Advanced Machining Processes
Prof. Shantanu Bhattacharya
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur
Week - 7
Lecture – 18
Mechanics of EDM - II

Hello, and welcome back to this lecture on Microsystems Fabrication by Advanced Manufacturing Processes. So, a brief recap about what we did in the last lecture, we were talking about a process called electro-discharge machining, and basically, this process is about a series of sparks or a series of small, short time discharges occurring between an anode and a cathode which would produce some kind of an impression on the anode surface. So, in this particular case, the workpiece is made the anode, and the spark discharge is actually the cathode, and the basic principle is that if the tool surface which is actually the cathode is propagated towards the workpiece surface, there is always a set of imperfections and discontinuities on both the surfaces which result in distance of shortest separation happening between such surfaces, between two corresponding peaks may be. And because of that the total electric field, which is also the voltage per unit distance, if it exceeds by in any chance the breakdown field of the medium which is separating them, there is a tendency of these stream of electrons to pass by, and these electrons actually go from the cathode to the anode surface thereby damaging the anode in terms of a local melt pool and a melting surface. So, we talked about how the spark is formulated in such a process, we also did talk about how the spark covers the whole workpiece space even though the spark itself is very-very small. So, the workpiece surface can be much-much more about maybe a couple of tens of thousands times larger dimensionally than the spark, which is actually a very small, which occupies a very small area.

So, the whole idea is that it is about, it is a dynamic process where the spark traverses over different such distances of close proximities between the workpiece and the tool and covers the whole surface in turn and melts the whole surface. So, that there is a uniform machining layer by layer of the workpiece surface in question. Today we will look into a little more depth about the various issues regarding the EDM process. So, as I have already illustrated before a negative of actually a very small, which occupies a very small area.

So, this is also the same dye sinking operation as happens in any other conventional, non-conventional process like ECM. So, here also whatever is the electrode shape negative of that would be exactly replicated in the workpiece surface or workpiece electrode which makes it actually a process amenable to microfabrication. So, here the whole goal, therefore, is to be able to produce the exact negative replica on the tool of the feature of the shape that you want to imprint on the surface in question at the micron scale. So, generally, the rate of material removal from the cathode is comparatively lesser than that from the anode and it is obvious why this happens. The main reasons why the material removal rate is always higher at the anode in this case is the stream of the momentum with which the stream of electrons strike the anode is much more than that due to the stream of positive ions impinging in the cathode, though the mass of an individual electron is less than that of the positive ion.

So, what is a spark? It is essentially a breakdown of the medium and once the electrons are generated from the cathode side and it comes out of the cathode, they are free electrons, and they would create a lot of collisions between the medium particles which are there in its path thus creating ions and more electrons. So, in principle, there are two different kind of electrons which are there one is the primary electron which is being emanated from the cathode and another is actually the secondary electron which comes because of the collision of those primary electrons with the particles of the medium. So, therefore, it is an avalanche really of the electron side. So, there is a tendency of the electron density to be much-much higher in this case than the number of ions per unit volume. So, the volume density of the ions may be lower, and particularly for the cases where the products of such dissociation reactions are multi-valent in nature, the electrons would definitely be much more in numbers than the number of ions which are produced.

So, that is one of the principle reasons why the total momentum which is transferred by the electrons onto one of the electrodes that is the anode surface is much more in comparison to the momentum being transferred by the cations which are produced in the medium because of such electron fission reactions or collision reactions. So, the ion flux is really lesser, and the ion flux is the principle removal mechanism for the cathode side and the electron flux is the principle removal mechanism for the anode side therefore, the anode has a much more erosion rate and then actually the cathode or the tool side. The other reasons which are responsible for this difference in the rate of removals between anode and cathode are principally the pyrolysis and particularly the pyrolysis of the dielectric fluid which is normally a hydrocarbon. So, as I have already illustrated many times before that in the EDM process there is always a medium in between the two the cathode and the anode and that is typically one with the high dielectric constant and is an insulating medium. So, the question of breakdown only arises when such high dielectric fluids are circulated, and hydrocarbons are the most prominent ones because they themselves do not participate in formulating oxides or nitrides or so on so forth of the material on the surface.

