Elements of Solar Energy Conversion

Prof. Jishnu Bhattacharya
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture - 21

Hello and welcome back to the series of lectures on Elements of Solar Energy Conversion. We are here at lecture number 21.

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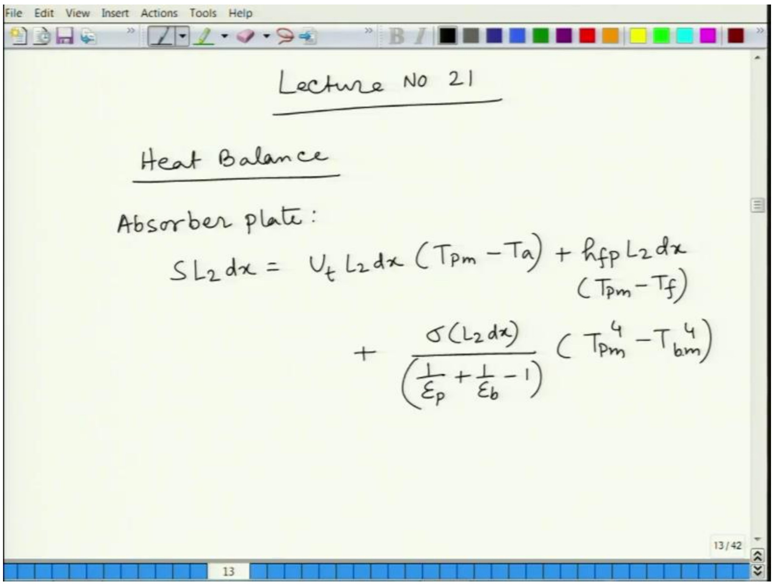

So, far we have started looking at the flat plate solar collector and the major version of it we have completed which is the liquid flat plate collector. And we have started in the last class the air heaters, ok. Which are significantly different in the design because it requires larger area of contact between the heat conducting fluid and the heated surface, ok.

And the design also requires involvement of a bottom plate other than the absorber plate, we need a bottom plate and that is why the design also becomes or the analysis also becomes little bit different that is what we have started looking at in the last class and we will continue looking at it today as well.

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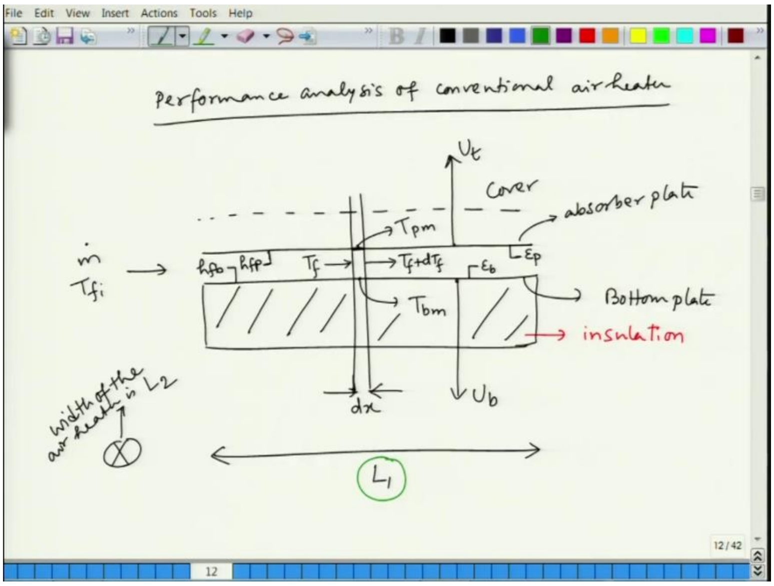

So, this is the figure we drew in the last class where we have seen that we have a cover and then we have the absorber plate and then we have the bottom plate here, ok. And the lower portion is insulated and we have seen that the resistance circuit, the thermal resistance circuit is little bit different in this case.

So, let us start analyzing it by doing the heat balance. Now heat balance we have to do individually for the plate, for the bottom plate and the fluid, ok. So, first we will look at the heat balance for absorber plate. So, the amount of radiation that it will receive in a small strip of dx length, this is the length dx. So, this is the absorber plate we are looking at and in the perpendicular direction to the board we have dimension L_2.

So, that is why the total amount of radiation that you will receive is SL_2dx and what will be the losses?

First one will be the loss towards the top; that means, $U_t L_2 dx\ (T_{pm} - T_a)$, where T_{pm} is the absorber plate mean temperature, and T_a is the ambient temperature. Plus the absorber plate will lose heat to the fluid or air which is flowing at the bottom and we have seen that the heat transfer coefficient we have designated it with h_{fp}; f stands for the fluid and p stands for the plate.

And the area remains the same and you have to multiply it with the temperature difference between the plate and the fluid, ok. Now another term we have here is the radiative heat transfer between the absorber plate and the bottom plate.

What is that? L_2dx (area term) is common to all the different heat transfer terms. And we have two emissivity terms ϵ_p and ϵ_b for the plate and the bottom plate and it will be multiplied by the fourth power of the difference in fourth power of temperatures between the absorber plate and the bottom plate, right. So, this is basically the heat balance for the absorber plate.

