Elements of Solar Energy Conversion
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Lecture - 29

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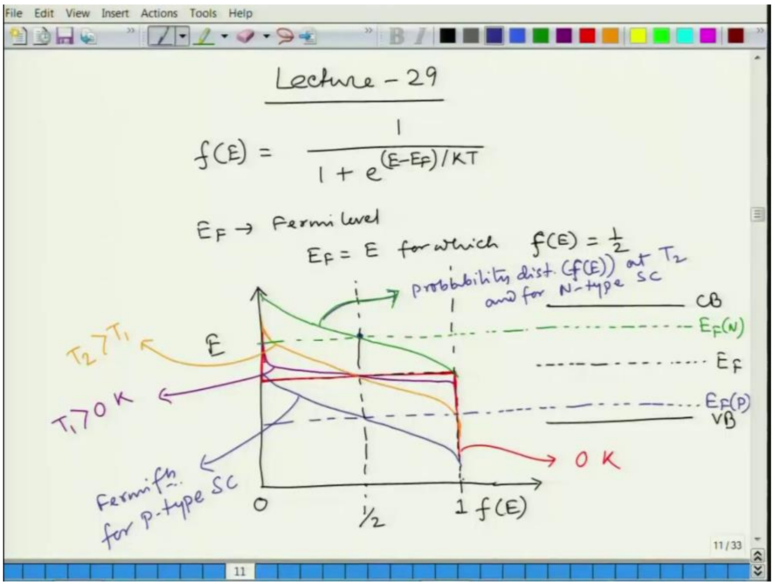

Lecture - 29

$$f(E) = \frac{1}{1 + e^{(E-E_F)/KT}}$$

$E_F \rightarrow$ Fermi level

$E_F = E$ for which $f(E) = \frac{1}{2}$

Hello everybody, welcome back to the series of lecture on Elements of Solar Energy Conversion. We are here at lecture number 29. So, in the last class we have started looking at one of the major and matured solar conversion technology which is based on photovoltaic.

So, we have started looking at how the semiconductors can act as a photovoltaic material, how the carrier concentration of electron and hole can be altered by doping. And two major concepts came towards the end of the class— one is density of state and the other one is Fermi function, ok. We will continue working on them and we will look at how they affect the carrier concentration.

So, we have looked at that the Fermi function for a particular energy level is of this form. So, first thing we can look at here is that it is a function of temperature. So, basically Fermi level is that energy for which Fermi function is half or the probability of finding an electron in that energy level is 50 percent, ok.

So, now, if we look at the effect of this in terms of the band structure. Let us look at that now. So, suppose we have this energy and Fermi function if we plot that, how will that change ok? So, for Fermi function you know, it is a probability. So, it can vary between 0 and 1. So, these are two hard limits for probability.

Let us say this is the band structure for a particular material and the Fermi level will be here, right (50 % occupancy probability). So, Fermi level will be just in between the conduction band and valence band, ok. So, if we draw the corresponding line here.

So, at 0 Kelvin, the probability will change between 0 and 1 abruptly, ok. So, in between you will have equal probability and here what you can write when the Fermi function is half; that means, here it is half between 0 and 1 and this is the Fermi level. So, this particular curve, it is for 0 kelvin when there is no excitation available, ok.

Now, we have seen that for the N type semiconductor or before I go to that let us first look at how the Fermi distribution will change with temperature. So, at 0 K, we see that these corners are sharp. So, it is kind of a step function. Now, with higher temperature what will happen, this thing will smear out but still it will remain cross the line at $f(E) = 1/2$, ok. So, this one is temperature $T_1 > 0\ K$, ok.

Now, at another level if temperature is even higher, then smearing will be higher. So, this one is T_2 which is greater than T_1. So, with temperature the smearing will be higher, but the crossing over will always happen at $f(E) = 1/2$. Now, if we say that for N type semiconductor, we have this Fermi level itself is shifted towards the conduction band, ok.

So, suppose the Fermi level itself is shifted to the conduction band for N type semiconductor. So, $E_F(N)$ is the modified Fermi level for N type semiconductor. Now, what will that mean? Let me increase that or extend that line and that will mean that for that particular material, which is a N type semiconductor, the crossover will happen at this point because that is the Fermi level it has to be $f(E) = 1/2$.

So, at that particular temperature what will happen your Fermi level of that material will be parallelly shifted and move upward. So, this is probability distribution which is also the Fermi function at temperature T_2 and for N type semiconductor, ok.

So, what we see that Fermi level is shifted upwards in the energy scale and here on the other hand if we have for a P type semiconductor then the Fermi level will shift downwards to the valence band ok because now the hole concentration will be higher. So, the same thing now you will have parallelly shifted, ok. So, for this one I should use another colour yeah. So, I should be consistent with the colour. So, let us say the colour will be this, ok.