It is a high temperature melting process of machining that we are talking about. So, what happens is that this dielectric fluid typically breaks down and because of the breakdown because of the fission reaction which is happening in between the both the electrodes there is always byproduct in form of hydrocarbon which are produced and as such these are the cations, and they rush towards the cathode they are positive ions, and they would create a thin film of carbon on the cathode itself. So therefore, this again gives a situation where already there is some kind of a layer or a coating which is present on the cathode side which is generating the electrons due to which again the bombardment of the anions or cations, I am sorry to that electrode is kind of shielded because of this superficial layer which develops or sacrificial layer which develops on the surface of such an electrode material. And there is another reason apart from the pyrolysis and deposition of a film and the electron flux being greater than the ion flux. And the third reason is basically the compressive force that is typically developed on the cathode surface primarily due to the cations.

So, when we are talking about cations in general, they are much more heavier objects because of the atomic mass of the species which is present and at least the cations that we are talking about is much more than the electrons in terms of their weight. So, this compressive force typically that is developed is because of the high momentum of the cations rushing towards the cathode. So therefore, if there is a high pressure because of the ion flux on the surface in general there is a tendency of the material which is melted away from the cathode to not go into the material medium, the dielectric medium because of that pressure. So, that pressure incidentally is not very high on the anode surface because the electrodes themselves are very lower in their weights and electron weighs about 9 point approximately 9.1 into 10 to the power of minus 32 kgs.

So, therefore, the compressive force is because of the cations is much more thus creating pressure, thus creating less chances of diffusion of the melt which is not true in the case of anions. So, these are three principle mechanisms as mentioned here for which are responsible for the differential removal rates across both the cathodes and the anode. And therefore, it is preferable that the tool be connected to the negative terminal of the DC source thus making it a cathode, and the workpiece be connected to the positive terminal thus making it an anode. So now, a situation has been obtained where there is an electric field, there is a potential and slowly the field is growing, so that it exceeds the breakdown field of the medium. So, if the tool is stationary relative to the workpiece, the gap would increase right as the material removal progresses.

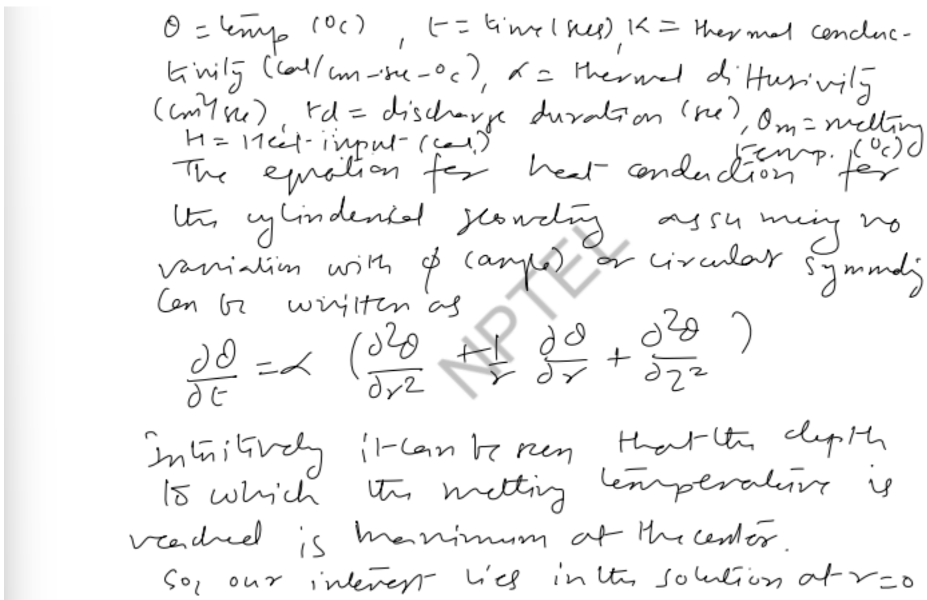
For obvious reason is that there would be a melting wherever the spark is generated and because of that there is going to be an erosion, the melt is going to come back into the dielectric fluid and one of the surfaces that is, in this case, the workpiece surfaces recedes away from the tool thus creating more distance and thus creating a situation where the electric field would no longer be equal to the breakdown field and the spark would simply cease to exist or a set of sparks would simply cease to exist. So, therefore, there are two ways that we can control this situation to make it a continuous, continued machining process. One is that we keep on increasing the voltage, so that even if the distances so generated are smaller or larger as the melting happens, the gradient of voltage dv by dx which is actually equal to the electric field, that gradient always keeps constant to the level of the breakdown field and a condition which is sufficient for the spark to happen between both the electrodes. So therefore, one way is to increase the voltage to initiate the spark and the other way really is to operate on the distance of separation between both the electrodes by moving the tool closer to the workpiece. So, if you have a situation where the tool is going closer to the workpiece, then also the distance d would decrease, and because of that the field would again be higher at a constant voltage value.

