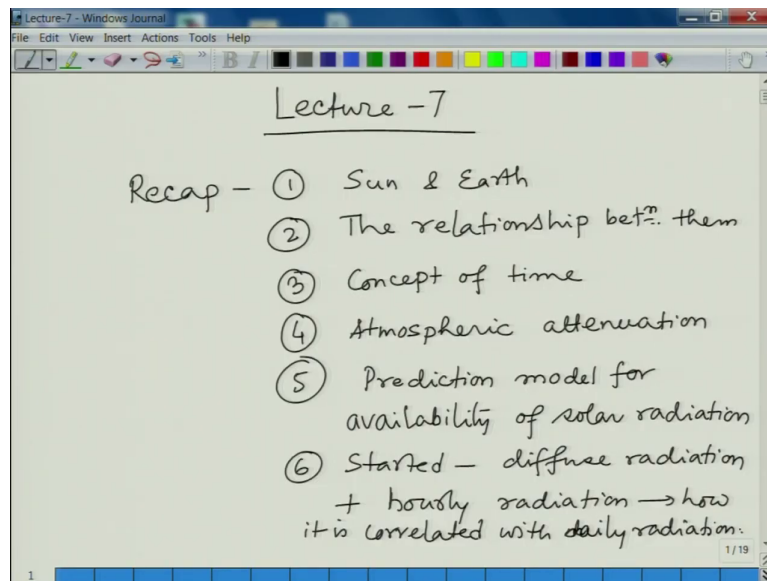


**Elements of Solar Energy Conversion**  
**Prof. Jishnu Bhattacharya**  
**Department of Mechanical Engineering**  
**Indian Institute of Technology, Kanpur**

**Lecture – 07**

(Refer Slide Time: 00:27)



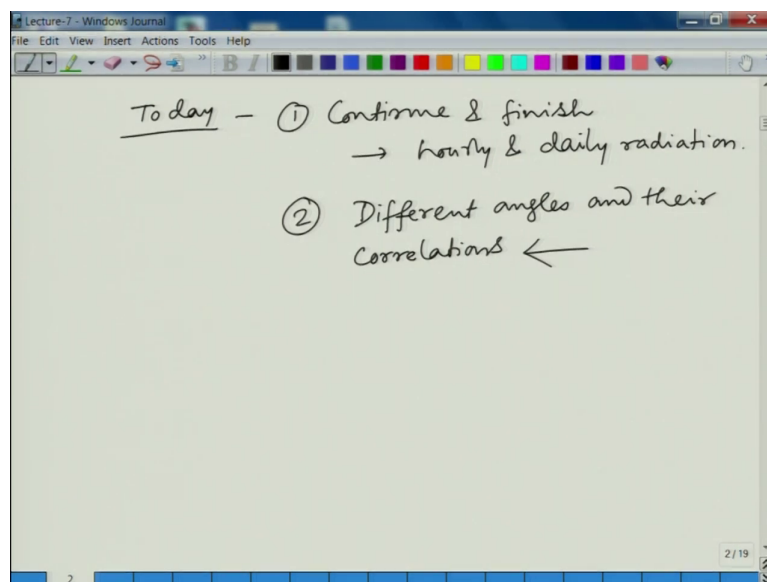
Hello and welcome back to this course on Elements of Solar Energy Conversion. So, far we have gone through 6 lectures, so today we are going to see the lecture number 7 ok. As for any other lecture, we will just give you the points which we already covered, so that you get from where we are coming.

So, the points that we have already covered in first 6 lectures are the basic concepts related to Sun and Earth ok, and then we covered the relationship between them ok. Then third thing we covered the concept of time, how the solar time and your clock time are related. Then we

have looked at how atmosphere plays a role, atmospheric attenuation that also we have looked at how the solar spectrum also get affected by the atmospheric components ok.

So, those things we covered. The other things that we covered is the prediction model for availability of solar radiation ok that we covered in the last lecture. And what we just started? We started to look at the diffuse radiation component of the total radiation available as well as the hourly radiation how it is correlated with monthly radiation or sorry daily radiation ok, because often we need at different hours how the radiation will be distributed ok.

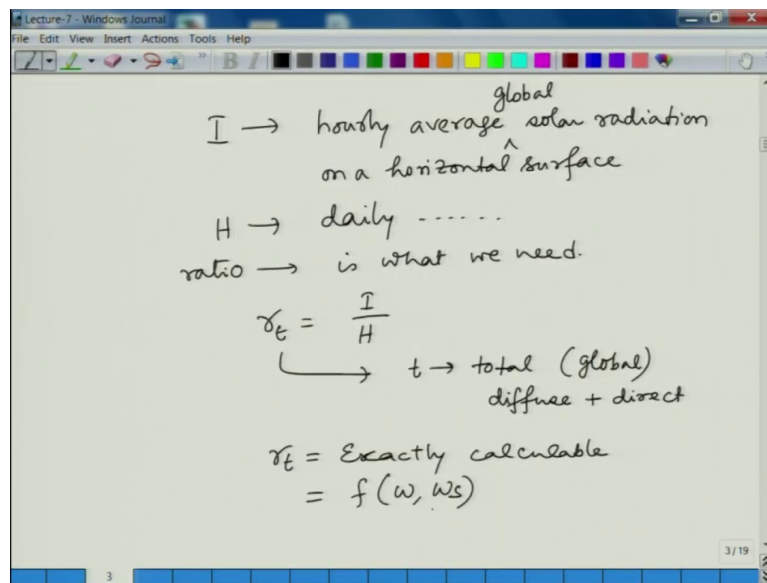
(Refer Slide Time: 03:30)



