Elements of Solar Energy Conversion
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Lecture – 09

Hello and welcome back to the series of lectures on Elements of Solar Energy Conversion. So, so far we have covered 8 lectures and today is the lecture number 9, ok.

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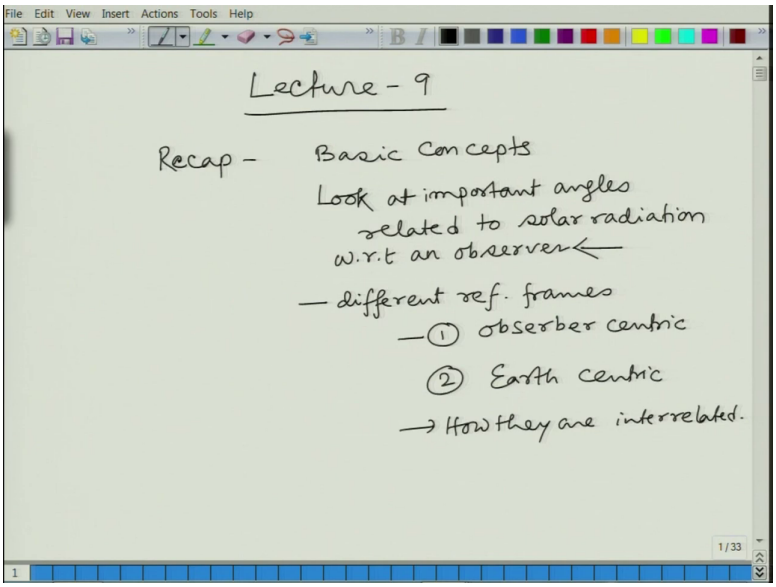

So, we are almost through about a 4th or a 5th of the whole content and the initial stage we go a little slow. So, that is why it took long, but as we go on our pace will increase a little bit and as you have the flexibility of pausing the video and watching it again. So, I do not think that will be a problem.

So, whenever you feel that things are not getting clear, please go back look at the older videos, or the videos of earlier lectures, and then you come back to the point, so that you understand all of it till that point and you can visualize. The most important thing I always stress that, you need to visualize all these different angles, solar positions, then only you will be able to grow your intuition. And, intuition is the thing that tells you that you have learnt a subject ok.

So, let us start with lecture 9. So, so far what we have covered is the basic concepts ok. Related to sun and earth relationship and also related to the solar radiation, it is spectrum how it varies the effect of atmosphere and all.

So, I am now telling all of those as basic concepts. And, now what we have started to look at important angles related to solar radiation with respect to an observer. Or rather we are interested in for a solar collector system how much radiation we can expect and all those things and that is why we are interested in this important angles and their correlations ok.

So, we have covered different reference frames majorly the observer centric reference frame and earth centric, which means the reference frame origin is at the center of the earth. So, how they are interrelated, that also we are looking at ok.

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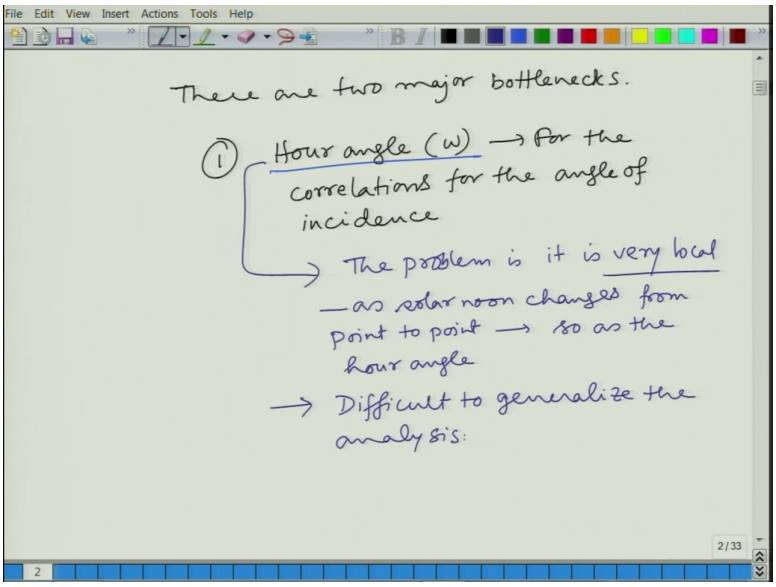

So, so far what we observed there are two major bottlenecks, that we observe where we stopped in the last lecture. First one is in all our relationship that we have derived so far we have used hour angle omega for the correlations, for the angle of incidence. As we discussed angle of incidence is a key angle, which determines the intensity that is available to that solar collector surface.

