Elements of Solar Energy Conversion
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Lecture - 25

Hello everybody, and welcome back to this series of lectures on Elements of Solar Energy Conversion. We are here at lecture number 25.

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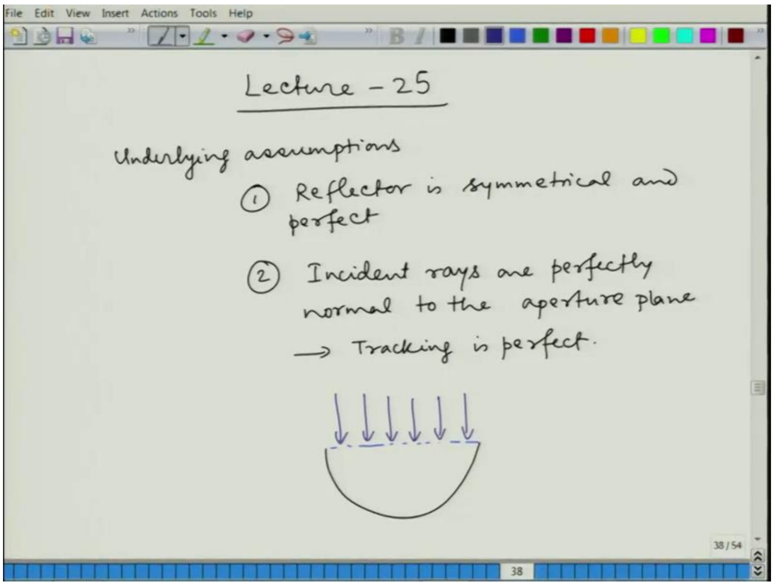

So, we have in terms of the solar conversion devices; we have covered the flat plate collector, its different configuration variations, then we have started looking at the concentrating collectors. And after giving the background of why concentration is required, what is the limit of maximum concentration possible for two-dimensional as well as three-dimensional conversion concentration devices.

Now, we have started the first major concentrating collector, which is the parabolic trough collector, which is a linear concentrator, and we have seen how to look at the optical properties of that concentrator.

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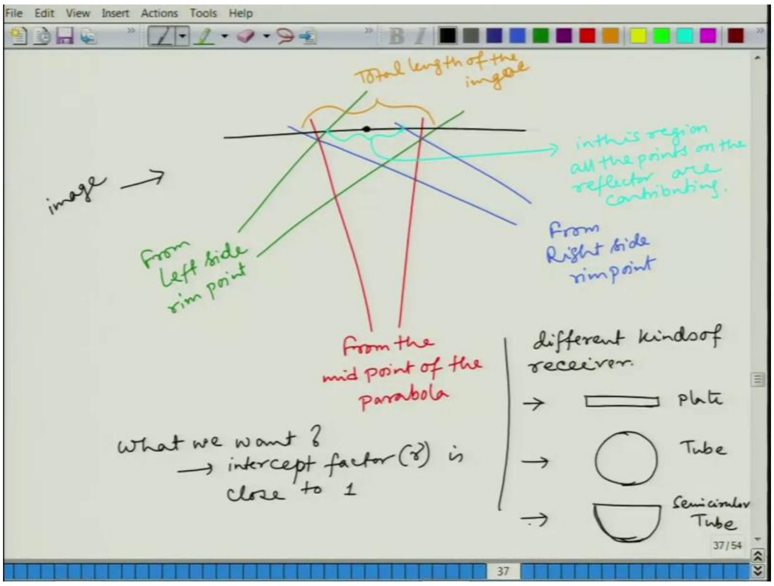

So, I will complete that discussion which we started in the last class. Here we have seen that the image that happens in the image the focal plane is somewhat distributed depending on the symmetry of the parabola, and the intensity is again not everywhere equal. So, all these things we have looked at.

And we started talking about different kinds of solar collectors or heat collectors, the receiver part, where we have looked at the plate we have looked at the tube, and we can also have the semi-circular tube.

So, how that how to determine the dimension of the receiver, which will have the maximum intercept factor? That means all the reflected rays that are coming from the parabolic dish or parabolic trough that should intersect with the receiver part. So, let us look at that.

So, first, what are the underlying assumptions? The first one is that the reflector is symmetrical because that will tell from one side how the rays will be reflected on the other. So, if it is symmetric, half of it, if we can analyze it, will be just the mirror image of the other half. And we also assume that it's a perfect reflector.

In real life, your reflector will have some surface defects, and it will not be exactly parabolic, and that is how you will have some defects in the image as well. But here, we are assuming it is a perfect reflector. And the 2nd assumption is that the incident rays are perfectly normal or perpendicular to the aperture plane.

When is this assumption true? When we have a tracking system, that is perfect. So, this means the tracking is perfect. So, basically, if you have this parabolic trough and this is our aperture plane, right. And what are we assuming? That all the rays are perfectly normal to this aperture plane, ok. And when is it possible? When the parabola can orient itself exactly looking at the sun, and then only this assumption will be true.

