Advanced Machining Processes
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Week - 8
Lecture – 19
MRR in EDM, EDM circuits and operating principle

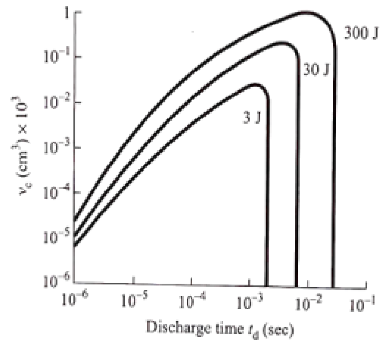
Hello, and welcome back to this lecture on Microsystems Fabrication by Advanced Manufacturing Processes. Let us quickly recap what we did in the last class. So, we studied about the mechanics of EDM process as just to for the sake of repetition, I would like to just say that this is all about the formulation of a spark between the tool electrode and the cathode, the tool electrode and the anode, the anode being the work material, tool electrode being the cathode material. And there is always a field in between which is created because of voltage gradient, and there is a dielectric medium in between, and it is all about breaking down or reaching the breakdown potential of that medium. So, that there is an electron flux which comes from the tool side, and ends into the anode side, the workpiece side, and in a way, it basically leads to a strong electron pressure, and that is a thermal wave, and basically, it creates a melt pool which also increases the distance between the two electrodes at that point, and the field goes down below the breakdown potential, and so the spark gets activated some other place of closest distance. And so this way the spark goes around, and that creates material to be removed.

So, we actually theoretically also estimated the material removal rate by zooming cylindrical axis system, where the spark was like a cylinder which was generating or which was a heat source, constant flux heat source on to a material of infinite slab like nature, and there we assumed how the heat transfer would take place from r equal to 0 to r equal to a which is the radius of the cylinder across which the spark would take place. We made some presumptions including zero heat flow and perfectly insulated substrate beyond the spark, and that way we were able to estimate the depth of melting temperature which was the Z value, and this depth of melting temperature would typically be a plot or an estimate of the crater which would eventually get formulated as wherever the temperature would reach the melting point there would be a melt pool which is generated. And this was helpful in estimation of the MRR or the material removal rate through thermal modelling. So, for example, if supposing there is a cylinder cylindrical nature spark like this, and there is an area formulated on the workpiece where there is the zone of melt.

So, this really is the Z value the depth of melting temperature value, and there are various such depths across the different points of the spark assuming the greatest temperature to have reached at the center of the spark corresponding to a equal to 0, and the radius having the minimum Z value at a equal to r, r being the radius of the spark, and that is how the depth of melting temperature was predicted. So, today we will look a little further into the various modalities associated with how the actual variation of the melting temperature depth happens with respect to the crater volume and also the discharge time. So, this graph right here illustrates how the volume of the crater varies with respect to discharge time. It is very, very obvious that as the total amount of pulse power, the total amount of spark power is increased to about 100 folds, there is a gradual increase in the way that the crater Vc would be formulated. So, crater volume is more if energy pumped is more.

Actual variation of Melting Temp. depth Z and crater volume Vc with discharge time td

- The figure on the right depicts the crater volume with td for different spark energies.
- One important feature which becomes evident from these results is that the material removal is very low for small discharge time and increases with td.
- Then reaching a peak value it suddenly drops to zero.



- It has been also established that the material removed per discharge strongly depends on the melting temperature of the material.
- The effects of cavitation in the mechanical removal process is also important.

So, that is in fact, intuitively understandable. And if you look at some other peculiarities in this graph one important aspect is that the volume removal V or volume of the crater is really very low at low discharge times. So, if this value of td is low, the low discharge times, the amount of volume removed as you are seeing is very low, which is for obvious reasons that if the spark is not of enough duration, the amount of transition time that it would take place for the heat energy to flow through the material will also be less. And because of that the amount of depth that the melting temperature can cover, which is also a function of the heat energy which it is able to release, the spark is able to release on the surface that will also be less because of that. You are not giving it enough amount of time at a certain hold point.

And even before the melting temperature-depth could reach its highest percolation threshold, you have close to the spark energy. So, therefore, the Vc value is low for lower discharge time. Then it suddenly peaks up. So, it reaches a peak value and finally, it drops down to zero as you can see here from these three places indicated by the spark. So, this area is really a sudden drop in the Vc, which is of again obvious reasons that once the material is formulated as a liquid pool, there is a tendency of washing away of the flux and there is a sudden de-formulation of the spark.

Because now as the melt has reached and the material has melted, the amount of potential gradient, which is available dV by dx, electric field is lower than the breakdown field. So, the spark has disappeared and whatever volume was initially formulated is typically the remaining volume and this thing goes away. So, therefore, this is actually indicative of the point where the spark is eliminated at a certain energy value. What is very important to be seen is that this value is almost varying with the amount of pulse energy. So, if the pulse energy were 3 joules, the criticality threshold here where the spark would be eliminated would reach faster.

So, the discharge time would be let us say in this case about close to 10 to the power of minus 3 or so. In this case, it is about 10 to the power of minus 2 which is at 30 joules and if the spark energy is even higher, the discharge time is again further higher 10 to the power minus 1. And so that is how this threshold is reached. It has also been established that the material removal per discharge strongly depends on the melting temperature of the material. So, if there were a high melting point material, this Vc value would automatically be lower and vice versa.

