Elements of Solar Energy Conversion
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Lecture - 34

Hello everybody, welcome back to this series of lecture on Elements of Solar Energy Conversion. We are almost closing to the end, and today we are going to continue talking about the photovoltaic conversion mechanism, and today here we are at lecture number 34.

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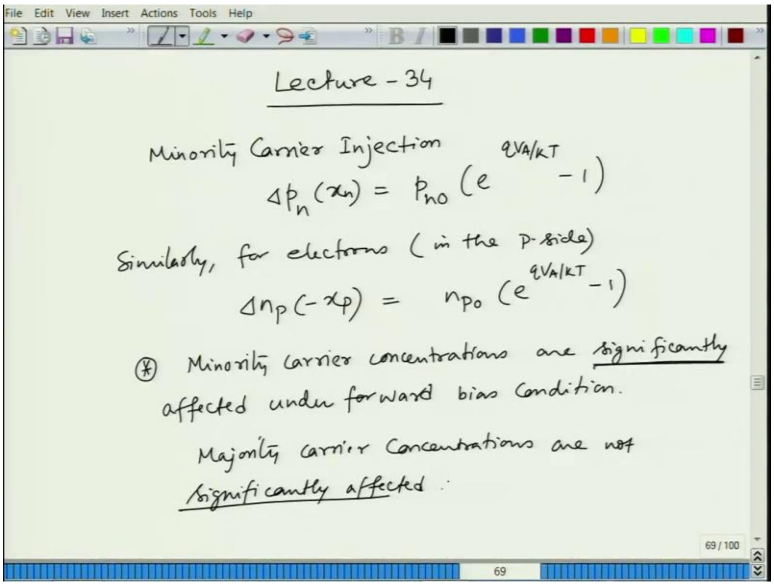

So, in the last class; what where we stopped? We have looked at the minority carrier injection right. So, due to the p n junction diode and if you apply a small potential ok, for forward bias potential, then you have some carrier injection in the minority side right; that is called minority carrier injection.

So, for the n side, you have an increase in the concentration of holes, and that happens at the space charge region edge. And for the n side, we know, or we have designated that location to be x_n, and that value is dependent on the equilibrium value of hole concentration in n side; multiplied by $e^{\frac{qV_a}{KT}} - 1$. So, this is the carrier injection that happens, and you can see that it depends on the applied voltage.

And similarly, for electrons, and of course, that will be in the P side where it is the minority carrier, there we will have this Δn_p, and the space charge region edge in the N side is $-x_p$.

So, that will be

$$\Delta n_p(-x_p) = n_{po}(e^{\frac{qV_a}{KT}} - 1)$$

So, I mean a similar procedure you can follow, and you can obtain this particular expression. And it is important to note that minority carrier concentrations are significantly affected; I stress on the word significantly affected under forward bias condition right.

But the majority carrier, of course, they are also affected and amount wise; they are equally affected because it has to be mass balance; the total number of holes or electrons are not getting changed, there is no generation, no recombination so far. So, only due to bias; what we have? We have the same amount of some amount of holes going from the P side to the n side.

And there, the minority carrier holes are significantly affected, but the majority of carrier concentrations are not significantly affected; we can say. Because the carrier concentration itself was higher to start with, and that is why a little bit of decrease does not matter ok.

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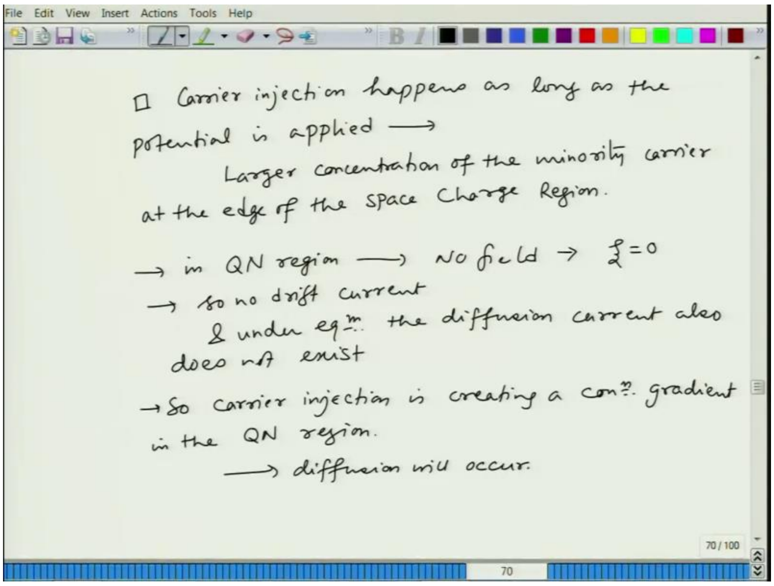

Now, you can see from the expression itself; you can say the carrier injection happens as long as the potential is applied right. If you take out the potential, then that carrier injection will go away.

