Elements of Solar Energy Conversion
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Lecture – 08

Welcome back to this series of lectures on Elements of Solar Energy Conversion. Now, we are here at lecture number 8 ok.

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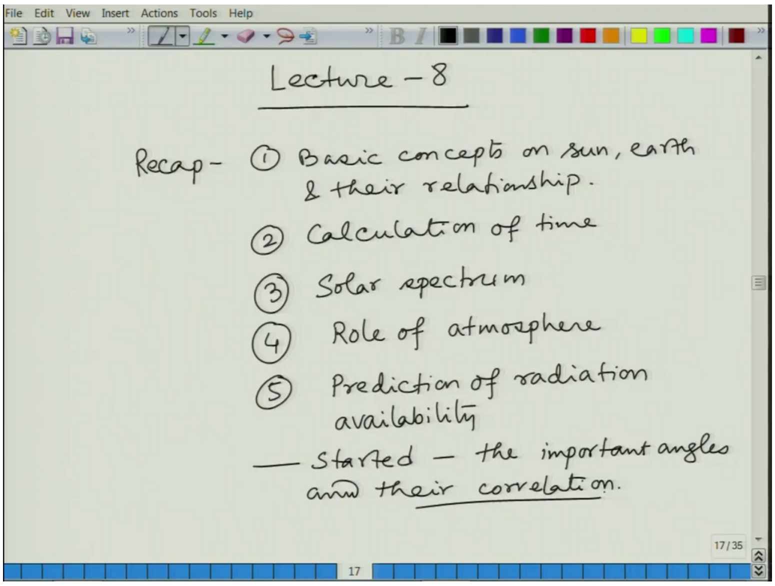

So, like any other lecture we will quickly tell you what is covered so far. So, in first 7 lectures we have covered the basic concepts on sun, earth, and their relationship ok. Then, we have covered the calculation of time, third major thing we covered is the solar spectrum, and then we covered the role of atmosphere.

Next, we started or we also covered this prediction of radiation availability. And, we have started in the last class, the important angles and their correlation ok. This is where we are at this moment and we will take it forward in this lecture.

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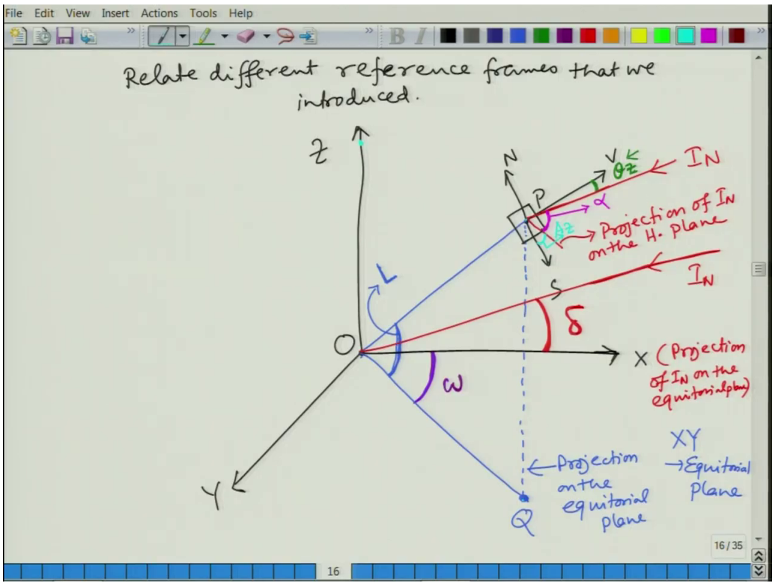

So, in the last class we stopped in this particular figure ok. So, what did we see? We have a horizontal plane at this point P and then the reference frame origin for this case is the centre of the earth, and then we have taken this X and Y, axis in such a way that X Y makes the equatorial plane. And, X is the; the X axis is the projection of the sun ray that is coming directly to the centre of the earth ok.

Now, we have looked at few angles in this combined reference frame ok. So, first one was the angle of angle between the projection of sun rays to the horizontal plane and the projection of the location of the observer ok. So, basically OX is the projection of sun rays on the equatorial plane and OQ is the projection of OP vector on the equatorial plane. So, angle between them is the hour angle ok.

And, we have also looked at the angle that the observer centre of the earth connector OP, that is making with the equatorial plane; that is the latitude L ok. What other things did we look at? Another very important angle we have looked at is the angle between the sun ray and its projection on the equatorial plane.

This is the declination angle that we have looked at delta ok. And, the fourth angle that we have looked at is the angle between the sun ray and the vertical line, or the normal to the horizontal plane, that angle is our zenith angle theta z ok. Now, another thing we can look at is the angle between the sun ray and its projection on the horizontal plane that makes with the south direction ok.

So, if we have ok. So, what we can say, that let us take the projection of the I N on the plane to be this one ok. If, this is the projection so, this is projection of the sun ray on the horizontal plane ok. So, that particular projection whatever angle it makes with the south direction what is that, you know that, right. So, please think and find out that angle before you proceed.