So, hour angle is what we have looked at ok, but the problem with this hour angle is the problem is it is very local ok. Because, hour angle tells you with respect to the solar noon what time it is right. And, solar noon changes from one point to the other with respect to the longitude right.

So, as solar noon changes from point to point so, as the hour angle right. So, if that is so, local then it is very difficult to generalize this particular analysis. So, difficult to generalize the analysis and that is why we need to make something which is more global.

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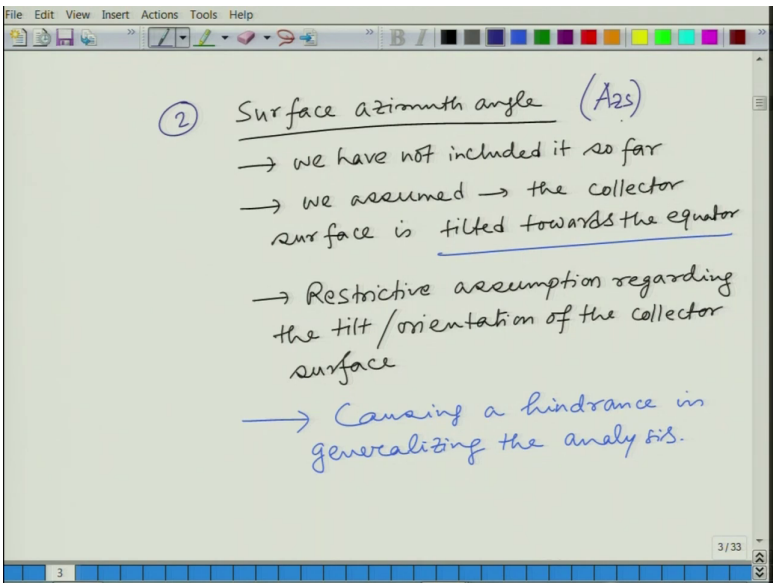

That is the first bottleneck we have seen and the second one is the surface azimuth angle ok. So, this surface azimuth angle we have not included it so, far. Why we need not include it so far?

Because, we assumed and in one case we also mentioned explicitly, that the collector surface is tilted towards the equator. So, this we assumed; that means, we have restrictive assumption. Assumption regarding the tilt of the tilt or orientation of the collector surface, again this is causing a hindrance in generalizing the analysis ok. So, how do we designate this

solar azimuth angle is designated with AZS ok. So, these two major factors we need to get rid of.

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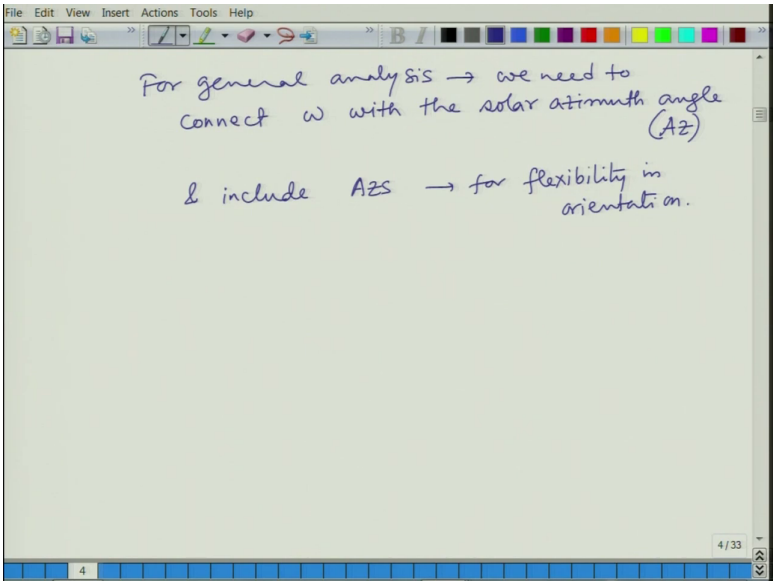

So, for general analysis we need to connect this hour angle with the solar azimuth angle. So, if you think closely, then both of solar azimuth angle is designated at as AZ ok. Now, if you think closely both of these quantities actually measure similar thing; that means, the time; time with respect to the solar noon.

That is what explicitly measured by the hour angle omega, but the solar azimuth angle, it measures the on the equatorial plane ok, with respect to the prime meridian, but in the equatorial plane, not in the observer plane.