So therefore, in order to avoid this problem of increasing the voltage, it is better to feed the tool with the help of a servo drive and this can be a control system wherein the gap is sensed and the magnitude of the average gap can be controlled and it keeps it constant by sensing the voltage and equating it every time to the breakdown field which is needed or breakdown potential which is needed of the particular medium. So theoretically, if we were to estimate the material removal rate values, we really need to consider the heat transfer process between one such spark which is happening over a certain surface and the way it gives the heat or delivers the heat energy to cause the material to melt and go out. So, let us now see how to estimate the quantity of the material removal due to such single discharge or single spark and this can be determined by considering the diameter of the crater and the depth to which the melting temperature would reach. So, if a crater like this let us say is formulated on a surface like this because of an arc which comes and strikes the surface, we really need to estimate what is the zone which gets affected from the surface and this zone which gets affected really is the zone where the melting would happen. So, this mass here, right here is actually a melt pool and we call this depth d, the depth of melting temperature.

So, we somehow need to estimate what this depth of melting temperature is for a fixed crater diameter, and in order to understand this model we have to make some assumptions and we really what we model for is the temperature in the zone of the spark. So, the assumptions that we make for estimating this is that the spark is a uniform circular heat source on the electrode surface and the diameter of this heat source is 2a of this circular source and it remains constant throughout the process of discharge. So, therefore, we do not vary the diameter in our model assuming that the approximate tentative diameter of striking on the surface is same even though the spark may have some characteristics of variability, but at least the heat transfer is taking place across a circular area if you consider the spark to be like a cylinder of constant size. We also assume that the electrode surface is a semi-infinite region thus making our problem simpler and which is obvious because the workpiece actually into question is can be very large even though the area or the zone which is affected is very small on the surface. So, it can be treated as a part of the semi-infinite surface and we also assume that except for the portion of the heat source, the electrode surface is insulated from all sides.

So, the only heat transfer which is going to take place is across this spot, and remaining everything else is totally heat insulated. So, there is no heat transfer which takes place away from the spot as one moves. So, these are some things which for simplification of the model you have to assume these. Also, we assume that the rate of heat input remains constant throughout the discharge duration which may not be really true because you know the discharge also is quite transient in nature it is time-varying, but then for sake of simplicity, we have to assume that the heat input rate here let us say H is the heat per unit time, heat energy per unit time supplied on to this surface or that remains constant throughout the discharge duration. We also assume that the properties of the electrode materials do not change, or they are not temperature dependent.

So, they do not change with temperature, and also that the vaporization of the electrode material is neglected. We assume the electrode is not really affected by the spark of electrons which are sent. So, these are the six assumptions that you make in order to estimate this model and really the model is quite simple now because we now need to assume certain things. For example, let theta be the temperature in degree Celsius of a particular zone in that circular region, t be the time in seconds at which theta is being monitored, K be the thermal conductivity of the anode which is also the workpiece over which this melt phenomena is happening. This can be assumed to have a units of calorie per centimeter second degree Celsius and let us assume a thermal diffusivity alpha again of the workpiece and as you all know that thermal diffusivity is basically the has the units of centimeter square per second cut.