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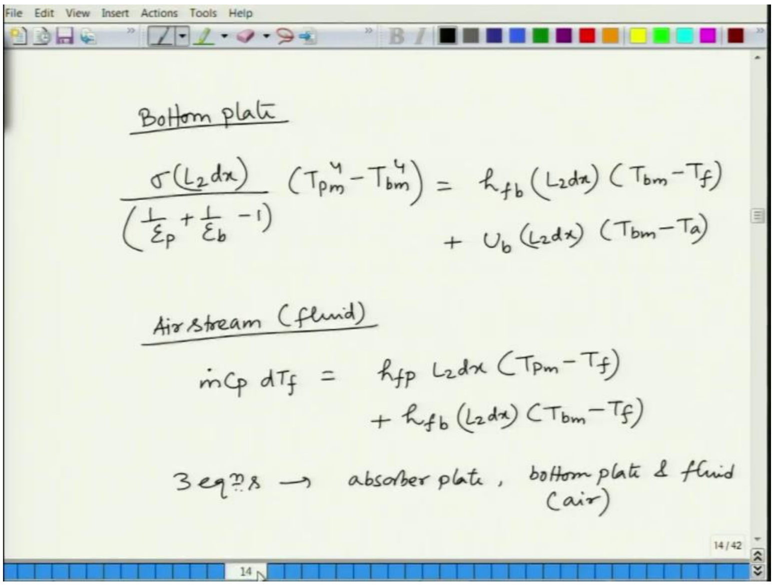

Now, similarly we can do that for the bottom plate, ok. So, for bottom plate what are the different terms? One will be the radiative heat that is coming from the absorber plate to the bottom plate and that is lost one through the fluid and other through the bottom portion or through U_b, right.

So, what heat gain it is doing from the absorber plate? The same thing that we wrote for the absorber plate, but this will be gain to the bottom plate not the loss to the absorber plate, but we are writing it as gain to the bottom plate, ok. So, this gain has to be equated to summation of two loss terms one is the heat that is being lost to the fluid. So, $h_{fb}\ (L_2 dx)\ (T_{bm} - T_a)$.

And we also have the bottom loss coefficient which is taking heat from the bottom plate to the ambient. So, this will be that term, ok. So, that gives us the heat balance for the bottom plate. Now similarly we can do that for the air stream which is the heat carrying fluid in this particular case, ok. So, here what we can write?

We can do the control volume analysis and we can write the amount of mass flow rate multiplied by the specific heat and that multiplied by the delta in temperature of the fluid in dx length and this amount of storage that is happening inside the fluid? That heat is coming from the top plate the other one is from the bottom plate, right.

So, there will be two terms one will be from the top plate or the absorber plate, ok. This is the term, this is the heat that is coming into that control volume from the absorber plate and the other term which is coming from the bottom plate in the same control volume of area L_2dx, but now the temperature difference is mean bottom plate temperature minus the fluid temperature, ok.

So, we have 3 equations, one for absorber plate, then for the bottom plate and the fluid which is air here, ok. So, these three equations will tell us how to go about this.

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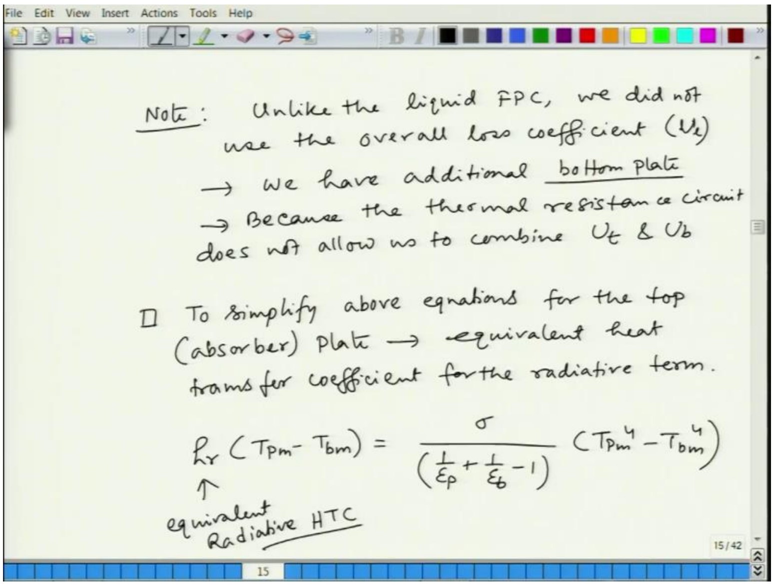

Now one first thing I want you to notice that unlike the liquid flat plate collector, we did not use the overall loss coefficient or U_l, ok. Why we could not use it? Because, earlier it was only the absorber plate and the fluid those were interacting between each other in terms of heat, ok.

But in this case, we have this additional bottom plate and that is why we cannot use a single term overall heat transfer coefficient, U_l. Because the thermal resistance circuit does not allow us to combine U_t and U_b. Top and bottom loss coefficient cannot be combined it can be combined if both are connected to the same node. Here we have two nodes one is for the absorber plate the other one is for the bottom plate.

For that reason, we cannot combine U_t and U_b ok, but we can of course, apply simplifications to this and we will see that shortly, ok. Now another thing that to simplify the equations that we have just written for the top or absorber plate. So, we will use the equivalent heat transfer coefficient for the radiative term, right. This we introduced in the liquid flat plate collector as well to combine the radiative term into the common ΔT form, ok.

So, what we will use? We will use this equivalent radiative heat transfer coefficient (h_r) and it will be multiplied by the temperature difference between the absorber plate and the bottom plate and that will be equal to the heat transfer rate, that is happening between the absorber plate and the bottom plate, clear. So, this is the equivalent radiative heat transfer coefficient, ok.

