Elements of Solar Energy Conversion
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Lecture - 18
Flat Plate Collector

Hello and welcome back. This is the 18th lecture of the series of lectures on Elements of Solar Energy Conversion. So, we have started looking at the most basic solar collector, which is a Flat Plate Collector.

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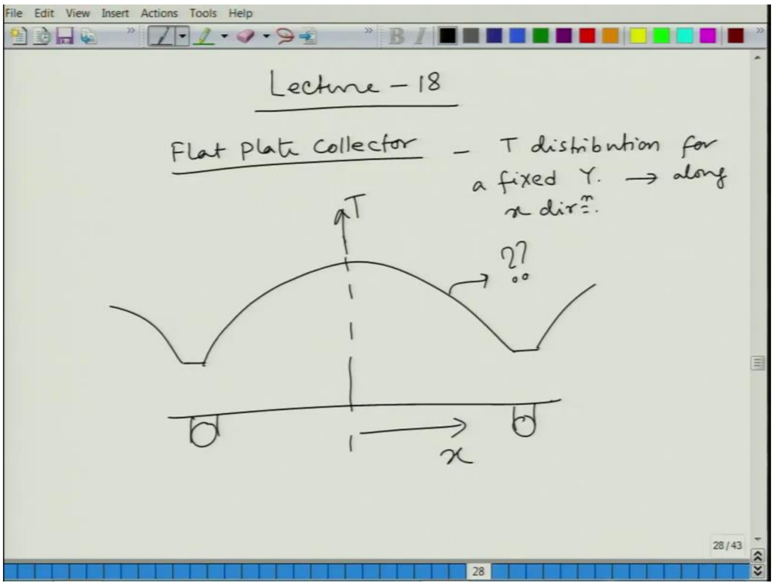

So, we have started looking at its thermal analysis, and in the last class, we have. So, we are here at the 18th lecture, and we are looking at the flat plate collector. So, in the last class, what we have seen is the temperature distribution for a fixed y, which means, along the x-direction.

So, this is the temperature distribution in the mid-plane between two tube locations, and we have seen that the temperature distribution will be like this. So, what would be the form of this function that we require to find out what will be the effective heat gain or loss from the plate?

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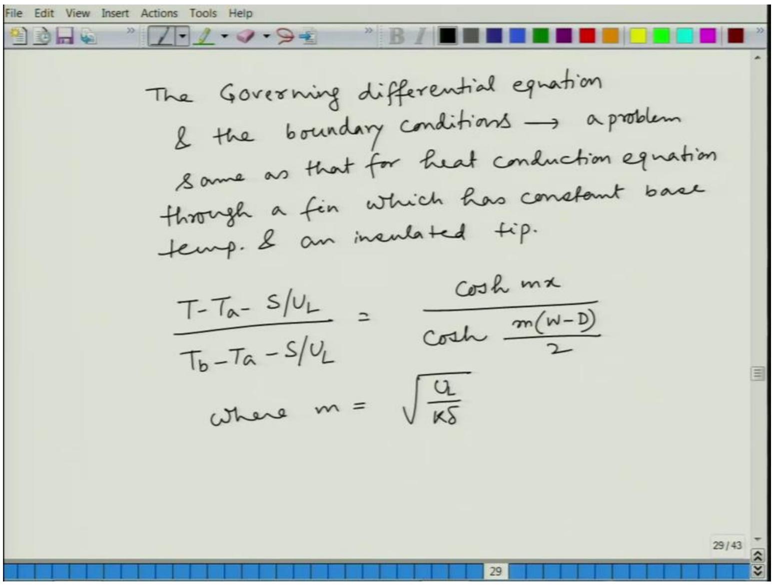

So, in the last class what we have seen that the governing equation governing differential equation and the boundary conditions give us a problem same as that for heat conduction equation through a fin which has constant base temperature and an insulated tip.