So, today what we are going to look at are the following. First is we will continue and finish first what we started in the last class that is the hourly and daily radiation how they are linked that we will finish today. And then a very important topic that we are going to start is the different angles and their correlations. This is a very important topic.

So, I would expect that you pay the maximum attention to this one because this will be required for all the rest of the component in this course as well as when you actually go for any design ok. So, these are the two things that we are going to talk about. So, let us start where we left.

(Refer Slide Time: 04:57)



So, we said that I this particular symbol is used for hourly average solar radiation and I should say global which means both diffuse plus direct radiation, hourly average global solar radiation on a horizontal surface ok. And H stands for daily and rest of the things are just same as the definition of I that means, daily average global solar radiation on a horizontal surface ok.

Now, what we are interested in is the ratio; ratio is what we need ok. And this ratio is designated by  $r_t$ , this I over H. Why  $t$ ? This subscript or superscript whenever you see that it

carries some meaning ok; so, here t stands for total; total which is equivalent to global here, which means this is diffuse plus direct radiation considered together ok that is what this t stands for ok.

Now, on the basis of several mathematical modeling as well as statistical correlations, people have come up with the value of r t ok. This r t is exactly calculable. And it is a function of the hour of the or the hour angle for the hour you are considering and the sunrise or sunset hour angle ok. So, and we will see what that function is ok.

(Refer Slide Time: 07:41)

$$r_t = \frac{\pi}{24} (a + b \cos w) \left[ \frac{\cos w - \cos w_s}{\sin w_s - \left(\frac{2\pi w_s}{360}\right) \cos w_s} \right]$$

*w* → hour angle for the particular hour  
 → it changes throughout the hour  
 → The mid point value is considered i.e.  
 Ex - 10am to 11am → w will be for 10:30am

derivation is beyond the scope of the present course  
 → RHS → w, w<sub>s</sub>  
 a, b → coefficients (Statistically)

$a = 0.409 + 0.5016 \sin (w_s - 60)$   
 $b = 0.6609 - 0.4767 \sin (w_s - 60)$   
 a & b → dependence on w<sub>s</sub>

I & H

So, r t is expressed by the following formula ok. So, this is the formula that expresses r t. And we should say that this derivation is beyond the scope of the current syllabus of the present course ok. So, you need to find this out if you are interested, you can go and find it out the

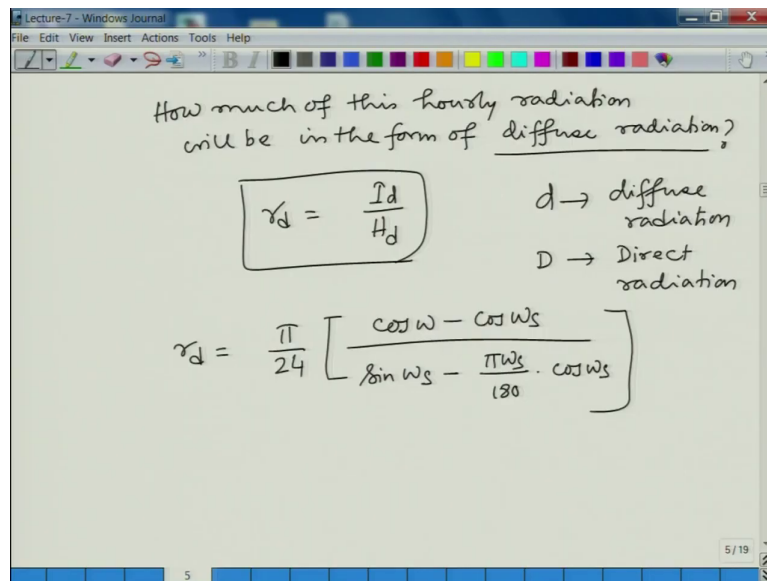
research paper where this derivation is done ok. But we need to use it; this is an important formula we need to use it. But how it comes that you have to dig up from somewhere else ok.

So, what so first you see that in the right hand side, we have two variables only; one is  $\omega$  or  $w$ , and the other one is  $\omega_s$  which is the sunset angle ok and other two things that we see  $a$  and  $b$ , these are coefficients ok. And these are actually obtained statistically ok. And the values of that  $a$  is this, and  $b$  is this ok.

Again you see that both  $a$  and  $b$  they have dependence on this sunset angle ok. The coefficients that goes into  $a$  and  $b$ , they are found out statistically over lot of data ok, so that is how we get it. And here you need to see that this  $w$  we see that this is the hour angle corresponding to a particular hour ok, but hour angle for the particular hour ok. But the hour angle itself changes throughout the hour right. So, it changes throughout the hour that we are considering. So, which value to take? Right.