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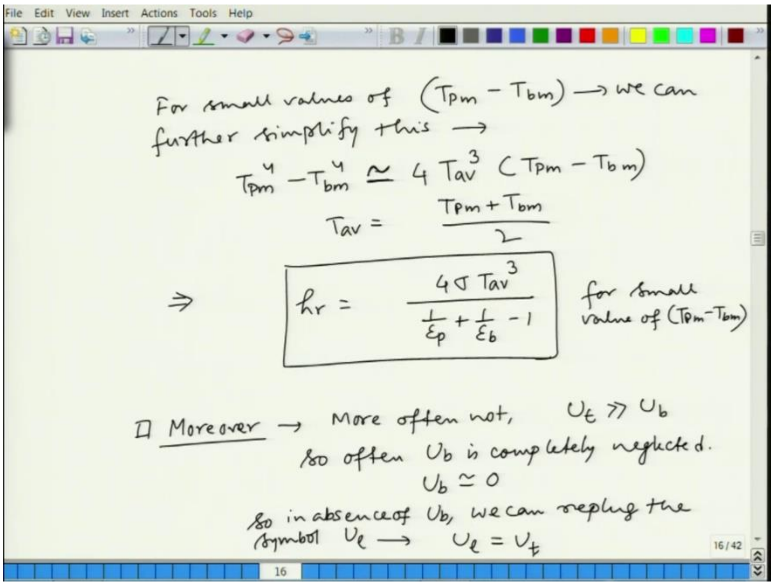

Now another simplification we can do, which is often true that for small values of the difference in temperature between the absorber plate and the bottom plate, ok. What we can do? We can further simplify this and what we can get?

For small values of $(T_{pm} - T_{bm})$,

$$T_{pm}^4 - T_{bm}^4 \approx 4\, T_{av}^3 (T_{pm} - T_{bm})$$

So, this is nothing, but a close approximation to the overall term and here,

$$T_{av} = \frac{T_{pm} + T_{bm}}{2}$$

which is very closely accurate if you have small $(T_{pm} - T_{bm})$ value, ok.

So, if we substitute that then this equivalent radiative heat transfer coefficient takes the following form, right. It is much more compact compared to the earlier one and let me write for small value of $(T_{pm} - T_{bm})$, ok. So, this is the equivalent radiative heat transfer coefficient. Now another simplification we can do.

So, more often than not this U_t is much much greater than U_b. What does it mean? It means that the top loss coefficient is dominating in the overall loss coefficient, ok. The bottom part what we have here? In the bottom part, we have this total insulation right and the loss through that part is really small compared to what is happening towards the top.

Towards the top we have the cover and the top plate which is heated. So, and we cannot like do away with the cover, ok. We have to have it and that is where the radiative and convection heat loss are also happening.

So, we cannot do away with it and that is why we are stuck with U_t, but U_b is our hand it is the thickness of insulation we can increase and often it is negligibly small. So, often U_b is completely neglected; that means, U_b is 0 that is what it is assumed, then one thing we can do that we only have U_t now and what we can do?

We can replace that with U_l. U_l stands for the loss coefficient and it is acceptable when you do not have any U_b. So, in absence of U_b we can replug the symbol U_l. It is convenient because it tells you about the loss it reminds you of the loss. So, what it means, that we will use U_l equal to U_t, ok. So, if that is the case what we can write?

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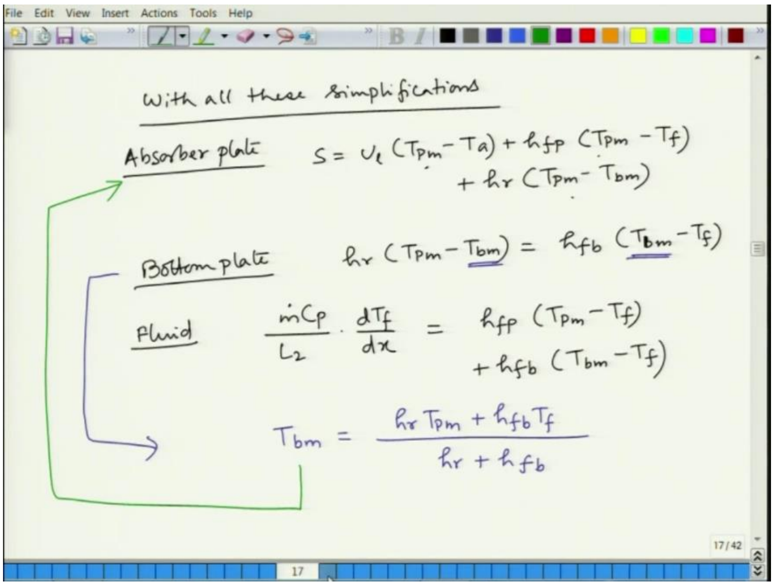

Now, with all these simplifications if we revisit the equations that we have written for the absorber plate. We can have the simple equation which is,

$$S = U_l\ (T_{pm} - T_a) + h_{fp}\ (T_{pm} - T_f) + h_r\ (T_{pm} - T_{bm})$$

So, S is the absorbed radiation per unit area and that absorbed radiation is being balanced under the steady state by three terms. The first term here is loss to the ambient, second term here is loss to the fluid or air and third term here is loss to the bottom plate, ok.

And for the bottom plate the heat balance equation takes even more simpler term because it has only two relevant terms, one is the radiative heat transfer between the absorber plate and the bottom plate that will be equal to whatever heat is being lost to the fluid from the bottom plate, right. And for the fluid also we need to relook at the balance equation, here we can see that this whole thing we cannot get rid of this L_2dx.

Let us write it in this form and this particular heat gain for that control volume for the air. The heat gain is happening due to 2 terms, one is from the top plate to the fluid and the other term is from the bottom plate to the fluid? So, you go back and look at the original equations that we have written and after you apply the simplification whether you get these three simplified equations.