And the form of a solution that we obtained is

$$\frac{T - T_a - \frac{S}{U_L}}{T_b - T_a - \frac{S}{U_L}} = \frac{cosh\ (mx)}{cosh\ (\frac{m(W-D)}{2})}$$

So, if you do not follow, please refer back to the last class and try to understand, and I would insist that you please derive this again as you have done for a fin problem.

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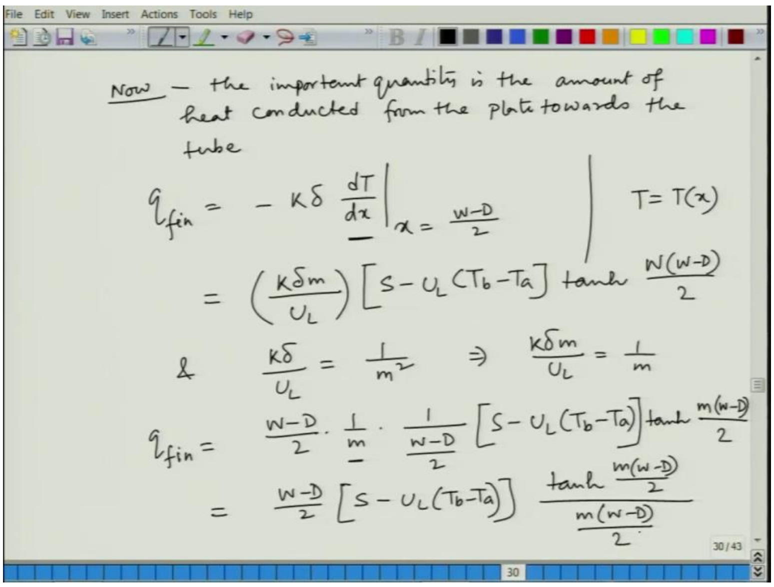

Now, what we are interested in here the temperature distribution is fine, but that is an intermediate step to ultimately find out what would be the heat loss from the plate to the, I mean, what would be the heat transfer towards the fluid. So, here is the fluid level, and what would be the heat transfer of that? So, what can we do?

The important quantity is the amount of heat conducted from the plate towards the tube, right whenever you have a temperature slope in this direction. So, heat will flow from high temperature to low temperature, and ultimately heat will come to the base of the fin, or that means, to the fluid here. So, what is that quantity?

If we say that it is q fin is

$$q_{fin} = -k\delta \frac{dT}{dx}$$

So, this is the simple Fourier law we have used, and now we have this T as a function of x that we just derived from the fin equation.

So, as we know that we can find this $\frac{dT}{dx}$ and the form that we are going to get. I am not going to derive it, but I am writing it. So, you can please check it after deriving:

$$q_{fin} = -\frac{k\delta_m}{U_L} * \left(S - U_L(T_b - T_a)\right) * \tanh(\frac{w * (w - d)}{2})$$

So, that is the amount of heat that is carried from the plate to the tube.

And, here we can make some more simplification because we know this $\frac{k\delta}{U_L}$ is nothing, but $\frac{1}{m^2}$. So, this is by definition of m. So, that means that this $\frac{k\delta_m}{U_L}$ is nothing, but $\frac{1}{m}$. So, what we can write this q_{fin} we can write to be.

$$q_{fin} = -\frac{(W - D)}{2 * m * \frac{(W - D)}{2}} * \left(S - U_L(T_b - T_a)\right) * \tanh(\frac{w * (w - d)}{2})$$

So, why we did that? We can now have we can now write this whole thing like this. So, please check by yourself whether this is true.

