maybe 20 to 40 nano micrometers in diameter and maybe 180 to 200 micrometers tall is made to make you know is developed to make sure that you can painlessly inject some drug or some moiety into the skin. So, the way it goes is that these needles are made on patches and there are you know tens of thousands of such needles which are suddenly increasing the concentration gradient of a particular drug molecule very near to the vasculature and then there is diffusion driven process that happens from where the injectable has been provided and how it is up taken from the vasculature side. Now the advantage of this microneedle is that you know this needle would not be able to it pinche into the skin, it does not disturb the pain receptors because there is no deformation damage on the pain receptors. You do not feel the pain as such, and the needle does its job to painlessly deliver a drug close to the vasculature.

I mean if you were to use a normal injection, of course, you know particularly in babies, and all the pain level and the experience is really horrendous. This is another example of microsystems in biology you know this is a silicon biochip which was developed at a program in Purdue where they looked at various food pathogens and how they grow, and it was a impedance assisted chip which would give you some importance you know of there are electrodes and they will monitor how the cell growth process happens in the medium etc. So, again the features and the structures which are being made in all these, this is these are set of micro cantilevers again made at IBM Zurich research which are used for doing DNA sequencing. So, all this kind of because they rhyme well with the sizes of the entities what they are playing with they are quite sensitive. And our job here is really not to look at how the sensing is carried out, but to look at how you could make or how you could manufacture the parts in that smallness which could correlate well to the biological world.

So, for that let us start beginning you know a little bit of introductory ideas of advanced machining processes. Of course, there is a class of materials which are used to electronic processing. I am going to come into this little more in lot of detail later on may be perhaps dedicate a few more lectures, but today I am, of course, going to start with the mechanical processes. So, therefore, I will get into the deep details of the USM process and try to pick from there. So, the materials that are typically used in the microsystems or the nanosystems they started off with silicon and some other microelectronic materials, but there are other materials which were needed along the way, particularly the biological chips, biochips as we call them.

MEMS materials

- Silicon and microelectronic materials
- Glass, Quartz
- Polymers
  – Poly (dimethylsiloxane) (PDMS)
  – Poly (methyl methacrylate) (PMMA)
  – Teflon, etc.
- Biological Entities
  – Cells, Proteins, DNA
  – Frontier of BioMEMS !

They need transparent covers, or you know something through which light can pass because a lot of transduction that is carried out in terms of you know change of signal which is machine readable is actually optical. So, therefore, transparency is a major concern sometimes. So, therefore, glass and quartz are very often used as covers. You have polymers of various classes polydimethylsiloxane being one, polymethyl methacrylate being one and these microsystems can be made with very small microcasting and molding processes as I will illustrate in a little more detail later on. But the idea is that polymers as a class of materials have really emerged well.

The processes which are involved in shaping these polymers are called soft lithography processes and they are actually today quite a bit in the industry from the standpoint of catering to the small diagnostic needs. The Teflon for example, which is a very hydrophobic kind of polymer you also have materials you know which are biological themselves and you can still carry out micro nano structuring through these biological materials. So, having said that let us now go to the first part of my lecture series which is about mechanical machining and here the idea that is kind of proposed is that you have not a direct have scratch machining per say, but a throw machining process where there is a small hard grain which creates an impact on to a surface. It breaks the surface; it fractures the surface and there is a small embrittlement created on the surface and that embrittlement goes away through the medium which brings in the small grain close to the surface. So, there are various types of mechanical advanced machining systems available today, but when it started off it really started off with two very broad class of machining processes.

One was ultrasonic machining process; we also call it as USM and another is water jet machining process which is otherwise called WJM. Now there are various variants of these processes for example, you may have a abrasive water jet or a ice water jet process. The idea is same that you are using some hard material thrown at a surface to create embrittlement and then carrying out or carrying away the embrittlement to make sure that there is machining action going on a surface. So, as I think illustrated the machining medium is solid grains, they are suspended in an abrasive slurry. In the USM process, for example, the work zone is flooded with that slurry and then there is a ram head which vibrates and impinges so that there is embedment of this grains on the surface which it is machining on.