And also one important factor is cavitation which I will just talk in the next slide, but there are effects of cavitation in mechanical removal processes, and it is extremely important because cavitation is a principle mechanism of removal of material in EDM as will be seen in the next slide. So, let us look at the role of cavitation in material removal in the ECM process. Refer to this particular figure here, it is between pressure of the zone of the spark which is plotted on this Y1 axis. And the material removal rate during a spark cycle Q is plotted on the Y2 axis with time scale on the x axis and very interesting thing comes into picture. So, if we look at the way that the pressure is varying on the top of the material and the way that MRR is varying, you can really see that the MRR is highest when the pressure that the medium would generate onto the surface in question is the lowest.

So, if the MRR is high which is indicated by this peak here, high MRR value in this range, the pressure at this point is the lowest as you can see here. So, it is even below than the atmospheric pressure. So, what is really going on if you really and this is a real plot, this is a real plot of the pressure versus the MRR happening on the tool surface. So, as you know that there is always a tendency of the electrons emanated from one of the electrodes that is cathodes in this case to give a sort of pressure wave or a wave of compression onto the anode surface which is the workpiece in this particular case. So, there is always an electron pressure which remains from coming from the medium, emanating from the medium, and coming onto the anode surface or the workpiece surface.

Role of Cavitation in Material Removal

- The MRR during a single spark plotted against time is as shown in the figure on the right.
- Clearly, the MRR is highest at pressures below pressures below atmospheric, showing the importance of cavitation.

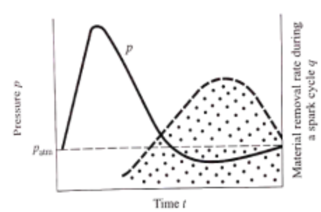


Fig. Role of cavitation in material removal.

- For arriving at a rough estimate, empirical relationships have been developed for the material removal rate during EDM.
- Since the size of the crater depends on the spark energy the depth and diameter of the crater are given by:

hc = (K1 $w^{1/3}$) cm and 2a = (K2 $w^{1/3}$) cm, where W is the spark energy (joules), K1 and K2 being constants. For Cu electrodes and kerosene as the dielectric medium, K1=0.4 and K2 = 0.045

- Vc becomes = $\pi/6\ K_1 \left(\frac{3}{4} K_2^2 + K_1^2 \right) W\ cm^3$

Now, supposing this pressure is there, there is of course, you know a compression and a rarefaction kind of longitudinal wave that we are talking about. And so, therefore, there may be one instance of time when the pressure is high and the next instance of time when there is a lower pressure or rarefaction is reaching the medium. So, it is like burst of electrons that is coming in that is how a spark is really composed of. So, if you assume that then there is a tendency of the pressure to go up which is being shown here with respect to time. So, the pressure is highest here meaning thereby that the electron pressure is highest here and this pressure is really in terms of kinetic energy that the electrons are providing onto the material surface.

So, there is highest kinetic energy as you see the graph here in this particular case the highest kinetic energy of the electrons on the surface. So, of course, this pressure wave then changes as the electron pressure has gone or it has eliminated, and the pressure slowly falls down. So, it comes to some atmospheric pressure value and then even goes below because there is a tendency of the fluid to move thus creating a velocity head. And so, typically the pressure head that would be available is lower in this particular case. So, here you can see the pressure going down than the atmospheric pressure.

So, this is the case where there is a tendency of material which has been formulated earlier in the weld pool to be attracted towards that low-pressure zone which is created. So, the pressure is falling below atmospheric pressure and there is a tendency of the material the melt pool to go into that low pressure zone. So, therefore, you can say that the debris is swept away more efficiently at that point where the pressure falls down the atmospheric pressure and that is the reason why MRR is the highest at low pressure. So, this effect itself is known as cavitation. So, this is a because of difference in pressure there is a transport of the liquid pool of material which is there on the workpiece surface more towards the low-pressure zone which has been suddenly created because of that rarefaction in the medium which has come.

So, if you can consider a series of these you know rarefactions and compressions coming there may be really points where there is oscillating pressure, and then the pressures whenever they hit below the atmospheric pressure is really the most efficient debris removal point for the system. So, therefore, MRR is just out of phase. So, if the pressure is the lowest the MRR is the highest there. So, let us now try to arrive at a rough estimate of an or of sort of an empirical nature for material removal rates during EDM operation. And initially, when some experiments have been performed by various people there are some things which we can find out intuitively one is that the size of the crater would depend on the spark energy as you can see here in this graph below before that as the spark energy is increased the size of crater of the Vc which is demonstrated by the Vc is also increasing more energy delivered means more melting.

And so it depends on the depth and the diameter the spark energy and the size of the crater really depends on the depth as well as the diameter. You remember how we calculated the Vc value to be equal to pi by 6 hc times of 3a square plus hc square this was what we derived last time as the crater volume the volume of that small crater of height hc and radius a. So, the Vc depends on the depth as well as the diameter a being the radius twice a being the diameter. So, empirically it has been found out that the way that the hc the crater height as well as the radius of the crater a varies is really dependent on the pulse power or the spark power or the spark energy. So, if I say W is the spark energy in joules empirically it has been found out that this hc the depth of the crater the depth of melting temperature that varies as the W to the power of 1 by 3.

So, it is the cube root of W with which the hc the depth of the crater is proportional. And similarly, so is the case with diameter the only difference in this particular case is that the constants of proportion are different in one case it is K1 other K2. So, this constant of proportionality is at two different values. So, hc is equal to K1 W to the power 1 by 3, and 2a is equal to K2 W to the power 1 by 3. And these case these K1 and K2 they are typically constant for a fixed set of electrodes and a fixed solution.