So, I insist that you go back and do it for yourself that will clarify the concepts, ok. Now if we do use this bottom plate heat balance equation, we can just find this T_{bm} in terms of the other terms,

$$T_{bm} = \frac{h_r\, T_{pm} \; + \; h_{fb} \;\; T_f}{h_r \; + \; h_{fb}}$$

So, there are two T_{bm} here. So, we have club them together and expressed T_{bm} in terms of the other terms that is what we did, ok. Now we can substitute this T_{bm} expression in the absorber plate equation, right.

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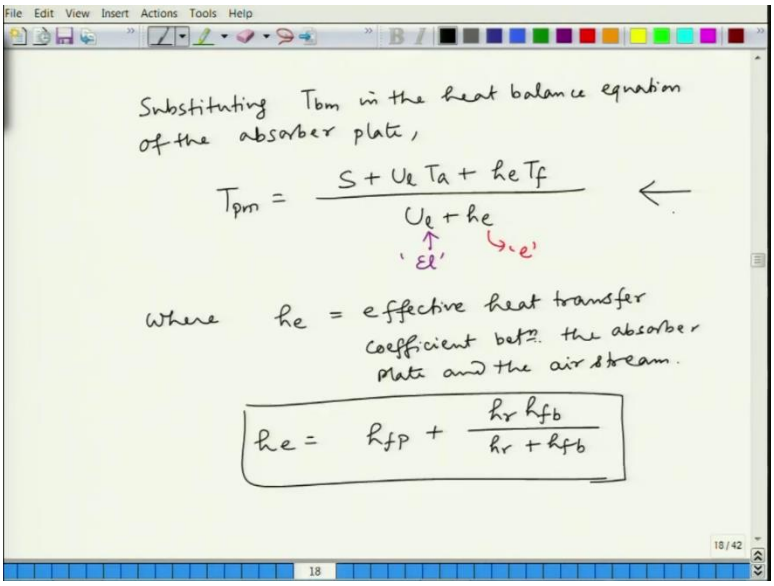

So, if we do that by substituting T_{bm} in the heat balance equation of the absorber plate, what we can write? We can now have a much simpler expression for the mean absorber plate temperature, ok. What is that? It is

$$T_{pm} = \frac{S + U_l\, T_a \; + \; h_e\, T_f}{U_l + h_e}$$

So, you can do this algebra, but we need to specify what we mean by h_e, ok. So, here h_e is the effective heat transfer coefficient between the absorber plate and the air stream and if you do the algebra what we will get?

This h_e is nothing, but the original h_{fp} which is the direct convection heat transfer coefficient from the plate to the fluid stream plus the combination of the radiative heat transfer from that absorber plate to the bottom plate and the effective convection coefficient from the bottom plate to the stream. So, that combination gives us this, ok. So, this is the effective heat transfer coefficient between the absorber plate and the air stream, ok.

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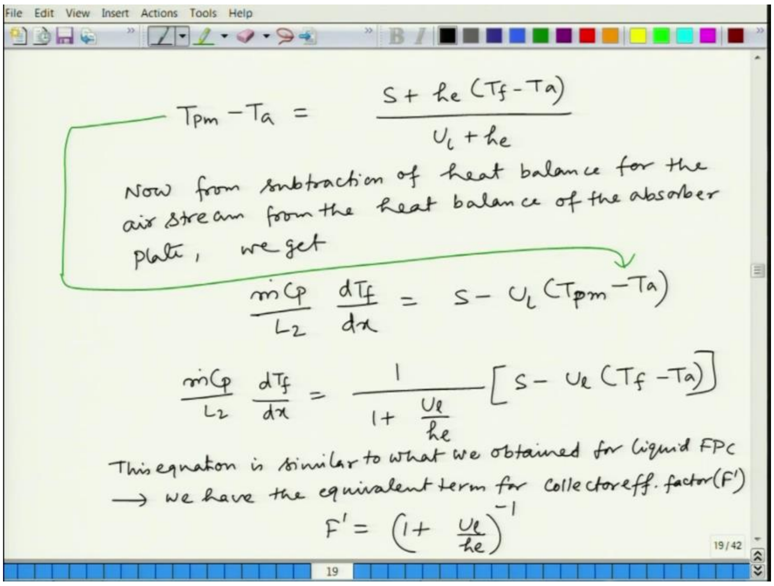

Now, what you can write for the $(T_{pm} - T_a)$. T_{pm} expression we have already written here right this is the expression for T_{pm}. Now, if you subtract,

$$T_{pm} - T_a = \frac{S + h_e\,(T_f - T_a)}{U_l + h_e}$$

So, you can do the algebra and you can find this. This is simply we have just subtracted the ambient temperature.

Now what we can do from the subtraction of heat balance for the air stream from the heat balance of the absorber plate. With all these different manipulations that we have done, what we get? We get the following,

$$\frac{\mathrm{m}C_p}{L_2}\frac{\mathrm{d}T_f}{\mathrm{dx}} = \mathrm{A} - U_l\,(T_{pm} - T_a)$$

Because this is the exactly the same equation, we have obtained for the liquid flat plate collector and when we substitute this $(T_{pm} - T_a)$. Here what we get?

$$\frac{\mathrm{m}C_p}{L_2}\frac{\mathrm{d}T_f}{\mathrm{dx}} = \frac{1}{1+\frac{U_l}{h_e}}\left[S - U_l\left(T_f - T_a\right)\right]$$

So, let me write it explicitly that this equation is similar to what we obtained for liquid flat plate collector and we have the equivalent term for the collector efficiency factor which we designated as F'.

Here,

$$F' = 1 + \frac{U_l}{h_e}$$

If you substitute that its thus exactly the same equation we obtained, ok.