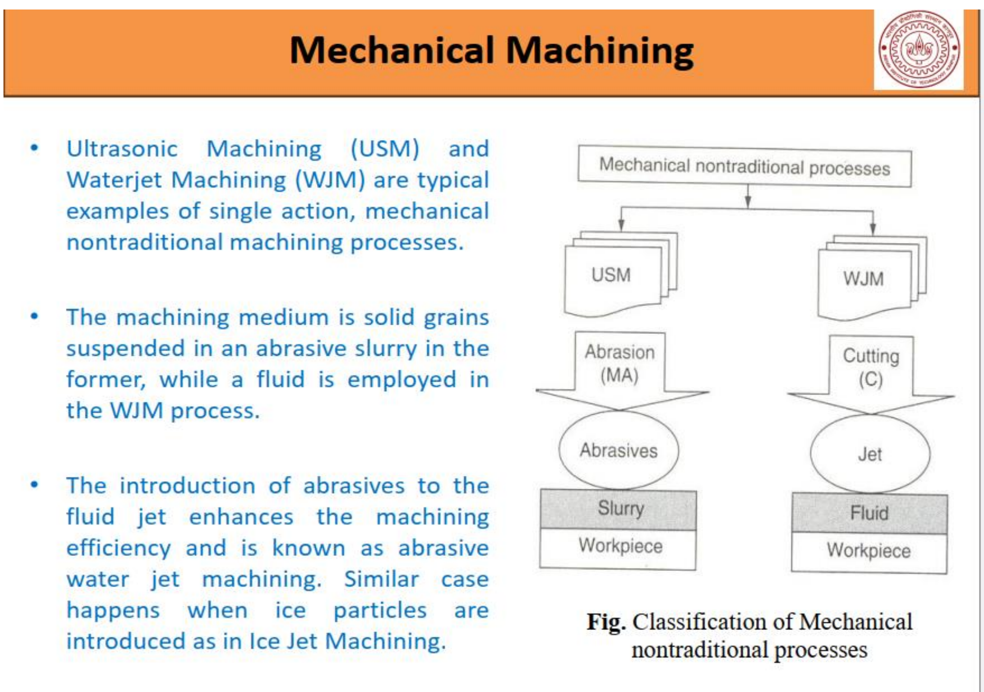


Fig. Classification of Mechanical nontraditional processes

So, the machining medium, of course, is a critical aspect in mechanical machining because the way you populate the medium, the density at which you are loading is of consequence as I will show later in a model that I will try to build. Of course, you know the nature of the grain, the hardness of the grain, the hardness of the surface, the relative strengths of the grain and the surface they are all critical, and also the frequency at which impact is made is also very critical for the ascertaining of how much material is being removed. Now, the introduction of abrasives to the fluid jet enhances the machining efficiency and you know the water jet machining has a variant called abrasive water jet machining based on it. So, you have not only water creating a pressure zone on a surface but also a thrown or impinged abrasive which is carried out with the water and creating pressure on a surface. So, having said that now let us look at you know the first process that I was talking of the ultrasonic machining in a slightly more detailed manner.

The use of USM was proposed by J.O. Farrer that was back in the year 1945 and why it is called ultrasonic is that you do involve a surface which does the impingement to move a surface. So, you have to have a at a frequency which is in the ultrasonic range. So, the first machine tool using the ultrasonic principle was designed way back in 1954 and basically, the process was developed because one needed to finish you know the components which were produced by an earlier developed process called you know the EDM or electro discharge or electro spark machining process. So, USM when it was developed was really a finishing process and people did not think that it will get into the mainstream for making you know making it to be a primary machining operation. It eventually happened today there are MEMS structures like pressure sensors etcetera which are being produced solely based on USM process also the industry.

The use became less important because it was a post-processing, but now it has become prominent because of that reason and of course, the processes which are mechanical or mechanically assisted in this advanced machining domain works very well for materials which are brittle like for example, you can have silicon or glass as materials you know they are brittle they will have some time they are electrically non-conducting also where other processes like EDM etcetera may fail to work. So, therefore, it is a very skewed window of materials which is considered as far as the USM process goes. So, what is the process let us see. So, in this particular process, the basic USM process you have the involvement of a tool as you can see here the tool it is made of a ductile and a tough material this tool and it vibrates with a very high frequency you know ultrasonic frequency 20000 hertz or you know more and there is a continuous flow of a slurry as I told the slurry contains perhaps some parts of abrasive grains which are dissolved in water or which are solvated in I mean in water. So, basically the grains are all suspended inside the water and the water grain slurry goes and affects the machining zone the machining region there is some sort of a you can say a operational head being created here on which there is a feed force which is initiated through the tool and this feed force again is initiated in a sinusoidal manner.