Now, this carrier injection; is causing a larger concentration of the minority carrier at the edge of the space charge region, is not it? And in the Quasi Neutral region, where there is no field ok, that means ξ is equal to 0. So, there will be no drift current, right. And under equilibrium, the diffusion current also does not exist because, in the Quasi Neutral region, there is no gradient of these concentrations; if there is no gradient, there will be no diffusion right.

So, under equilibrium, it does not, but once you increase the space charge region edge concentration, so there is a gradient that you are creating. So, the carrier injection is creating a concentration gradient in the Quasi Neutral region right because one edge, you have altered the concentration for ok. So that means diffusion will occur; let us first try to understand this qualitatively, how this carrier injection is going to affect the currents ok.

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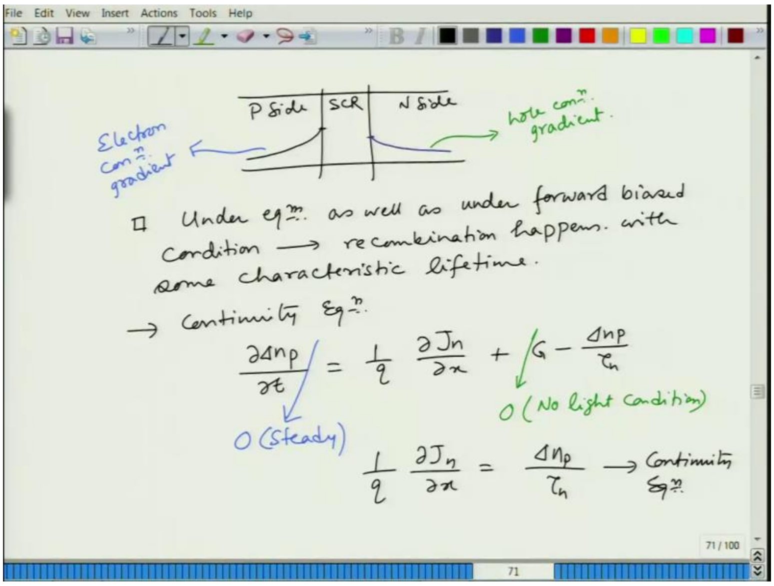

So, you can qualitatively say that if this is the p n junction where this is the space charge region, you have this P side, N side, and here you have the space charge region.

Then, on the P side, where the electrons are minority carriers, you have a larger concentration here. So, you have a concentration; gradient created at the edge of the Quasi Neutral region, ok. So, this is the electron concentration gradient ok created, and similarly, you can have some other hole concentration; it does not have to be equal, and in the N side, you have this hole concentration gradient right; this is the result of the carrier injection.

So, whenever under equilibrium itself; under equilibrium as well as under forward biased condition; under forwarding biased condition, this recombination happens ok; with some characteristic lifetime, this we have seen before. So, in the continuity equation, the general form of continuity equation is this, if we write it for the minority carrier in the P side, that means, for the electrons right; this we have seen earlier, we are just restating it.

So, under steady-state; this term will be 0 right, and if there is no generation; that means no light is falling on the diode, that term will also be 0; so under no light condition, no light or any form of energy going in; so that will not generate anything.

So, what terms are we left with! This term right. So, we will be

$$\frac{1}{q}\frac{\partial J_n}{\partial x} = \frac{\Delta n_p}{\tau_n}$$

τ_n is the characteristic lifetime for an electron, ok. So, this is the continuity equation form, so this is the continuity equation for minority carriers on the P side.

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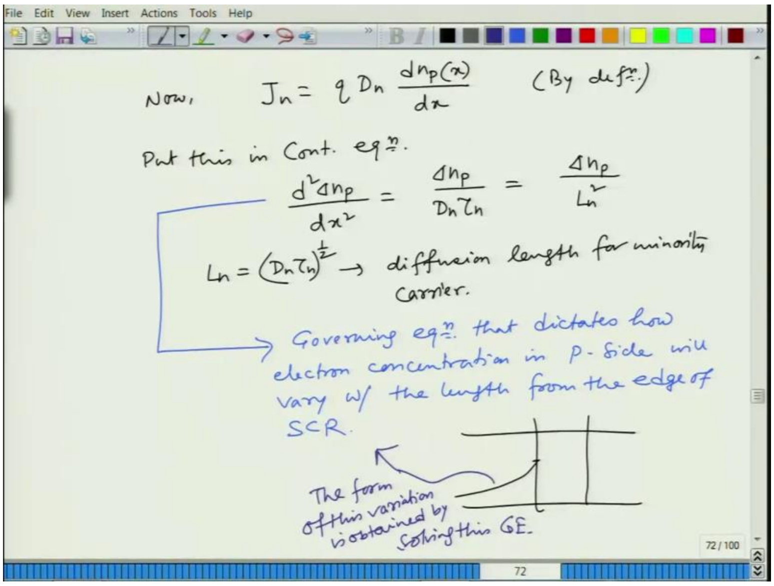

Now, just by definition of

$$J_n = qD_n \frac{dn_p(x)}{dx}$$

Everything we are talking about on the P side, only the same thing can be done for, the other side ok and this is by definition right.