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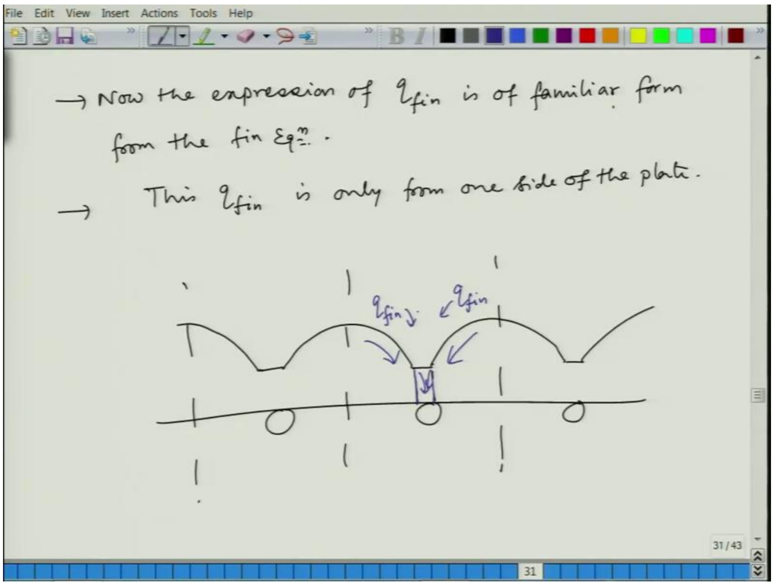

So, why did we change this by multiplying it with $\frac{(W-D)}{2}$ and dividing it again by $\frac{(W-D)}{2}$? Because now the expression of q_{fin} is of familiar form from the fin equation because whenever you have the hyperbolic tan of something divided by the same thing, that is the familiar form of a heat fin equation. So, that is why we did this conversion.

So, that is the first point; the second, this q_{fin} is only from one side of the plate right. What do I mean by that? So, if you have a series of these tubes so, the profile of temperature will look something like this, right. So, now, if you consider what heat is coming, what heat is coming down to this region. So, you have to take heat coming from both directions.

So, from one direction, you have this $q_{fin,}$ and from the other direction, you will also have the same q_{fin} right because of symmetry.

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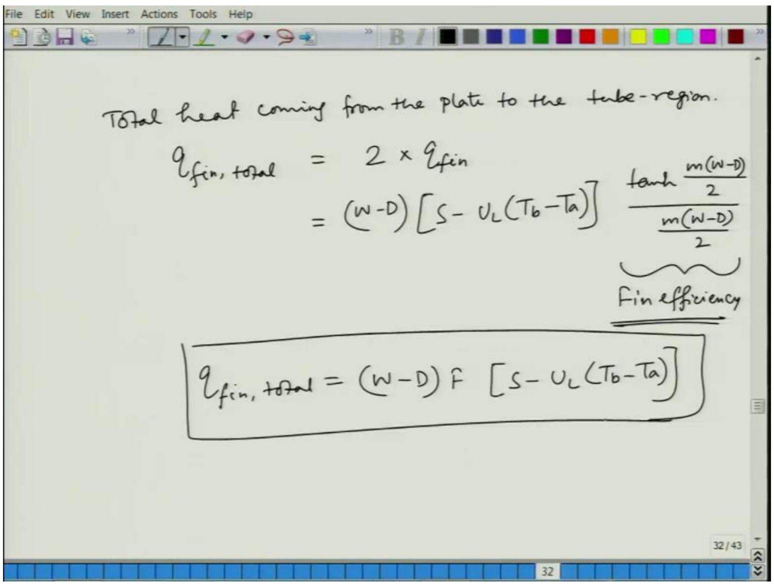

So, what you need to do? You need to find the total heat coming from the plate to the tube for the tube region is if we say q_{fin} total, that will be two multiplied by whatever we found, twice the quantity that we have found.

$$q_{fin,total} = -\frac{\tanh(\frac{w*(w-d)}{2})}{m*\frac{(W-D)}{2}} * \left(S - U_L(T_b - T_a)\right) * (W - D)$$

So, this one is a familiar form as we are saying this is called fin efficiency. Fin efficiency you are familiar with from your knowledge on fin heat transfer.