So, we consider the midpoint value; midpoint value is considered. What do you mean by midpoint value? So, if we see an example that if we are considering the hour between 10 am to 11 am. So,  $w$  will be for 10.30 am, so that is a single value that you can take which will represent the whole hour between 10 am and 11 am ok. So, that is how we get the relationship between  $I$  and  $H$  ok.

(Refer Slide Time: 12:28)



How much of this hourly radiation will be in the form of diffuse radiation?

$$r_d = \frac{I_d}{H_d}$$

$d \rightarrow$  diffuse radiation  
 $D \rightarrow$  Direct radiation

$$r_d = \frac{\pi}{24} \left[ \frac{\cos \omega - \cos \omega_s}{\sin \omega_s - \frac{\pi \omega_s}{180} \cdot \cos \omega_s} \right]$$

5 / 19

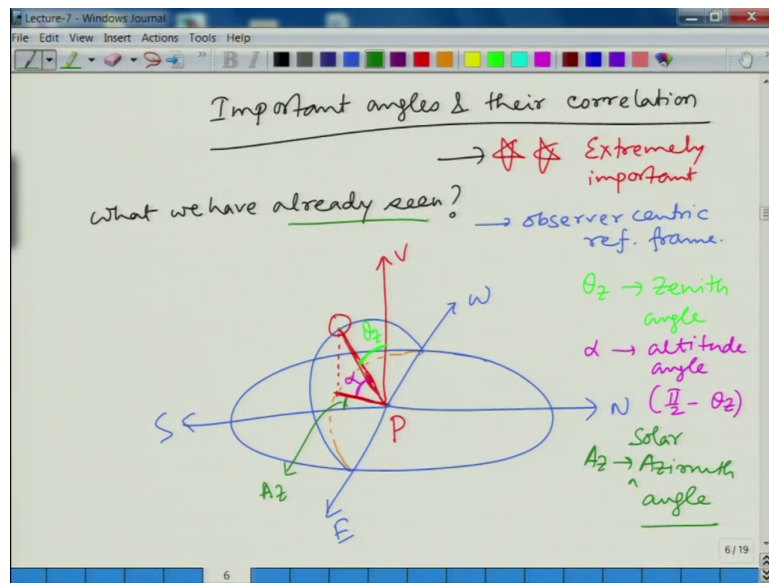
Now, one small piece of information is still left which is how much of this; how much of this hourly radiation will be in the form of diffuse radiation right that is a legitimate question to ask. And the complementary part will be given or complementary part will be the direct radiation right.

So, this question if we want to ask, again we will use some formulae which we will not be deriving here, but we will use them because that is important. So, in this case, we call it  $r_d$  ok. Again I am saying that  $d$  small  $d$  stands for diffuse radiation; and whenever you see capital  $D$  that stands for direct radiation ok.

So, of course, the  $r_d$  or the ratio between the hourly and daily radiation on a horizontal plane will be  $I_d$  over  $H_d$  ok. And how we get this  $r_d$  is through this particular formula ok. So, that brings us to the end of the discussion about the predictability of the hourly and daily radiation

at a particular location which we can relate to the hour angle as well as the sunset and sunrise angle. So, these things we have covered so far. Now, next what we are going to talk about is the very important part of the course is the important angles and their correlation ok.

(Refer Slide Time: 15:03)



I should super emphasize this; this is extremely important. So, I demand your absolute attention on this part ok. Now, we have already seen some of these. So, let us recap a little bit of them, so what we have already seen? Ok. So, first thing is we have seen the observer centric reference frame ok.

So, we have seen that if the observer is standing on the middle of a flat ground, and this is North, South, East, West. So, sun will be tracking a path somewhat like this, and projection of this semicircle on the horizontal plane will be this. So, if sun is here, sorry, if, ok, if sun is

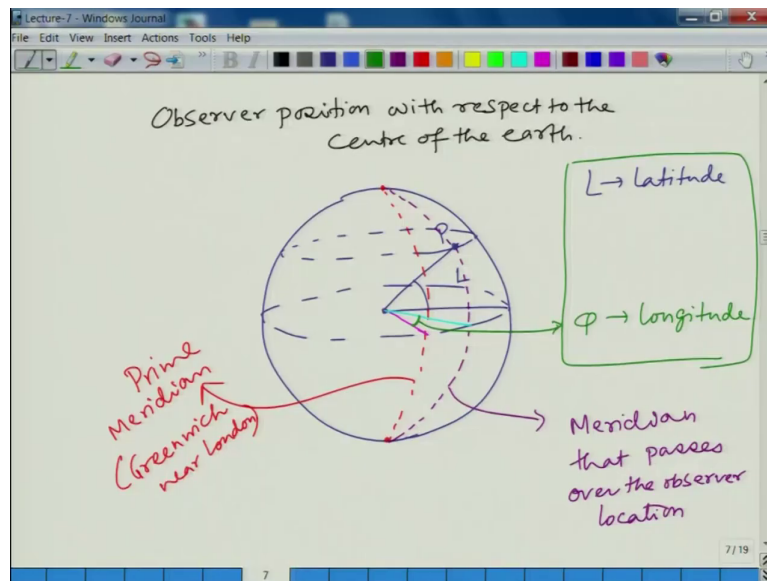
here, then this is the projection and let us say observer is at P. So, we have seen that this will be the vertical direction ok.