Now, so, if we put this in the continuity equation, then what we get? We get the governing equation for the minority carriers electrons on the P side. And this $D_n \tau_n$, this is the diffusion coefficient multiplied by the characteristic lifetime; you can see that it gives us a unit of length squared ok. If you look at the units, so you can as well write it in terms of ${L_n}^2$Ok.

So, L_n is what? L_n is basically nothing, but this $(D_n \tau_n)^{1/2}$and it is called diffusion length for the minority carrier right. So, what we obtained here is the governing equation. So, this equation is the governing equation that dictates how electron concentration in the P side of the diode will vary with the length from the edge of the space charge region right.

So, earlier what we have seen, that if this is the space charge region, you have some concentration, and it will vary. So, this governing equation tells us the; so, basically, the form of this variation is obtained by solving this governing equation ok; that is the purpose, and that exact form is important, because then only we can quantify it. The concentration gradient form will tell us; how the diffusion happens. So, it is important to get hold of it to exactly quantify the diffusion current.

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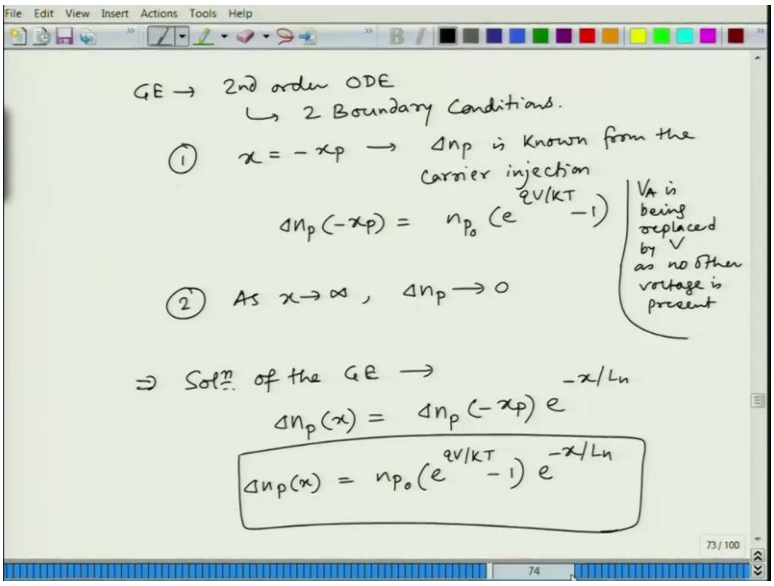

Now, this governing equation is second-order; ODE, Ordinary Differential Equation right. You have only one dependent and one independent variable; the independent variable is the length x, and the dependent variable is Δn_p. So, the second-order ordinary differential equation will require two boundary conditions right; this you are familiar with your math background.

So, what can we have? The first boundary condition that we have is at the space charge region itself. So, and that is x equal to ($-x_p$) because we are talking about the P side and the space charge region stands where x is equal to ($-x_p$), and it is known right rather Δn_p is known from the carrier injection, right.

We have a quantifiable or quantified value of Δn_p at the space charge region edge ok. And what is that value? Δn_p is equal to at minus x_p is equal

$$\Delta n_p\left(-x_p\right) = n_{po}(e^{\frac{qV}{KT}} - 1)$$

So, let us not use V_A anymore; we do not have to say applied explicitly, that is the only voltage; so we are going to use V. So, here a small note that V_A is being replaced by V, as no other voltage is present in the expression; so, there is no confusion; earlier we had to do it because we had to differentiate that applied voltage from the built-in voltage ok.

So, now we do not have that built-in voltage anymore in the expression, so we do not have to differentiate by explicitly telling it is V_A ok. So, this is the space charge region Δn_p and the second boundary condition that we have is as x tends to infinity; that means, deep into the quasi-neutral region; what you have? You have, this tends to 0 because there will be no effect of the carrier injection deep into the quasi-neutral region; so that you can say x tends to infinity, your Δn_p tends to 0.

So, the boundary conditions you have, the governing equation you have; so you can find the solution of the GE; the governing equation under this boundary condition, which is $\Delta n_p(x)$; that means it is a general variable x now, and that is equal to $\Delta n_p\left(-x_p\right)$ and then this exponential function of x divided by L_n.

So, you can try this by yourself, and you can see whether you are getting this solution or not. Very straightforward, from your high school days differential equation understanding, you will be able to solve it ok. Now, we can put this value of the pre-exponential factor ok, which

$$\Delta n_p(x) = n_{po}(e^{\frac{qV}{KT}} - 1)e^{-x/L_n}$$

So, that is the complete solution of that concentration gradient, so it gives us the form of how that variation happens.