So, let me now tell you the angle to be the solar azimuth angle A z ok, this angle is A z. And, another angle of interest here is the angle between the sun ray and its own projection. What is that angle? That angle is alpha or solar altitude angle ok. So, now, we have identified all these angles and now we are in a position to interrelate them ok.

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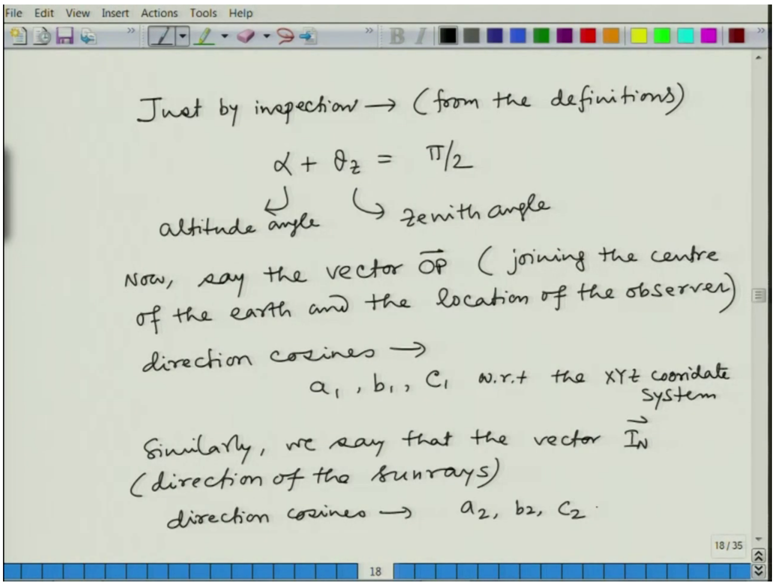

So, let us go to the next screen. And, see that just by inspection or from the definitions actually from the definitions. What we can see is the solar zenith angle and the solar altitude angle; these two are complementary to each other ok. So, alpha plus theta z they will make you a right angle ok.

This is the altitude angle, and this is the zenith angle. So, here I want to stress that all these symbols that we are going to use like, alpha, theta z, A z, L phi all those things they signify a particular angle right. So, for all the derivation we will not see or we will not say that this is the altitude angle or that is the zenith angle and all these things ok. So, what will we do?

We will assume that you recognize that alpha means altitude angle, theta z means zenith angle. And, for that to be true that assumption to be true you have to do this go through these notes and go through these lectures, many times, do examples on these and then only you will internalize all these naming system, as well as you will grow intuition in terms of visualizing these 3 dimensional thing and interconnection between two reference frames and that is very very important ok.

So, that is the first relation we get alpha plus theta z equal to pi by 2. Now, say the vector OP ok, this is the vector joining the centre of the earth and the location of the observer ok. So, if you look at this vector and if you write the corresponding direction cosines ok, for this vector OP. Then, what we can write? Let say the direction cosines are a 1, b 1, and c 1 ok. With respect to the X Y Z coordinate that we have shown earlier coordinate system ok.

And, similarly we say, that the vector I N which represents the direction of the sun ray ok, which is actually direction of the sun rays ok. So, let say corresponding to that vector the direction cosines are a 2, b 2, and c 2 ok. So, if that is the case now we have to find out what are the values of a 1 b 1 c 1, a 2 b 2 c 2 ok.

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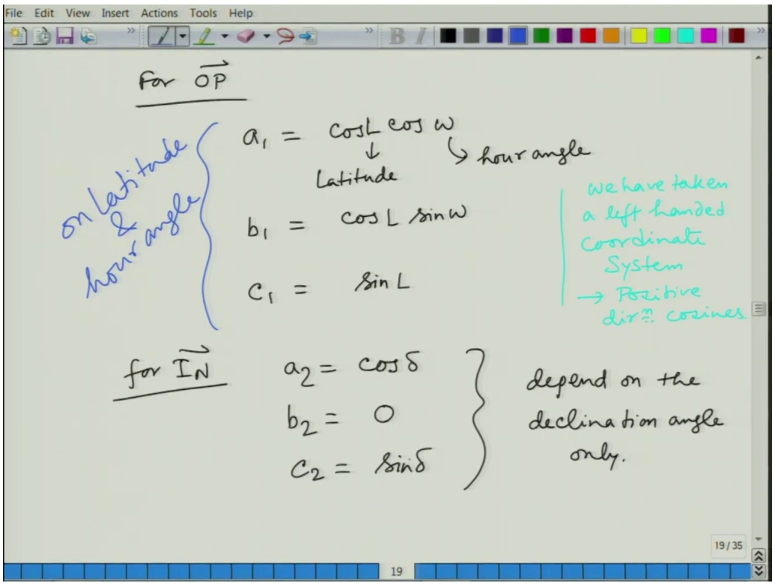

So, for OP vector what we can write this a 1, which is the direction cosine in terms of the X axis. What we need to do? We need to first bring it to the equatorial plane X Y plane that is for that we have to multiply it by cos L, the latitude of the observer ok. And, then within that X Y plane, if we want to project it on the X axis, then what we need to do? We need to multiply it with cos of omega that is the hour angle.