So, you have the force between some 0 value to some maximum value to again 0 value depending on how the tool is positioned and how it squeezes grains which are close to the surface. So, the frequency of the tool in this particular case which gives the force cycle is about 20000 hertz and the motion of the tool is for very small amplitudes typically 15 to 20 microns. The abrasive grains are about anywhere between 25 to 50 microns and there can be grains made out of silicon carbide. So, boron carbide which is B4C so on so forth. Now there is a continuous flow of this slurry which is initiated in this small gap between the tool and the workpiece where the force field comes, and the forcing ram comes.

And of course, as one squeezes you know there are two different cases which happens one when the grain strikes the moving ram and impinges onto the surface through a bouncing action. The other cases where the ram is very close and started squeezing a particle which comes in between the surface and the ram. So, these are two different kinds or approaches of machining that happens. But the whole idea is that the impact of the hard abrasive grain fractures the work surface which is otherwise quite hard or brittle. And the embrittlement which comes out is kind of removed through sort of a medium which flows along with the abrasive grain.

Basics of the USM process

- The basic USM process involves a tool (made of a ductile and tough material) vibrating with a very high frequency and a continuous flow of an abrasive slurry in the small gap between the tool and the work piece.
- The tool is gradually fed with a uniform force.
- The impact of the hard abrasive grains (Aluminum oxide (Al_2O_3), Silicon carbide (SiC), Boron carbide (B_4C), Diamond) fracture the hard and brittle work surface, resulting in the removal of the work material in the form of small wear particles.
- The tool material being tough and ductile wears out at a much slower rate

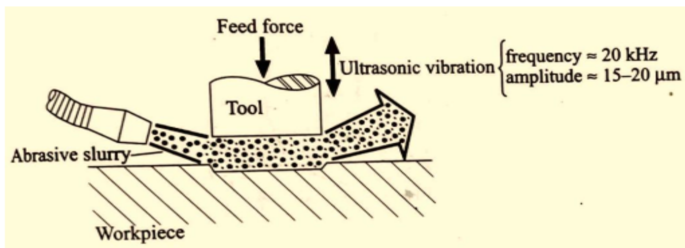


Fig. Ultrasonic Machining

This results in the removal of the work material in form of smaller wear particles etcetera. And of course, the tool material is tough and also ductile, and idea is that you choose the tool material in a manner with an appropriate hardness ratio. So, that the wear on the tool side is relatively lesser in comparison to the wear on the workpiece side. So, that is how the whole USM business or the USM process happens. Now, I am going to get into a little more of the model the physical model associated with USM to be able to give you some perspective of what is the rate at which the material is being removed you know in such a process.

Now, if I looked at the mechanics of how the material would be removed as I already kind of illustrated. There are three or four different reasons for such material removal to happen. One is, of course, the hammering impact of the abrasive particle on the work surface by the tool. So, this is the squeeze case. You can say that the tool has actively hammered on a abrasive grain pushing it into the workpiece surface.

You have the impact of a free abrasive particle on the work surface that is like you can say the grain throw crease where the grain goes all the way to the vibrating ram. The ram is still not in a position to squeeze because it is slightly farther away, and the grains are smaller in you know diameter in comparison to the gap that is there between the workpiece and the ram. In such a case, if when you are throwing the slurry to a surface that going to be impacts which will create a rebounds, and abrasive grains can be rebounded of the surface and go and strike the other surface that is the approaching ram surface. And of course, there is a momentum transfer because of which this can come and create an embedment or an impact on to the workpiece surface.

So, that is impact from free abrasive grains. You could have erosion due to cavitation. Now, this is something I would like to explain to you little more detail. You see when there is a moving membrane or in this case moving surface which is doing so at an extremely rapid pace. If you talking of ultrasonic frequency, it is about 20000 times in a second. So, when such a thing happens, of course, the slurry which is there on the other phase of the ram that is the phase between the ram and the workpiece.

That slurry being water would have its own inertial delays and supposing the withdrawal of the surface is sudden, the slurry may not be able to really follow that high a frequency of withdrawal. Of course, it is going to create a low-pressure region and you know the low-pressure region can be infused through air or some other moieties which are there. Some air can be out bleeded by the slurry and this gap which is there is able to get covered in some time delay. Therefore, the air inclusions start happening on the medium because of such a process very fast-moving process, and that creates bubbles. These bubbles are responsible for a lot of material transport because these bubbles are pressure balls you can say within it is a two-phase flow now that you have water from the slurry and the air inclusion which is there from the atmosphere.