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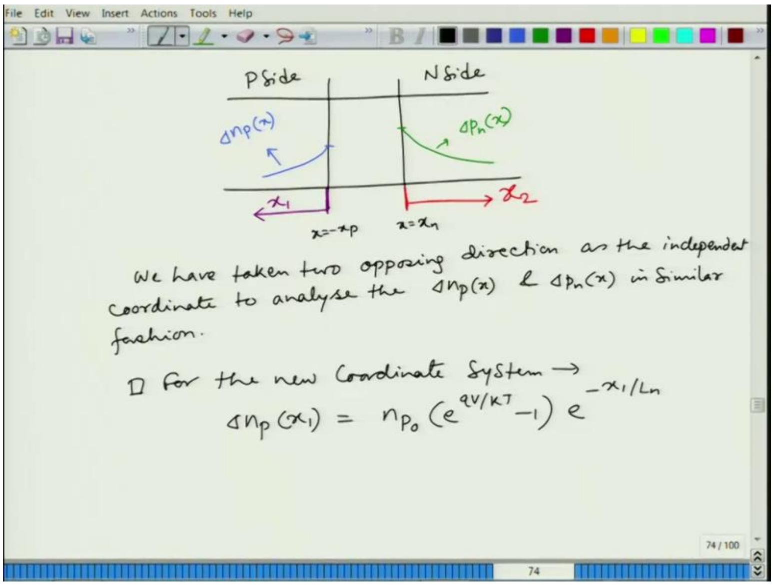

Now, let us go back to the P N junction diode; let us look at it again. So, again this is the space charge region ok, and we can see that this is x equal to $(-x_p)$ and this is x equal to (x_n). Now, for the P side, so this is the P side, and this is the N side. So, for the P side, the Δn_p happens like this, ok. And for the N side delta p n; so, this is basically this is delta p n as a function of x, is not it? And this one is Δn_p as a function of x.

Now, you can see the direction of these two changes for Δn_p and Δp_n are in opposing direction right. So, if you fix your like fix your origin here for the Δn_p; then, let us say that the x is increasing in this direction. So, let us use a new symbol x_1 which is opposite to the direction of x, and on the other hand, if you fix origin here, you name this coordinate system x_2, ok.

So, this is just to ensure that we have taken two opposing directions; as the independent coordinate to analyze the $\Delta n_p(x)$ and $\Delta p_n(x)$ in a similar fashion; otherwise, we always have to bother about the sign. So, here the difference between x_1 and x_2, it is ensuring that as x_1 or x_2 go up in magnitude, the Δn_p or Δp_n also I mean, it goes down ok; that kind of assurance is obtained if we have a different coordinate system. That is the only reason; why we are choosing x_1 and x_2, instead of a single x ok.

So, now, in this new coordinate system, what can we write? So, for the new coordinate system; what we can write is Δn_p, now x_1, you note that we did not use x, but we have used x_1; this is

$$\Delta n_p(x) = n_{po}\left(e^{\frac{qV}{KT}} - 1\right)e^{\frac{-x1}{L1}}$$

So, please note that x_1 is introduced, and you do not have to bother about the sign because that is the sign you have used earlier, ok.

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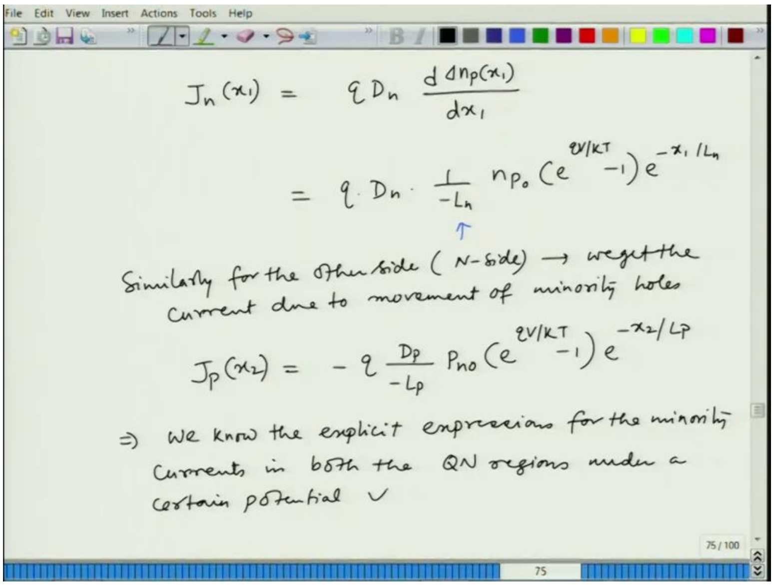

So, if we put these in the expression of the current due to minority electrons in the x_1 direction, we have this

$$J_n(x_1) = qD_n\frac{d\Delta n_p(x_1)}{dx_1}$$

This is the definition of J_n. So, now, we can know the Δn_p variation with x; we quantifiably know. So, what we can write,

$$J_n(x_1) = qD_n\frac{1}{-L_n}n_{po}\left(e^{\frac{qV}{KT}} - 1\right)e^{\frac{-x_1}{L_n}}$$

So, this L_n comes from the differentiation ok. So, we just straightforwardly differentiated $\Delta n_p(x)$ expression with respect to x_1, ok.