So, this is so for few times initially I will repeat what angle is what ok. And, this is hour angle ok. So, please go back to that figure and ensure that you understand each of these direction cosines, how we are getting it? So, a 1 is cos latitude multiplied by the cos of hour angle. What is the other direction cosine?

That means, towards the Y axis. What would be the direction cosine? There we will find again we have to bring it to the equatorial plane or X Y plane by multiplying it with cos latitude. And, then to get the Y component, now we have to multiply it with sin omega ok, instead of cos omega, now we have sin omega.

Now, the third, the Z axis direction cosine ok; for that, we only have only one angle dependence sin L. So, if we go back and look at this figure, we can see that for the X for the vector OP, if you want to come to the or if you want to project that to the equatorial plane XY you get this OQ right.

Now, OQ from this you can to get OQ you have to multiply it with cos L ok. By cos L you get OQ and then if you want to project to the X axis, then you have to multiply it with cos omega. And, for the Y axis you have to multiply with sin omega ok. And, all these things are happening in the XY plane.

Now, if you look at the plane that is making this if we say this point Z, Z and then P Q O in this particular plane, Z P Q O plane, the Z axis projection would be sin L ok, or the sin of latitude angle. That is how we are getting the direction cosines. And, here I should mention that we have taken a left handed coordinate system. And, that is why we get all of these direction cosines in positive value; so, positive direction cosines ok. If, you take right handed one of them will be negative ok, fine.

Now, the other important vector is the vector I N which represents the direction of the sun rays ok. So, now, if we consider the corresponding direction cosines, sorry this will be a 2 ok. This particular direction cosine will be in the with respect to that declination angle delta right. So, we will have it cos delta.

Because, that is how the X axis is chosen. We have chosen the line, which is projection of I N on the equatorial plane to be our X axis. So, this is straight forward cos delta ok. And, the second direction cosine b 2 will be 0, because that is how the X, Y, Z coordinate system is chosen, if you are cutting one axis, completely in one plane, the same plane the other axis will be 0 and c 2 will now be sin delta.

So, this is much simpler in terms of it just depends on single angle ok. So, depend on the declination angle only, that makes sense right. So, declination angle tells you with respect to the axis of the rotation of earth, where the sun is ok, at any given point of time. So, that make sense that the direction cosines the with respect to the reference frame, which is centered at the centre of the earth will only depend on the declination angle.

And, here the dependence is on latitude and hour angle. Here, also it make sense right. So, here we are looking at the observer location with respect to the centre of the earth. So, it does not depend on anything that is related to the sun, but it does depend on how much with respect to the prime meridian it moved or in rotation direction. So, that is the hour angle and where it lies in terms of the latitude ok. So, that observer position that matters ok. Now, once we have these two direction cosine sets for these two vectors.

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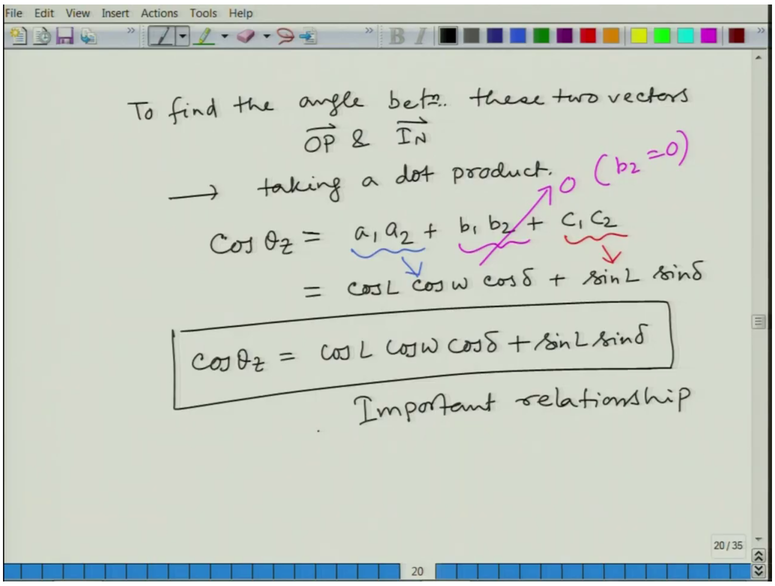

Now, we are in a position to find the angle between these two vectors one is OP, and the other one is I N ok. So, how to find that? By, so, by taking a dot product, ok; so, you know the dot product will give you the angle between them in terms of cos. And, let me go back to this figure where the angle between I N and OP we can see ok, let me take this colour.

So, you can see here that OP when it is getting extended we are getting this P V vector right. When OP is getting extended we are getting this P V vector. So, the direction of OP and P V are the same ok. So, whatever angle that P V vector is making with respect to I N that is our angle between these two and what angle is that? That is theta z right? That the that is the zenith angle.

So, now, if we see that then we can directly write this cos theta z, which will be which we will obtain in terms of in the form of dot product, and that is by definition direction cosines multiplied and added right. So, a 1, a 2 plus b 1, b 2 plus c 1, c 2 ok, that is how we get the cosine of theta z. Now, you take the values from values that we have obtained and you directly can write this.