These would like to create some impact on the surface and create some erosion on the surface. This process of formulation of bubbles and what it does to the surface is also called cavitation and there is a lot of erosion because of cavitation which happens because of this delay of the medium to follow the rapid membrane or the rapid ram head as in this case so quickly. There is also another impact which comes from chemical action. Sometimes the slurry itself is designed in a manner that you may have some chemical erosion component also happening. So, that chemical action associated with the fluid is also responsible sometime for material removal.

So, the four principal reasons for the USM to sort of shelf the material are illustrated here as individual hammering action, the impact of free grains, the erosion due to cavitation, and the chemical action associated with the fluid use. Now a number of researchers have already tried to model the USM process, but what we better know in terms of its ability to map the process correctly is one which is proposed by M.C.Shaw. The model is called Shaw's model, and it has some basic assumptions.

It is a very basic model which I am getting into to explain how you could relate properties of the surface, properties of the tools like hardness, etc, and the grain in order to create an expression for material removal rate. So, let us look at the Shaw model. Now in this model, the direct impact of the tool on the grains in contact with the work piece is taken into consideration. Now one can say that the rate of work material removal which would happen because of such impact embrittlement and carrying away is proportional to the volume of the work material per impact that is removed. You know, so let us say there are n grains on a surface and each of them have embedded into a harder workpiece surface and some volume you know is created because of such embedment.

So, wherever the grains have embedded, and the embrittlement has happened, and the surface has broken, that volume is typically equal to the pinch volume of the surface. So, we assume that to happen and then we say that the rate of work material removal is really proportional to the volume of the work in one impact. So, you have to consider a ram coming and approaching another surface and impacting the surface if there are several grains which are creating pinch points and several volumes removed, several volume fragments removed. So, that is very important to map that how many are in a way getting squeezed together on a particular you know finite area ram surface. It is important to also record the number of particles making impact per cycle.

USM process

- Thus, volume of work material removal rate (Q)

  $Q \,\alpha\, v Z \nu$

where, v = volume of the work material removal per impact

Z = number of particles making impact per cycle

ν = frequency

So, the volume of the work material removed per impact, number of particles making impact per cycle, and also at what rate the ram is approaching. So, what is the frequency at which the ram is approaching? So, number of times that it approaches in a second is very critical for because we are talking of rates. Rates are in time and so time aspect has to be there and there are certain things which we assume from the M.C. Shaw model. One of them is that we assume all the impacts to be identical and we also assume that all the abrasive grains are identical and spherical in their shape. So, that is how we approach the problem, and let us now start doing the basic model to see what or how do we estimate the overall removal rate, material removal rate. So, the material removal rate Q here is related to V Z nu. These are the parameters discussed in the last slide, volume of the work material removed per impact, number of particles making impact per cycle, and the frequency. Each of them is, of course, having a proportional relationship to material removal.

The more would be the volume of work material removed per impact. The more would be the removal rate. The more would be the number of particles making impact per cycle. The more would be the removal rate. The more would be the operating frequency.

The more would be the removal rate. Look at a impinging grain and how we can correlate the material parameters of the grain as well as the work surface to that impinging grain model. Let us assume that there is a surface here, there is a work surface here and there is a grain which comes and pinches on this work surface. Of course, the diameter of the abrasive grain is small d and one can assume that it has created a crater on the surface, which is capital D, which is actually a circular crater as can be visible from this example right here. This is circle of diameter capital D and in this kind of a case, we need to correlate how this pinching process etcetera is happening and is there any geometric relationship you know between this. Now, we also assume that as the abrasive grain has impinged into the surface it has traversed about h depth within the surface and that is how the diameter D has come into existence.

So, when the grain was touching on the top right here D was 0 and as the grain is going from depth 0 to depth h the diameter of the cross-section of the grain which is mapping on the surface or getting formulated on the surface is increasing from 0 to capital D. So, the figure on the right is showing the indentation process caused by the impact of an abrasive grain. If capital D is the diameter of the indentation at an instance when the impingement depth is h we can calculate a geometric relation which corresponds to square of capital D by 2 equalling square of small d by 2 minus d by 2 minus h that means this much square. Now, of course, I mean this results in a relationship D square equals 4 small d h minus 4 h square and as the h is quite less in comparison to the diameter of the grain one can easily approximate D as twice root of D h neglect h square 4 h square. So, having said that now we assume that the volume of material dislodged per impact is proportional to the cube of diameter.