So, similarly for the other side; that means, for the N side; we get the current due to movement of minority holes right, and we can write the similar expression J_p; now, we will have x_2 right is equal to now, as it has a negative or the expression; the charge is negative for holes what value you will get is

$$J_p(x_2) = -q\frac{D_p}{-L_p}P_{no}\left(e^{\frac{qV}{KT}} - 1\right)e^{\frac{-x_2}{L_p}}$$

So, you agree to this? We just use the same thing; x_1 and x_2 are consistent with the current direction, ok. So, now, we know the explicit expressions for the minority currents in both the

Quasi Neutral regions and that under a certain potential V ok. So, for the forward bias, V is positive; for the reverse bias V is negative, but the expressions everything stays just as is. Now, that is the expression we obtained for the Quasi Neutral region.

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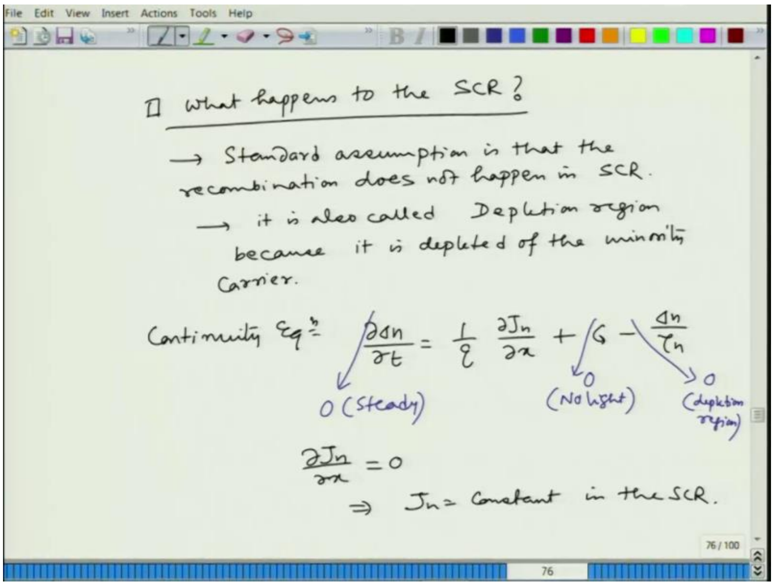

Now, what happens to; what happens to the space charge region, right? Because if current flows through the diode, it has to pass through the space charge region as well, right; so? What happens there? So, the standard assumption is that the recombination does not happen in the space charge region.

Why is it the standard assumption? I mean, why can we take this assumption? Because the space charge region is also called depletion region right and depletion region, why? Because the minority carriers are less in number in the space charge region so, it is also called depletion region because it is depleted of the minority carriers right.

Why is it depleted of? Because you have a potential there, ξ is not equal to 0, and all the minority carriers that will come into that space charge region will be sucked into the other side right. So, that is why the depletion it is called the depletion region and that is why; if you have less number of or no number of minority carrier, how can the recombination happen?

So, even the recombination term, you can neglect from the continuity equation. So, now, if we look at the continuity equation ok, this Δn, again we are writing it for the electrons. And similar you can write for the hole right this is the form of the continuity equation we have used earlier also.

So, now, we are; we do not have this term which is due to steady-state ok, and we do not have this term, due to no light condition ok and this term is also 0, because it is depletion region right. So, what we are left with; is just an only single term, and that is equal to 0. So, what we have this $\frac{\partial J_n}{\partial x} = 0$, what does that mean? That J_n is constant in the space charge region, is not it? So, we get a very simple expression; it is constant, and what constant is it? That is very easy to find as we see next.

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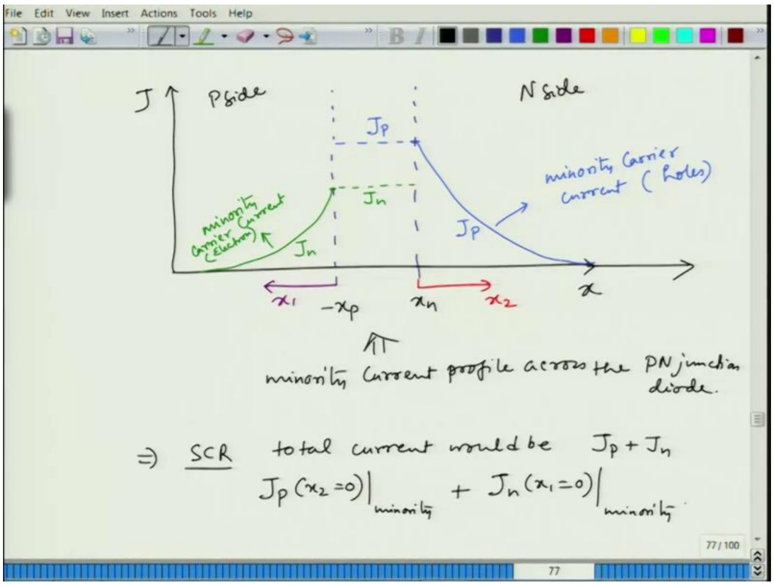

Now, you think of the P_n junction diode like this, ok. Let us say in this axis we are just plotting J and that J may be due to and J is for the minority current ok; minority carriers, so we will see that right now.