So, that is why we can now write. So, that is q fin total. That is an important relationship that we have obtained from getting the temperature distribution along with the plate along the x-axis for a fixed y, and then we are converting it, or we are making it simple in terms of the fin efficiency.

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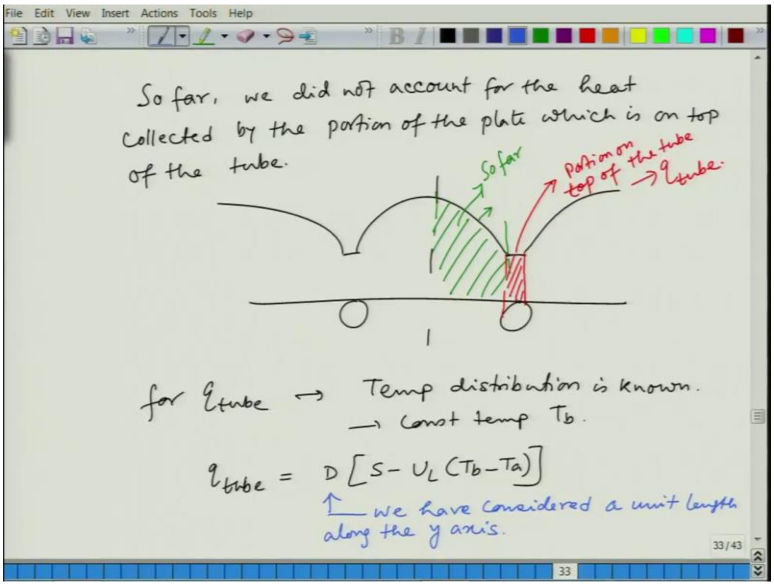

Now, so far, we did not account for the heat collected by the portion of the plate which is on top of the tube that is carrying the fluid. So, that means, what do I mean? We have this plate; we have these tubes; we have seen that the temperature profile will be somewhat like this and what we have accounted for is the heat collected by this portion. This we have accounted for.

Now, what are we going to do? We are going to include this portion as well. So, if we do not take this account into account, that will be causing some error. So, the portion on top of the tube gives us another quantity which we call q_{tube}. Now, this is because we do not have to worry about the temperature.

For q_{tube} temperature, distribution is known, right that causes all the problems, and that is why we had to solve the differential equation, along with its boundary conditions for the portion here, but for the tube portion, we do not have to do that because temperature distribution is known and that is constant temperature T_b.

So, what would be that? This q tube you can write simply

$$q_{tube} = D[S - U_L(T_b - T_a)]$$

Here please note that this represents the D which is the diameter of the tube that represents the area for that region because we have considered a unit length along the y-axis, and that is why it represents the area, and it is because we have a single temperature T_b.

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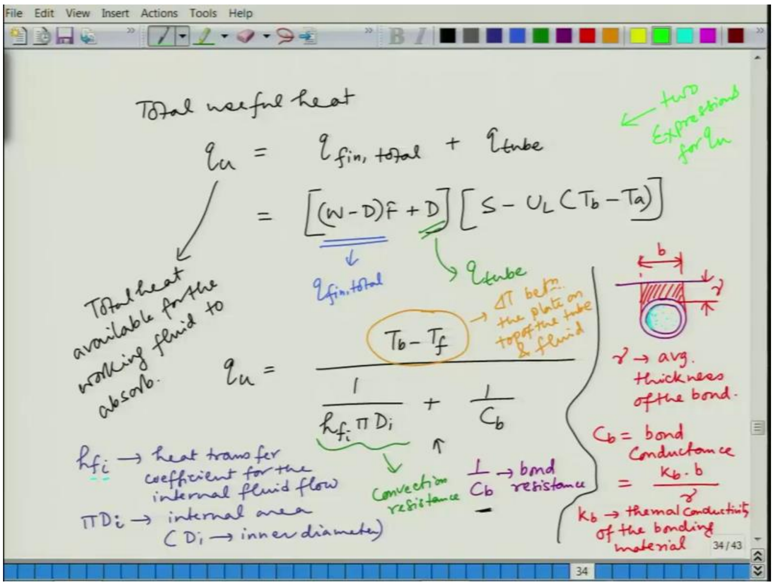

So, what can we write? Now, the total useful heat that is heating up the working fluid that is

$$q_u = q_{fin,total} + q_{tube}$$

That means, from both the directions of the plate across the tube from both the directions, whatever heat is coming from the plate plus the heat accumulated by the portion of the plate which is just above the tube.