This will be cos L cos omega cos delta plus sin L sin delta ok. So, this term actually giving rise to the first term ok. And, the third term is giving rise to the second term ok. And, this particular thing is contributing to 0, because your b 2 is actually 0 ok. So, that is how we get and this is a very very important relationship.

So, let me write it again to put emphasis on it. So, this is a, and this is an important relationship. This we will use again and again in many occasions and actually we have used it already. You please look back we have used similar equation already ok. So, now, we have derived that equation as well.

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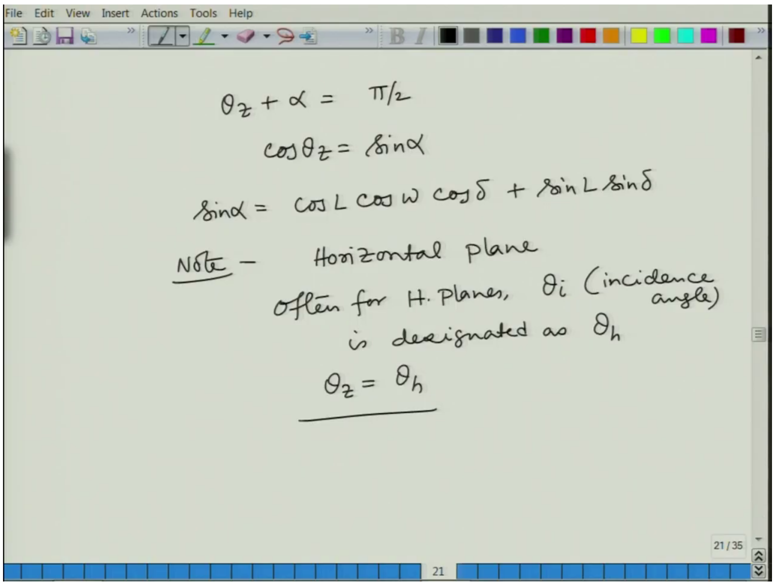

So, now if we look at this theta z, we already know that theta z plus alpha is equal to pi by 2, this we have seen right. So, you can write and what does that mean? That cos theta z equal to sin alpha right. And, if that is the case then you can write sin alpha equal to cos L, cos omega, cos delta, plus sin L, sin delta ok. So, both the altitude angle and zenith angle you can obtain by this formula.

So, this is very straight forward, but very important relationship ok. And, another thing I should note here, that whatever we are talking so, far in this particular figure, we are talking of a horizontal plane right. And, that is why the normal is actually towards the vertical direction, the normal is making an angle of zenith that is theta z with the sun rays ok.

So, often for these horizontal planes theta i which is we have seen that it means incidence angle is designated as theta h ok, h stands for horizontal plane. Now, so for horizontal plane this theta z is equal to theta h ok.

So, sometimes it is given that your zenith angle is this and or something is given you have to find out the zenith angle, but it is very important to know, that zenith angle is the same thing as the angle of incidence for a horizontal plane ok. That is why I am putting this note here. Now, we will look at several other important relations. First is if we now apply a tilt angle ok.

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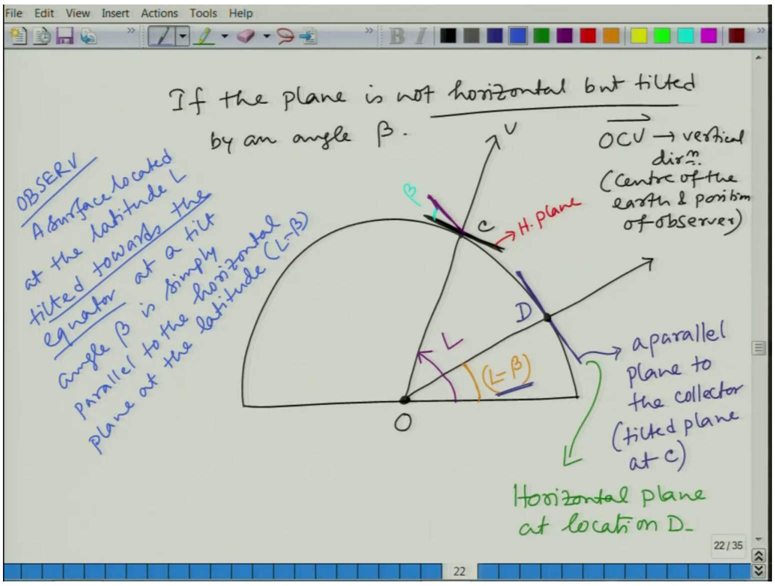

So, if the plane is not horizontal, but tilted ok. And, if it is tilted by an angle beta ok, this beta stands for tilt angle all the time. So, again you have to remember that whenever you see beta it means tilt angle. Tilt angle means, the angle the plane makes with the horizontal plane ok, or you can also say in terms of the surface normal. So, the angle the surface normal of the tilted plane makes with the vertical direction that is the angle beta ok.