And, so, basically they are connected and we have to establish that connection ok. And, include this surface azimuth angle AZS in the analysis. So, that any orientation can be analyzed throughout the I mean through this formulation ok for flexibility in orientation ok. So, first let us look at the physical picture ok, what do this AZ and AZS mean? Ok.

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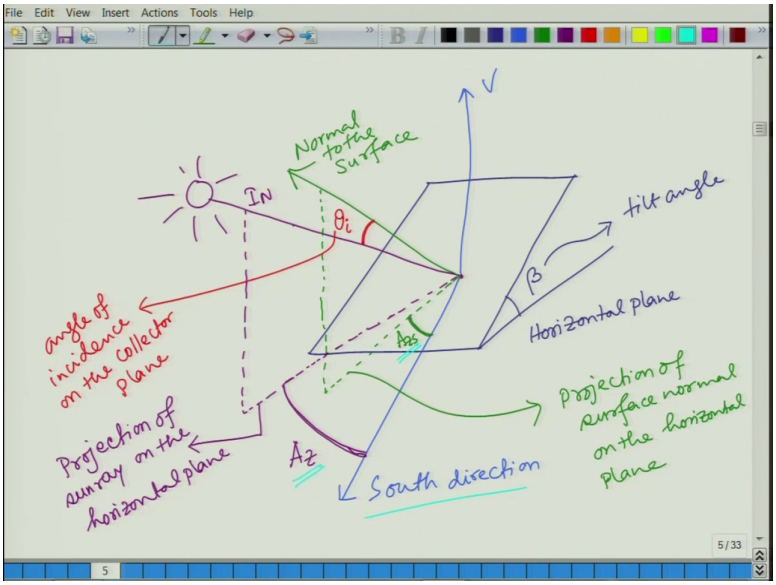

Suppose, we have a plane ok, which is tilted by an angle beta with the horizontal. So, if this is the horizontal plane, this beta is the tilt angle right. So, these designations are very standard. So, please do not use any designation, then it will be very difficult to handle several of these angles. So, whatever you may use some other designation as I am using, but I used urge you to use the same, so that there is no confusion between different angles ok.

So, if that is the case, let us take a point on this particular plane ok. And, with respect to that point we can draw vertical ok. And, we can also draw a surface normal, this is normal to the

surface. So, this is the surface normal vector ok. And, we can also draw the direction of sun ray ok.

So, let us say this is the direction of sun ray here we have apparent location of the sun ok. Now, what you can do? Ok. Another thing we need to mention here is the south direction ok. As we have talked about the azimuth angles are always measured with respect to the south direction right that is how the solar azimuth angle is measured.

So, let us say this is our south direction ok. Now, what we can do? We can draw projection of these two important vectors; one is I N, the other one is the surface normal, on this particular horizontal plane. So, if we project, the surface normal here and if that projected point is connected to the observer point, then the angle it makes ok.

So, let me use the same colour coding, so that you can understand. So, the angle it makes this projection on the horizontal plane with the south direction, these angle is our surface azimuth angle AZS right. And, the same thing we can do, we can project the direction of sun or sun rays to the horizontal plane and connect it to the observer location ok.

So, now again that particular connecting line. So, basically this particular line is the projection of the sun rays on the horizontal plane ok. Let me write that explicitly. This is the projection of sun ray on the horizontal plane ok. And, the other one this one is the projection of surface normal on the horizontal plane ok.

Now, the angle this projection of sun ray that makes with the south direction, that we call the solar azimuth angle A z. So, in this figure you can see that the tilt angle and both the solar and surface azimuth angles are put together, so that you can visualize the whole picture ok.

Now, one important angle we want to show here, so that the relationship is even more clear. So, what is the angle between the surface normal and the sun ray, can you tell me? Please think of it and now I am writing this is the angle of incidence right. So, this is the angle of incidence on the collector plane ok.

So, now, both these angles; both these angle and the solar azimuth angle both are now oriented or measured with respect to a single reference frame, which is global south ok. Global south means from wherever you are you have to direct the vector towards the south pole of earth ok.

So, that is a unique thing and you have to I mean it is general ok. Because, that is a global south direction and you can find it anywhere in the earth, wherever you are interested in ok.