So, if we put the values that we have found so far, this will be multiplied by $[S - U_L(T_b - T_a)]$. This is coming from the q_{fin} total, and this is coming from the q_{tube} ok. Now we can say that this is the total heat available for the working fluid to absorb. The working fluid is absorbing this heat, but again until it reaches the working fluid, it has to pass through a few resistances.

So, the same thing we can write in terms of the difference between the base temperature or the tube temperature and the fluid temperature at that location. So, let me first write, and then I will explain each term. So, this is the thermal resistance and what we are assuming here is that the tube, of course, will have a certain thickness, and it will be bonded to the plate with some bonding material.

I am exaggerating it, but it will have some thickness, and if we say that, the average thickness of the bond is Υ. So, Υ is the average thickness of the bond, then the C_b, which is the bond resistance that will be equal to K_b multiplied by the base length b into Υ.

So, what is b? This is the base length that may or may not be equal to the outer diameter of the tube and Υ is the average thickness, and K_b is nothing, but the thermal conductivity of the bonding material if that is the case, then C_b will be the bond resistance and $\frac{1}{C_b}$ will be the reciprocal of the resistance or conductivity.

And, what would be the other part or rather this $\frac{1}{C_b}$. So, C_b is this; this is what I should say? This is bond conductance and this $\frac{1}{C_b}$ is the bond resistance. Now, let us look at the first term, what it contains this h_{fi} this is nothing, but the heat transfer coefficient for the internal fluid flow.

So, when fluid is passing through this tube, then there will be some internal heat transfer coefficient depending on the velocity of the fluid and other things, and there in the inside boundary, whatever heat transfer coefficient you will get that will be your h_{fi}; i stands for internal, and f stands for fluid.

And πD_i is the internal area for the tube that is the area for the fluid for the heat transfer because D_i is the inner diameter. So, that will be the convection resistance, you can say. The first term here will be the convection resistance, and we have this $\frac{1}{C_b}$ which is bond resistance.

If we sum them up, then we have total resistance, and what we have written here is the delta T between the plate on top of the tube and the fluid between the plate and the fluid; this is the temperature difference. So, now we have two quantities. So, now, we see that we have two expressions for q_u. So, one expression or we have two expressions for the usefully q_u.

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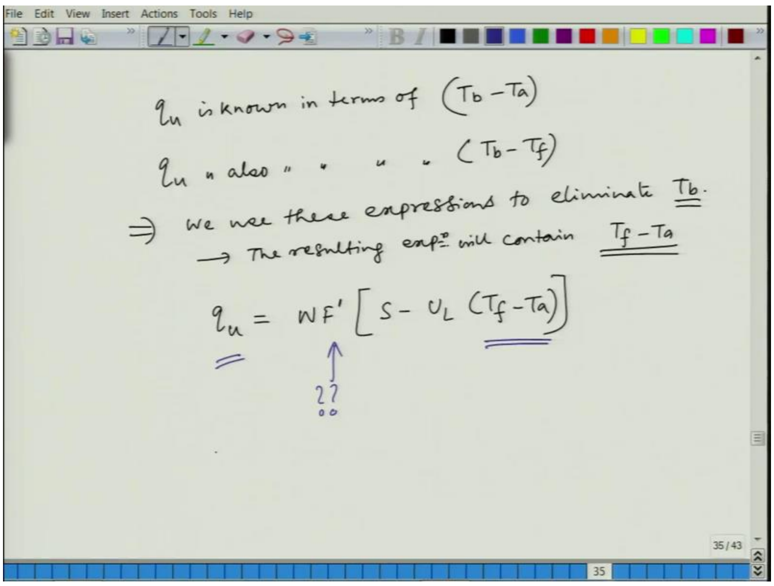

So, now we can write this q_u is known in terms of this $T_b - T_a$ that is one expression that we have obtained, and q_u is also known in terms of $T_b - T_f$. So, in terms of the temperature difference between the base and the ambient and temperature difference between the base and the fluid.