Now, very interesting thing we can observe now, let me just draw a figure of a cut in the earth ok. So, let say this is the centre of the earth O ok. And, let us say that we have at certain location C we have a plane, which is. So, this is the horizontal plane which; that means, the plane that is perpendicular to this O C line right. So, OC and let say V, this particular vector is the vertical direction. It connects to the centre of the earth and position of observer right.

Now, whatever plane is perpendicular to that direction that is our horizontal plane right. So, this is horizontal plane right. So, suppose we have a collector, which is a tilted surface and it makes angle beta with the horizontal plane; so, let say that we have a plane which is making an angle beta here with the horizontal plane ok. So, now, we have to find all the relationship with this tilted surface right.

So, it is little difficult, but we can make it super easy. How? Let say that we draw a parallel to this wherever it takes in some other location you draw a parallel line to this tilted plane ok. So, this is what a parallel plane to the collector which is a tilted plane, tilted plane at located at the position C ok. So, somehow somewhere on the earth you will actually doing this. And, what is that position? Let us name them, name that point to be D.

So, similar to what we did at C what we can draw? At D we can draw a line that extends along OD ok. Now, you can imagine that this particular parallel line, wherever it is cutting this is the horizontal plane at location D right. So, the same plane; so, the same plane is also a horizontal plane at location D.

Do you agree? Right. So, if that is the case, then what can we say. We can say that, a surface located, so, this is what we observe, ok. So, a surface located at the latitude L ok. So, let say the latitude here, here of the original position C is L and the latitude here for the point where the horizontal is parallel to the collector that is L minus beta right.

Because, you can see that if this particular plane, this particular collector plane is parallel to the horizontal here at D, then the latitude has to be L minus beta, they are on the same plane ok. This whole thing is not a 3 dimensional thing, but you can think of all the things are lying on the plane of this board ok. So, this angle is L minus beta, this is very important.

Now, so, a surface located at the latitude L tilted towards the equator, this is a an important clause we will see what it means later. So, tilted towards the equator at an angle or I should say at a tilt angle beta is simply parallel to the horizontal plane at the latitude L minus beta true. So, that is a very significant observation, which is which will make our life much easier, when we talk about the tilted plane.

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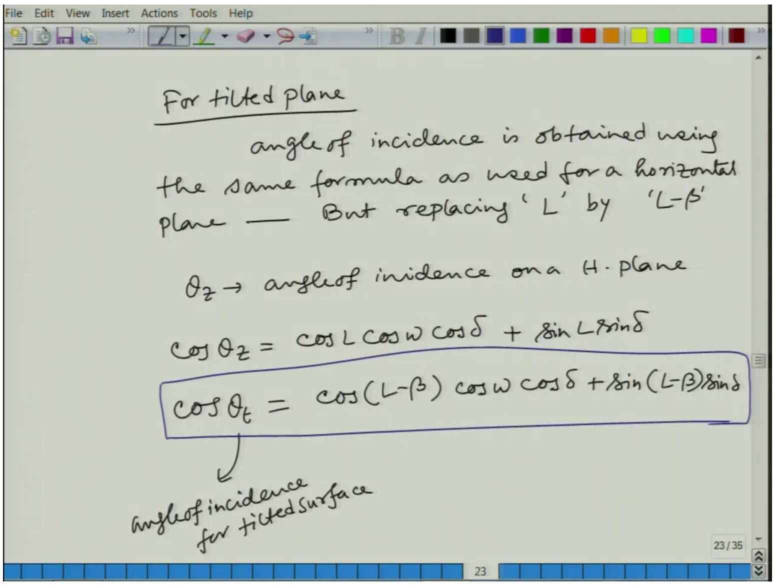

So, for tilted plane, what we can write that the angle of incidence is obtained using the same formula as used for a horizontal plane ok. But, the catch is by replacing 'L' by 'L minus beta ok; how easy is that right. So, we have seen earlier that theta z is the angle of incidence on a horizontal plane ok. And, we have also seen that this cos theta z is your cos L, cos omega, cos delta, plus sin L, sin delta right.

Now, you want to find the this angle of incidence, let say that is theta t t stands for tilted. So, this is angle of incidence for tilted surface. So, what we can write? The same thing cos theta t now we can write and we will replace this L by L minus beta ok; everything else tells the same. And, here also we have 1 L.

So, we have to replace that as well ok. This is super easy right just by a simple observation you did not have to go through all the different manipulations; you just change the latitude by L minus beta ok. So, as if it is a correction factor ok, so far so good. Now, from this relationship we are in a position to make some important observations already ok.