So, this one basically is the Fermi function for N type semiconductor and this one the lower one is the Fermi function for P type semiconductor, ok. So, that is how the probability distribution or the Fermi function will change. You have seen that for temperature, it will be smeared more and more smeared as you go up from the from absolute zero and for the doping it will be shifted parallelly.

The whole thing will be shifted parallelly, for N type semiconductor it will be shifted up and for P type semiconductor it will be shifted downwards, ok.

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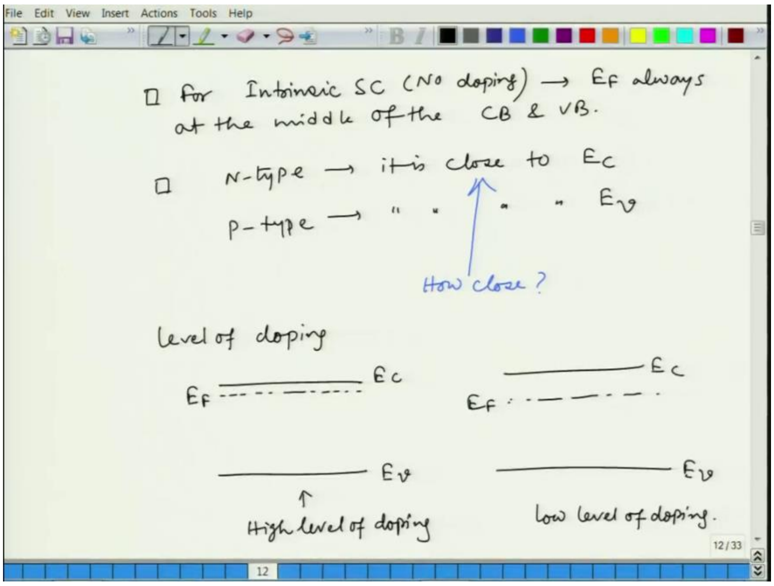

☐ For Intrinsic SC (No doping) → E_F always at the middle of the CB & VB.

☐ N-type → it is close to E_C

P-type → " " " " E_V

How close ?

Level of doping

High level of doping

Low level of doping.

So, few observations we can write, one is that for intrinsic semiconductor that means no doping case. In that case, E_F always at the middle of the conduction band and valence band or the in the middle of the band gap. Now, for N type it is close to conduction band and for P type it is close to valence band and we have talked about this close to, right. But how close, that will depend on the level of doping, ok.

So, you can say that if this is conduction band and this is valence band and E_F (the Fermi level) is really close to the conduction band; that means, it is high level of doping and for low level of doping you will have closer to the conduction band, but not that close this is low level of doping, ok.

So, that will give you the qualitative understanding how the Fermi function is affected by doping as well as with temperature, ok.

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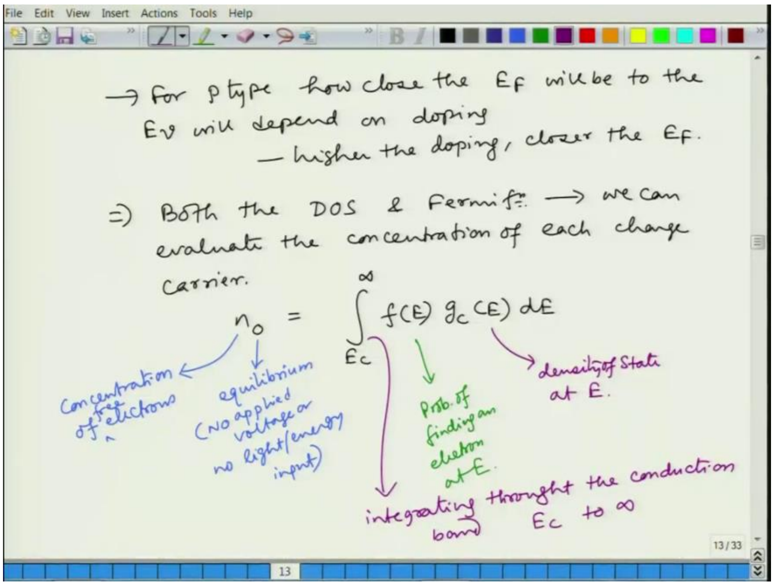

→ For p type how close the E_F will be to the E_V will depend on doping

— higher the doping, closer the E_F.

⇒ Both the DOS & Fermi fn. → we can evaluate the concentration of each charge carrier.

$$n_0 = \int_{E_C}^{\infty} f(E)\, g_C(E)\, dE$$

Concentration of free electrons

equilibrium (No applied voltage or no light/energy input)

Prob. of finding an electron at E.

density of State at E.

integrating throught the conduction band E_C to ∞

So, similarly for P type, how close the Fermi level will be to the valence band will depend on doping, higher the doping closer the Fermi level, ok. And by doping I mean introducing impurity element in the bulk lattice of the pure intrinsic semiconductor, ok.