Mechanics of USM

Let us now consider the impact of a rigid, spherical abrasive grain of diameter 'd' on the work surface.

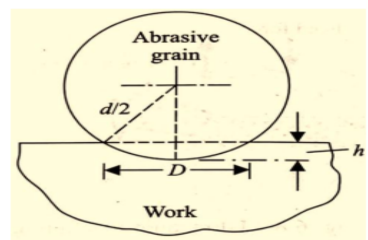


Fig. Scheme of idealized grain indentation

The figure on the right → shows the indentation caused by such an impact. If 'D' is the diameter of the indentation at any instant and 'h', the corresponding depth of penetration we get

$$\left(\frac{D}{2}\right)^2 = \left(\frac{d}{2}\right)^2 - \left(\frac{d}{2} - h\right)^2 \Rightarrow D^2 = 4dh - 4h^2$$

As $h << d$ (diameter of the abrasive grain)

$$\therefore D = 2\sqrt{dh}$$

So, we get Q is proportional to D h to the power of 3 by 2 z times of nu where the z and the nu have their own obvious meanings. Since the mean speed of the tool is low the mean static force let us call this force F applied to the tool can be equated to the mean force experienced by the grains. One can assume that you know we are only going a few microns even if it is 20000 times in a second. So, the overall velocity of the tool's average velocity is quite low, and one can assume that in such a condition one may just hover around the same region with very small movement creating almost a constancy of presence. So, one can assume the force translated between the ram head and the grain to be over large duration and that can probably mean that your inertial component associated to the impulse created because of tool motion because of the absolutely small amplitude can safely be neglected.

And it does not have any impulse anymore so that the force does not go to the you know twice its value, but you can assume the force at which the tool is being vibrated is typically the force which is being translated onto the grain. So, we have to make this assumption as a part of the Shaw theory of course, there is a problem that you know one can have an impulse of the force and sometimes there may be grain crushing or grain breaking also in the process. So, we will discuss those details later on once the simplistic model is immersed in a proper manner. So, when the duration of an impact is let us say delta t and the maximum value of the force is F_i, force F_i is F_i maximum. The nature of variation of this force with time is little tricky because you have to understand that the ram is moving it is freeing the grains for a substantial portion of the cycle and it is only pressing for only a small portion of that cycle.

It is a simple harmonic motion so obviously, there is one section of the motion which is kind of interfering in nature while executing the push on the abrasive. The remaining part is kind of idle because the tool is going away from the abrasive. So, this kind of a situation has to be mapped in terms of you know the force-time characteristics and you have to understand that why the force-time characteristics is in that particular desired manner. So, it is kind of shown here so this is how the force-time characteristics is for the grain pressing craze. As one can see and let us understand from a cyclic point of view that assuming this whole time duration starting from here to here as the cycle time or time for one cycle only a very small portion of that cycle is experienced in terms of a force on the grain.