So, we see few of the angles here. So, first one is the sun ray; so this is the sun ray ok. The angle it is making with it with the particle that is the zenith angle ok. So, this angle we have seen earlier zenith angle. And the other angle is the altitude angle which is just  $\pi/2$  minus the zenith angle. So, this  $\alpha$  is the altitude angle which is nothing but  $\pi/2 - \theta_z$  ok. This we have seen.

And the another important angle which the projection of the sun ray is this one right, projection of the sun is this line what angle does it make with the south direction. So, this particular angle is we call A Z; A Z is the azimuth angle or solar azimuth angle actually ok. But whenever it is called the azimuth angle, then we only I mean even if it is not mentioned solar we mean that to be solar azimuth angle ok. So, this we have already seen.



(Refer Slide Time: 19:20)



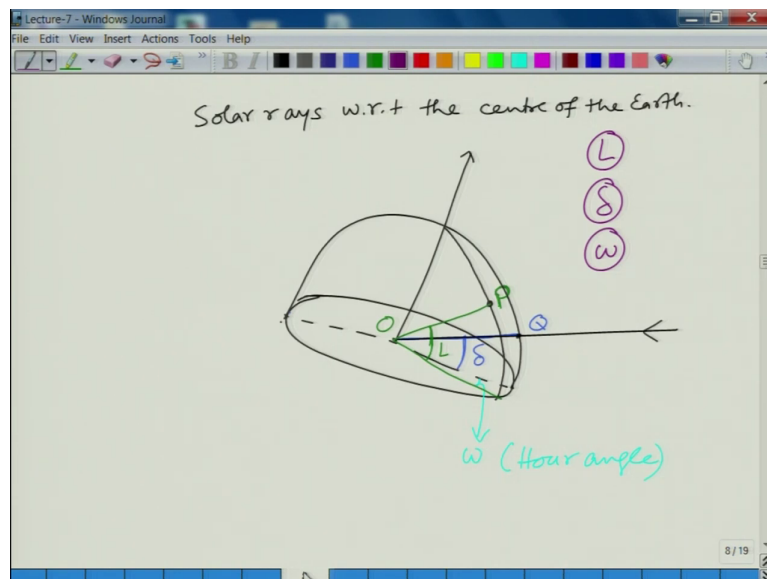
So, what else have we seen is the observer position with respect to the centre of the earth ok. This we have already seen that if this is earth ok and if we have this is the centre and we have this equatorial plane. So, if observer is here ok on this particular position, let us say this is P, so the angle this particular line makes with the equatorial plane that is our latitude. So, L is the latitude.

The other angle that we need is, so if these are the poles you can draw these half circles along the surface of the earth, and one of them is our prime meridian right. So, we fix one line to be prime meridian against which all other points are put I mean measured whether east or west to that particular line; this is the one that runs through Greenwich near London. So, you have heard of GMT or Greenwich Mean Time this is the line that tells you what would be the local time of that particular location, anyway.

So, this purple line that we see this is the meridian. So, meridians are the lines that are passing from one pole to the other through the surface of the earth ok. So, P is the meridian that passes over the observer location ok. So, the angle between these two, angle in the so angle between these two lines, so basically it crosses the equatorial plane here, the other line crosses the equatorial plane here. So, the angle between these two ok is our phi or longitude ok.

So, if we know the latitude and longitude of one place, we know exactly where the observer is lying in terms of the reference frame which is located at the centre of the earth ok. So, these are the another two angles that we have seen earlier ok.

(Refer Slide Time: 22:58)



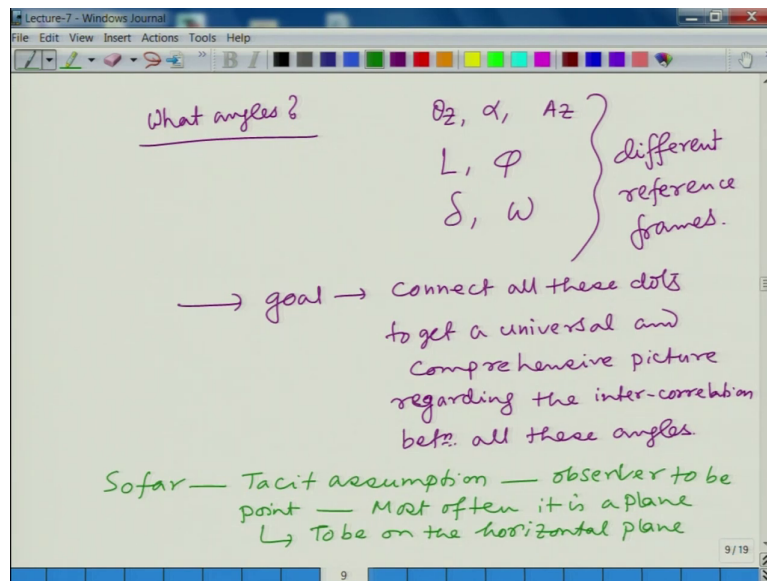
And then another set that we have learnt is the solar rays with respect to the centre of the earth ok. So, we have seen that the earth rotation axis is at certain angle with the orbital plane.