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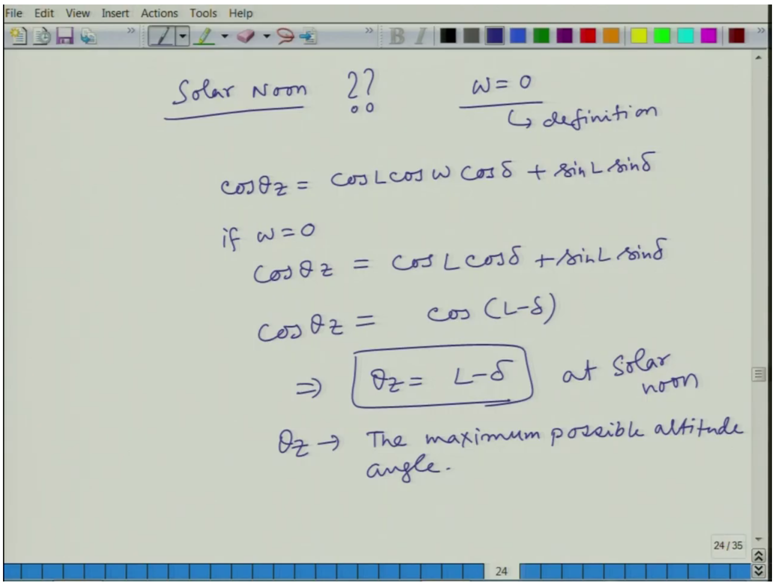

So, first of all we have talked lot about solar noon right, while we talked about the solar time, clock time, and all those things, we talked about solar noon. So, what is that? Solar noon is what we call, when the sun is at the top most possible position for that particular day. And, how it is defined? It is defined when the hour angle is 0 right.

So, that is how it is defined, ok; now, in this particular equation, if we put hour angle to be 0. So, we have this sorry we have this ok. So, if this omega hour angle is 0, then what we can write? This cos theta z is nothing but cos L cos delta plus sin L sin delta; because, cos omega is 1 now, ok.

So, is the right hand side familiar? It is right. What is that? It is cos of L minus delta right, cos a cos b plus sin a sin b; so, cos a minus b. So, both sides are cos so, what you can write theta z to be L minus delta, neat right. So, at solar noon this particular zenith angle will be equal to L minus delta.

And, you can note here, that here so, it tells you zenith angle is what? The it correspond to the maximum possible altitude angle right. That means how much you have to look up to look at the sun directly, starting from the horizontal plane how much you have to look up ok.

So, here we can see that it the zenith angle depends only on the latitude with what it signifies the position of the observer and the declination angle that is the day of the year. So, for a particular location suppose you are at Hyderabad, for that particular location, for a particular day of the year, you know exactly how much the sun will go up in the sky. So, it depends only on the location and the time of the year or day of the year.

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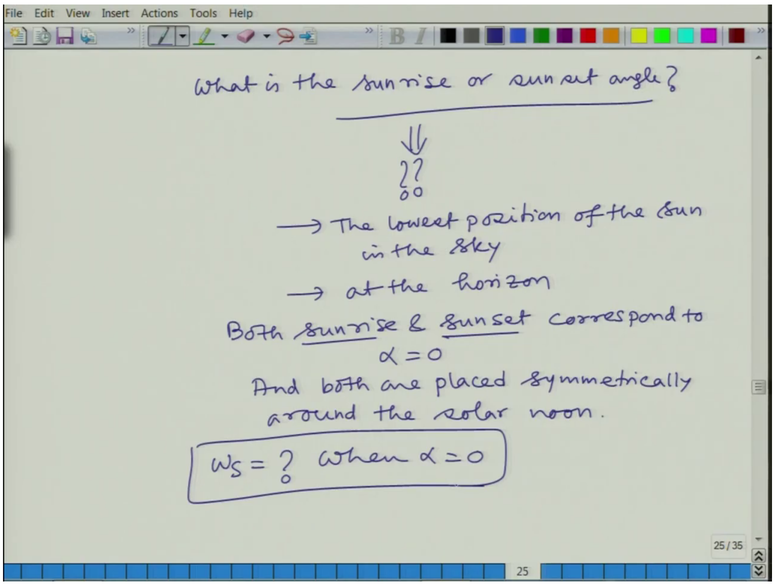

Now, the other thing that we can note here is that, what is the sunrise or sunset angle ok. These we have used right, when we talked about the prediction of solar radiation availability. Do you remember we have used this s a prime, s a bar, and s p bar all these quantities they all depended on this w s and w right.

So, the sun rise and sunset angle first of all what do they mean? They mean the lowest position of the sun in the sky of the sun in the sky right. Lowest position means what? The horizontal position. Horizon; not horizontal, but at the horizon, that is the lowest possible position right. So, sun rise means at the horizon it is moving up and sunset means at the horizon the sun is going down ok.

So, both sunrise and sunset correspond to alpha equal to 0 right. When you are at horizon or the sun is at horizon, then the altitude angle is 0 right, you do not have to look up at all. So, both of them that is in two different directions, but both correspond to alpha equal to 0.

And, both are placed symmetrically around the solar noon ok. So, if we want to find out this sunrise or sunset angle, what we need to find? Why we need to find? How much is this when alpha equal to 0? Right. That is the question we are asking ok.