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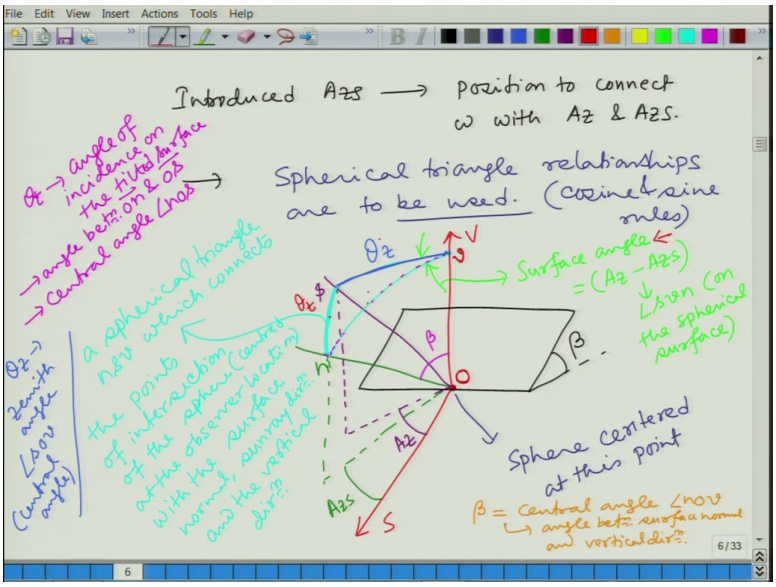

Now, as we introduced AZS or surface azimuth angle. Now, we are in a position to connect this hour angle with AZ and AZS ok. And, now we are going to use the mathematical relationship that we have taken from the three dimensional geometry. That is the spherical

triangle relationships are to be used ok. You remember the cosine and sine rules that we learnt couple of lectures ago I guess ok.

So, if that is the case let us look at this particular figure once again. So, let us say that this is the plane, that we are interested in this is the tilt angle beta with the horizontal. And, suppose we are here at this point, here is the vertical direction, here is the south direction and this is the suppose this is the surface normal and this is the sun ray direction ok.

So, we can have a sphere. So, let us say that we have a sphere centered at this point ok. Then, if we have that sphere, what will it do? It will have some intersection with these three lines; one is with the vertical line. So, let us say this is the point V small v let us say ok. And, for the surface normal let us say this is the point n ok. And, for the solar radiation direction let us say this point is S ok.

So, this particular cn triangle is a spherical triangle nsv, which connects the points of intersection of the sphere, which is centered at the observer location. So, point of intersection with the sphere of the sphere with the in the same order.

So, to the surface normal and then sun ray direction and the vertical direction. This is I am writing it explicitly, so that you can visualize and think that, what is this solar or what is the different angles, that will be relevant to this particular solar this spherical triangle ok.

So, now we will try to locate few angles here. So, one thing is again we can drop these projections ok. So, this one will be our AZS the surface azimuth angle and then we can also drop a projection here ok. So, this angle will be the solar azimuth angle AZ ok. And, now what we can see here, this one you have to visualize that this particular angle ok, which is surface angle ok.

So, this surface angle is nothing but your AZ minus AZS. That you have to visualize and make sure you understand that that surface azimuth angle will be the difference between the solar azimuth angle and the surface azimuth angle ok. So, which angle I am meaning here?

This is the svn angle on the spherical surface ok. What other angle can we see here is ok. Let me know, I have to show all the angles here itself ok. One thing I forgot I need to mention this is the origin of the sphere or the center of the sphere, which is the observer location O.

Now, another thing we can write here is the again the, this one you have to visualize, that beta is nothing but the central angle nov ok. So, what is the central angle nov? This is the angle right. So, this is the line that is making a central angle between no and v right and this angle is this and it is nothing, but beta. Because, it is the angle between the surface normal and the vertical direction ok; so this is the angle between surface normal and vertical direction. So, that is the tilt angle right.

So, that is one angle we got the other angle that we can see here is this one ok. This is again a central angle. So, that angle is our theta right, the angle of incidence. Let us say theta t; t stands for the tilted surface. So, angle of incidence on the tilted surface ok. And, what angle is that? That is the angle between n or on and os.

So, the surface normal direction and the sun ray direction that is the angle of incidence right. And, on this particular case it is the central angle delta or angle nos right; that is also a central angle theta t. And, the other angle that we can see is this angle ok that is also a central angle and angle between the vertical and the solar direction. So, that is the zenith angle right.

So, this theta Z is zenith angle and it is the central angle sov that is a central angle. So, please differentiate between the central angle and the surface angle. So, here we have talked about one surface angle, which is this one ok, that is AZ minus AZS and we have talked about few of these zenith angle, the theta t here and here it is beta. So, three central angles we have talked about and one surface angle ok.