So, let us say this is our earth ok, semicircle, one hemisphere of the earth. Now, if sun ray is radiating normally at certain point P let us say this point to be Q ok. And if we have if we drop a or rather this particular sun ray that is going through normally to the through the surface of the earth what angle it makes with the equatorial plane ok.

So, let us say this is the equatorial plane line. So, the angle this one makes we called it declination angle right. And on the same figure what we can write, we can drop meridian ok that is passing through this point P which is the observer location. So, that particular line is what it is the angle it is making like this is the point joining the centre of the earth and the observer, and the angle it is making this is the latitude right.

Now, what we can see here that between these two we have certain angle like the projection of QO on the equatorial plane and projection of PO on the equatorial plane, we have certain angle between those two projections on the equatorial plane and that is what we call the hour angle ok. So, these three, so here we are getting few angles the latitude, declination angle, and hour angle ok. So, what are the angles that we are getting so far? So, from this figure we got theta z, alpha and A z.

(Refer Slide Time: 26:10)



So, what angles we have seen so far? Theta z, alpha, A z. And from the next figure, what we got the latitude and phi or the latitude and longitude ok. And then from the other figure we have seen another two angles; one is declination angle and hour angle. So, one is declination angle and hour angle ok.

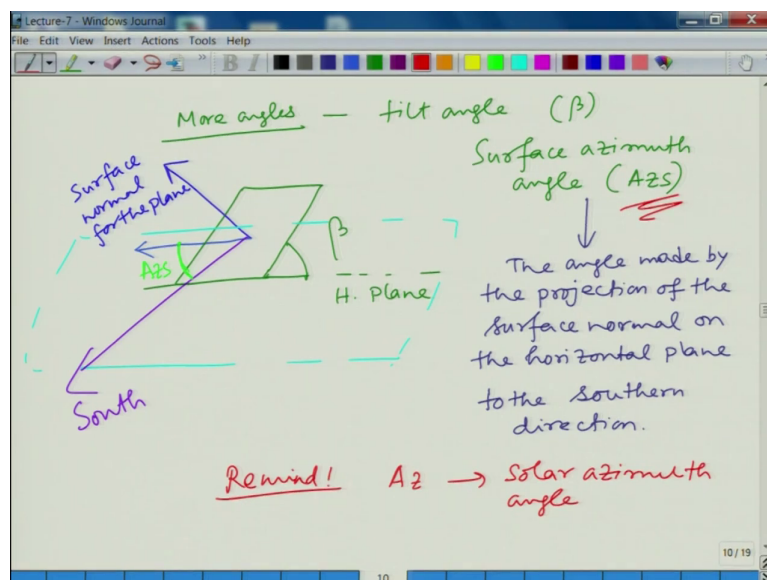
So, you have seen that all these things are connected to different reference frames ok. So, we have obtained these from different reference frames ok. So, our goal is to connect all these dots, all these dots to get a universal and comprehensive picture regarding the inter correlation between all these angles ok, so that is our goal that is what we are going to do now.

Now, before I proceed I should also mention here that. So, far we have made a tacit assumption ok. What is that tacit assumption? We have taken the observer, observer to be a

point ok. It is just a point that is what we considered. But it can be a plane most often it is a plane right. If you place a solar photovoltaic panel on your roof top that is a plane that is not a point ok. But most often it is a plane, so that assumption we have taken.

And also whenever we have talked about point we have taken to be on the horizontal plane ok, that is also not always valid. Whenever you take it not as a point, but as a plane, then it can be at any other position than horizontal position ok.

(Refer Slide Time: 29:31)



So, more angles that we need that are needed is the tilt angle right which we call beta. So, what is that if you have a planar like collector, what angle does it make with the horizontal? If this is the horizontal plane ok, the angle it makes is called the tilt angle. So, that is one important angle we need to consider. And also whenever you have a plane, then you can draw a surface normal for that plane right. So, this is surface normal for the plane.

And whenever you can draw that vector, you can project it on the horizontal plane. So, suppose this is our horizontal plane. If you project this surface normal, then it will have a direction; it will have an orientation right. So, if in the horizontal plane, this is the southern direction south ok.