θ = temp (°C), t = time (sec), K = thermal conductivity (cal/cm-sec-°C), α = thermal diffusivity (cm²/sec), td = discharge duration (sec), θ_m = melting temp. (°C)
H = Heat input (cal)
The equation for heat conduction for the cylindrical geometry assuming no variation with ϕ (angle) or circular symmetry can be written as

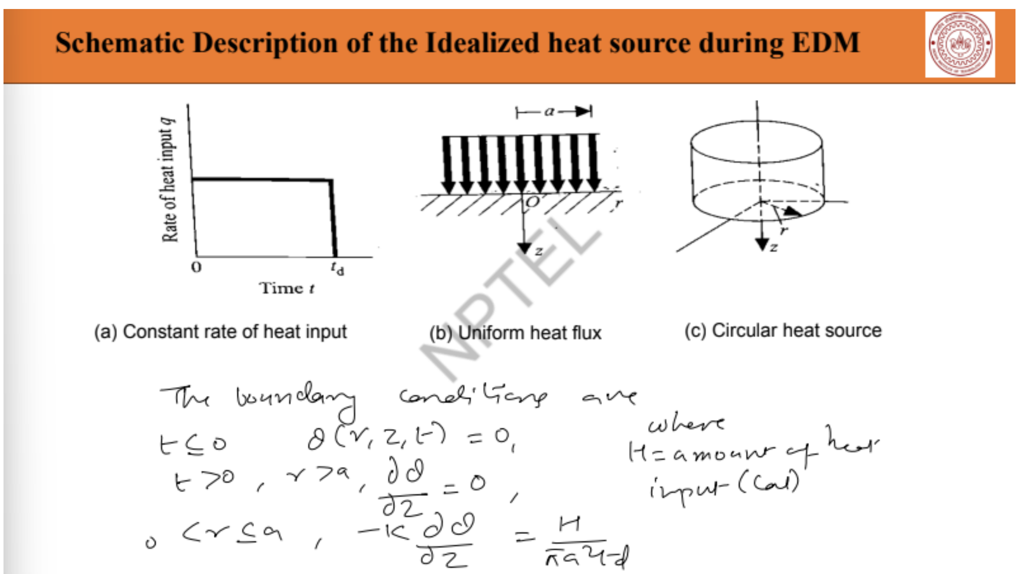
$$\frac{\partial \theta}{\partial t} = \alpha \left(\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r}\frac{\partial \theta}{\partial r} + \frac{\partial^2 \theta}{\partial z^2} \right)$$

Intuitively it can be seen that the depth to which the melting temperature is reached is maximum at the center.
So, our interest lies in the solution at r=0

Also, we assume td to be the discharge duration in seconds and theta m to be the melting temperature in degree Celsius and these are all related to the workpiece. So, this for example, is thermal conductivity of the workpiece or anode, same is true for this thermal diffusivity of workpiece and this is melting temperature of the workpiece. We also assume H to be the input heat rate in calories. So, input heat which comes to that small area where the spark contacts, let this heat be in calories, unit of energy, and the job now is simple because we really need to estimate what the equation for heat conduction for the cylindrical geometry is. This is the geometry that we are considering on a semi-infinite workpiece surface.

So, therefore, if we assume no variation with the angle phi, phi is this angle. So, we consider circular symmetry. Then the way that temperature behaves in time d theta by dt is represented as thermal diffusivity alpha times of the variation of the second derivative of theta with respect to the temperature, the radius, radial distance plus 1 by r del theta by del r plus del 2 theta by del z2 and z is basically the variation in this direction, this is the z direction, this really is the r direction and this exactly is the angular variation. So, the way that temperature, let us say this is the melting zone that is formulated. So, the way that the temperature varies in this melting zone is really a function of the rate of variation of temperature with respect to z, z being this depth direction and it is also a function of the rate of variation of temperature with respect to the radial direction r from the assuming a circular symmetry of this particular heat source.