Now, when we have both the density of state and Fermi function, we can evaluate the concentration of each charge carrier—electron and hole. We can we evaluate them, but before that it is important to grasp the qualitative concept first and then you can always use quantitative formula to evaluate it exactly, ok.

So, for electron you can find it by multiplying the probability of finding an electron which is given by the Fermi function and the number of available states. And you can integrate it to get the complete number, ok. So, let me first write it and then I will explain, why I have written it, ok.

So, first thing is that this is concentration of free electrons because it is not just the electrons; electrons in the conduction band which are able to carry electricity or charge, ok. And this 0 or o these stands for the equilibrium case, ok. So, n_o means the equilibrium electron density or electron concentration; equilibrium means no applied voltage or no light or any energy.

Now, we see that what we are doing? This is the probability of finding an electron at energy level E and this one is the density of state at energy level E. So, if you multiply them that will give you at that particular energy level what is the concentration, probability of finding an electron and how many states are there that will give you the concentration.

And you are integrating it throughout the conduction band or throughout the energies which are corresponding to the conduction band; that means, conduction band edge energy is E_c and from there all the way upward to infinity ok. So, that is how the total number of electrons that are available in the conduction band at equilibrium is expressed in this form, ok.

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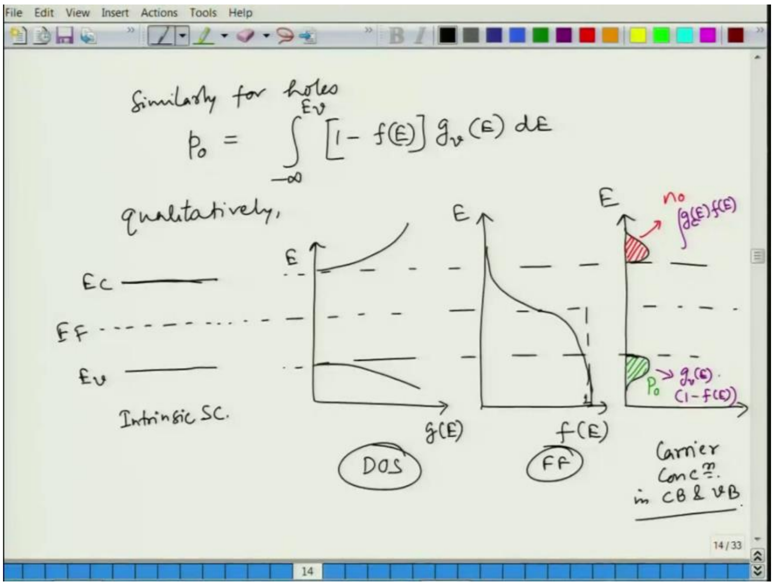

Similarly for holes

$$p_o = \int_{-\infty}^{E_v} [1 - f(E)]\, g_v(E)\, dE$$

qualitatively,

And similarly for holes you can write p_o; p stands for hole every time and n stands for electron. Now, p_o again— o is for the equilibrium case what you will have. Now you will have to integrate it over the lowest level which is minus infinity to the top of the valence band edge, ok. So, that is the integration limit.

$$p_o = \int_{-\infty}^{E_v} [1 - f(E)] g_v(E)\, dE$$

Now, for the probability of finding a hole will be 1 minus the probability of finding an electron right, either it will be a hole or will be an electron. So, probability of finding an electron will be 1 minus the Fermi function, ok. So, that is the tricky difference that you have to note. It is not the Fermi function itself, but 1 minus that into the density of state in the valence band, ok. So, by similar logic you will be able to find it.

Now, qualitatively if you look at how these integrations will give you the quantity. Qualitatively, what you can draw that if you have the conduction band, valence band and this is intrinsic semiconductor ok, so that means, this will be the Fermi level and let me continue this one as well, ok.

Now, here we can plot three things: one is the density of state which we have seen that it will be somewhat like this. In the band gap, there will be no density of state and as you go up in the conduction band density of state will be increased and as you go down in the valence band the density of state will increase and it will follow a square root law, right. So, this we have seen this is the density of state.

Now, other thing that is important here is the Fermi function, ok. So, we have seen that for a particular temperature if this is one, it will be somewhat like this, right. So this is the Fermi function. Now what we are doing by integrating? We are multiplying these two and then summing up (integration means summing up), right.

So, ultimately for the electron density or electron concentration in the conduction band and the hole concentration in the valence band will be somewhat like this, ok. So, this is n_o which is for the conduction band and this is p_o for the valence band. So, n_o how we are getting it, we are getting it by this $\int g_c(E) f(E)$. So, this n_o will be basically the area under this curve and p_o will be area under this curve, ok.