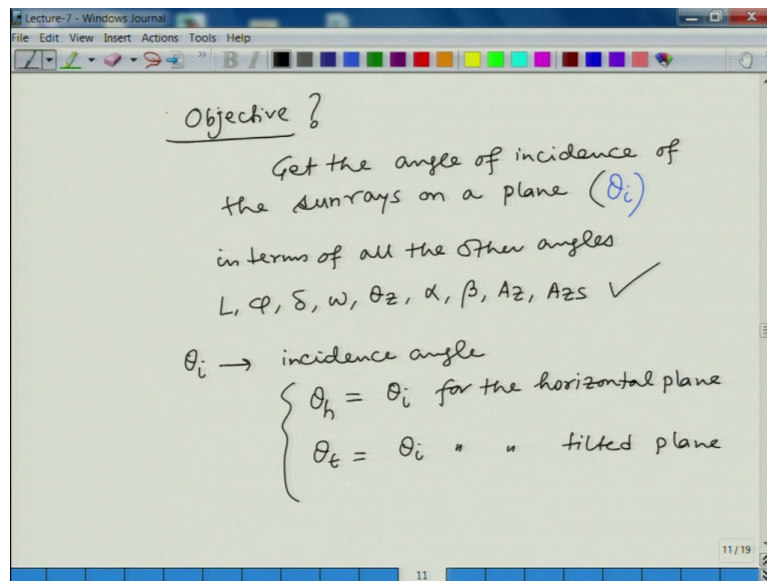
So, what angle does this thing make? The projection of the surface normal on the horizontal plane what angle it makes with south direction that is called AZS ok. So, that angle is called surface azimuth angle or AZS. So, here we have another angle which is surface azimuth angle which is designated as AZS ok.

So, here two important other angles we obtain whenever we consider it to be a plane not a point, we have its tilt angle and the surface azimuth angle. So, let us define this surface azimuth angle as well. So, the angle made by the projection of the surface normal on the horizontal plane made by this projection to the southern direction ok, that is called the surface azimuth angle.

Now, here you see that we have a solar azimuth angle as well; so, just to remind that you need to differentiate between these two that we have A Z right. Right now, we have seen this AZS, but we have seen earlier one A Z which is solar azimuth angle ok; it is the angle between the projection of sun ray on the horizontal plane with the southern direction ok.

So, azimuth angle whenever we talk about we define it with the reference of the south direction, whether it is solar azimuth angle or it is surface azimuth angle ok. So, it is very important that you note the difference between these two ok. So, with all these angles that we have seen, what is the goal?

(Refer Slide Time: 33:51)



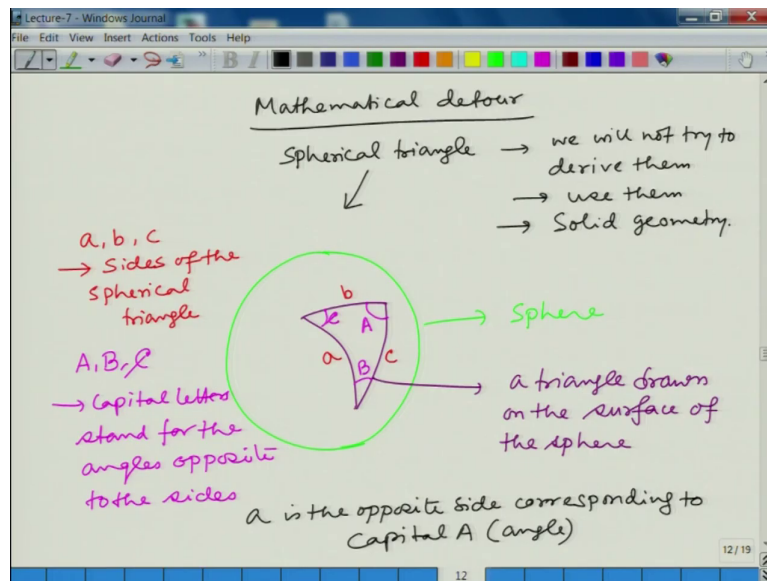
What is the objective here? Objective is to get the angle of incidence of the sunray or sunrays on a plane ok. So, whenever you have a photovoltaic panel it critically depends what will be the angle of incidence at any point of time by which the solar angle solar radiation is hitting that surface right. So, that angle of incidence if we designate it with theta i, i stands for incidence then that angle is a critical angle and we need to know it exactly ok this is what we are after. And how will we get this? We will get it with respect to the other angles.

So, we need to find this theta i in terms of in terms of all the other angles. And what are all these other angles? We have latitude, we have longitude, we have declination angle, we have hour angle, we have zenith angle, we have the altitude angle, we have the tilt angle, then solar azimuth angle, surface azimuth angle and all ok. So, these are the angles that we have and we need to find theta i with in terms of these values ok that is the next task we are going to take.

Now, whenever we call theta i, so this is the incidence angle or angle of incidence whatever you want to call it, but the thing is it can be theta h as well ok. Whenever we talk about the horizontal position of the collector, so this is theta H is equal to theta I for the horizontal plane ok and when we call theta t that we usually call for the tilted plane.

So, these are two particular designations that we often use theta h and theta t which are for horizontal plane and the tilted plane ok. So, now, we have set up the problem nicely. And now we are going to take a short detour in terms of I mean before we actually approach the actual real problem, we need to take a short mathematical detour.

(Refer Slide Time: 37:39)



This detour will equip us with some instruments, mathematical instruments which will be used for the derivation of this inter relationship ok. And this mathematical detour will be on spherical triangle. Again we will not try to derive them; we will not try to derive them, we



will just use them. And if you are interested, you can look up the solid geometry from which this relationship will come ok.