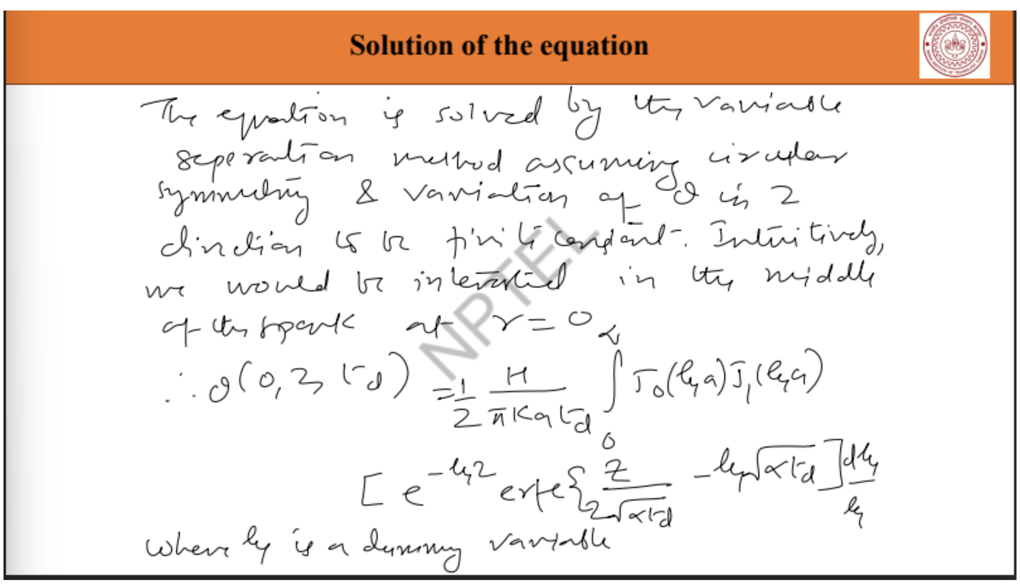
So, intuitively it can be seen that the depth to which the melting temperature is reached would be maximum at the center. So, that is considering that it produces a hemispherical crater, this depth here is the maximum, and as it moves away from the, let us say r equal to 0 region to the outside the temperature varies quite a bit, and also the melting temperature-depth varies quite a bit based on that. So, our interest principally lies in this region corresponding to r equal to 0 because this gives you the maximum depth up to which the melting zone reaches thus giving you an estimate of the crater volume that you are able to remove using that maximum depth and using a circular diameter of an equal to 2 r on the surface of the workpiece. So, let us now assume boundary conditions. So, in our particular case in this example, we can assume the following more boundary conditions to solve this particular equation.



So, the various boundary conditions are that corresponding to time before the time of observation that is at point of time 0 in our reference frame, we do not consider any temperature around the region. So, in fact, theta which is a function now of the radius, the depth, and the time t is considered to be equal to room temperature. In this case, we consider room temperature to be in our reference frame 0 temperature. We then assume that at time t greater than 0 as the process has started for any r which is greater than a, a being the total radius of the spark as you can see here, this is the zone which is heat affected you assume this other to be fully insulated region. So, you assume that for r greater than a there is no variation of theta as such in the z direction right.