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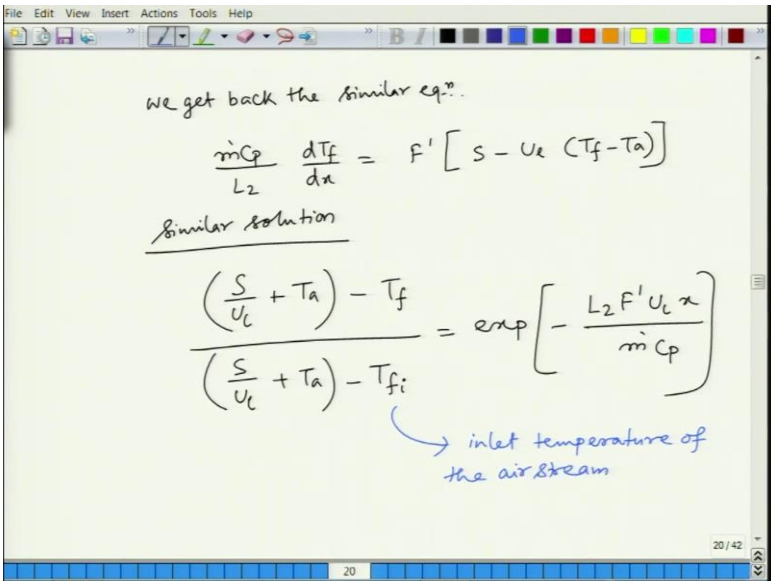

So, here we get back the similar equation because not exactly same because we did not introduce the area of the collector yet. So, that is why there are some external dissimilarities, but mathematically they look exactly the same, ok. So, this is the governing equation for the heat gain of the air stream in the air heater, ok.

So, we will if we have the similar equation, the solution will also be similar right there is no question of any dissimilarity. So, what we can write? We without doing anything just by following the solution of the liquid flat plate collector, we can straight away write it in this form.

So, T_{fi} is the inlet temperature of the air and we can write,

$$\frac{\left(\frac{S}{U_l}+T_a\right)-T_f}{\left(\frac{S}{U_l}+T_a\right)-T_{fi}}=exp\left(-\frac{L_2\,F'U_l x}{\mathrm{m}C_p}\right)$$

Just by following what we obtained for liquid flat plate collector, we can write the same equation here where this particular quantity we have introduced which is the inlet temperature of the air stream, ok.

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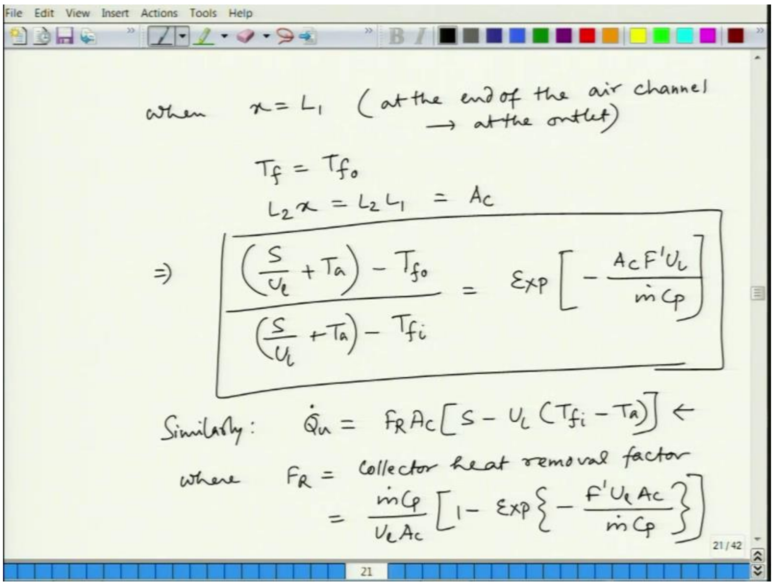

So, now from this solution when we have x $= L_1$. So, this was our L_1. L_1 means the length along the flow direction, ok. So, x $= L_1$; that means, at the end of the air channel (at the outlet).

So, we will have T_f will be T_{fo} right that will be the outlet fluid temperature ok and this L_2x $= L_2L_1$ and what is L_2L_1 ? L_1 is the along the flow that is the length and L_2 is the across the flow that is the length.

So, L_2L_1 will be the total area of the collector right, Ac. So, what we can write that,

$$\frac{\left(\frac{S}{U_l} + T_a\right) - T_{fo}}{\left(\frac{S}{U_l} + T_a\right) - T_{fi}} = exp\left(-\frac{A_c\, F' U_l}{\mathrm{m} C_p}\right)$$

So, this is the final equation which is obtained by the thermal analysis of the air heater, ok. So, similar way as the liquid flat plate collector what we can write, the rate of useful heat gain by the air stream will be,

$$\dot{Q_u} = F_R A_c \left[\mathrm{S} \; - \; U_l \; (T_{fi} \; - \; T_a)\right]$$

So, here you note that I am not going through all the derivations again you can just draw parallel between the air heater and the liquid flat plate collector and that is how you can directly write these equations.

The solution of the differential equations and then all the subsequent ones you can directly write when you see this parallel between these two, ok.

So, again we have written this equation directly and here the F_R which is nothing, but the collector heat removal factor that is,

$$F_R = \frac{mC_p}{A_c\, U_l}\left[1 - exp\left(-\frac{A_c\, F' U_l}{\mathrm{m} C_p}\right)\right]$$

Just drawing the parallel between the air heater and the conventional liquid flat plate collector you can write all these things, ok.