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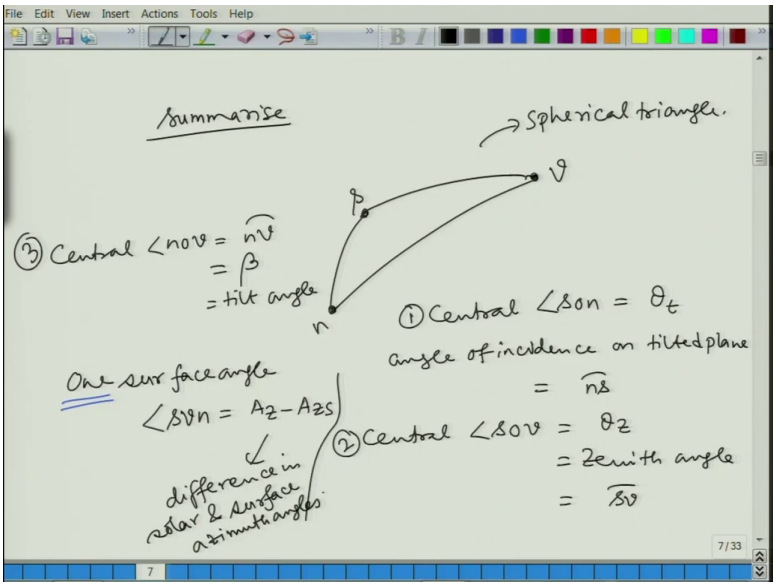

Now, let us go to the next page and let us try to say this. So, just to summarize from that mess, that we have this and this spherical triangle ok. So, here what we have is so, this is the spherical triangle, we are interested in and here the central son is our theta t or the angle of incidence on the tilted plane.

And, then so, this is angle of incidence on tilted plane ok. And, this is same as the curve ns ok. So, that is the arc that we are subtending on the surface of the sphere ok. Similarly, the central angle sov is our zenith angle theta z ok, and that is again the arc that we subtend that is sv ok. Now, another central angle that we see here central angle nov, that is our arc nv and that is beta or the tilt angle. Just we are clamouring all these information from the last page figure.

So, if you do not understand any of these bits please go back to the last page figure and look at it whether you agree with that ok. And, one surface angle we have so, three central angles; central angles are this 1, 2 and 3 and here we have just single one surface angle, which is our svn and that is A Z minus AZS ok. That means difference in solar and surface azimuth angles ok. So, now we are in a position to apply the spherical triangle rules. So, apply spherical triangle rules.

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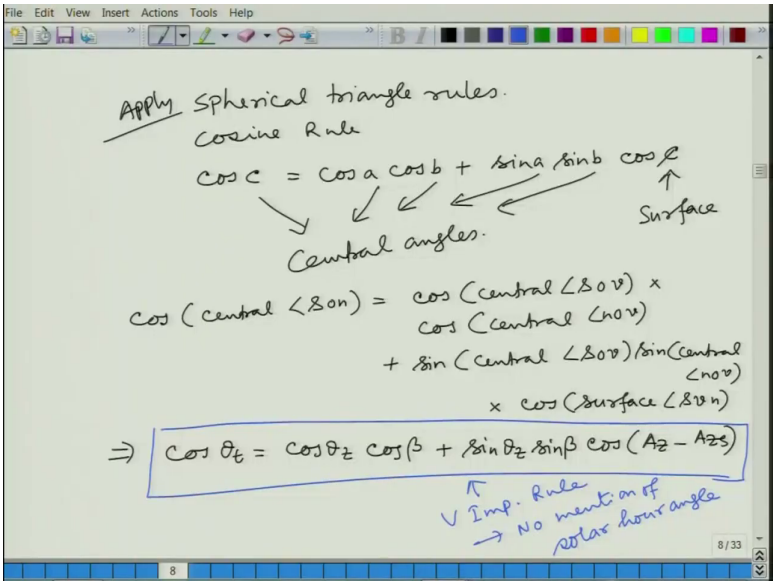

So, the cosine rule is we have seen it. So, here we can write or cosine rule, what is that? That is this rule right ok. This is a surface angle and all other a, b, c all these are central angles right.

So, what we can write for this particular case? We can write cos of central son will be equal to cos of central sov, multiplied by cos of central nov. So, for the first time I am writing all

these different angles with central or surface angle, because that way your intuitions will grow. First case you have to see very very lucidly ok.

So, that is why I am doing it, later I would not mention this central or surface as we use them and plus sin of central sov multiplied by sin of central nov ok. And, this multiplied by cos of surface. So, only one surface angle is there in the cosine rule, so that is angle svn ok. Now, we will just substitute what those angles are ok.