And typically for copper electrodes and kerosene as dielectric medium the K1 value is treated as 0.4 for K2 it is 0.045 if it changes to some other like iron and the medium may be something like let us say some other silicon oil the K1 K2 may as well change. So, it is really a property which depends on what materials are being machined what material is being used for the electrode and what material is the dielectric fluid. And therefore, if we just put this these two formulations empirically obtained hc and twice an into the spark volume estimate which is represented here by equation 1 Vc then becomes equal to pi by 6 times of K1 times of 3 by 4 K2 square plus K1 square W to the power of or W centimeter cube.

So, that is how you would characterize Vc just substituting these two values. So, as you know this is equal to pi by 6 times of hc times of 3a square plus hc square and we just substitute the values obtained in both the cases. So, you have pi by 6 times of K1 W to the power of 1 by 3 which is the hc value times of 3 by 4 K2 times of W to the power of 1 by 3 square and plus instead of hc you can have K1 W to the power of 1 by 3 square. So, in other words, this can be represented as pi by 6 K1 times of 3 by 4 square of K2 plus K1 square and we get W outside the bracket centimeter cube. So, that is how the volume can be estimated empirically out of the various powers that the system is using in joules W.

So, this is the spark power or spark energy and the various constants K1 and K2 depending on what is the medium, what are the two electrodes so on so forth. So, a rough estimate of the MRR can also be arrived at based on what is the melting temperature of the material in question. So, if theta m let us say is the melting temperature empirically there may have to be a relationship between the material removal rate and that particular temperature. So, that relationship has also been defined in literature as if theta m were the melting temperature in degree Celsius of the material, then the amount of material removal rate in terms of mm cube per charge of the material, millimeter cube per amount of charge of the material is given by this expression here 4 into 10 to the power of 4 theta m to the power of minus 1.23. It, therefore, indicates that if theta m for material is higher the material removal rate should be lower and vice versa. There is a minus sign on this superscript minus 1.23. So, we of course, have assumed to come at this particular you know formula we have assumed that only an average sparking condition meaning thereby that we see the spark to be dancing all round and the total removal rate Q is an average effect because of such sparks going around. It is not really questioning a single spark and the way it behaves, it is multiple sparks over the whole surface and how the average material removal rate Q may be governed by that multiple sparks formulating at different points of time within the electrode and the workpiece.

So, the MRR also typically depends on the circulation of the dielectric fluid. So, if the kerosene oil or silicon oil which is there inside is flowed at a higher rate there will of course, be a more velocity head and lesser pressure head thus forcing more material to go into this fluid and so MRR would go up and without a forced circulation the wear particles you know they continuously kind of melt and reunite with the electrode which also results in problems. So, therefore, you are melting the same material again and again because there is no forced circulation. So, circulation ensures that whatever debris is formulated is being nicely packaged into the flowing stream and goes out along with that stream. If the particles were there in that zone in that region, they would really have a chance when the spark has the spark time has the discharge time has ended to again because the temperature the heat transfer at that scale is very high the particle may as well reunite back and deposit as debris again.

So, typically you would be again and again removing the same material layer and thus the MRR would come down in that particular manner. So, two things we have understood today is that the point where the MRR is the highest is really the point when the pressure is below the atmospheric pressure and number two is that there is a huge utility for the material to be removed higher with higher flowing streams of the dry electric fluid. So, velocity of circulation is a very critical parameter for determining the MRR in this particular method or of machining. So, in fact, if you look at this trend here it actually mentions that with and without forced dry electric circulation what are the repercussions. So, the material removal rate without forced as you are seeing is you know it really goes down as the electrodes advance towards each other.

So, this is the inter-electrode gap that we are talking about IEG. So, the material removal rate is going down fast you know without the circulation, but it is kind of arrested it does not go that fast if there is forced dielectric circulation. So, one good point that also can be concluded is that after discharge is complete let us say the spark settles down the dielectric medium around the last spark should be allowed to deionize fully. So, the iron column which is formulated should be able to get deionized and that would be faster if supposing there is a forced dielectric circulation. Otherwise, there is a tendency of the spark to get reformulated every time at the same zone and then the overall rate of removal would reduce because of that.

Because already there is a melt pool which has been formulated in the zone where the spark was there before. So, it also kind of makes this the circuit between the pool and the electrode the work piece to relax a little bit it is the iron path which has been formulated how soon you can dissuade that how soon you can diffuse that into the remaining medium is also of critical importance. You have to ensure that the second spark which formulate should be at a different place it should not be in the same if the conductivity of that particular region is higher because of stagnant electrolyte there is a tendency of the spark to get reformulated there itself or at least the electron flow to be there itself. And it may not be very efficient material removal process. So, therefore, circulation of dielectric will also that way critical apart from the fact that the debris removal is also there you do not hit upon the same iron column twice because of forced circulation of the dielectric fluid.

So, there is also something that you have to ensure the voltage across the gap must be kept below the discharge voltage until this deionization is complete. And this would ensure that the current does not flow in the same iron path which is highly conducting now. And at least that diffusion time for the ions to go out or emanate out is provided by the system. So, therefore, the time required for complete deionization depends on the energy released by the preceding discharge. And if the discharge were larger in terms of energy there would be a longer deionization time needed and with forced circulation, this would go down further and further.

So, that is what the role of melting temperature would be. Now, we will come to a very different aspect of the problem that we are talking about a case where we have to keep on feeding voltage in sparks. And somehow the frequency at which energy is being applied should also be optimized with the frequency at which the spark is formulated. Otherwise, typically we are losing energy if we give a continuous supply of energy and let the process follow by itself. So, the point of time when the spark is formulated it should really be, it should match with the frequency at which the energy is being supplied.