So, what we mean by a spherical triangle? So, you can think of a sphere ok. And on top of that, you can, so this is a sphere not a circle, but a sphere. And on the surface of the sphere, you can draw a triangle right. Why I am drawing it like this? Because even if you draw a straight line on the surface of the sphere in three dimension, it will be it will look like this ok. So, this is what this is a triangle drawn on the surface of the sphere ok.

So, now, you can think that ok these are the. So, let us say this is  $a$ ,  $b$  and  $c$ , these are the lengths of these the arms of the triangle. And the corresponding angles let us say this is  $A$ , this is  $B$ , and this is capital  $C$  let us say this is capital  $C$  ok. So, here this  $a$ ,  $b$ , and  $c$  small, they represents the sides of the spherical triangle ok. And the capital  $A$ ,  $B$  and  $C$ , the capital letters stand for the angles opposite to the sides.

So, note that the small  $a$ , so small  $a$  is the opposite side corresponding to capital  $A$ , capital  $A$  is the angle that is how we denote the angles and sides of this particular spherical triangle ok. Now, when we have this spherical triangle, what we can do with it? So, we will use two rules ok.

(Refer Slide Time: 42:05)

The image shows a digital whiteboard with the following content:

- Title:** Rules for spherical triangles
- Section:** Cosine Rule & Sine Rule
- Equation:**  $\cos C = \cos a \cos b + \sin a \sin b \cos C$
- Annotations:**
  - Red arrows point from the text "Small letters (sides of the Spherical triangle)" to the terms  $\cos a$ ,  $\cos b$ ,  $\sin a$ , and  $\sin b$  in the equation.
  - A purple arrow points from the text "Cap C is the angle for the spherical triangle" to the  $\cos C$  term in the equation.
  - Another purple arrow points from the text "Cosine Rule" to the entire equation.
  - A red arrow points from the text " $\cos a \Rightarrow$  the cosine of the angle the straight line a is making at the centre of the sphere." to the  $\cos a$  term.
- Diagram:** A small diagram of a sphere with a green circle representing a spherical triangle. One side is labeled 'a' and an angle is labeled 'C'.

So, rules for spherical triangles. So, rules for spherical triangle is one is called cosine rule and the other one is sine rule. What is the cosine rule is this one  $\cos c$  ok, we will talk about it in a while  $\cos c$  equal to  $\cos a \cos b$  plus  $\sin a \sin b$  and  $\cos$  capital C ok. So, what is happening here, what is going on? These are all small. So, let me say this, this, this, this and that are all small, small letters ok.

That means these are sides of the spherical triangle ok. Only this one, this capital C is the angle for the spherical triangle ok. So, now, one question arises how can we say that  $\cos$  of a side what does it mean? Cosine is always associated with the angle right. How can we say that it is a side?

So, if you look at the spherical triangle, again let me just quickly draw it here and let us say we have certain length on this on the surface of the triangle, surface of the sphere. Now, we

have a centre of the sphere also right. So, any side or any length will actually subtend an angle at the centre right.

So, if this is the length a, the angle it will subtend we can represent it with the angle a ok. So, what we mean by this cosine of a length? It is the cos a is representing the cosine of the angle the straight line a is making at the centre of the sphere ok, so that is what it means.

That the cosine of an angle cosine of an of a length is actually cosine of the angle that particular length is making at the centre of the sphere ok, so that is what we get to be the cosine rule. This is the cosine rule which we will use again and again in the derivations.

(Refer Slide Time: 46:08)

Sine Rule

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$$

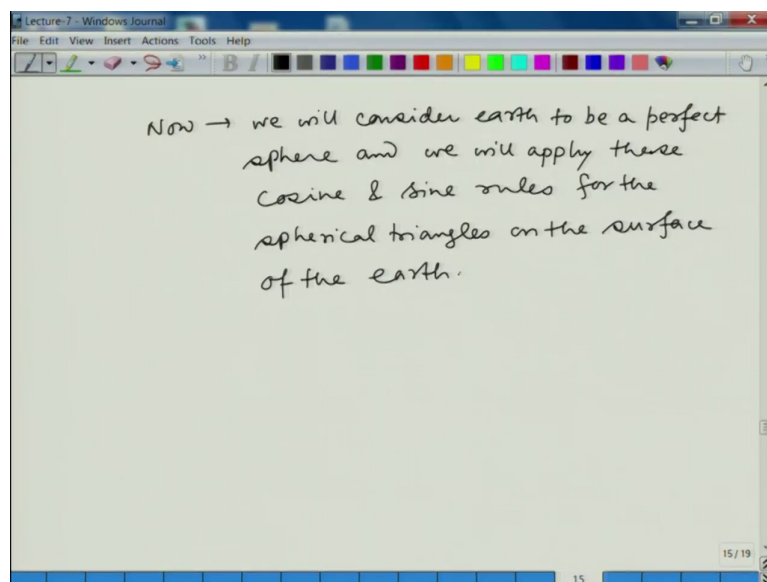
A, B, C → Surface angle  
a, b, c → Central angle.