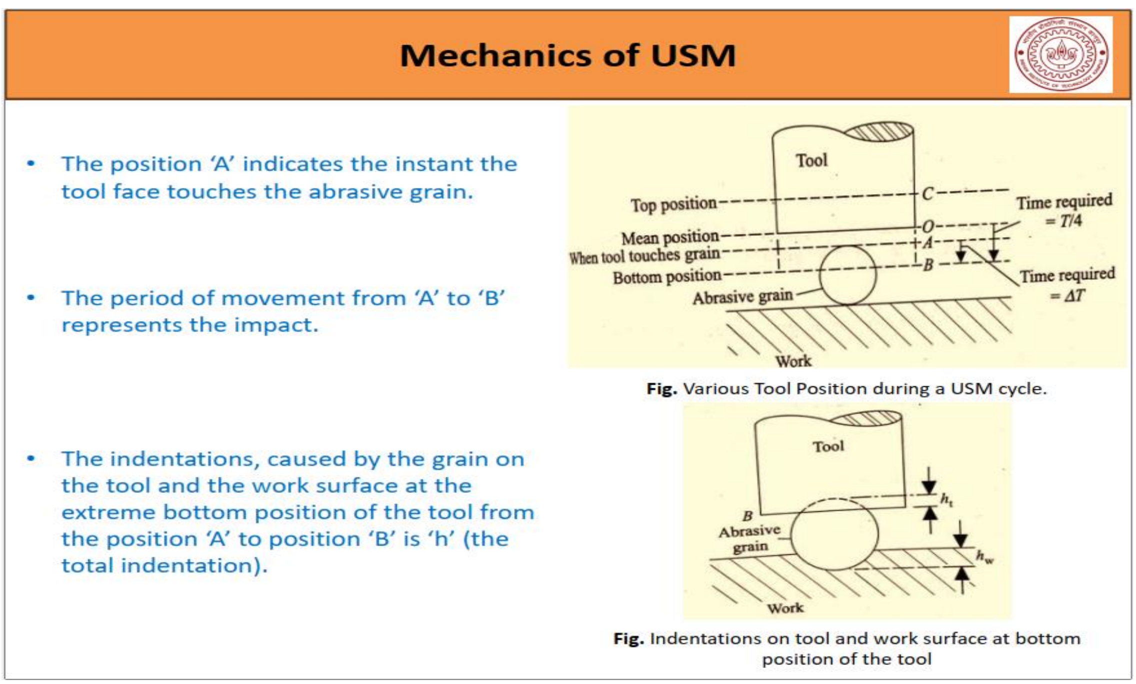
So, the portion for which the tool head presses and unpresses the grain is kind of illustrated here through this delta t that is how we assume that for a period of delta t, there will be contact between the vibrating ram and the grain surface. While the grain is being impinged the grain is pushed up to a certain depth let us say that maximum depth is H. So, the if I look at the force characteristics the maximum force which has been translated onto the grain from the ram which is this F_i maximum is corresponding to that point beyond which the vibrating ram does not press anymore and starts reverting its motion back. So, that there is slower release of the force eventually and there is loss of contact which results in force 0. So, that is how this whole cycle can be explained with a maximum F_i max corresponding to the pushing of the grain.

Of course, the remaining portions on this whole time period of the vibrating ram is no contact of the ram. So, you have 0 force on the ram which kind of illustrates very well the overall force-time characteristics. Now, the average force if I were to look at which is of importance can be provided through the integral 0 to T F i t dt and one can have the average by looking at the time integral of force divided by the total time period T. Now, of course, T is the time period of each cycle. The duration of an impact among this T time period is delta T as one has recorded here in this particular illustration.

Of course, the maximum force is recorded as a F i max. So, owing to the sharpness of this curve and the fact that although there is some curvature to it, one can it may not be too erroneous if one can estimate this whole area under the curve as a triangular area which is the height of F i max and total width or total base size of delta T. So, let us try to calculate this on that average basis. So, let us write that it will not be erroneous to assume the nature of variation of the force be triangular, therefore, F average can be approximated as 1 by T times the area under the triangle which is half base altitude.

So, half times of F i max times of delta T. In other words, this is Fi max by 2 times of delta T by T. So, let us call this one. This is the average force of the ram on the abrasive particle. Let us get an idea of what positioning are we referring to. So, you have a tool which is coming from the point 0 where it was cantered earlier and it comes all the way to half its motion touches the abrasive grain somewhere around A, pushes the grain from A to B at a height delta h or a depth delta h for the time required delta T.