So, here one thing we have to note that we have assumed the heat transfer coefficients between the air stream and the plates are known, right. h_{fp} or h_{fb} all these things are known that we have assume, but how to know that, right.

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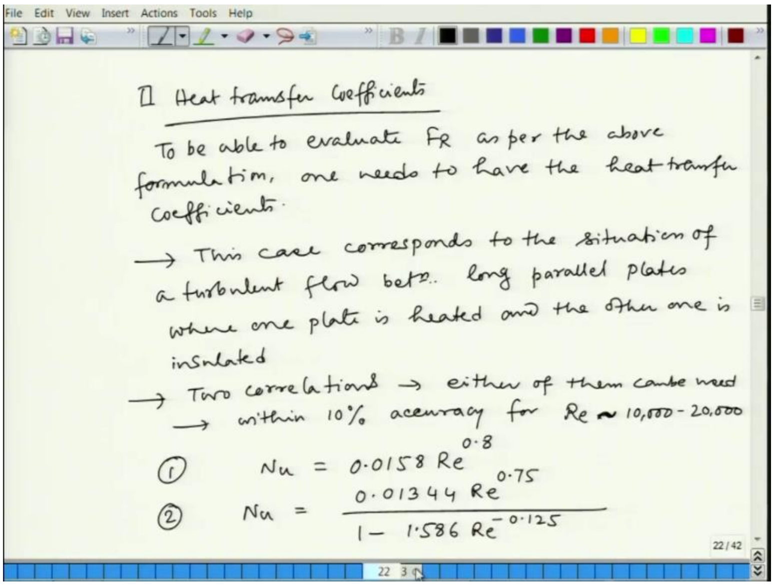

So, a note on the heat transfer coefficients, ok. So, to be able to evaluate F_R as per the above formulation, one needs to have the heat transfer coefficients right and this case actually corresponds to a known case from the convective heat transfer coefficient or convective heat transfer theory. This case corresponds to the situation of a turbulent flow between long parallel plates where one plate is heated and the other one is insulated, right.

So, in this case our absorber plate is being heated by the solar radiation and the bottom plate is insulated right and for such case we have two correlations you know from the convective heat transfer. These correlations are obtained by dimensional analysis and then fitting large number of experimental data point to obtain certain equations and those are called correlations that are used to find out the heat transfer properties under a particular flow situation, ok.

So, two correlations exist and either of them can be used. Both gives you within 10% accuracy for the Reynolds number range of 10,000 to 20,000. So, that is the typical range that you will find for this air heaters and it serves quite accurately you will get the h, ok. So, what are the correlations? One is this Nusselt number,

1. $$Nu = 0.0158 Re^{0.8}$$

So, these are the non-dimensional numbers you know them I am not going into detail of that. I am just providing the correlation so that you can use them while you solve the numerical problems.

2. $$Nu = \frac{0.01344 Re^{0.75}}{1 - 1.586 Re^{-0.125}}$$

And for evaluating the Reynolds number you know that you have to use the equivalent diameter, right.

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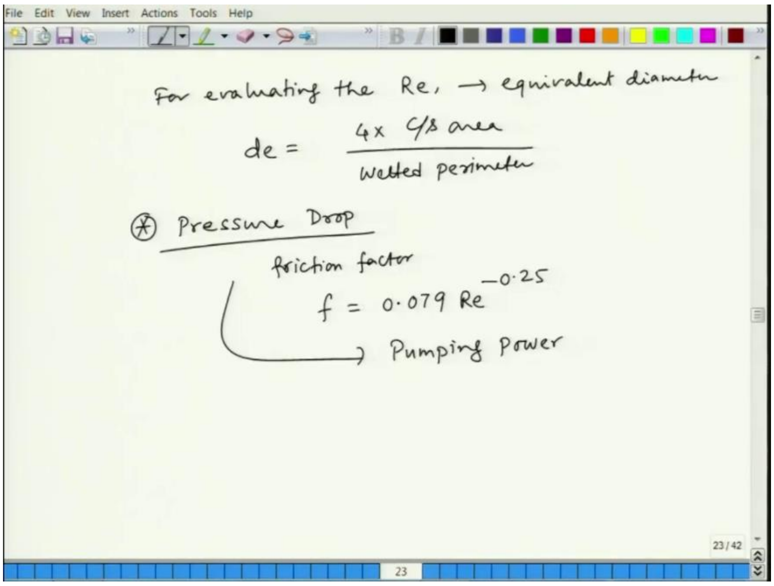

So, for evaluating the Reynolds number you, find the equivalent diameter of the channel area (cross section). So, equivalent diameter is:

$$d_e = \frac{4 \times cross\ sectional\ area}{wetted\ perimeter}$$

So, you have to evaluate that for the air channel where the flow is happening and then use that equivalent diameter in evaluating the Reynolds number and then you can use the correlation to get the Nusselt number, from the Nusselt number you will have to get the h by using the thermal diffusivity, ok.

And another important aspect of this thing is, you need to know the pressure drop, right. So, your flowing air through a channel and that is why a pressure drop will happen which will tell you how much power your compressor or blower will consume, ok. So, this pressure drop is given through the friction factor and this friction factor,

$$f = 0.079Re^{-0.25}$$

And this you require to obtain the pumping power, ok. So, once you have identified the equivalent diameter from there you get the Reynolds number, it will give you two quantities, one is the pressure drop (friction factor) and you will also get the heat transfer coefficient which will give you the collector heat removal factor through different other inputs you have to put and that is how you get how much useful heat that air heater will give you, ok. So, that is the whole analysis we have done for a conventional air heater, ok.