So, the packet comes, and it gets delivered and then there is another packet which gets delivered. So, there are several different electrical circuit designs which are available to provide this pulsating DC across the work tool gap. And though the operational characteristics are different in almost all such circuits typically a capacitor is used for storing the charge particularly before that discharge takes place in the gap. So, it is used as a charge storing device, it is a device which would give a, which would charge up to a certain point of time and then try to outburst the charge at all at one go as a packet. And it intuitively seems reasonable that we should include this in the part of the circuit for talking about the equivalence of frequency in which supplying of the energy from a source to the electrode is happening with respect to the way that spark is getting transported between the electrodes.

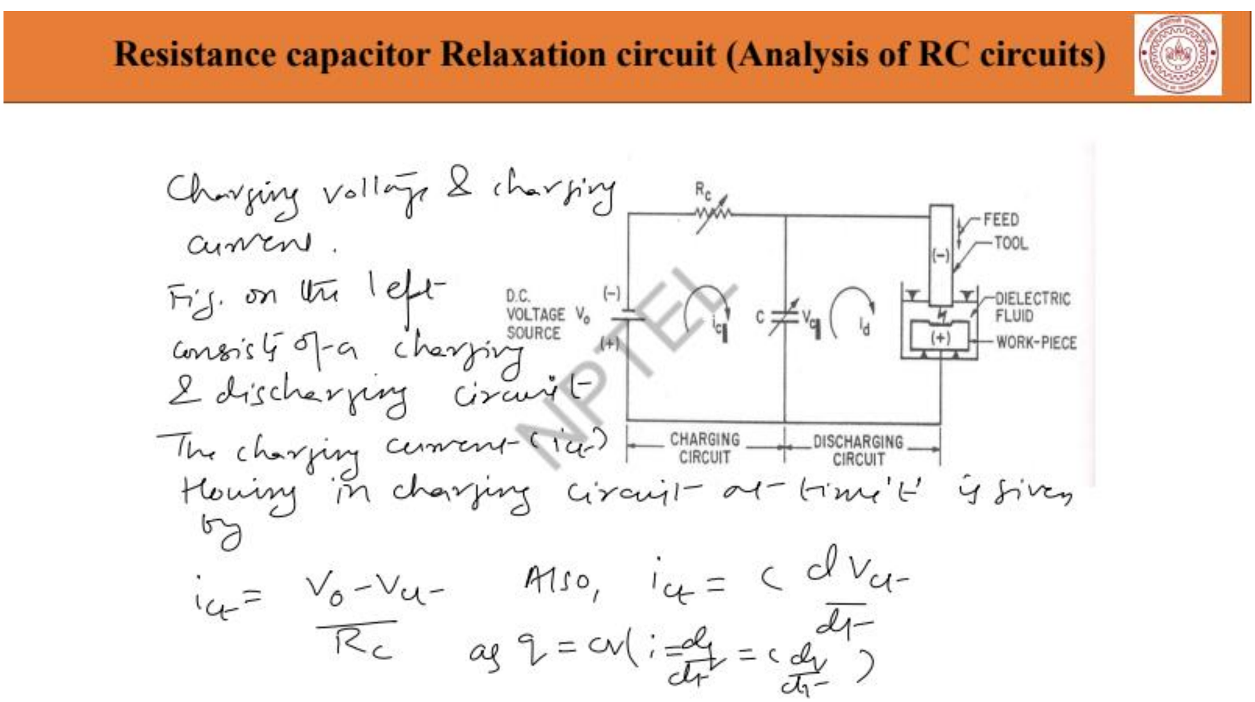
So, of course, the suitability of a circuit highly depends on machining conditions and the requirements. And there about three different principles which are used for supplying pulsating DC. One is the simple RC relaxation circuit, resistance capacitance relaxation circuit and then you can, of course, modify and make it rotary impulse generator or controlled pulse circuit so on so forth. So, these are the three principle mechanisms with which you can supply a pulsating DC signal in such cases. So, let us look at all these techniques one by one.

And the idea is that from circuit design itself, we should be able to predict somehow what is the spark energy that we are sending. And because spark energy is highly correlated to the volume removal of the material per unit time, it should also be able to give us an estimate of what is the MRR from the basic operating conditions of the circuit. So, it is in a way to sort of in a predictable manner remove material by fixing the various voltage-current relationships that you are providing on one side. So, power is Vi as we know in order to remove some material per unit time on the other side. So, let us actually look at the first basic such circuit that is the resistance capacitance relaxation circuit.

So, this estimates, in fact, such a circuit. So, there are two parts of the circuit, one is of course, where the DC voltage source is connected, and this side of the circuit is called the charging circuit. This is the common capacitor which is there between the charging and the discharging side meaning thereby that in the first go, in the first instance, there is a tendency of the current to charge the charging part or charge the capacitor through the charging part of the circuit. So, because as you know that capacitor is an open circuit to DC therefore, typically this side of the capacitor would get positively charged and this would get negatively charged and there is a huge charge containment inside this capacitor. Now, if we assume the other part of the circuit that is the discharging circuit to have the tool electrode combination. So, this is the workpiece, this guy here, right here, this is the tool and so this is the tool electrode combination and there is a dielectric fluid in between and typically this is what the EDM setup would look like.