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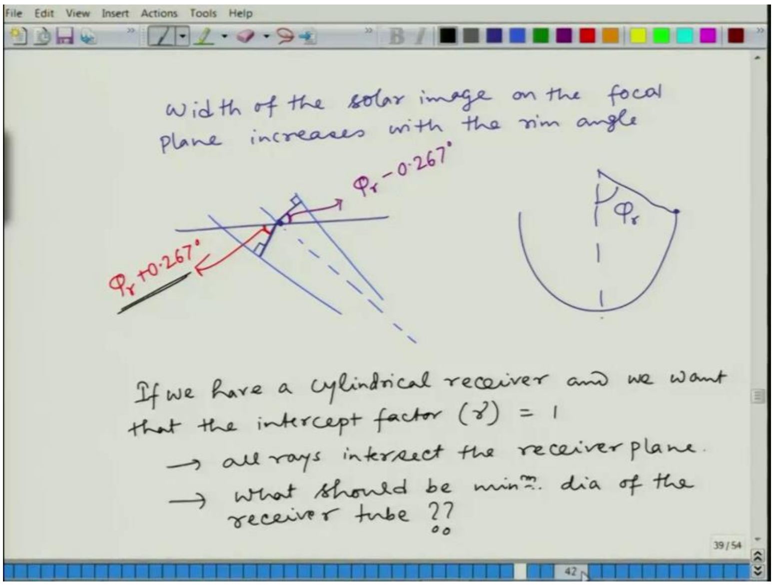

Now, under this assumption what we can observe that the width of the solar image on the focal plane increases with the rim angle, right. So, basically, if you have this parabola. So, rim angle means what? The angle that the rim point makes this is phi r. So, rim angle if increases, that means, the size of the parabola increases that also mean the image size the width of the image will increase.

So, if we look at the focal plane where this is the focus focal point, so we can say that from any side, the reflected ray will make a cone that we have seen on one side it will intersect shorter distance, and on the other side, it will intersect larger distance, right. And what can we do here? We can see what the angles are.

So, if you draw a perpendicular from the focal point to the one edge of the cone, and so, let us say that this is a perpendicular we are dropping. So, this angle, let me just remove these two symbols, ok. So, now this angle you can see to it, and you can find that it will be ϕ_r minus the half cone angle, which is 0.267˚.

Similarly, if we drop one perpendicular on the other edge of the beam, so if this is a perpendicular, then this angle also you can find out, and this angle will be ϕ_r plus the half cone angle, which is 0.267 degrees. So, these two you can find, and we can also see that if we have a cylindrical receiver and we want that the intercept factor gamma is equal to 1. That means all rays intersect the receiver plane.

If we want that, then what should be the minimum dia of the receiver tube? So, that is a legitimate question to ask, right. We want to know the dimension of the tube so that all the rays are intersected.

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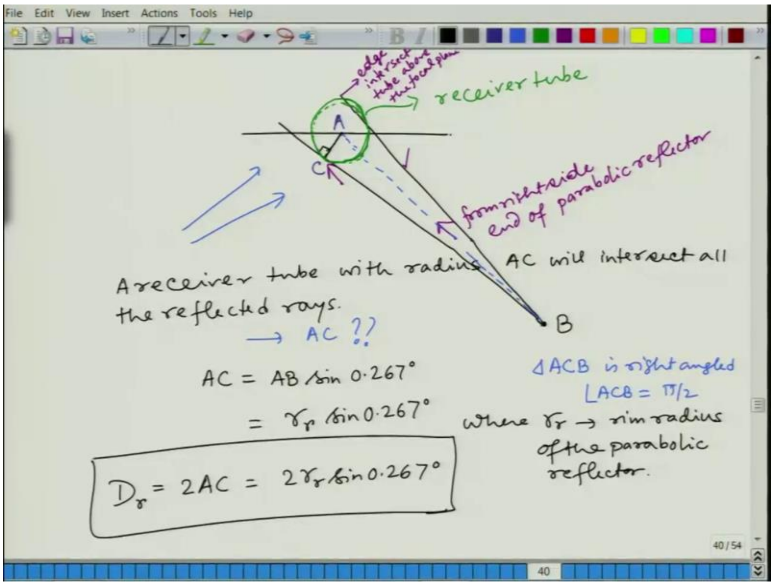

So, now again, if we look at it, then from one side, we have this cone, and you can see that if you have A, this is the perpendicular. So, if you have a tube with this diameter, so this is the receiver tube with a radius of this. So, let say this point is the focal point is A, and this perpendicular point is C. So, a receiver tube with radius AC will intercept or intersect all the reflected rays from one end. So, this is from the right end, from the right side end of the parabolic reflector.

So, from one end, you see that this C point is the limit. And the other edge, which is this edge, will anyway intersect in the other part of the tube, which is above. So, this edge intersects the tube above the focal plane. So, that is also minimum radius that you need is AC. And from the other side also you will see from another side; that means, from this side, the cone that you will find will intersect this particular receiver tube completely.

Now, so the task is what AC is? That we want to know, right. That will tell us the receiver tube geometry. So, now, what we can write this AC, let say if we what is the origin from where this is getting reflected let say this point is B.