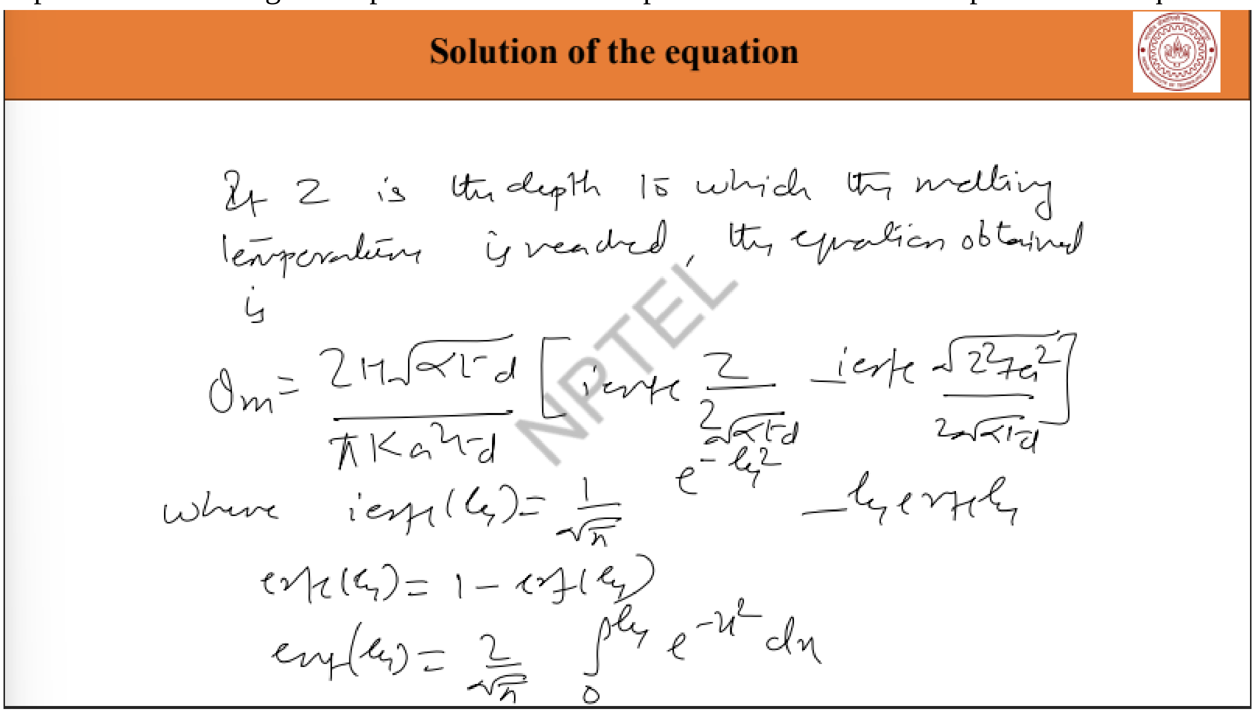
So, basically, the crater formulates like this and here there is no melting point which has been reached similarly here in this zone there is no melting point which has been reached. And finally, we have for the region r between 0 and a. So, typically it means bulk of the circular area of the spark between 0 and a here. We assume that the heat transfer is basically governed by the equation K delta theta by delta z is equal to the total amount of heat per unit area per unit time. So, it is basically the heat rate coming into the area per unit time.

So, is the flux rate pi a square time of discharge td. We already have assumed H to be the amount of heat input in calories. So, with these three boundary conditions, we kind of limit ourselves to a very simplified you know solution of the equation which has been given in the earlier step where the space-time relationship of temperature in the melting zone has been sort of correlated. And the final solution which is arrived at from this equation which we can actually do by either variable separation method or by assuming a combined parameter method. So, we have the variation of theta in z temperature theta in z direction with an assumption that we do everything at r equal to 0 where the depth is the maximum as represented by theta at 0 for different z values for the whole duration of the spark discharge td to be given by 1 by 2 pi times of K thermal conductivity a is the diameter times of H and divided by time of discharge td integral 0 to infinity J0 zeta a J1 zeta a times of e to the power of minus zeta z some error function of the coefficient z by twice root of alpha td minus zeta root of alpha td del zeta by zeta.



So, in fact, this is what the estimation for temperature would be at point r equal to 0 at the center of the spot in question. Zeta here is just a dummy variable, is a variable of integration that we are concerned with. And this actually can be a combined parameter you know. So, it can be having or containing the radius, it can be containing the time, or it can also be containing the z value. So, with this combined parameter method you can actually get this estimate at the point the center of the spark at r equal to 0 corresponding to all z's and all the depths which are obtained and all the time of discharge which is treated to be constant by our assumption in this particular case for the spark.

So, having said that we now have a relationship between the temperature and the depth in the melting zone. So, if this is equated to the melting point of the material you can find out a plot for the z values from this relationship. So, let us see that if z is the depth to which the melting temperature is reached the equation can be simplified as theta m equal to twice h root of alpha td by pi thermal conductivity a square td times of the error function 3 of the coefficient z divided by twice root alpha td minus error function of the third kind Ierfc of the coefficient root of z square plus a square divided by twice root alpha td. And the way that this error function of the third kind is defined is that this Ierfc of the variable zeta is defined as 1 by root of pi e to the power of minus zeta square minus zeta erfc zeta where erfc again is the second error function defined as 1 minus erfc of zeta and erfc of zeta as we all know is the numerical integral 2 by root pi 0 to zeta e to the power of minus x square dx. So, therefore, we have a very concrete way to interrelate the depth of melting temperature with depth z from this particular equation.