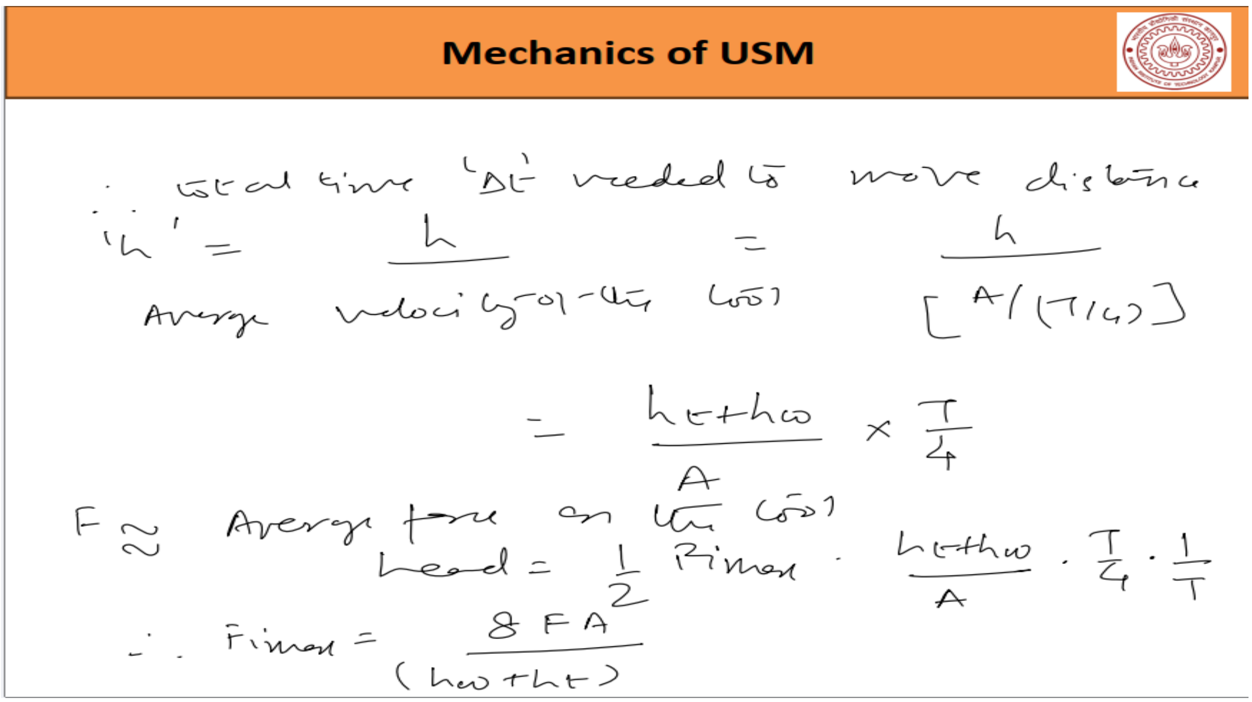
The total time that is needed for the whole cycle being T meaning thereby that the simple harmonic part of the tool executes 0 to B plus B to O plus O to C plus C to O in time T. Therefore, one can average out the time of motion for just the path OB as T by 4 and really the position A and B defines what is the height. So, let us say position B minus position A is the total depth of indentation in this particular case and I think you kind of get a feel of what is going on and for what instance the force is applying to the surface of the position a. Now, having said that the important aspect here is to be able to calculate this relationship of the various depths which are there which are related in fact inversely to the hardness of the materials to this whole process the velocity, the frequency, the amplitude and some of the other parameters. So, if I assume that in a particular impinging operation there is a depth h_w moved by the grain towards the workpiece side and a depth h_t moved by the grain towards the tool side.



Then the total indentation span is represented through ht and hw of course, this corresponds to work side indentation, and this corresponds to tool side indentation. So, if we assumed the amplitude of oscillation as A, the average velocity would be recorded as A divided by T by 4.

Remember we had illustrated about how moving one amplitude motion you know OB corresponds to T by 4 time period. So, that is what the average velocity of the tool is 4A by T. The total time needed to move the whole indentation depth or indentation span as I would say.

Let us assume that to be delta T. This is really equal to the total h that has been moved which is ht and hw combination divided by the average velocity which from the last instance was found out as 4A over T. This is average velocity of the tool. Thus, the average force on the tool comes out to be half Fi max times of ht plus hw by A times of T by 4 times of 1 by T or in other words one can have the Fi max to be equal to 8 times F average times the amplitude A divided by ht plus hw. Now, this kind of indicates a relationship between the average force, the amplitude, the indentation depths.



In a way, indentation depths are a function of the hardness. So, if the surface is harder the depth would be lower or vice versa. So, there is an inverse relationship which is available and the purpose of all this analysis is to sort of create a sort of a relationship with the material properties on one hand and the parametric properties like velocity or frequency. So, that one can estimate a generic relationship or way to find out the actual material removal rate. I think I will like to close this topic today.

I will perhaps continue from here in the next lecture. In the next lecture, we will see even the grain throw case and then try to evaluate whether this case of squeeze is better, or grain throw is better. Of course, from these parametric relationships emerge a MRR model and then we will try to use that to estimate machining time etcetera for USM cases. So, as of now thank you very much for attending this lecture. Thank you.