So, AC and you can join it with one point AC. So, this ACB is a right angle triangle where the angle ACB is the right angle. So, now, if we want to know what AC is, that is nothing but AB sin of this half angle which is 0.267 degree.

And AB is what? AB is the rim radius of the parabolic reflector, where this r_r is the rim radius of the parabolic reflector. So, basically, the D of this receiver will be twice this minimum radius which is AC; that means, $2r_r \sin(0.267^{\circ})$.

So, that is the minimum radius that we have to use for the receiver diameter, or the minimum diameter that we have to use for the receiver tube is twice r_r; that means the rim radius multiplied by the sin of the half-angle 0.267˚.

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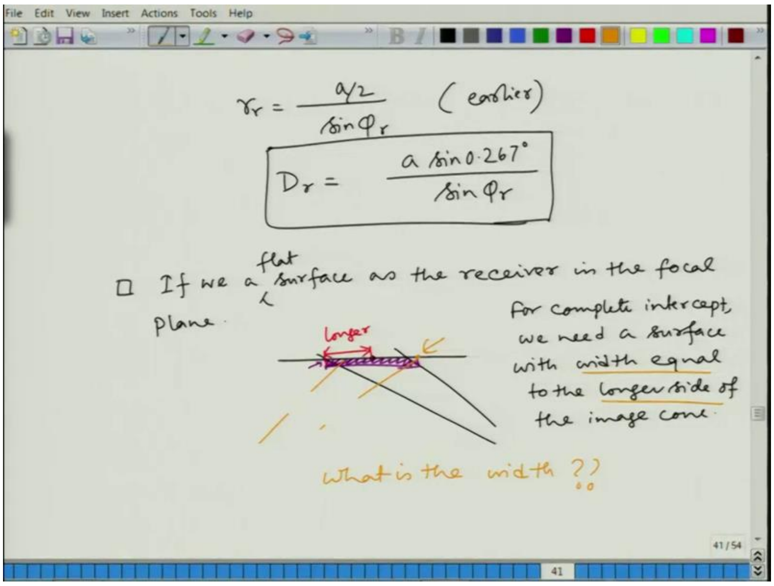

And we can also write from the same figure that r_r is nothing but aperture half aperture length divided by $sin(\phi_r)$; that means the rim angle. So, this you have seen earlier. In the last class, we have derived that. So, basically, D_r

$$D_r = \frac{a * \sin(0.267^\circ)}{\sin(\varphi_r)}$$

In terms of the aperture half or aperture area, aperture width, and the rim angle, you can write the minimum diameter that you require for the receiver tube.

Now, the next question is if we have a surface or rather a flat surface as the receiver in the focal plane. So if this is our receiver plane and we have the cone just like this. Then, what we need to have for complete intercept, we need a surface with a width equal to the longer side of the image cone. That means this is the longer side.

So, if we have the receiver of a width of this longer side, then from one end, you will have interception here, and for the other end, you will have the interception here. So, the other end interception will be at this edge. So, basically, if we have the longer side width, that means it will cover the complete range. So, what is that? What is the width that we need to find? So, let us go back to geometry again.

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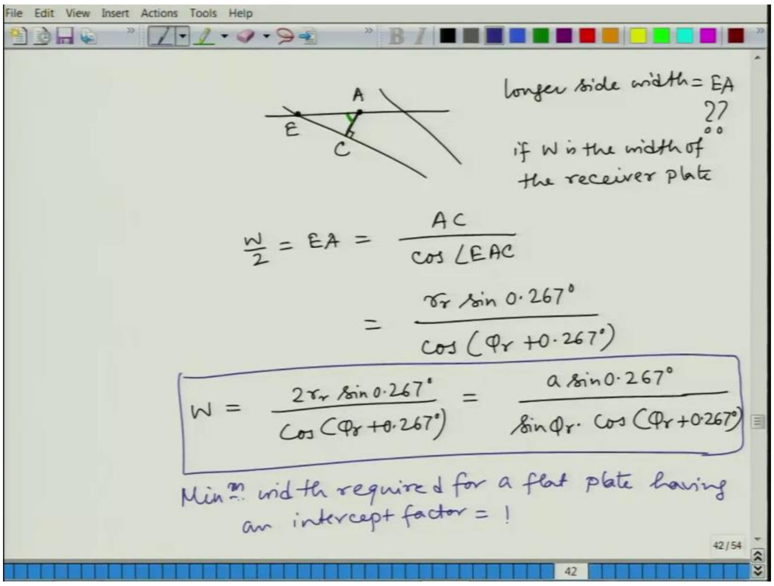

So, you can say that earlier, we have seen that this is the longer side, and this point we said A, and we have dropped a perpendicular here at point C. Now, if we say the longer side width is EA, so longer side width is equal to EA, and that is what we want to know, and that is half-width.