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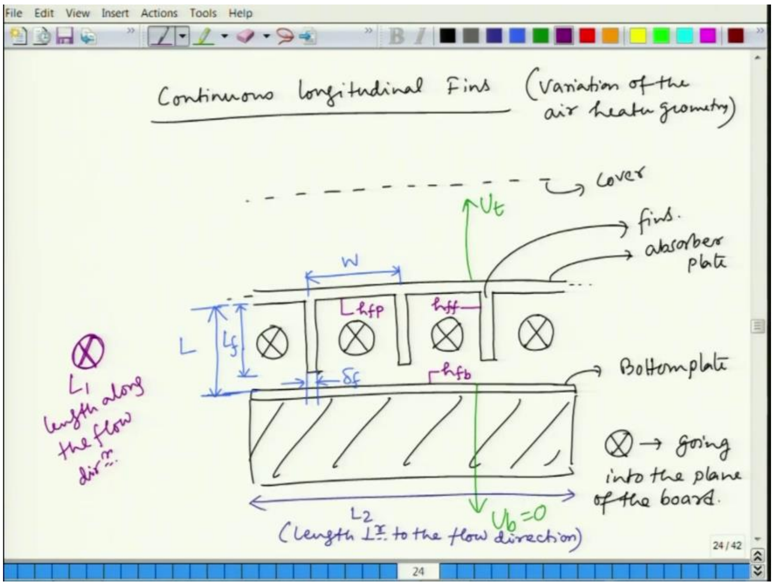

Now, we are going to look at one other very common variation of this air heater where to increase the heat transfer area what we do? We use fins to increase the area and those are continuous longitudinal fins, ok. So, this is kind of a variation, but very common variation of the air heater geometry, ok. So, what do we mean by these continuous longitudinal fins?

You can think of this absorber plate ok. Now instead of a flat plate, we are having few fins which are protruded into the airstream of equal length and at equal distance, ok. So, this way it is going on, ok. This direction also it is continuously going on and we have the bottom plate here. So, let us say this is the bottom plate and the lower portion of that is insulated, ok.

So, this is the bottom plate and these are the fins and this is the absorber plate. Now you need to visualize how air is flowing. Air is not flowing. So, the drawing that I am showing right now is not similar to the earlier air heater drawing that I have shown you it is in the perpendicular direction.

Because if air flows from this direction then there will be huge amount of pressure drop air channels are very small, ok. So, air is flowing here in this direction perpendicular to the plane of the board. So, this is the air stream that are going, ok. So, this particular symbol means going into the plane of the board, ok. That is how air stream is going and of course, we have a cover which is topping the absorber plate. So, this is the cover.

Now, if we put some dimensions here, let us say that in between these two plates we have mid plane length which is of length L. So, not here, it is the bottom of this plate and this is the length between top and bottom plate and the length of the fin is L_f. And here from mid plane to mid plane the distance between two consecutive fins is w and each fin has dimension of δf, ok.

Now, we have few different surfaces for which the heat transfer coefficient is important. One is this particular surface let me use another color, this particular surface let us say heat transfer coefficient is h_{fp} and from this surface it is h_{fb}, as we have used in earlier case also. Now we have another vertical surface which is giving us heat transfer coefficient h_{ff} now second f is for fin, ok.

And then the usual stuff is there, here you have the top loss coefficient and from the bottom plate you have the bottom loss coefficient which we again assume 0, ok. Now, the other dimension that we have that this is now perpendicular to the flow direction. So, let us say this dimension is L_2, this is again the same convention we are using as we did for the conventional air heater without fin, ok.

So, this L_2 is length perpendicular to the flow direction and along the flow direction let us say length is L_1. So L_1 is the length along the flow direction, ok. So, now we will do the analysis, but we do not have to go through each individual step which is very similar. So, I will go little bit quickly.

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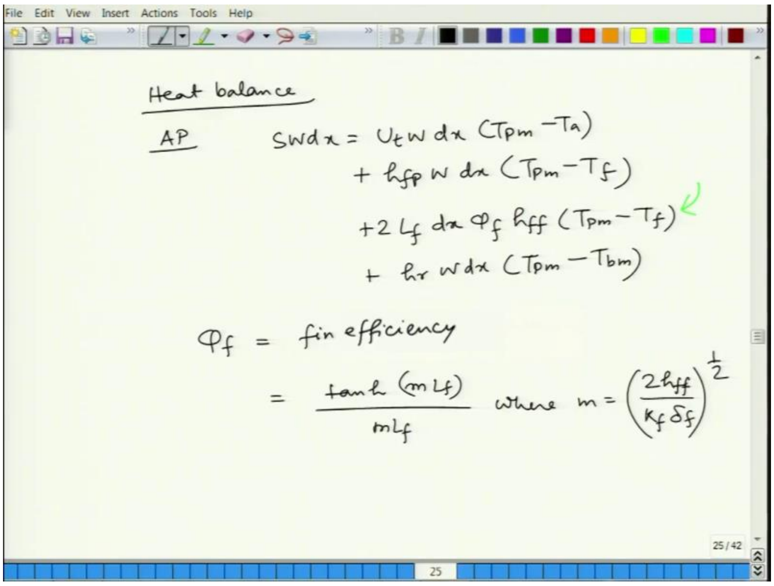