So, if we substitute then we get cos theta t will be equal to cos theta z, cos beta plus sin theta z, sin beta and cos of this difference between solar and surface azimuth angle ok. So, this is very simple right, we just use the cosine rule on this particular imaginary circle that is located I mean that centered at the observer location.

So, this is a very important rule ok. And, note that no mention of solar hour angle. So, omega is not at all mentioned. So, this is general right. All these angles are measured with respect to something universal like, global south or horizontal plane at particular location, these are universal things and we are talking with respect to that ok.

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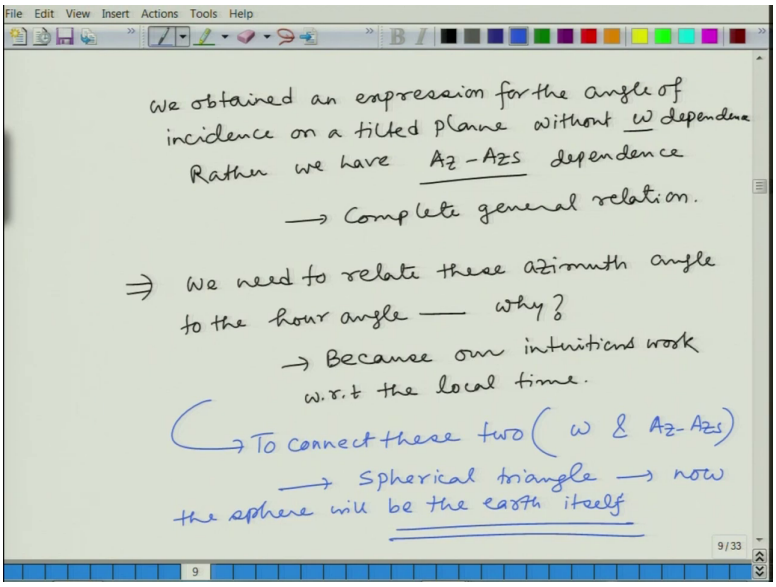

So, what we can say that we obtained an expression for the angle of incidence on a tilted plane without hour angle dependence rather we have the azimuth angles, both solar and surface azimuth angle dependence ok. So, this is completely general relation.

Of course, we will see that we will need different forms of this general relation, because not whatever is given or measured that you can find from this relation. So, we need to have different forms, but this particular form is indeed very general right ok.

So, now what we can say, now we have seen both the sides, but we have to connect the dots right. So, we need to relate these azimuth angles to the hour angle right. Even if we do not need it, but we need the inter relationship between them, why? Because, whenever we talk

about a solar collector, we have to talk about the time the local time of that right that is how our intuitions work.

We do not think in terms of the global scale we think of the watch that we have we can at least connected to the solar time and then for that local solar time we have to connect to what those azimuth angles mean, because our intuitions work with respect to the local time ok. So, that hour angle dependence has to be there. So, to connect these two; that means, this and the difference between these two angles azimuth angles ok.

So, for connecting again we will involve spherical triangle, but now it is not the sphere will be the earth itself ok. So, last time the spherical triangle we have taken the sphere was an imaginary spheres centered around the observer location right.

But, here we this is not imaginary surface sphere anymore this is the earth itself and of course, centered around the surface of the or center of the earth ok. So, that is the major difference ok. Let us see how we can use this spherical triangle formula, when our sphere is the earth itself.

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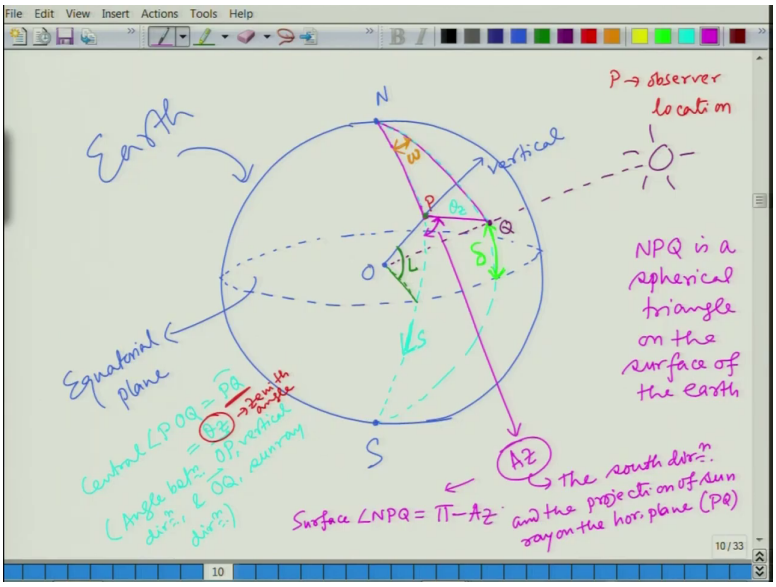

So, now, let us draw an earth good enough. Now, we have the center here and let us say this is the equatorial plane ok. First this is our earth and this is equatorial plane ok. And, here you can draw the North Pole, let us say the center of the earth is O ok.