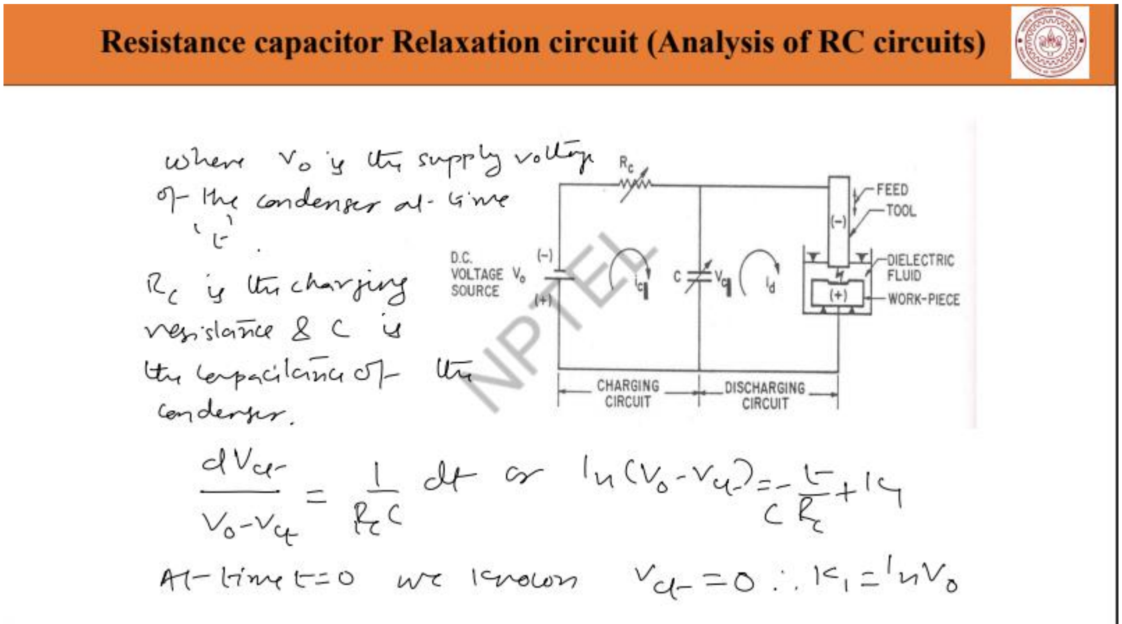
This is the EDM part, the electrode discharge machining part of the setup, and supposing we have a feed in the downward direction provided to this tool. So, that that condition of breakdown happens and there is a spark which is created. So, as the tool is moving slowly towards the workpiece and the dV by dx is increasing, the electric field is increasing there would be a point of time when whatever charge has been stored in the capacitor here in this particular circuit should typically get discharged because there is a spark being formulated by breakdown of the medium which is between the tool and the electrode, between the tool and the electrode. So, all this charge should be discharged here through the EDM and that would in fact, cause the MRR to happen, the material removal to happen, and therefore, this part is called the discharging part of the circuit. So, you really have a relationship between the way that the capacitor is charged through this DC voltage and then the way that the capacitor is discharged which does not go into the voltage source, but actually goes into the discharging part of the circuit.

So, that you have a spark. So, let us look at some analysis here. So, charging voltage in this particular case is V0 given by the source voltage here, and let us assume that charging current in this particular case is some value ic. So, we call it ict because there is, of course, a time component it is a transient process the way that the charge transfers from the voltage source to the capacitor here. And so therefore, the charging current ict flowing and charging circuit, let us say a time instance t is of course, given by the expression V0 minus Vct by Rc. Let us assume there is a voltage Vct being formulated across the capacitor.



Now, of course, this voltage would be in opposition to the voltage which is formulated here. So, the voltage formulated here is let us say in this direction, this other voltage would also be in opposition to this direction. So, this is Vct, this is V0. So, Vct, V0 minus Vct is the drop and this drop is across the variable resistance the variac Rc and that is what ict is. So, the way if we consider the way that the capacitance charges, let us say if q t is the amount of charge that is coming into the capacitor.

So, q t can also be given by the fixed capacitance value C times of Vct, where Vc is the voltage on the charge on the capacitor as a function of time. So, Ict is really nothing but q dot t. So, you can simply have it as c Vct by dt and therefore, you can say that this V0 minus Vct by R which has also been predicted from the earlier equation 1 is actually equal to C Vct by dt. So, we can solve this equation and here we know that Rc is the charging circuit resistance and C is the capacitance of the condenser in question, this condenser. So, dVct by V0 minus Vct is 1 by Rc C dt or you know if you integrate it in time, you get ln V0 minus Vct is actually equal to minus t by Rc times c plus k1.