So, the heat balance first for the absorber plate which is,

$$SWdx = U_t\, Wdx\, (T_{pm} - T_a) + h_{fp}Wdx\, (T_{pm} - T_f) + 2L_f\varphi_f h_{ff}\, (T_{pm} - T_f)$$

$$+ h_r Wdx\, (T_{pm} - T_{bm})$$

Now, what is this φ_f? It is nothing, but the fin efficiency. Again, you know this from the conductive heat transfer coefficient. You have derived it in this course as well which is,

$$\varphi_f = \frac{\tanh\,(mL_f)}{mL_f}$$

where, m = $\sqrt{\frac{2h_{ff}}{k_f\delta_f}}$

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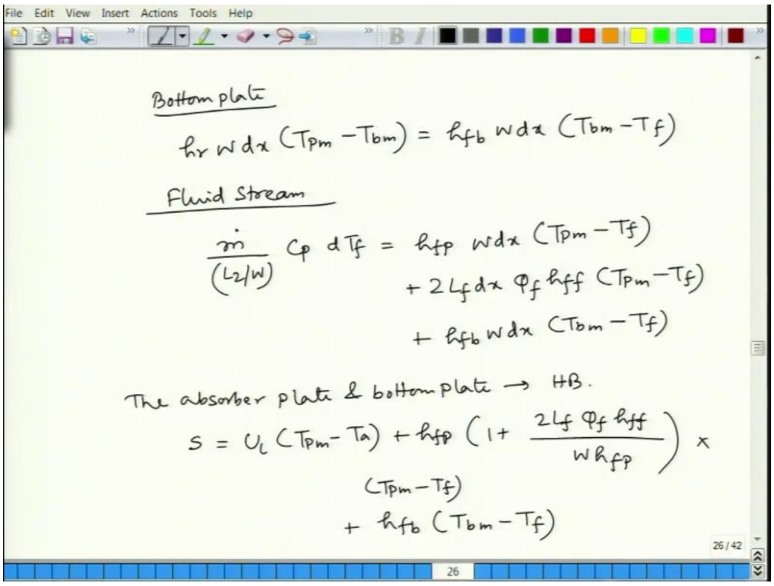

Similarly, if you do the bottom plate heat balance, it will be,

$$h_r W dx (T_{pm} - T_{bm}) = h_{fb} W dx\, (T_{bm} - T_f)$$

It is the simplest of all three heat balances because it has only two terms, one is to the fluid the other one is radiative transfer from the upper plate, ok.

For the fluid stream: For the fluid stream now you can see that we have few channels, ok.

So, for those number of channels we have to incorporate in terms of the total mass flow rate. So, what we will do? The total mass flow rate basically if it is $\dot{m}$ that will be divided by the number of channels, i.e.,

$$\frac{\dot{m}}{L_2/W} C_p \mathrm{d}T_f = h_{fp} W dx\, (T_{pm} - T_f) + 2L_f dx \varphi_f h_{ff}\, (T_{pm} - T_f) + h_{fb} W dx\, (T_{bm} - T_f)$$

So, this derivation you have to do by yourself and if you are stuck anywhere you can look at this video back again, but the way I am writing it will be difficult for you to follow. So, I insists that you do it on your notebook the same steps. So, now, the absorber plate and bottom plate these two heat balances if you put together what you get?

$$\mathrm{S} = U_l\, (T_{pm} - T_a) + h_{fp} \left(1 + \frac{2L_f \varphi_f h_{ff}}{\mathrm{w}\, h_{fp}}\right) (T_{pm} - T_f) + h_{fb}\, (T_{bm} - T_f)$$

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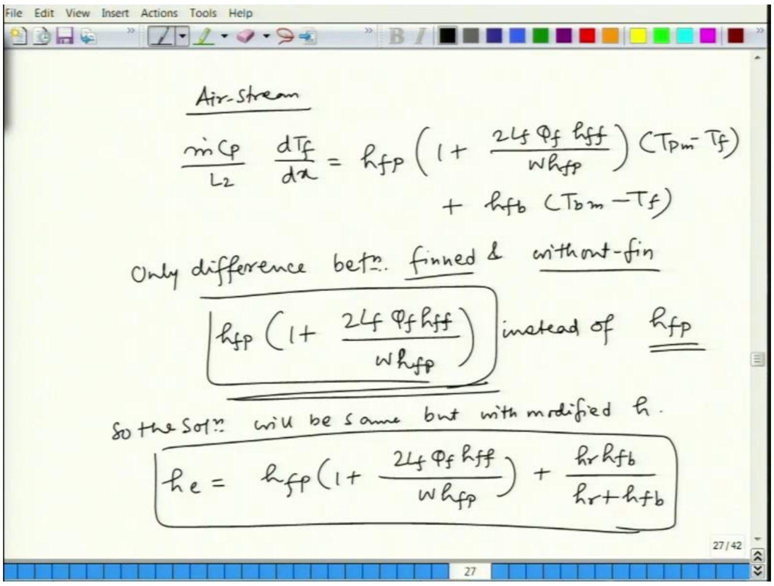

Now if you do the air stream part you can write,

$$\frac{\dot{m}C_p}{L_2}\frac{dT_f}{dx} = h_{fp}\left(1 + \frac{2L_f\varphi_f h_{ff}}{\mathrm{w}\, h_{fp}}\right)(T_{pm} - T_f) + h_{fb}\,(T_{bm} - T_f)$$

So, just note that only difference between finned and without fin is that this h_{fp} is now modified to $h_{fp}\left(1 + \frac{2L_f\varphi_f h_{ff}}{\mathrm{w}\, h_{fp}}\right)$ instead of h_{fp}. Without fin what you had? You had only this h_{fp}, now for finned case you have modified this h_{fp} by this, ok. So, the solution will be same, but with modified h and the effective heat transfer coefficient h_e that will also be modified in the following manner, ok. So, these are two major modifications that the geometry of fin is introducing in the analysis, rest of the thing are exactly the same.

So, in this lecture we have looked at the thermal analysis of air heaters both the conventional version as well as the longitudinal fin version, ok.

So, thank you very much for your attention.