So, any value will be $g_c(E) f(E)$ for the conduction band and for the valence band what we have is $g_v(E)[1 - f(E)]$, ok. And for intrinsic semiconductor, you can see both of them will be same. So, this is the carrier concentration in conduction band and in valence band, ok. Now, how these things will change if you dope? That becomes very interesting.

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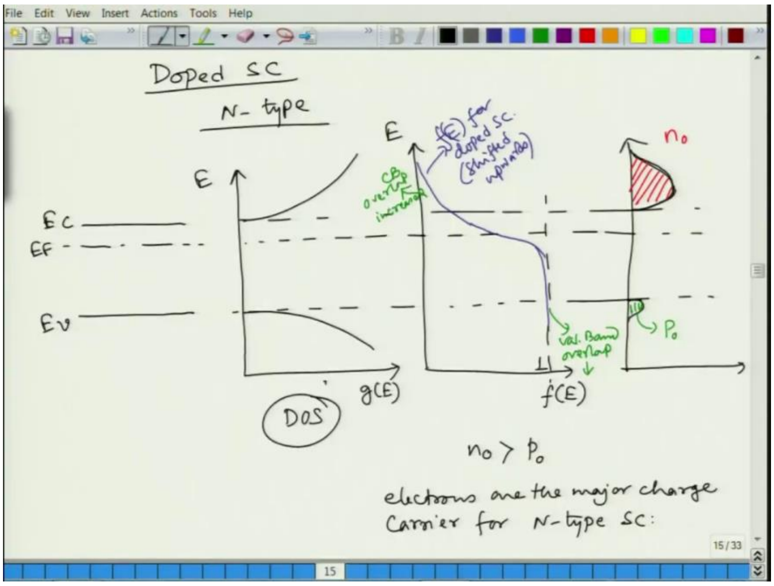

$n_0 > P_0$

electrons are the major charge Carrier for N-type SC:

Now, let us look at doped semiconductor. First we will look at N type. N type means there are more number of electrons than holes, ok. Now, N type also means that the Fermi level has shifted towards the conduction band, ok. So, let us do the same exercise here. So, if we have E_c, E_v and now we have Fermi level which is closer to the conduction band edge, ok.

Let me extend these lines then do the same exercise here. Here we will have the density of state and it will be the same. Density of state is unaffected by the shifting of the Fermi level. Only the Fermi function is getting affected not the density of state. So, density of state just stays the same as we have seen for the intrinsic semiconductor ok, but the Fermi level will not stay the same. What will happen? So, it will cross over the whole thing in the Fermi level, ok.

So, of course, the whole thing will be shifted; that means be there will be lot of intersection in the valence band and there will be little intersection in the conduction band, ok. So, this one is Fermi function for doped semiconductor which is shifted upwards, ok. Now, what will be the effect? So, here you see that in the conduction band the overlap increases because it is shifted upwards, right.

So, this is valence band. In the valence band the overlap is less. Now, what will be the effect of both of these changes into the carrier concentration, because the overlap increased here you will have a thicker bulb at the conduction band and a very tiny bulb here at the valence band. So, this thing is the n_o for equilibrium condition what is the electron concentration in the conduction band and this is the p_o which is the hole concentration, ok.

So, because of the shift of the Fermi level upwards what you are getting, you are getting more overlap in the conduction band and that is why the electron concentration will be higher there at the cost of the hole concentration because the Fermi level has shifted upwards you have less amount of overlap in the valence band and that is why the hole concentration is lower.

So, what we see here that n_o is significantly higher than p_o, which means electrons are the major charge carrier for N type semiconductors is not it.

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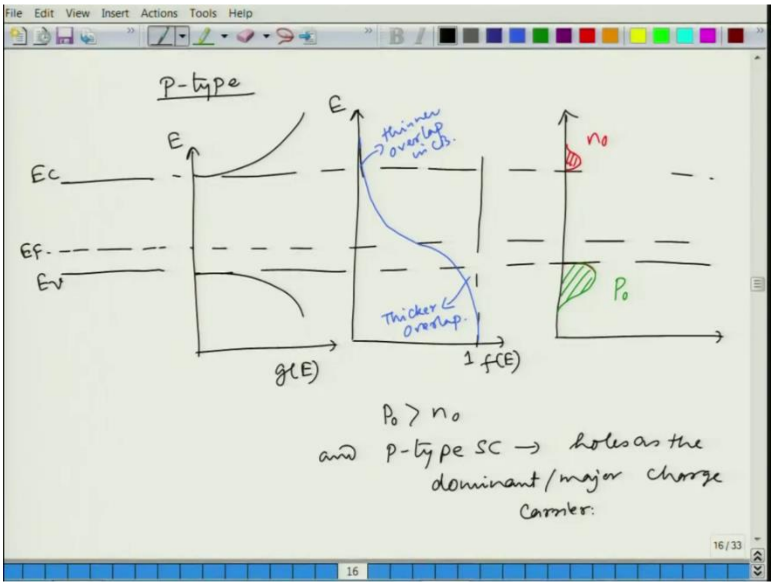

$P_o > n_o$

and p-type SC → holes as the dominant / major charge carrier.