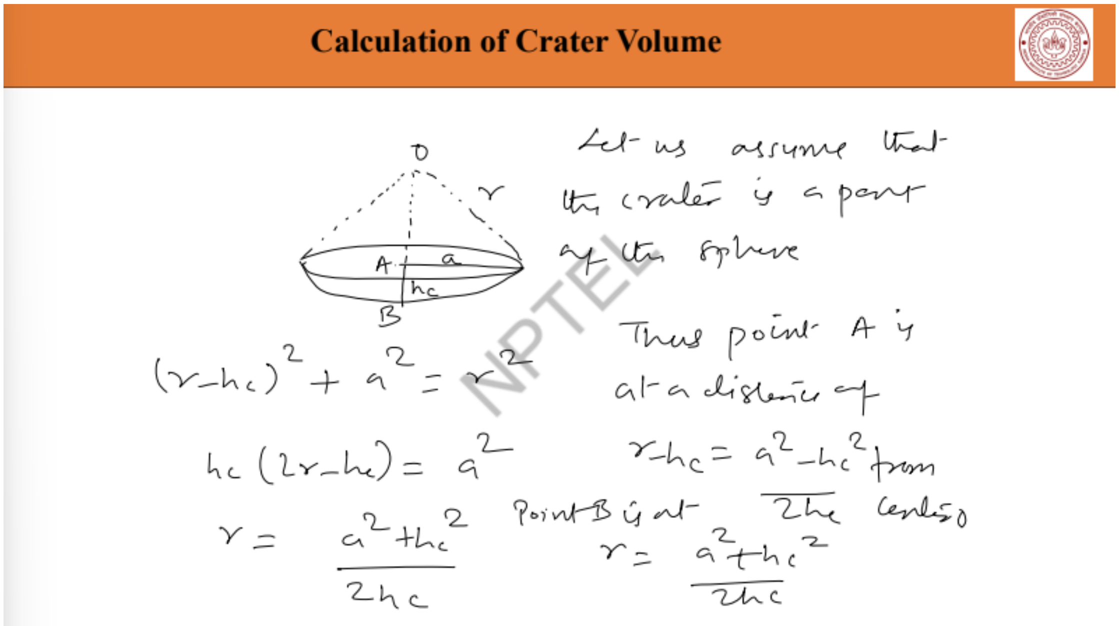


So, once the value of this depth of melting temperature is arrived at the question is that if supposing we define the amount of heat energy in a little different manner particularly you have to accommodate for the fact that the material is melting and there is some kind of heat which goes into the mix without really getting registered on the temperature scale and so that amount of heat is also known as the latent heat. So, if we somehow assume that heat to be not recording or recorded in terms of a temperature rise then that heat needs to be taken off from the final heat equation. So, with that kind of an assumption, the amount of heat would actually be used for the temperature rise whether in the molten state or in the solid state of the material would be quite different by neglecting the latent heat of melting in this particular case. So, we have to modify the equation quite a bit and the rate of heat input per unit area per unit time would then be given by the equation the total heat H total which comes from the spark into the workpiece minus the latent heat per gram of the material which is Hm assumed to be Hm in this particular case times of density times of the volume of the material which is removed which is again the volume we assume this volume to be cylindrical for our estimation times pi a square z divided by of course, because it is a heat flux per unit area per unit time pi a square td. So, H total here is actually total heat energy in calories we have already defined this earlier in our earlier assumption and Hm is the latent heat in calorie per gram rho is the density of the molten material and of course, 2a is the crater diameter.

So, typically we are assuming as you have seen here cylindrical crater although in actuality it is actually a hemispherical or a quadraspherical crater. So, where this 2a the spark diameter or maybe you can call it the diameter of the crater assuming it to be the same as the spark diameter is actually given by an empirical relationship times of W to the power of coefficient n1 times of td to the power of coefficient n2 and this is actually in centimeters. So, this is purely empirical from experimental experimentally obtained results W here is the total pulse energy, and normally this energy is given in joules. We already know that td is the time in this time of discharge in seconds and the coefficients n1 and n2 and K are all constants, and they really depend on the property of both the electrodes and the medium. So, the K the constant here n1 the first coefficient the power coefficient n1 of the total pulse energy W, and the power coefficient n2 of the total time of discharge they all depend on the tool electrode tool or workpiece material as well as the dielectric medium which is between them.