So, if W is the width of the receiver plate, in that case, we can write W by 2, we need it to be EA, and that is $\frac{AC}{\cos(angle\ EAC)}$. And you can you have seen that AC is $r_r * \sin(0.267^{\circ})$ and EAC also we have seen a couple of pages ago, here this angle is $\varphi_r + 0.267^{\circ}$. So, here this will be $\cos(\varphi_r + 0.267^{\circ})$.

So, what we can write,

$$W = \frac{2 * \ r_r * \sin(0.267^{\circ})}{\cos(\varphi_r + 0.267^{\circ})} = \frac{a * \sin(0.267^{\circ})}{\sin(\varphi_r) * \cos(\varphi_r + 0.267^{\circ})}$$

And that is the minimum width required for a flat plate having an intercept factor equal to 1. So, these are the two most common geometries, a tube geometry or a flat geometry.

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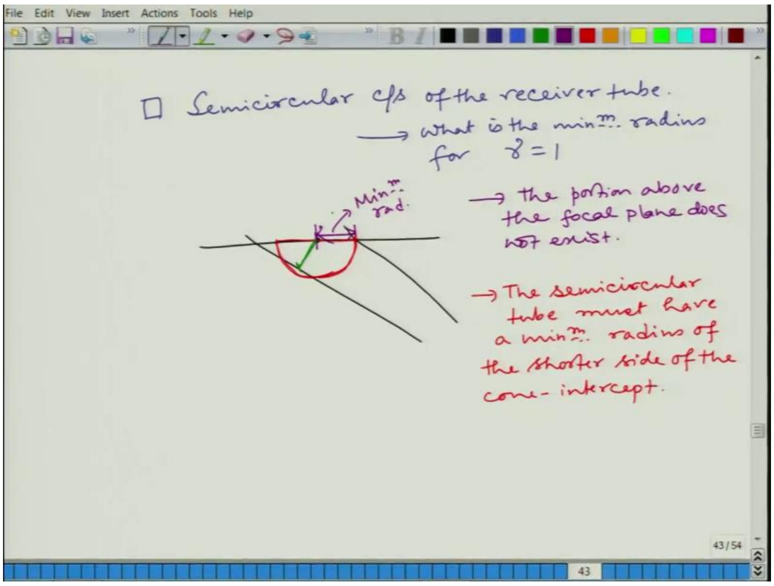

Another possible geometry is the semi-circular, semi-circular cross-section of the receiver tube. There also, we need to know what would be the minimum radius. So, what is the minimum radius for the intercept factor equal to 1?

So, here what is happening? Here, suppose we have this cone, now; for the circular one, we have taken that this will be the radius of the tube because this is actually giving you an intersection here, and the other one is getting intercepted somewhere above the focal plane.

But in this case, for the semi-circular one, the above portion or rather the portion above the focal plane does not exist, is not it? So, what we need to have? That the minimum radius would be the shorter side which will anyway intersect the longer side edge of the cone. So, here the semi-circular tube must have a minimum radius of the shorter side of the cone intercept; that means the minimum radius would be this length; this would be the minimum radius.

So, now, we can find what is the minimum that particular length we can find, and in a similar way, we can find what would be the minimum diameter for the semi-circular tube.

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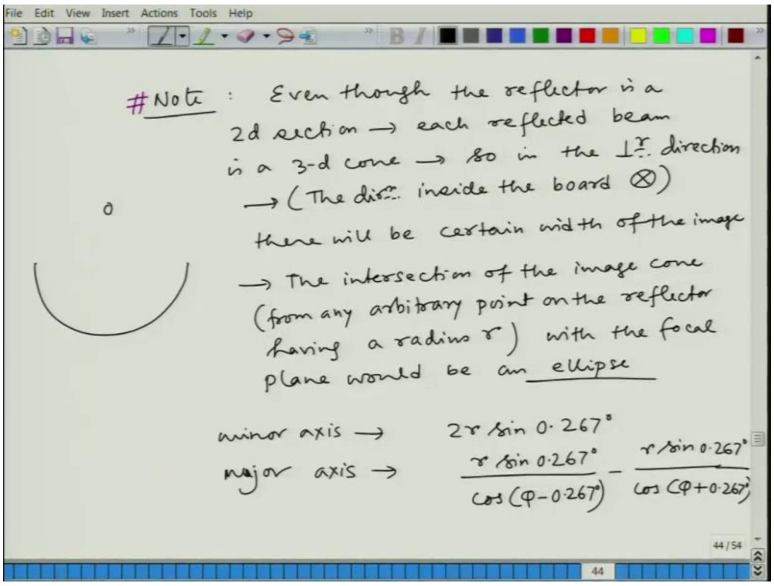

Now, here I want you to note that even though let me write it so that it is in front of you, which is a very important parameter. So, I would insist that you please imagine the situation. Because this is a two-dimensional collector, but the ray that is coming is in the form of a cone which is a three-dimensional geometry.

So, even though the reflector is a 2d section, each reflected beam is a 3d cone; any beam that is falling on the reflector and then getting reflected to the receiver is a 3d cone, right. So, in the perpendicular direction, that means, that means if this is the parabolic trough, this is the receiver.

So, the perpendicular plane or perpendicular direction that is going inside the board, so that means, the direction inside the board, ok which we often designate in this fashion.