Now, what happens for the P type? Just the opposite of course, for P type semiconductor what we have, we have again conduction band, valence band and now the Fermi level is shifted towards valence band. It is close to the valence band, right.

So, first thing the density of state and again it will stay unaffected then the Fermi function. Now the overlap will be very thin here while the valence band overlap will be much thicker, ok. So, thicker overlap here in the valence band and thinner overlap in conduction band, ok.

Now, if you find out the carrier concentration it will be just the opposite. You will have a small bulb here at the cost of a larger or thicker bulb here, ok. So, here p_o will be much more than n_o. You see that this is happening only because the Fermi function is getting shifted downwards for the P type semiconductor and the density of state stays exactly the same.

So, here p_o is higher than n_o and that is how the P type semiconductor has holes as the dominant or the term that we use is major charge carrier, ok. So, that is how the Fermi energy and density of state are playing roles in this carrier concentration.

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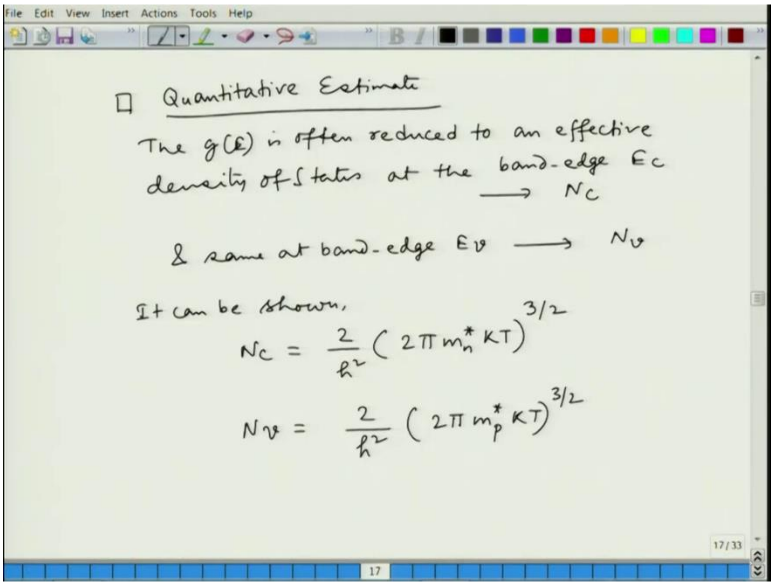

☐ Quantitative Estimate

The $g(E)$ is often reduced to an effective density of states at the band-edge E_c $\longrightarrow$ N_c

& same at band-edge E_v $\longrightarrow$ N_v

It can be shown,

$$N_c = \frac{2}{h^2}\left(2\pi m_n^* KT\right)^{3/2}$$

$$N_v = \frac{2}{h^2}\left(2\pi m_p^* KT\right)^{3/2}$$

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This is well and good in terms of qualitative concept, how it is affect getting affected we know. But now we need to have some quantitative estimate of this n_o, p_o everything, right. So, for quantitative estimate, we need to have more useful terms such as the band edge density of state, ok.

So, the density of state is often reduced to an effective density of state at the band edge. Edge is the most important thing as you can see, everything is happening at the edge that is the interesting part of the whole semiconductor structure, right.

So, for the electrons, the conduction band edge which is E_c is important, ok. So, $g(E)$ can often reduce to an effective density of state at the band edge E_c and we name it N_c, that effective value we name it N_c and same at band edge E_v that means, that is relevant for the holes and we name it N_v, ok.

So, it can be shown again this derivation is beyond the scope of this syllabus. So, this effective value you can write,

$$N_c = \frac{2}{h^2}(2\pi m_n^* kT)^{3/2}$$

The meaning of every term you know h is the Planck's constant, m_n^* is the effective mass of electron and T is the temperature and k is the Stefan constant, ok.

And similarly N_v we can write of course, the expression will be same only thing we have to use the effective mass of hole. So, this gives us what would be the effective value at the band edge, ok. So, instead of doing the integration what we can do we can just multiply that effective value with this effective density of state at the band edge.