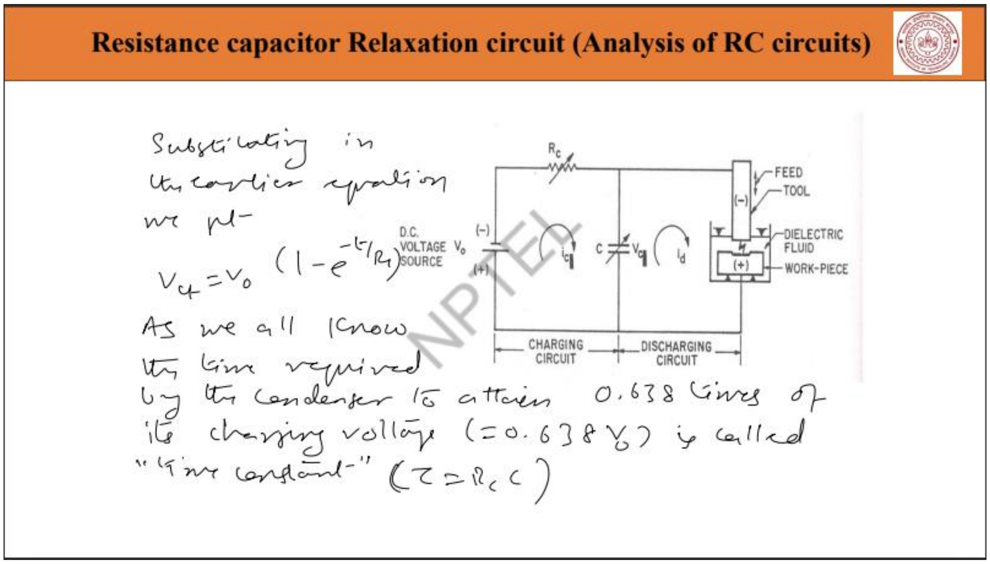


We know that at time t equal to 0 the amount of charge which has been developed here the Vct is also equal to 0. So, Vct equal to 0 at time t equal to 0, because hardly any charge has flown from the voltage source. We can assume a switch actually in the circuit and we can say that the switch closes at time t equal to 0. And therefore, we know that K1 for t equal to 0 Vct equal to 0 becomes ln V0. So, if we substitute in the earlier equation which was actually this right here equation A, we would get ln V0 minus Vct equals minus t by Rc c plus k1 value which is ln V0 or we can say V0 minus Vct by V0 ln is minus t by C Rc.

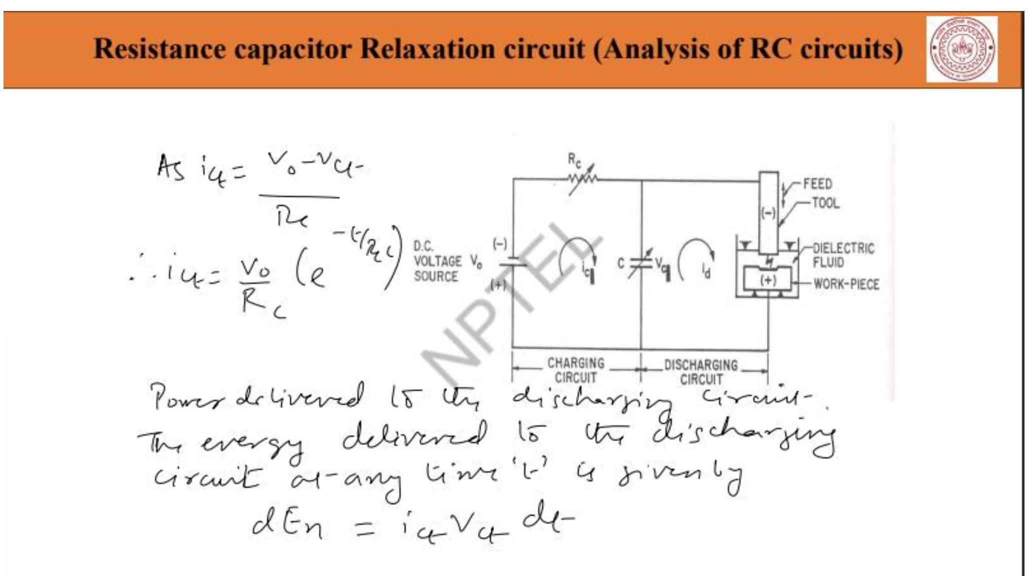
In other words, the Vct value is actually equal to V0 1 minus e to the power of minus t by C Rc. That is how the Rc circuit typically the derivation is. So, you have Vct is actually equal to V0 1 minus e to the power of minus t by Rc c. So, this is called the time constant of the Rc circuit. And we also know that the time really which is needed by the condenser to attain about 63 percent charging voltage is equal to Rc c right.

So, this is one time constant from the definition of a Rc circuit. So, in other words, it means that this Vct is actually 0.63 V0 at tau equal to Rc, tau equal to one time constant. As Ict is actually equal to V0 minus Vct by Rc, we can find this out. So, we already have the value of Vct from the previous equation as V0 1 minus e to the power of minus t by Rc C by Rc.

And therefore, we can write this down as V0 by Rc e to the power of minus t by Rc C. So, that is the value of the current in time. So, the power delivered to the discharging circuit is basically Vi right, power as you know is Vi. So, we can write this as Vct Ict.



So, this is the function of time right. And so, the amount of total energy delivered to the discharging circuit at any time t is given by dEn equals Ict Vct dt that is the amount of energy which is discharged at a certain instance of time. So, let us now actually calculate the amount of energy which is needed by substituting the values of the Ict Vct etcetera in this particular equation. dEn therefore, can be written as the value of Vcd Icd V0 by Rc e to the power of minus t by CRc times of Vcd which is V0 1 minus e to the power of minus t by CRc dt that is what dEn is. So, if we integrate equation 1 here to find the overall power with time, we have overall energy I am sorry in time, we have En equals V0 square by Rc times of minus tau e to the power of minus t by tau plus tau by 2 e to the power of minus 2 t by tau plus K2. Here the tau is the time constant which is RcC has been assumed to be RcC.



So, at time t equal to 0, we know that the total amount of energy which has been delivered to the capacitor is also 0, the process is not yet started on the charging circuit which is we have we have assumed this kind of we have made similar assumptions in earlier expressions. And so, if we put the value of t equal to 0 as

En0 here, K2 comes out to be equal to V0 square by twice Rc times tau putting the value of t equals 0 in this particular expression. And therefore, the overall En total energy delivered in time as the process of charging of this capacitor C happens is given by V0 square by Rc times tau times of half plus half e to the power of minus twice t by tau minus e to the power of minus t by tau, where tau is actually equal to Rc times of C. So, the average power can be obtained by taking a ratio of the total energy during the total charging time. So, if we assume that the total charging time is equal to tc, power should be the total energy per unit time, so charging time is tc, so En by tc.