So, in that perpendicular direction, there will be certain width of the image, right. The image is not just a line image is not a line on the focal plane, but it is a three-dimension with some spread in the z-direction. So, the intersection cone, intersection of the image cone from any arbitrary point on the reflector having radius r with the focal plane, would be an ellipse.

It is not a line; it is not a circle but an ellipse. So, for that ellipse, it can be shown I am not going into detail, but the minor axis will be $2r * \sin(0.267^{\circ})$ and the major axis would be $\frac{r*\sin(0.267^{\circ})}{\cos(\varphi-0.267^{\circ})} - \frac{r*\sin(0.267^{\circ})}{\cos(\varphi+0.267^{\circ})}$.

So, every point on the reflector will make an image in the form of an ellipse on the focal plane. And all these different images will be superimposed to give you the overall image for the two-dimensional parabolic trough. So, that completes the optical portion of the analysis of the parabolic trough.

We have seen how the images will be formed on the focal plane or different kinds of receiver geometry; what would be the minimum radius for that for complete interception. So, all those things we covered. Now, what we are going to see now is the thermal analysis of a parabolic trough.

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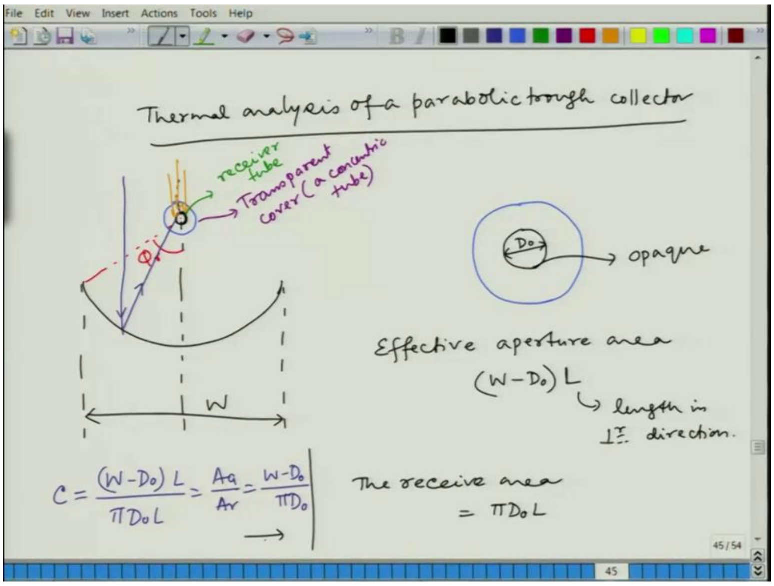

We will see that there will be some similarities with what we have done for the flat plate collector, and there will be some significant dissimilarity which we need to note especially. Now, let us have this kind of parabolic geometry. Of course, this is symmetric; this is the axis of symmetry which is also the focal direction, and somewhere here, we will have the receiver tube.

Now, so far, we have not considered anything related to the cover of the receiver tube. If you have a naked tube, then the losses will be so high because it is a concentrating collector. So, losses will be higher because of higher temperatures. So, similar to the flat plate collector, what we have? We have a transparent cover.

So, let us say that this is the transparent cover which will also be another tube, which will be a concentric tube. And let us say let me just say that this is the receiver tube, and the top one is the transparent cover which is also a concentric tube. The tube is the most common geometry of the receiver; that is what we are going to analyze that.

Now, let us say that any, so from this center, this angle is nothing, but our rim angle ϕ_r. So, the assumption is still valid that the aperture plane is perpendicular to the sun ray direction. So, any plane that is coming perpendicular to this plane will be reflected in this tube. And let us say this is the width of the aperture, which is W.

Now, you can see that if we zoom this part, this is the tube, and then this is the transparent cover, and let say that the diameter of this one diameter of the receiver tube is D_o. Now, we can say that the effective aperture area is now $(W - D_o)L$, because D_o is this portion that is opaque. If it is opaque, any radiation will not pass through that region to reach the reflector.

So, the effective aperture area is not $W * L$, L is the length in the perpendicular direction, perpendicular to the board. So, it is not $W * L$, but $(W - D_o)L$. And the receiver area is what? That is $\pi D_o L$, πD_o is the radius of the circumference of this receiver tube, and you multiply that with L in the perpendicular direction.

So, now the first thing we want is the concentration ratio that is nothing, but this effective absorber area divided $\pi D_o L$ the receiver area that we have seen basically is $\frac{A_a}{A_r}$. That means this is $\frac{(W-D_o)}{\pi D_o}$. For our tube geometry of the receiver, we have the concentration ratio, which is $\frac{(W-D_o)}{\pi D_o}$.