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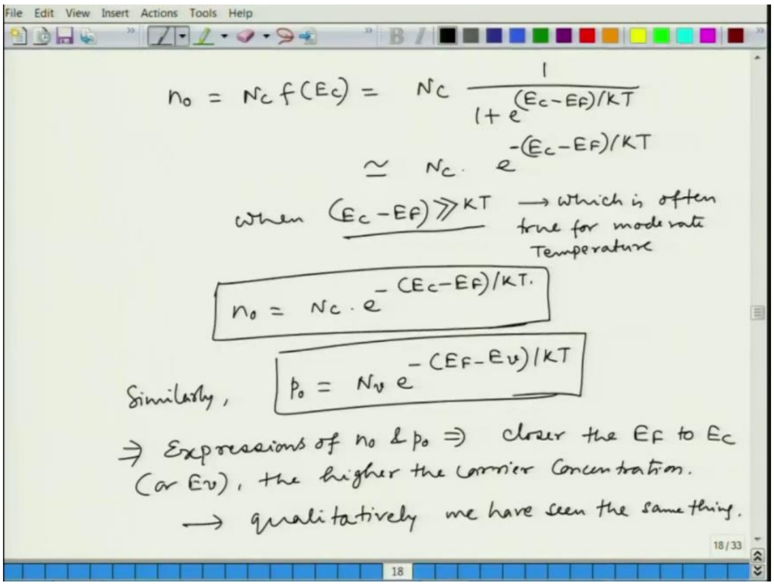

$$n_0 = N_c f(E_c) = N_c \frac{1}{1+e^{(E_c-E_F)/KT}}$$

$$\simeq N_c \cdot e^{-(E_c-E_F)/KT}$$

when $\underline{(E_c-E_F) \gg KT}$ → which is often true for moderate Temperature

$$\boxed{n_0 = N_c \cdot e^{-(E_c-E_F)/KT}.}$$

Similarly, $\boxed{p_0 = N_v e^{-(E_F-E_v)/KT}}$

⇒ Expressions of n_0 & p_0 ⇒ closer the E_F to E_c (or E_v), the higher the carrier concentration.

→ qualitatively we have seen the same thing.

So, now if we find the equilibrium concentration of electrons that will be N_c which represents the effective density of state multiplied by the Fermi function and again the Fermi function will be at the edge, ok. So, here you can write N_c,

$$n_o = N_c f(E_c) = N_c \frac{1}{1 + e^{(E_c - E_F)/kT}}$$

So, you have just used the definition of Fermi function and this you can simplify with this expression,

$$n_o = N_c f(E_c) \approx N_c . e^{-(E_c - E_F)/kT}$$

So, this is an approximation, and it is valid when,

$$(E_c - E_F) \gg kT$$

which is often true for moderate temperature. If you go for really high temperature then of course, this approximation is not valid.

So, if big number when the $(E_c - E_F) \gg kT$, then you have a valid approximation, ok. So, basically from this what we get is,

$$n_o = N_c . e^{-(E_c - E_F)/kT}$$

Now, we have some quantitative estimate of the carrier concentration in the conduction band for electron. And similarly, we have the hole concentration which will be,

$$p_o = N_v . e^{-(E_F - E_v)/kT}$$

Now, it is other way round because the energy is increasing or rather decreasing as you go deeper into the valence band, ok. So, please pay attention to that, ok. So, what it shows both the expressions of n_o and p_o shows that closer the Fermi level to E_c or E_v the higher the carrier concentration.

So, the exponent if it goes bigger, the value becomes lower. So, the E_F closer it is to the conduction band you have n_o maximum and E_F closer it to the valence band you have p_o maximum, ok, that is how it goes. And this is the same observation we have seen when we have looked at qualitatively.

So, the schematic concept should be clear in your head as well as you should know what is the quantitative estimate of that, both quality and quantity are important, ok.

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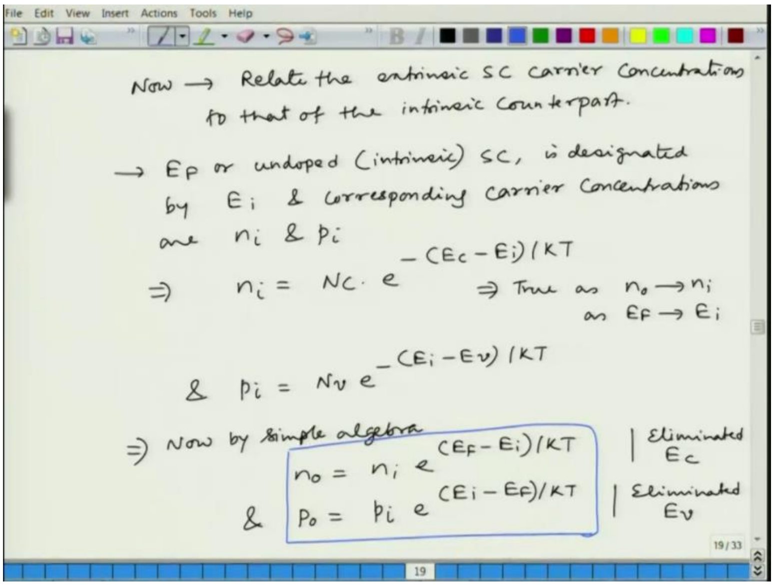

Now → Relate the extrinsic SC carrier Concentrations to that of the intrinsic Counterpart.