And, let us say that, the observer location is here let me use a different colour. So, the meridian that goes through the observer location is this right. And, this will be south ok. And, let me write this is the observer location P; P is the observer location ok.

Now, if that is the observer location. And, now suppose we have the solar direction or solar radiation is coming through this point ok. So, sun is somewhere here ok. And, normal direction of sun ray that is going directly to the center of the earth, that point let us name it Q ok. So, another meridian we can draw through this Q point right ok.

So, now one vector can we draw here, that through the observer location. If, I draw a line through the center of the earth what will we get, please before I proceed please stop and think what that direction will be then you switch play the video again ok. So, this angle or this direction is the vertical direction right.

Let me write it explicitly this is vertical ok. And, so, here you notice that we have a spherical triangle already. So, think of this spherical triangle, do not we have ok. So, this NPQ is a spherical triangle on the surface of the earth ok.

Now, on this spherical triangle now we have to find out, what will be different angles? Do you know them; do you recognize them, that is what our task will be ok. So, first thing that very easily we can recognize one angle which is the angle between the, so, if you connect these 2 points, what angle is this?

This is the latitude right, latitude of point P, because that is the angle it makes with the equatorial plane ok. That is easy the other thing is that similar way just for the location of location of the observer, we have said that that is latitude. But, if we think of the latitude of Q what is that? That angle is so, basically this particular angle. So, basically whatever angle this particular line will make at the center O.

So, what is that angle that is our declination angle right delta? So, these two angles we obtain very easily. And, the angle between the meridians that one meridian is going through the observer and the other meridian is going where sun is radiating normally; that means, Q. That angle is our this angle, angle between these two meridians that is our hour angle right omega.

So, few of these triangles or angles we have directly determined ok. Now, one thing we will determine here is the which is little tricky so, please pay attention yeah. So, this particular angle what is this? Let me first write it this angle is our solar azimuth angle, why do you say so. Because, you see this is the angle between the south direction and the projection of sun ray on the horizontal plane.

So, this projection of sun ray on the horizontal plane is nothing, but your PQ ok. So, PQ line that is making to the south direction and can you see that this line is nothing, but going to south. So, this is our south direction. So, whatever angle it makes with the south direction that is our AZ, which is the solar azimuth angle ok. So, lot of these angles we have obtained. So, basically now you can see that this surface angle NPQ is nothing but pi minus AZ. Is not it?

So, it is a straight line NS is a straight line on the sphere surface and so the NPQ, this particular angle is your pi minus AZ ok. Another angle we have here is the zenith angle. What is that? The zenith angle is the let me just explain why we get that ok. So, our central POQ or the arc it makes on the sphere surface that is the zenith angle right, why it is zenith angle?

Because, it is the angle between angle between OP ok, which is the vertical direction and OQ, which is the sun ray direction right. So, this PQ ok, this PQ, which is the arc on the surface is equivalent to the zenith angle theta Z let me write that zenith angle as well ok. Because it is the angle between the vertical direction OP and the sun ray direction, which is OQ that is why it is the zenith angle.

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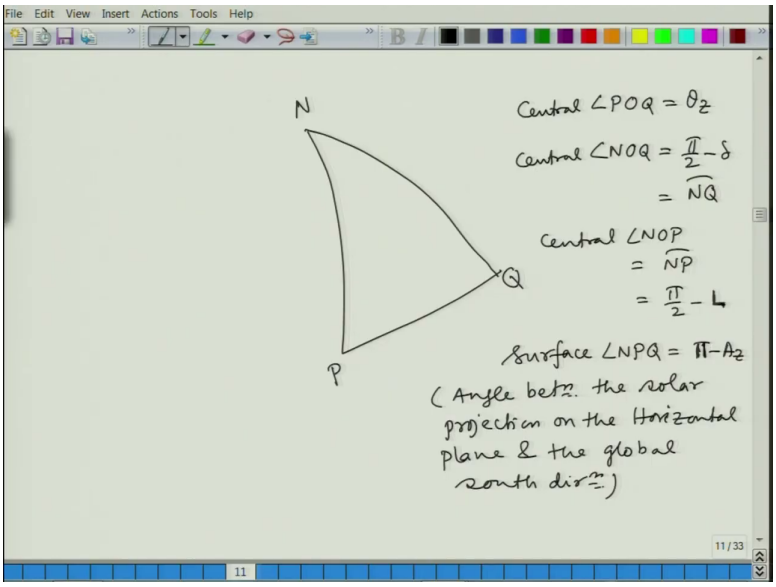

So, again we have obtained let me go to the next page and let me draw this spherical triangle again as we did in the last case. So, this is our north pole, this is our observer location and this is the point where sun is radiating normally ok. So, what we obtained is that this central angle POQ is our theta Z.