So, they are all dependent they are essentially constants and are all dependent on the tool-workpiece and dielectric medium. We found out last time that the amount of heat of latent you-know nature is estimated by assuming a cylindrical size of the cavity although it is actually a quadraspherical size of the cavity. So, let us actually also try to estimate the volume of such a crater. So, that more accurate form can emanate out of the total heat which is needed for the depth of melting temperature analysis. So, here let us say that if the melting temperature depth is Z and the crater diameter is 2a, and at r equal to 0 that means radius equal to 0 the maximum melting temperature depth Z maximum is given by hc which is actually the crater depth for all practical purposes in centimeters and the Z max of course, is proportional to Z hc is proportional to Z.

So, the total volume of the material Vc which removes, or which is actually contained in the melt pool of this particular crater size of maximum depth hc and the total diameter 2a is given by a very simplistic expression and we will try to arrive at it from just by using an elemental analysis of the sphere. So, it is pi by 6 hc 3a square plus hc square in centimeter cube. Let us see how supposing we have a crater-like this of circular nature and the crater has maximum depth hc at the center and the radius of the crater is we need to calculate what is the volume Vc of this particular crater. So, let us extend this center all the way up to the center of the sphere of which this crater is a part of and we can always arrive at that this is this being hc and this being we have let us call this O, this a, this B and this some point c here. So, we have that if we assume the total radius to be r here of the particular sphere assuming the master sphere's radius to be r of which the crater is a part of by using Pythagoras theorem in the triangle OAC we have r minus hc square plus a square equals r square which essentially means that r is equal to a square plus hc square by 2 hc.



So, the point b on the same scale can be identified as r which is actually given by this a square plus hc square by twice hc, and point A on the same scale can be identified as r minus hc assuming O to be the origin this is the origin and so therefore, this can be a square minus hc square by twice hc r as you already know is given by this a square plus hc square by twice hc. So, then it just boils down to a simple problem of integration let us assume this crater to be present like this. We have just twisted the sphere upside down this guy this right here is the full depth of the crater hc and it is a part of this sphere here with center O and basically we can actually consider a very small element like a cylinder in this particular area and then try to see what is the volume of this and integrate such cylinders starting from the point A to the point B and we already know that B is defined by r which is a square plus hc square plus by twice hc a is defined as a square minus hc square by twice hc. So, then it becomes a very, very simple problem if this distance from the center were y. So, we would be left with the radius here as r square minus y square whole under the root which would be true because in triangle OAC you have to apply Pythagoras theorem to obtain this expression here.

And therefore, if the radius of this cylinder here this small cylinder here is given by this r square minus y square whole under the root the total volume dv that the cylinder would have is pi r square minus y square and let us assume that this small length here is dy times of dy and integral dv is really for particularly for the crater is from r varying between point a that is h square minus or a square minus hc square by twice hc to the point b a square plus hc square by twice hc of dv. So, this comes out to be pi by 6 hc 3 a square plus hc square centimeter cube. So, we obtain that the amount of crater volume in this particular case comes out to be equal to this pi by 6 hc 3 a square plus hc square and this volume can be effectively used for predicting the amount of heat lost because of melting of the material in terms of latent heat of melting. And the net heat which is available now is the total heat H total minus hm times of rho the density of the material times of this crater volume. So, it is H total minus Hm times of rho times of pi by 6 hc 3 a square by plus hc square, and this you can, of course, make per unit area per unit time because it is the heat flux essentially.

So, that is what you can use as a boundary condition for solving for the zone between 0 and a of r where minus K d theta by dz is actually equal to this value. So, we are now being able to predict very accurately the relationship between the depth of melting temperature and this plot of the crater really which is the various z's at different values of r from the center of the particular crater. So, we are towards the end of this lecture, and I would just like to recall that we have been today able to theoretically sort of predict the whole zone of melting in for one spark. Just to recall there are several such sparks which occur at a certain frequency and one way of estimating the material removal may be really to look at the spark frequency and seeing for one particular discharge duration how much material gets removed in terms of a crater. So, you can get a ballpark figure of the estimate of the MRR or material removal rate there.

So, in the next lecture, we would like to extend this further and try to predictably answer some of the questions as to what the MRR would be from various methods. Thank you.