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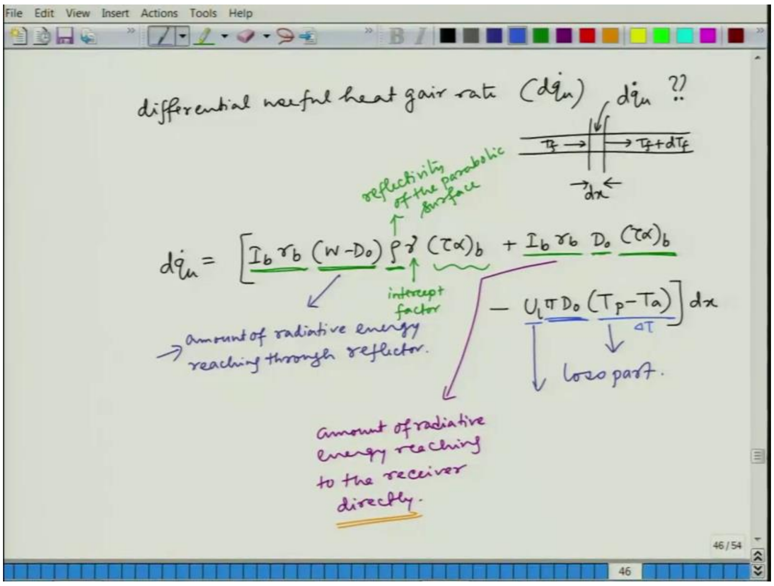

Now, what we can write is the differential useful heat gain rate or $d\dot{q}_u$. What is that? Why do we call differential? If we look at the tube, the receiver tube at one small portion of it of length dx, what can we have? That we have some temperature T_f coming in some increase temperature $T_f + dT_f$ going out. So, for that, what would be the $d\dot{q}_u$? That is what our question is.

So, that will be the balance of heat which was coming in from the solar radiation and subtracted; the subtraction has to be of the factor the loss that is going out to the ambient. So, this $d\dot{q}_u$ this will be let me write this expression first; then I can explain $T_p - T_a$ this whole thing multiplied by dx. So, what are the terms here? So, the first term, if we look at this, is the heat, the amount of radiative energy reaching through the reflector.

So, if you see, let me just compartmentalize these things; this is the total amount of radiation coming in terms of the beam, not as diffuse because concentrating collector beam radiation is the major one, so diffuse one we can completely neglect. So, $I_b r_b$ is nothing, but r_b is the ratio between the tilted plane intensity to that of horizontal plane intensity. So, $I_b r_b$ is giving you what is on the aperture plane what is the intensity of the solar radiation.

And then, the next part is the width of the aperture. So, that the effective width is $W - D_o$. So, that is the total amount. Now, what is ρ? ρ is the reflectivity of the parabolic reflector. Now, all the rays that are falling on the parabolic reflector a portion of it will reach the reflector because reflectivity is not 100 percent.

Whatever coating you use, whatever material you use, the reflectivity will be less than 100 percent; that means ρ will be less than 1. And then what is this? This is the intersect factor.

Let me write this. This is the reflectivity of the parabolic surface. And then this ϒ is the intercept factor, so it will tell how much of the reflected beam will be intercepted by the receiver tube. And what is this? This you know this is for the effective transmissivity absorptivity product for that system of the receiver tube and the transparent cover and that also for beam radiation. So, that is why this whole first term is giving you the amount of radiative energy reaching through the reflector.

Now, what is the second term? The second term is nothing but the amount of radiative energy reaching the receiver directly. That means, if we look back at the geometry, you can see that here some rays from this direction, some rays will reach the receiver directly. It will not be reflected through the parabolic surface but will reach it directly. So, this is the part, the direct portion of it.

And here again, what we see? That $I_b r_b$ will be there, which is the tilted surface intensity, and then D_o is the width of that particular receiver tube. So, that is why D_oAnd then again, the $(\tau\alpha)_b$ which is the effective transmissivity absorptivity product. So, that is the direct portion.

So, first and the second term gives you the total input. And the third term is giving you the loss part. So, you can see that this loss coefficient is there; you have the circumference πD_o and then the temperature difference. So, this is basically the area, the loss coefficient, and this is the ΔT part. So, that gives you the effective loss from the receiver tube.

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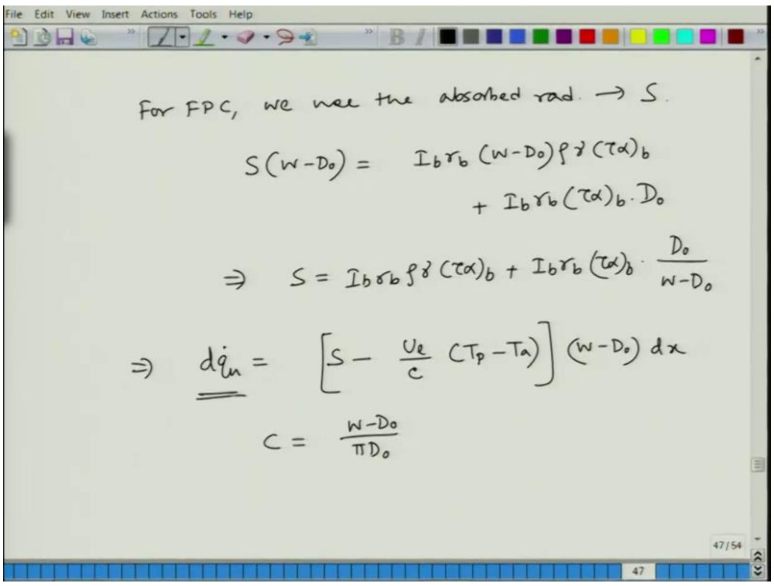