Now, central angle NOQ is our pi by 2 minus delta ok. So, one thing I need to mention here that ok. This is not pi by this is pi by 2 ok. Let me just write it again, this is pi by 2 minus AZ ok. Because, the central angle that makes is you know when it goes to the southern hemisphere, then this whole sign convention changes and we only talk about with respect to the equator ok. So, that is why this is pi by 2 minus AZ.

So, this angle is also pi by 2 minus delta ok, this is nothing but the arc NQ on the surface. Now, other central angle we have is angle NOP and that is our curve NP and this is pi by 2

minus L ok. So, I need to go back ok, this is fine ok; I am making a mistake here. So, this is pi minus AZ, but here it is pi by 2 minus L ok.

Now, surface angle NPQ is pi minus AZ ok. And, this angle is what? This is the angle between the solar projection on the horizontal plane and the global south direction so, yeah. So, that was a mistake I made. So, please correct it this is pi minus AZ and for the angle NOP, which is the central angle that is pi by 2 minus L ok.

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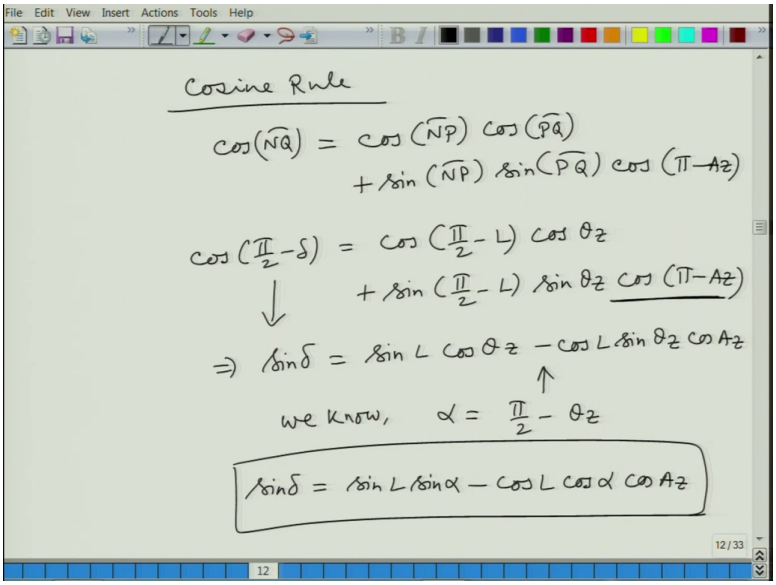

Now, let us apply the cosine rule ok. If, we apply the cosine rule, then this particular surface central angle will be cos of NP cos of PQ. So, this arc on the surface; that means, it is a central angle plus sin of NP, sin of PQ multiplied by the surface angle, which is cos of pi minus AZ ok.

Now, if we substitute these values cos pi by 2 minus L latitude cos of zenith angle theta Z plus sin pi by 2 minus L, and sin theta Z cos of pi minus AZ the solar azimuth angle ok. So, from here this is nothing, but sin of delta right cos theta is equal to sin pi minus theta. So, that way we can write cos L sin theta Z cos AZ ok. This minus sign is coming because of this term cos pi minus AZ is nothing, but minus cos AZ and that is why we are getting this ok.

Now, what we can write is we know that alpha which is solar altitude angle that is pi by 2 minus theta Z right. So, if we substitute that, what we get is sin delta equal to sin L sin alpha, minus cos L cos alpha cos AZ ok. That is another important angle that we obtain from the spherical triangle on the surface of the earth ok which involves all different angles, but it still does not involve the hour angle, which we will see very soon ok.

So, let me stop here for this class. And, in the next class we will continue using this spherical triangle on the surface of the earth, and we will connect the hour angle with these azimuth angles. And, get a completely general derivation that will be applicable anywhere on the earth for any surface and that is our goal ok.

Thank you very much for your attention.