→ E_F or undoped (intrinsic) SC, is designated by E_i & corresponding carrier Concentrations are n_i & p_i

$$\Rightarrow \quad n_i = N_C \cdot e^{-(E_c - E_i)/KT} \quad \Rightarrow \text{True as } n_o \to n_i \text{ as } E_F \to E_i$$

$$\& \quad p_i = N_v \, e^{-(E_i - E_v)/KT}$$

⇒ Now by simple algebra

$$\boxed{\begin{aligned} n_o &= n_i \, e^{(E_F - E_i)/KT} \\ \& \quad p_o &= p_i \, e^{(E_i - E_F)/KT} \end{aligned}}$$

| Eliminated E_c

| Eliminated E_v

Now, we need to relate the extrinsic semiconductor carrier concentrations to that of the intrinsic counterpart, ok. So, how far we moved by doping, by making it extrinsic semiconductor, how much carrier concentration we could tweak? So, that is important to quantify.

So, E_F for undoped or intrinsic semiconductor is designated. Let us designate it by E_i, i stands for intrinsic and corresponding carrier concentrations are n_i and p_i, ok. If that is the case then we can write that n_i is,

$$n_i = N_c . e^{-(E_c - E_i)/kT}$$

So, we have done nothing, in case of E_F we have written E_i and in case of n_o we have written n_i, ok.

So, basically from the extrinsic part we have used the same expression just for intrinsic we have named that E_F to be E_i, ok. So, this is true as n_o tends to n_i as E_F tends to E_i, right. So, this expression we can write for intrinsic semiconductor and for p_i what we can write,

$$p_i = N_v . e^{-(E_i - E_v)/kT}$$

The same thing again we have done E_F we have replaced by E_i and p_o we have replaced by p_i, ok.

Now, by simple algebra you can show that this n_o you can write as,

$$n_o = n_i.\, e^{(E_F - E_i)/kT}$$

So, what we have done from n_o and n_i expression, we have eliminated E_c. So, we have eliminated E_c. So, we are left with E_F and E_i, E_F is the doped semiconductor Fermi level and E_i is the undoped semiconductor Fermi level. And similarly, p_o will be,

$$p_o = p_i.\, e^{(E_i - E_F)/kT}$$

So, here what we have done, we have eliminated E_v, right. So, that is giving us how the carrier concentration is changed as you dope compared to its intrinsic value.

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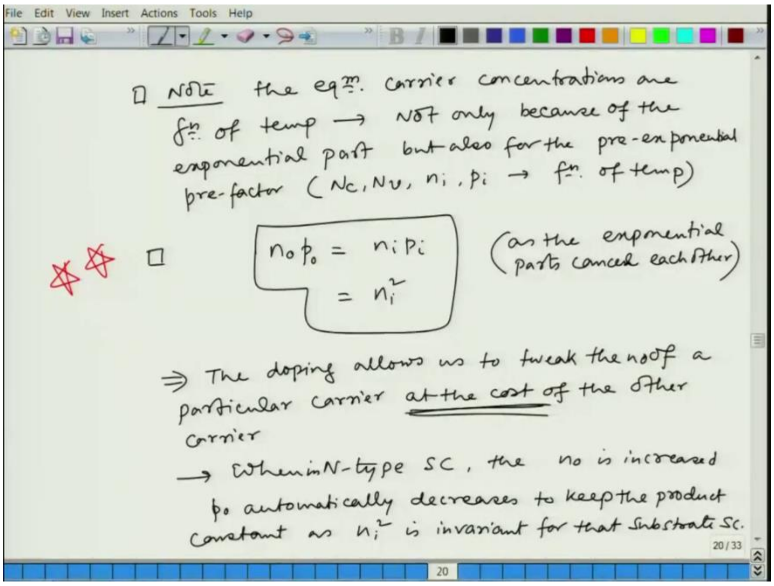

☐ **Note** the eq^m. carrier concentrations are f^n. of temp → Not only because of the exponential part but also for the pre-exponential pre-factor (N_c, N_v, n_i, p_i → f^n. of temp)

☐ $$\boxed{n_o p_o = n_i p_i = n_i^2}$$ (as the exponential parts cancel each other)

⇒ The doping allows us to tweak the no of a particular carrier at the cost of the other carrier

→ When in N-type SC, the n_o is increased p_o automatically decreases to keep the product constant as n_i^2 is invariant for that substrate SC.

So, note that the equilibrium carrier concentrations are function of temperature, right. Everywhere we have this kT thing, so that means, it is a function of temperature. So, it is not only because of this kT (because of the exponential part), but also for the pre-exponential factor that means, N_c, N_v, n_i, p_i, everything they are functions of temperature, ok.