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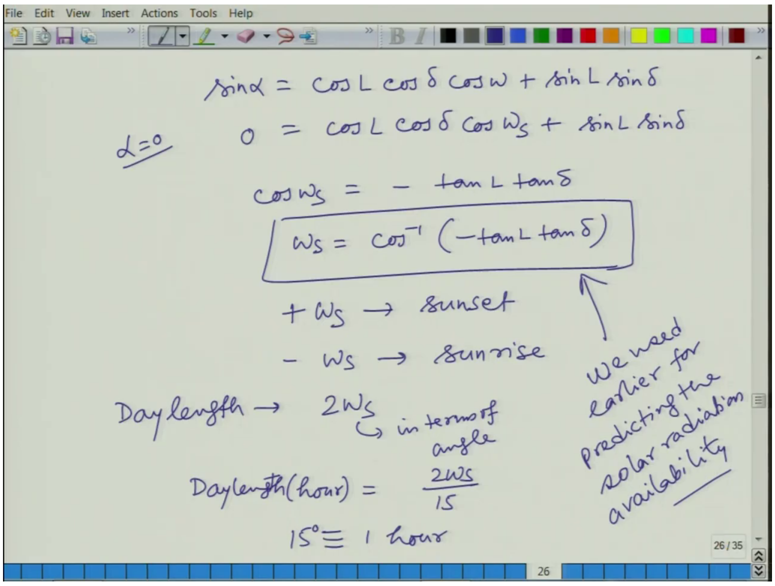

Now, how to find this, again let us take the equation. So, we know in terms of the altitude angle, we have this equation sin L equal to cos L, sin alpha equal to cos L cos delta cos omega plus sin L sin delta right. This we have seen earlier. So, now, for alpha equal to 0 sin alpha equal to 0 ok. So, we have cos L cos delta. And, now we will write this omega to be omega S, because it represents either sunrise or sunset right plus this does not change ok.

Now, if you bring this in one direction I mean this, if you find out simplify this equation to find out cos omega S, what you get is minus tan L, tan delta right. So, what you can write is cos inverse minus tan L tan delta. And, for a particular location L and a particular day of the year delta you will have 2 values of sorry 1 value of this omega S. And, positive one, positive omega S will mean sunset and negative omega S will mean sun rise right.

And, now we know exactly how to find this. This formula also we used earlier for predicting the solar radiation availability ok. And, we also know we would like to know the day length right. Day length will be what it will be just the difference in time between the sunrise and sunset. So, it will be just 2 into omega S ok. Now, this is in terms of angle. So, if we want to know day length in hour then we will have 2 omega S divided by 15, because 15 degree is equivalent to an hour ok.

So, that is how we get the day length. Again we have use this day length in solar radiation prediction and we have derive them now from first principles. When we combine these two reference frame one is centered at the observer, the other one is centered at the centre of the earth, we obtain the inter relationship and we derive these equations ok.

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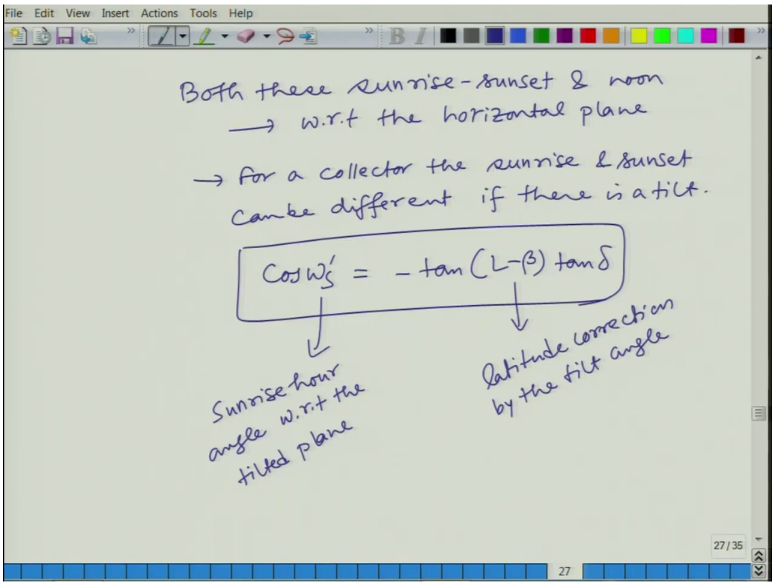

Now, one thing so, both these sunrise, sunset ok and, solar noon we have talked about with respect to the horizontal plane right. We have assumed that we are on a flat ground or field and it is horizontal and we want to see the sunrise sunset from there. But, for a collector the sunrise; that means, when the sun radiates on it, starts radiating on it, that is sunrise and when it stops radiating at it, which is sunset can be different, compared to the horizontal plane, can be different if there is a tilt ok.

So, with respect to the collector plane, the sunrise and sunset we have to reevaluate or we have to make the correction for the tilt ok. So, what we can write? We can write the same thing and similar way we will correct it by correcting the latitude ok. So, this is a very simple relationship we just obtained from a simple observation by correcting the latitude.

So, this is omega S prime is the sun rise hour angle with respect to the tilted plane ok. And, this is the latitude correction by the tilt angle and the same formula, tan latitude into tan declination. This formula has been used to obtain the sunrise hour angle for a tilted plane as well ok.