Resistance capacitor Relaxation circuit (Analysis of RC circuits)

$$\therefore dE_n = \frac{V_0}{R_c}\left(e^{-t/R_cC}\right) V_0 \left(1 - e^{-t/R_cC}\right) dt$$

integrating on both sides we have

$$E_n = \frac{V_0^2}{R_c}\left[-\tau e^{-t/\tau} + \frac{\tau}{2} e^{-2t/\tau}\right] + K_2$$

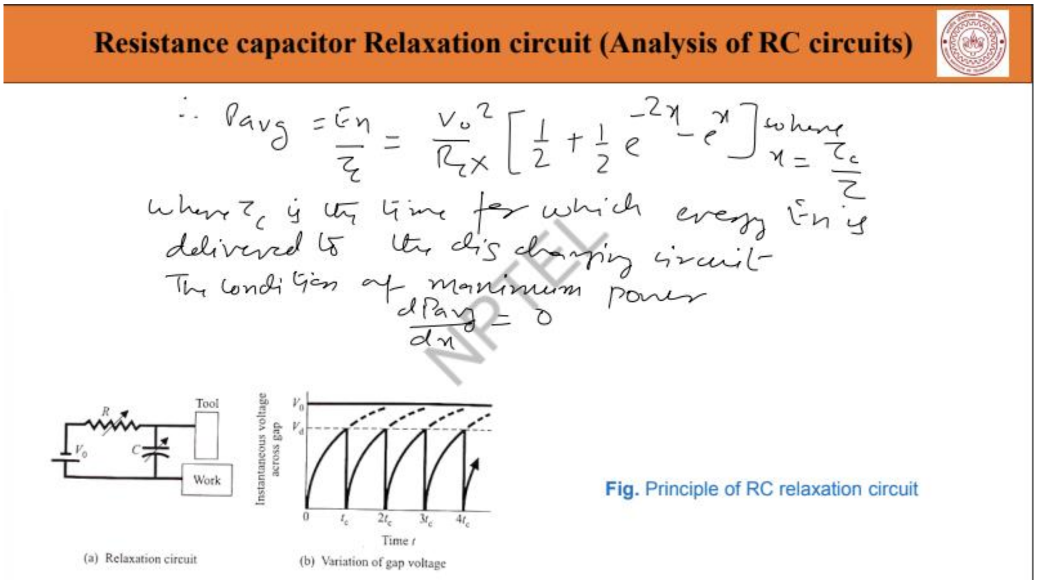
where $\tau = R_c C$

at $t = 0$ $E_n = 0$ $\therefore K_2 = \frac{V_0^2 \tau}{2R_c}$

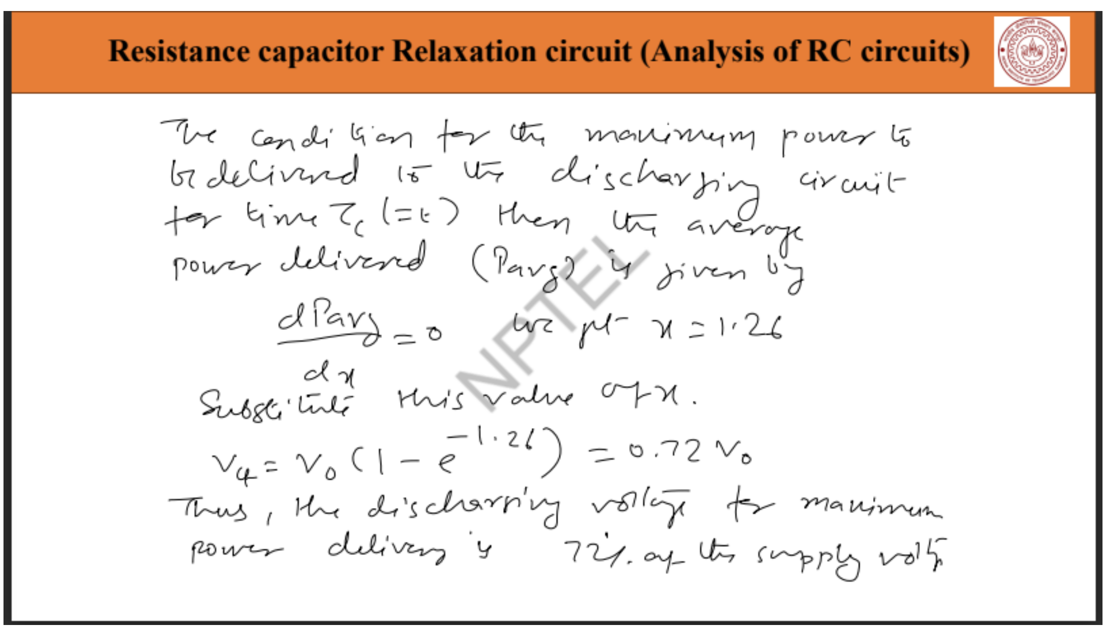
$$\therefore E_n = \frac{V_0^2 \tau}{R_c}\left[\frac{1}{2} + \frac{1}{2} e^{-2t/\tau} - e^{-t/\tau}\right]$$

So, let us assume parameter x here is equal to charging time per unit time constants. In other words, how many time constants contribute to the charging time is the new parameter x that we have introduced. So, if in terms of x if you want to represent power average, the average power can be represented as V0 square by Rc times of x times of half plus half e to the power of minus twice x minus e to the power of x. x is again this how many times the time constant is really the total charging time. So, we can actually optimize this power, because that is the whole goal behind the charging-discharging.