So, let us draw the space charge region here, ok, and this is the direction where x is plotted ok. So, here again, just like earlier, this is $-x_p$, this is x_n and we have seen that we have used x_1 in this direction and x_2 in this direction, ok. So, from the given expression that we obtained after solving all those equations, first the continuity equation, first the governing equation for the concentration profile, and then the continuity equation. The current expression that we obtained, we can draw in this fashion.

So, so as the concentration profile flattens, your current will go to 0 right. So, the current is going to 0 deep into the P side. So, this is the P side, and this is nothing but the minority carrier current, and in this case, the minority carrier is the electron. So this is the electron current. What can you write then? You can write it to be J_n right.

And on the P side, you will have some other value; let us say this, and you will have the same thing. So, again it will go to 0; you can extend this axis, and again it will go to 0 as you go deep into the quasi-neutral region. In this case, this is N side, ok, and this is the minority current; minority carrier current and this case, they are holes; the minority carriers are holes ok.

Now, the space charge region; we have not touched yet right. So, what will happen? We do know that it will be; it will be constant throughout the space charge region. Now, whatever constant happens at the edge, we can extend that ok; this will be the constant throughout the space charge region, then only the boundary condition is matched right. So, that is the J_n in the space charge region as well, and similarly, you can find the same constant value corresponding to J_p; so this is J_p, ok.

So, both J_p and J_n are constant, but they are different constants ok in the space charge region, and those constants, they get smaller and smaller as you go deeper into the quasi-neutral region; both for the P side and N side, and that is how the minority current looks like. So, this in the above picture is the minority current profile across the P N junction diode right.

Do you agree to this? Now, if you look at the space charge region only ok; space charge region, you have the that is all the carrier you have right; you do not have any other carrier, because those have drifted out. So, in the space charge region, the total current would be this $J_p + J_n$. And more precisely, what we can say J_p at x_2 equal to 0; that means the J_p in the minority side and J_n, which is x_1 equal to 0, and again, that is also minority current right.

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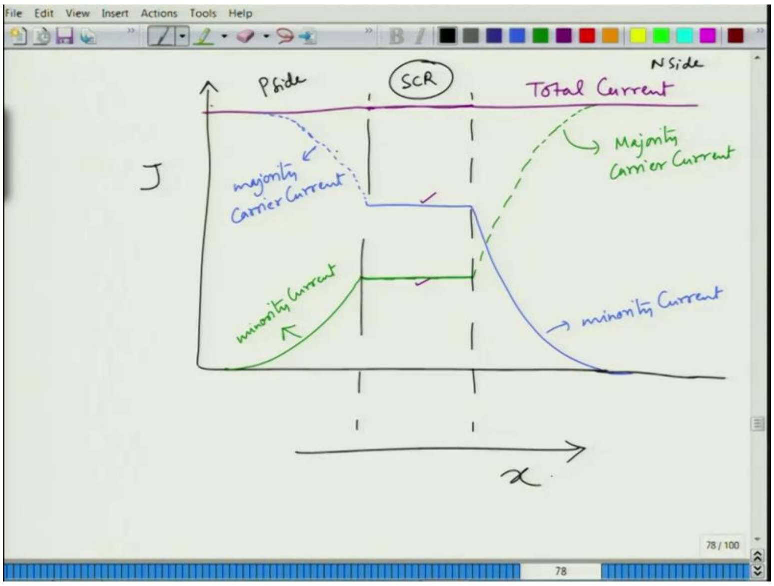

So, now, if we redraw that thing on a bigger scale and now, I am not using; so let us say that this was the or I should I can use this current also. So, this was the case for the electrons in the P side right. So, and this was the case for holes in the N side, ok. Now, the total current in the space charge region will be the summation of this and that. So, let us say it is somewhere here, ok.

So, if that is the total current and total current cannot be different in the different regions, right; it has to be equal everywhere. So, you can extend this line on the P side and on the N side to get the total current across the P N junction diode. So, this is total current ok, and these are; this is minority current right, and this one is also minority current, here we are plotting J, and different plots will give you different J.

Now, what we are left with is the majority current; majority current does not apply for the space charge region right; so, majority current only in the quasi-neutral region. So, what you can write here that if you subtract the minority current from the total current in the quasi-neutral region, you will get the majority carrier current, ok. So, this will be ok; this is the majority carrier current because this is now on the N side, ok. And similarly, we can draw that for the holes ok, so this is the majority carrier current in the P side.

So, that gives us a complete picture of how different carriers are contributing to the total current across the P N junction diode. So, on the left-hand side, you can see the majority carrier is shown by this blue plot ok and in the this is the P side; let me explicitly write it, this is P side, this is N side, and this is space charge region ok. So, the majority carrier is important, and it is carrying the maximum amount of charge, and on the P side, it is hole ok, and electron is the minority carrier on the P side.

In the space charge region, that minority carrier contribution or majority carrier contribution does not exist. So, minority carrier contribution just stays the same ok, and from there, we continue in the other side of the quasi-neutral region ok. So, now, if we; so that is the qualitative picture we obtained.