Now, usually for flat plate collectors, we have seen that we use the absorbed radiation in terms of S, which is the common symbol that we use. So, if we write the expression of $d\dot{q}_u$ in terms of S, then what can we write? The S into the effective aperture width will be equal to this $I_b r_b (W - D_o)\rho\gamma(\tau\alpha)_b + I_b r_b (\tau\alpha)_b D_o$

So, there are the first two terms in the expression of $d\dot{q_u}$. And that is the input that is coming, and we can write it in terms of S into the effective aperture width. So, basically, S you can find out to be

$$S = I_b r_b \rho \gamma (\tau\alpha)_b + I_b r_b (\tau\alpha)_b \frac{D_o}{W - D_o}$$

So, now the equation of $d\dot{q_u}$ is simplified in terms of $[S - \frac{U_l}{C}(T_p - T_a)](W - D_o)dx$. Here we have introduced C; C is nothing, but you have seen that C is $\frac{(W-D_o)}{\pi D_o}$. So, everything else is the same. I just replaced that complicated expression in terms of S, which is then absorbed radiation per unit area, and we have introduced the concentration ratio, which is C.

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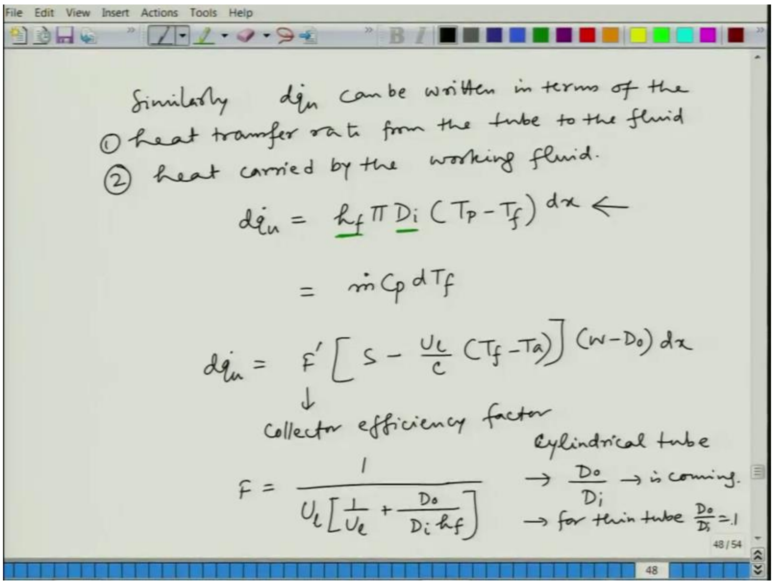

Now, similarly, the $d\dot{q_u}$ can be written in terms of the heat transfer rate from the tube or receiver tube to the fluid. That we can do in terms of this and we can also do in terms of heat carried by the working fluid, everything will be $d\dot{q_u}$. So,

$$d\dot{q_u} = h_f \pi D_i (T_p - T_f) dx$$

So, here is the inner diameter of the tube; this is the heat transfer coefficient for the fluid inside the tube. And the same thing we can write in terms of the increase in temperature of the fluid flow.

So, $d\dot{q_u} = \dot{m} C_p dT_f$ that also we can write. So, now, if we use the same expression, this expression of $d\dot{q_u}$ and this particular expression of $d\dot{q_u}$ what we can write,

$$d\dot{q_u} = F' \left[S - \frac{U_l}{C}(T_p - T_a)\right](W - D_o)dx$$

So, F is again the collector efficiency factor, and F you can write as

$$F = \frac{1}{U_l[\frac{1}{U_l} + \frac{D_o}{D_i h_f}]}$$

So, here you note that we have a cylindrical geometry, cylindrical tube, and that is why this factor $\frac{D_o}{D_i}$ is coming. This you have noted from the conduction knowledge. And for thin tube, this is basically $\frac{D_o}{D_i}$ is equal to 1. For a thin tube, you do not have any difference between the outer and inner diameter.