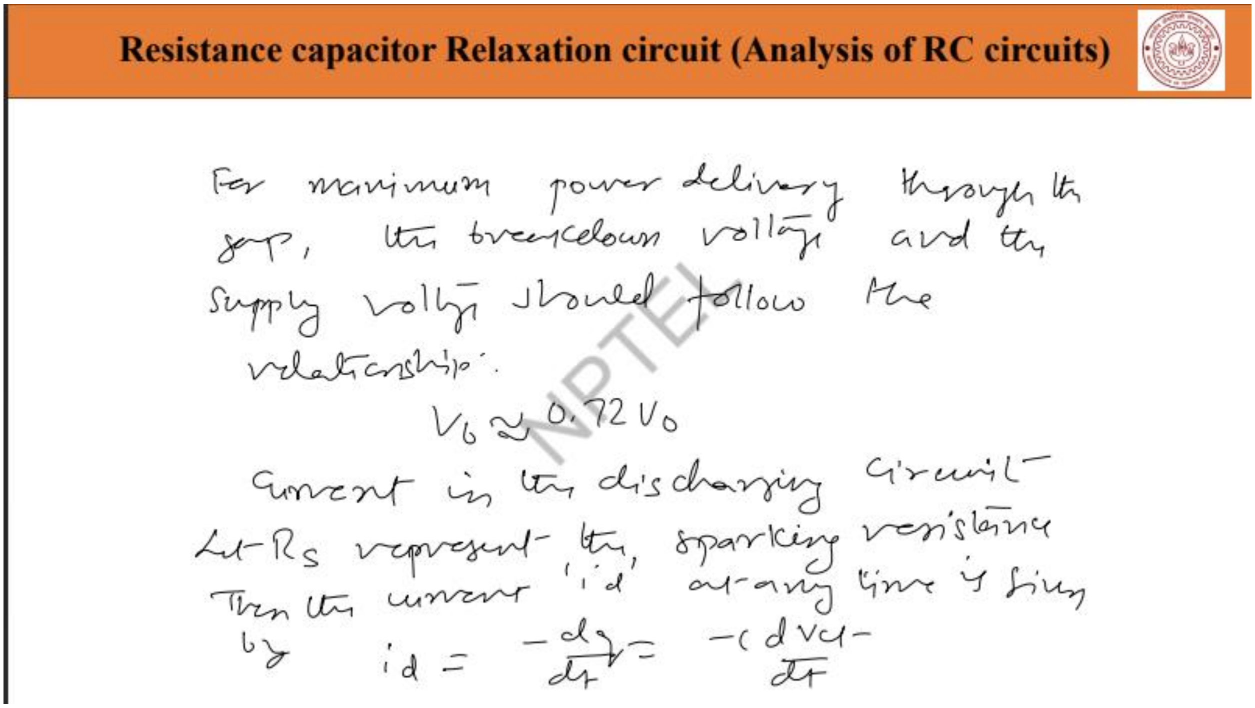
So, optimizing of average power is carried out by differentiating this with respect to the parameter x. So, we will really have to see at what point or at what x value or at corresponding to what ratio between the total charging time and the time constant tau would the optimum power be delivered. So, it gives you an idea of the frequency right, because one charging time is indicating of or is indicative of one charging-discharging cycle. So, if the total charging time is more than that, that means there is more than one charging-discharging cycle which is there. And therefore, dP average by dx equal to zero would give you that condition, and it has been obtained that this is only possible for a value of x equal to 1.26. So, therefore, if the charging time is 1.26 times the time constant of the capacitor, this corresponds to the optimum power delivery from the voltage source to this charging capacitor C in question. So, if you put the value of this 1.26, we can find out what really is the Vct, and what percentage of the voltage rise we need in the capacitor for this optimum best performance at x equal to 1.26.



So, this can be represented as V0 1 minus e to the power of minus t by RcC. We are considering this t to be equal to tc charging time. Therefore, V0 1 minus e to the power of minus x. If you put the value of x equal to 1.26 here, Vct would come out to be equal to 0.72 V0. 0.72 is 1 e to the power of 1 minus e to the power of minus 1.26. In other words, it means that the total amount of charge of the capacitor in time is actually 72 percent of the output power. So, that is about the time that you have to let the charge to rise in the capacitor before the discharge would happen, and that kind of also determines the frequency at which the capacitor should charge and discharge for optimum power delivery from the power source on one side to the discharging circuit or to the electrode discharge machining circuit on another side.



So, that is what this optimum point of 72 percent comes out to be. So, this is kind of represented in this figure here.



constant of the capacitor may be somewhere here, which is 63 percent of V0, and it is going above the operating point here for the power graph to rise to its maximum. This is the power condition corresponding to dP average by dx equal to zero, x equal to 1.26. So, in other words, we really can say that the amount of discharging voltage for maximum power delivery is 72 percent of the maximum source voltage or supply voltage. So, this is one aspect of the system that we have considered today, and then we will consider the discharging part where whatever has been now supplied to the capacitor would further be discharged into the medium in terms of spark. And from that, we can have a correlation between the material removal rate which is because of this discharging process, and the source voltage that you are using with the ICT. So, you have a control on the operational parameters to estimate predictably what would be the material removal rate. So, we will complete this in the next lecture. Thank you.