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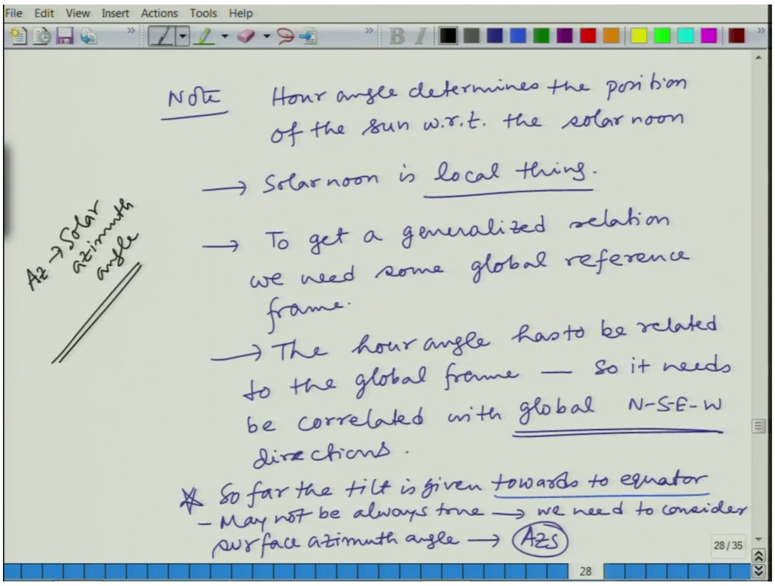

So, now few notes I should mention that, the hour angle determines the position of the sun with respect to the solar noon right. So, that is fine, but solar noon is a very local thing right, is a local thing. Because, we have seen that as the solar noon are different at all the different points, that is why we had to take one standard latitude against which all the clocks will be calibrated ok. So, that local thing we cannot make something very generalized.

So, to get a generalized relation we need some global reference frame ok. So, this omega or hour angle has to be in terms of the global reference frame ok. So, the hour angle has to be related to the global frame and what is that global frame? Is the north or south direction or east or west direction these are global thing. Even, if you are here when you take tell you are looking at south; you are looking at the south pole of the earth right. So, that is the direction of south. So, that is the global reference frame.

So, it needs to be correlated with global North-South, East-West directions ok. In that case it will be a general relationship; you can use it for any location. So, that is a very important requirement that we have. Another thing we need to mention here is that, so, far the tilt is given towards the equator only. So, if we look back where we mentioned it first, yeah.

So, here you note that, I mentioned the tilt is given towards the equator ok. So, this particular assumption is there, but it may not be always true; may not be always true. So, we need to again generalize it, we need to consider the surface azimuth angle right. The surface azimuth angle is represented through A zs ok.

Again I remind here that A z is your solar azimuth angle. So, please do not get confused between these 2 A z means solar azimuth angle and A zs means surface azimuth angle ok. And, both of them contain S solar also contain S and surface also contains S. So, that is why it is important to remember what we mean exactly by A z or A zs ok.

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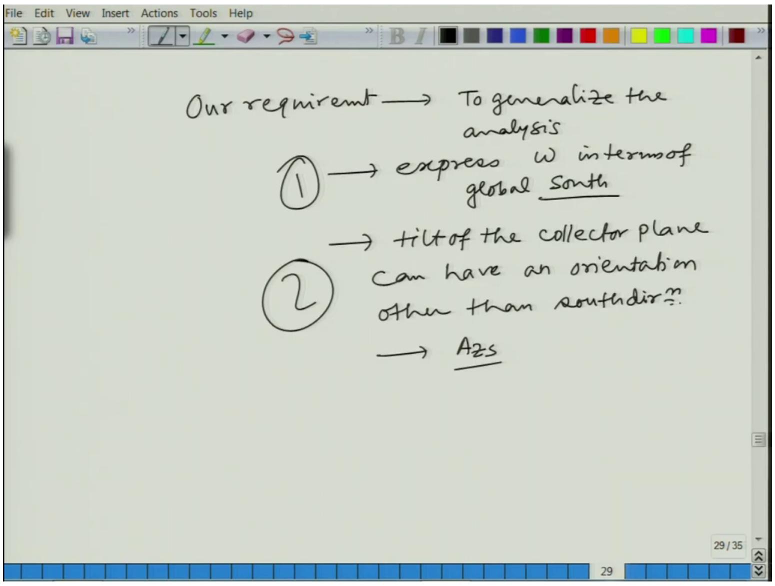

So, our requirement is to generalize the analysis and express omega in terms of global south. Actually, if you fix it with global south, then the whole reference frame is fixed you just do not need to fix it with global north east west all those things just one direction is fine and we have chosen that to be south.

The other one is the tilt of the collector plane can have an orientation, other than south direction and hence it can have a value of surface azimuth angle. So, with these two goals one is this, other one is this; with these two goals we will now delve into a problem where we will completely we will do a completely generalized analysis.

And, this is little more complicated than what we saw in today's class. And, here I stop and we will start this new analysis in the next class.