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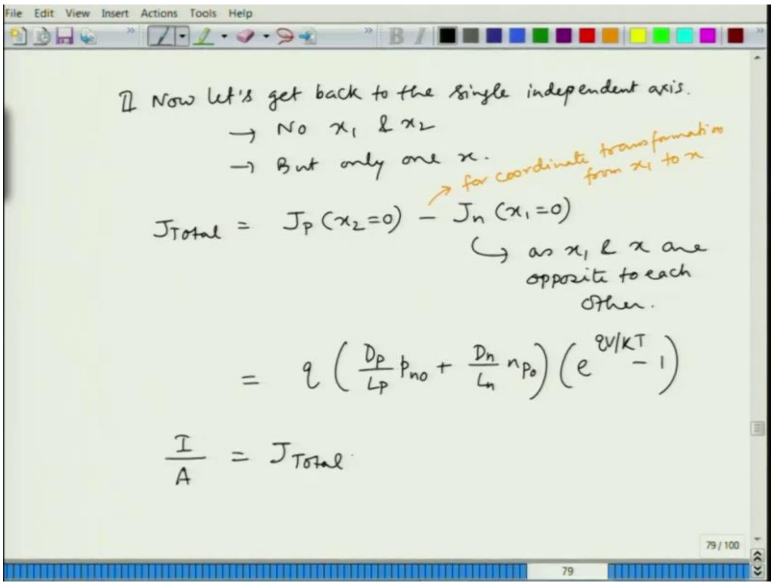

Now, let us get back to the single independent axis in a sense no, x_1 and x_2, but only one x. So, for that, we need to make a little bit of adjustment in terms of coordinate transformation right. So, what we can write; that J total is equal to J_p at x 2 equal to 0, and we have to use minus for x_1 because now, x_1 and x are opposite to each other.

So, basically, what we did? We have used a single x here, ok. So, this single x is giving us this adjustment of the minus sign, ok. So, this is for coordinate transformation from x_1 to x ok, nothing else. And we have obtained the expression for J_n and J_p. Now, if we put those values, what we get? We can simplify. So,

$$J_{total} = q(\frac{D_p}{L_p}p_{n0} + \frac{D_n}{L_n}n_{po})(e^{\frac{qV}{KT}} - 1)$$

Do you agree to this? I mean, please do the algebra by yourself and make sure that you get this expression, ok. Now, this j total is nothing but your current density, right. So $\frac{I}{A} = J_{total}$

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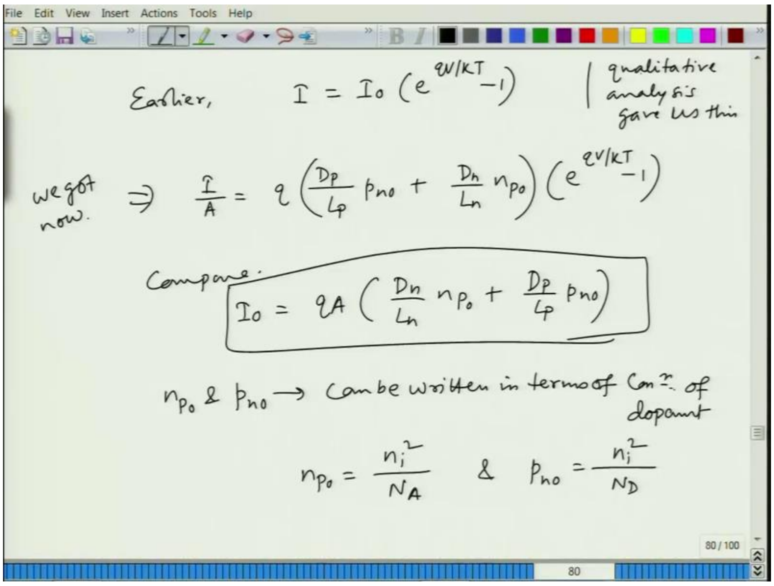

Now, earlier we have obtained; earlier we obtained that $I = I_o(e^{\frac{qV}{KT}} - 1)$, this we obtained before going quantitative analysis gave us this right. So, now, what we got?

$$\frac{I}{A} = q(\frac{D_p}{L_p}p_{n0} + \frac{D_n}{L_n}n_{po})(e^{\frac{qV}{KT}} - 1)$$

Now, if you compare, so if you compare what you get? I_o is nothing,

$$I_o = qA(\frac{D_p}{L_p}p_{n0} + \frac{D_n}{L_n}n_{po})$$

You can further write in terms of the concentrations ok; n_{po} and p_{no} can be written in terms of concentration ok; of dopant.

What I mean is n_{po} is nothing but the intrinsic carrier concentration squared divided by the concentration of acceptor elements ok and p_{no} is $\frac{{n_i}^2}{N_D}$ which is the dopant concentration ok.