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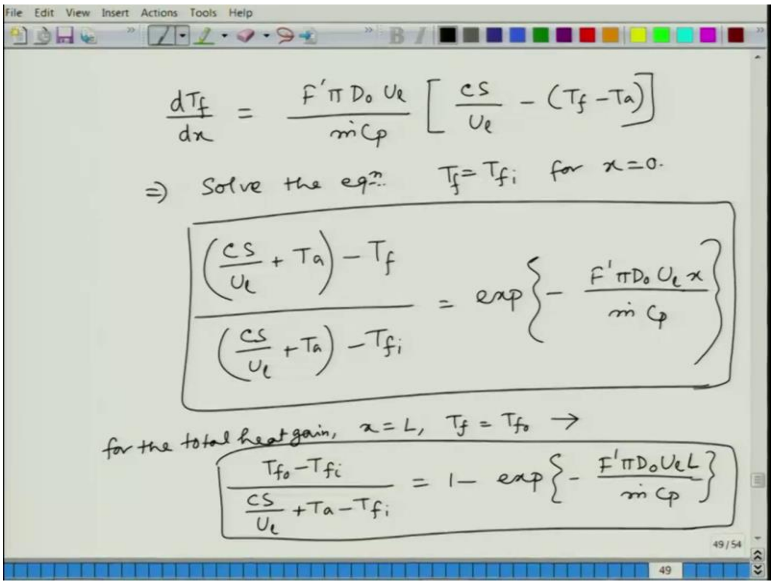

Now, in terms of the fluid temperature increase with the length $\frac{dT_f}{dx}$ you can write, so here the procedure is very similar to what we did for the flat plate collector. So, I am not going into detail about every bit of equations from where it is coming from everything, but the major equations that you are going to get I am going through that steps.

So, this will be

$$\frac{dT_f}{dx} = \frac{F'\pi D_o U_l}{\dot{m} C_p}[\frac{CS}{U_l} - (T_f - T_a)]$$

Similar equation we also got for flat plate collector now, if we solve the equation with the boundary condition that $(T_f = T_{fi})$ for x equal to 0, ok.

If you solve that, what will you get?

$$\frac{\left(\frac{CS}{U_l} + T_a\right) - T_f}{\left(\frac{CS}{U_l} + T_a\right) - T_{fi}} = \exp(-\frac{F'\pi D_o U_l x}{\dot{m} C_p})$$

So, this is the solution that will tell you how the fluid temperature will increase from the inlet towards the outlet. So, for the total heat gain, this x equal to L, and we have $T_f = T_{fo}$.

So, what can we write

$$\frac{(T_{fo} - T_{fi})}{\left(\frac{CS}{U_l} + T_a\right) - T_{fi}} = 1 - \exp(-\frac{F'\pi D_o U_l x}{\dot{m}C_p})$$

So, this expression tells you tells what would be the overall temperature rise of the working fluid as it passes through the receiver tube.

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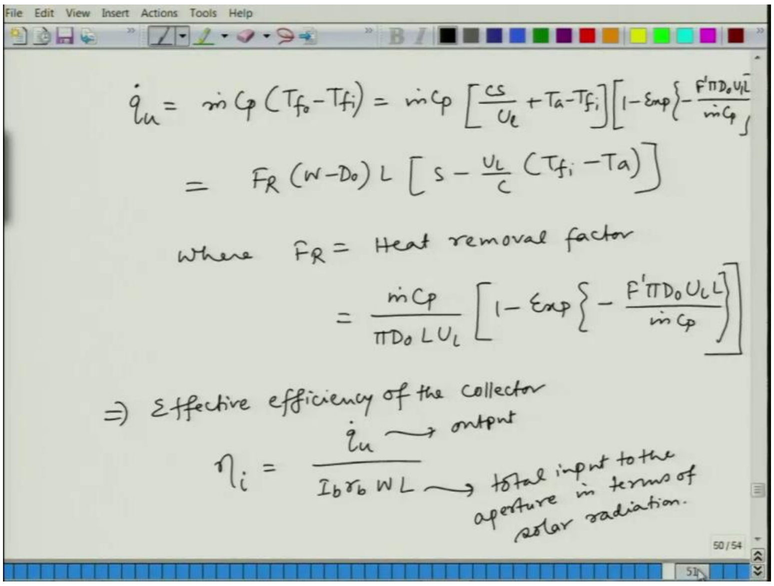

So, what can we write?

$$\dot{q_u} = \dot{m}C_p(T_{fo} - T_{fi}) = mC_p\left[\frac{CS}{U_l} + T_a - T_{fi}\right][1 - \exp\left(-\frac{F'\pi D_o U_l x}{\dot{m}C_p}\right)]$$

And the same thing we also have seen that we can write in terms of the heat removal factor F_R,

$$\dot{q_u} = F_R(W - D_o)L[S - \frac{U_l}{C}(T_{fi} - T_a)]$$

Where F_R is our heat removal factor, so, in each step, you should draw parallels to the analysis that we have done for the flat plate collector that will help you understand the physical meaning of each of these terms.

$$F_R = \frac{\dot{m}C_p}{\pi D_o L U_l}[1 - \exp\left(-\frac{F'\pi D_o U_l x}{\dot{m}C_p}\right)]$$

So, that is the that completes the analysis, and what you can find, you can find the effective efficiency of the collector. Efficiency means what? What you are getting out at the cost of the input.

So, output by input is your efficiency.

$$\eta_i = \frac{\dot{q_u}}{I_b r_b W L}$$

This is the total input to the aperture in terms of solar radiation, ok.

That is how the efficiency of a parabolic trough collector is computed. So, that completes the thermal analysis. But we will need to discuss few things about how to find this U_l and other things which will complete in the next class here we stop for this class.

Thank you for your attention.