So, that means, it is a strong function of temperature. Exponential parts are anyway changing largely by a small change in temperature, they are more sensitive functions and the pre exponential thing also it has this temperature dependence, ok.

Now, another very very important observation that you can see is if you multiply these two carrier concentrations, one is electron carrier concentration and the hole concentration. If you multiply that you will see that it will be same as the multiplication what you get for the intrinsic value. If we go back to the expression here you can see that if you multiply n_o and p_o then the exponential parts they are reciprocal to each other.

So, they get cancelled out, ok. So, what you are left with is $n_o\ p_o = n_i\ p_i$, ok. So, as the exponential parts cancel each other, ok. And for $n_i\ p_i$ we know that is ${n_i}^2$, right. Because for intrinsic semiconductor the number of electron and number of holes will be the same, right.

So, $n_i = p_i$ that means, $n_i p_i$ gives you ${n_i}^2$. So, that is a very important very very important observation you should make. Physically what it means is that the doping allows us to tweak the number of a particular carrier at the cost of the other carrier. So, it is very important to note this at the cost of.

So, if you want to increase the number of electrons in the conduction band, the cost is coming at the reduced number of holes in the valence band. So, their multiplication does not change. n_i is a material property, right. So, n_i you cannot change, even by doping you cannot change it. So, if you cannot change n_i that means, the product of n_o and p_o you cannot change. So, if you increase $n_o,\ p_o$ will has to decrease.

So, one carrier is increased, or the number is modified at the cost of the other carrier, ok. So, when in N type semiconductor the n_o is increased p_o automatically decreases to keep the product constant as ${n_i}^2$ is invariant for the substrate semiconductor. Of course, here I should mention that if you do heavy doping, then you cannot call it a substrate semiconductor anymore because it will form a kind of an alloy.

So, if you put lot of aluminium in silicon substrate then it will form a silicon aluminium alloy or some inter metallic. So, you cannot just tell it to be a doped semiconductor, in that case the material itself got changed, so, n_i will also change then you can have this $n_o\ p_o$ to be some other value, but that is never happens. In reality you dope much much smaller quantity than the substrate. So, now we are going to start or going to look at the motion of this carrier.

So, ultimately when the carriers are in motion then only you will get this photovoltaic effect, right.

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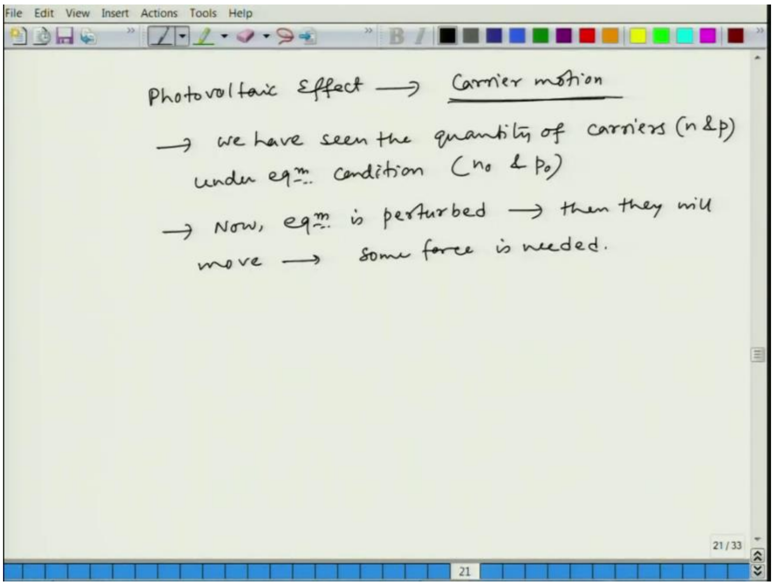

Photovoltaic Effect → Carrier motion

→ we have seen the quantity of carriers (n & p) under eq$^{\underline{m}}$ condition (n_o & p_o)

→ Now, eq$^{\underline{m}}$ is perturbed → then they will move → some force is needed.

So, photovoltaic effect is the result of this carrier motion if they do not move then you do not have any current. So, not the concentration but the motion is important, ok. So, I will just start it today and in the next class we are going to elaborate on this, but let us just start what do we mean by carrier motion.

So, one thing is that we have seen the quantity of carriers that means n and p under equilibrium condition and that is why we call them n_o and p_o; naught means equilibrium. Now, when equilibrium is perturbed then they will be set in motion, then they will move. Under equilibrium nothing can move there is no driving force for any carrier to move.

But when you perturb the equilibrium then the carrier motion will change, I mean the carrier will start to move. So, some force is needed for them to move, ok. So, that can be either the sunlight which is causing increase in energy of individual carriers and it can be the voltage if you apply from outside that will also perturb the equilibrium and that will set the carriers in motion, ok.

So, we will look at these different kinds of carrier motions in the next class.

Thank you for your attention.