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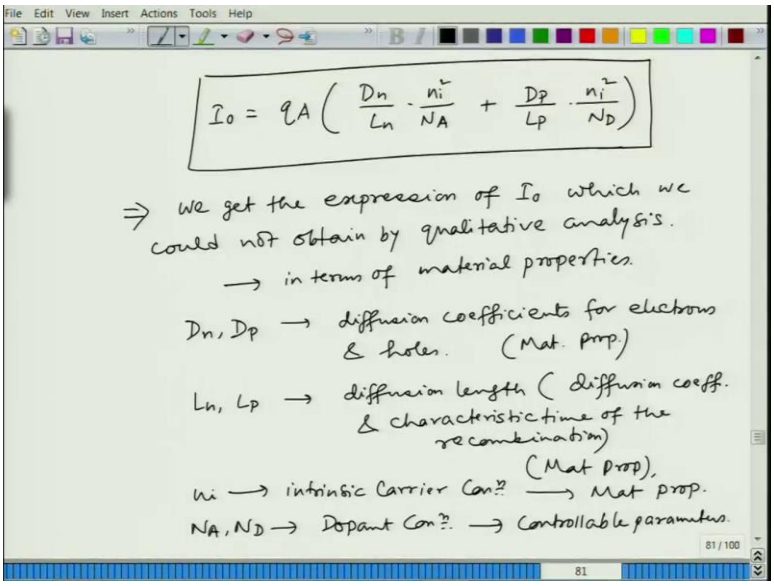

So, if we replace those values, what can we get? This I_o

$$I_o = qA(\frac{D_n}{L_n}\frac{{n_i}^2}{N_A} + \frac{D_p}{L_p}\frac{{n_i}^2}{N_D})$$

Now, we have it, so we get the expression of I_o which we could not obtain by qualitative analysis right.

So, now, we have obtained in terms of material properties; so what I mean by material properties? Because all these $D_{n,}$ which is the diffusivity or diffusion coefficient of electrons in the material ok; if it is silicon, then silicon so, that is the diffusion coefficient of the electron.

D_p is the diffusion coefficient of holes ok; D_p and D_n; they are diffusion coefficient for electrons and holes right. And L_n and L_p are called the diffusion length, which has the diffusion coefficient and characteristic time of; characteristic time of the recombination right, which is also called life span.

So, the diffusion length takes into account that information. So, L_n and L_p; are also material property; so this is a material property, these are also material property, and n_i is, of course, the intrinsic carrier concentration; intrinsic carrier concentration that is also a material property, is not it? And N_A, N_D are also material property in terms of those are; those you can control ok.

The amount of dopant that you put in P side or N sides; those are controlled. So, these are dopant concentration, controllable parameter, and q is the charge of the electron, A is the area; the area is, of course, controllable that gives you I_o. So, there is a complete skeleton; how we can obtain this I_o is obtained now.

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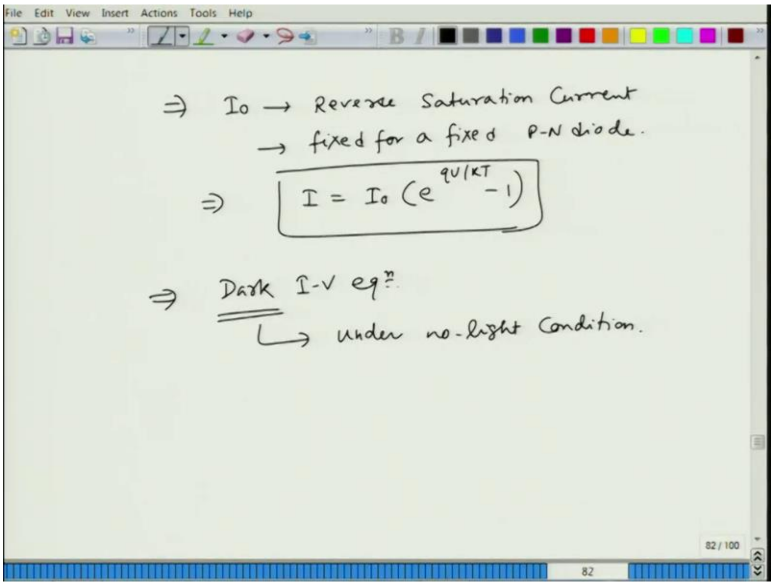

And this I_o is nothing but the reverse saturation current right. So, it is fixed for a; fixed P N junction diode. So, the P N junction diode, I mean the P and N part, is obtained by a particular dopant by doping those elements. So, once you have fixed those dopants, there is no way you can change this reverse saturation current. That is all it depends on, ok.

And this particular expression; is called the dark I V equation. I V is for the current-voltage equation; for any device, any electronic or electrical device, this V I characteristic curve is very important. So, we are trying to get there, and this V I equation is only applicable; when there is no generation or any light falling on that diode, right. So, that is why it is called dark, under no light conditions.

So, if you follow how we proceeded, we proceeded with first understanding the equilibrium where there is no voltage. If you just bring them P and N together, what happens inside the band structure and all, and that is how we got the equilibrium. Now, once you get the voltage applied, so forward bias, reverse bias; how does that affect? That is what we got, and this is the pinnacle of that analysis which gives us the dark I V equation.

And in the next class, we look at when you apply light, how the photovoltaic effect generates, and how you get some current. So, let us stop here in this class and continue from here in the next class.

Thank you very much.