Next one is the sine rule ok. So, what is that sine rule? Sine rule is sin capital A divided by sin small a will be equal to sin of capital B divided by sin of small b, and similarly that will

be equal to sin of capital C divided by sin of small c ok. So, just to remind this is capital and this is small ok. Now, here you do not need any explanation because now you know what sin capital A means that is the surface angle, and sin small a, that small a is the central angle ok.

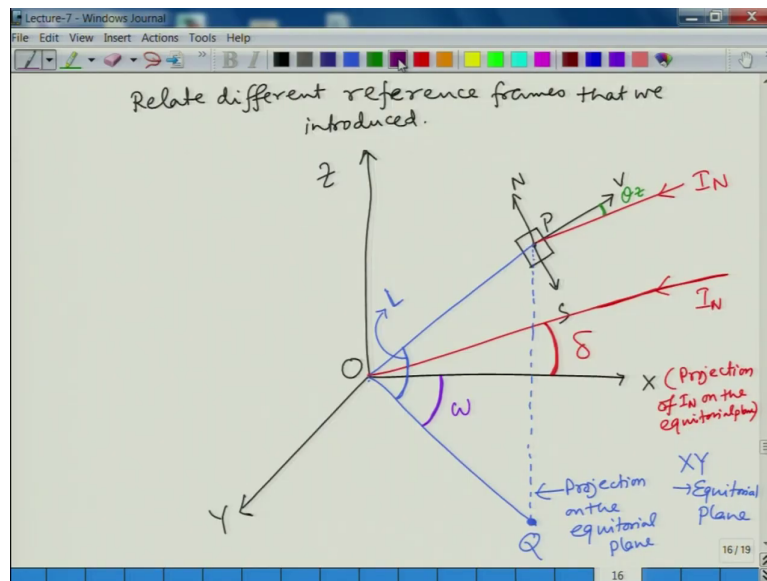
So, let me just write that again, so that no confusion persist these are surface angle; and a, b, and small c is the central angle ok. So, these are the two rules that we are going to use. One is sine rule; the other one is this cosine rule ok. Now, let us see what we can get out of this mathematical detour ok.

(Refer Slide Time: 47:57)



Now, let us go to the portion of now we will consider earth to be a perfect sphere which is closely true, but not exactly true ok. So, we will consider earth to be a perfect sphere, and we will apply these cosine and sine rules for the triangles or rather I should say spherical triangles on the surface of the earth ok. So, we will see how we can apply it.

(Refer Slide Time: 49:22)



But before that what we need to do? We need to explain or we need to relate different reference frame reference frames that we introduced ok. We have introduced several of them, several reference frames, now we have to combine them in one picture ok. So, let us say and please pay close attention to this. So, let us say we have a reference frame which is centered at the centre of the earth. Let us say this is X, this is Y, and this is Z ok.

Now, consider location of the observer ok P. And now we are not considering it to be a we are considering it to be a point, but now we are considering that point to be part of a plane which is horizontal plane ok. So, let us say on that horizontal plane, we will have a north direction and a south direction ok. Now, and on that horizontal plane, we will also have a vertical direction.

Now, let us say sun is radiating from such an angle that the sun ray that is radiating directly on the surface of the earth will reach the centre of the earth like this. So, let us say this is the direction of solar radiation  $IN$  ok. And as all the radiations we can consider to be parallel to each other, one of them will a parallel one will also be also be going through the observer location ok.

Now, what we can see? We can see that there is one angle that you can readily see, this angle that the vertical line or rather the normal to the horizontal plane makes with the solar radiation direction that is our zenith angle right. Another angle which you can locate very easily is by connecting this centre of the earth to the observer location ok.

So, what would be, now what you can do? You can project this to the horizontal plane ok. So, this  $X$  and  $Y$  you can consider  $XY$  to be the equatorial plane ok. Now, if you project this  $OP$ , the line joining the centre of the earth and the observer location if you project that you will get this line ok. So, you have projected this projection on the equatorial plane ok.

So, what angle this one will be, can you please try? Before, I tell you pause the video here and think what angle will this be. The projection on the equatorial plane and the vector  $OP$  ok; now, you can switch on, and see that this angle will be the latitude  $L$  ok. And this particular angle another one which you can say that this,  $IN$ , this particular solar radiation direction that is making with the equatorial plane this angle will be your declination angle right  $\delta$  ok.

And the declination angle I mean this  $x$ -axis here is the you can say that this is the projection of  $IN$  on the equatorial plane ok. So, now, so this  $OX$ , and let us say this is let me name this point because this will be used, let us name this  $Q$  ok. This is on the equatorial plane, let us name this  $Q$ . So, the angle here between this  $OX$  and  $OQ$ , what is the this angle? Again I would urge you please stop it and try yourself.

What angle this will be? This will be your hour angle ok. So, now this will be the hour angle. And what other angle do you see here? Ok. So, let me stop it right here. Let me give you a

chance to look at these angles, what other angles you can fit in this reference frame, and we will start from right here in the next lecture.

Thank you very much.