So, now what we can do we use these expressions to eliminate T_b, and that is the common link T_b. So, we can completely remove it, and we can get a relation between that ambient temperature and the fluid temperature. So, the resulting expression will contain this $T_f - T_a$ And that is what we require.

We do not care about the base temperature because that is an intermediate temperature that is useful to know, but ultimately what we are interested in that what is the loss that is happening from the fluid to the ambient. So, if we do that, this elimination what we can write, so, again, I am not going to derive this elimination process from simple algebra; you can do it by yourself. What I am writing is the final expression.

So, let me write it first. So, this is the expression, is it not cool? Because we have now this useful heat as a function of this $T_f - T_a$ and that makes sense intuitive sense because that is what we are interested in, but in the meantime, what we have interested or what we have introduced is this F' which we do not know right and that we have to explain.

So, what is this value F'? So, let me write that expression first; let me write that on the next page.

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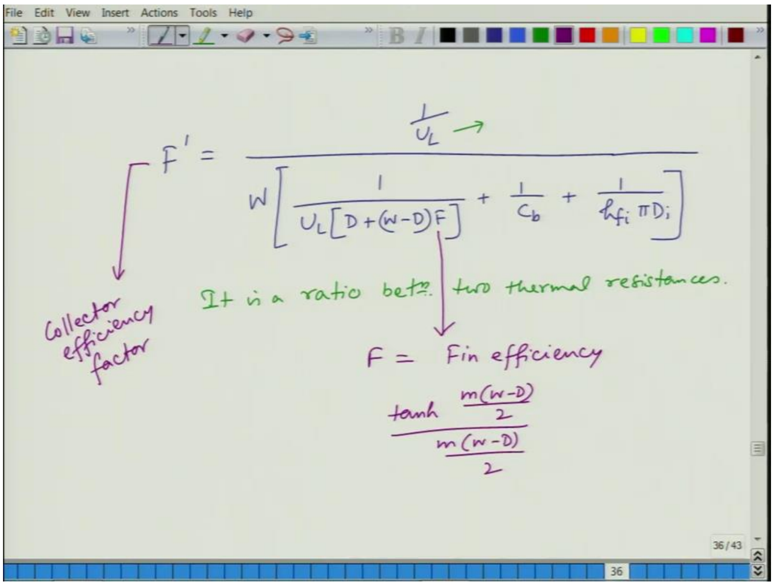

So, if we write that F prime expression, it is little involved. So, in the numerator, you have $\frac{1}{U_L}$ and in the denominator, you have series of terms. So, F prime is what you can see that it is a ratio between two thermal resistances. What is U_L? U_L is your overall heat loss coefficient from the plate to the ambient.

So, $\frac{1}{U_L}$ is the resistance. So, it is a ratio that is the first observation; it is a ratio between two thermal resistances, and in the denominator also you can observe that all of these are actually thermal resistances, and we are summing them up. Now, here in one of the terms, you can see that we have this F. F we have seen earlier that is called fin efficiency.

You remember that is nothing, but $\frac{tanh\frac{m(W-D)}{2}}{\frac{m(W-D)}{2}}$. So, that fin efficiency is now included in this F'. So, what is this F'? The name we call it collector efficiency factor. So, this particular quantity collector efficiency factor includes the effect of fin efficiency and all other kinds of thermal efficiency or thermal resistances it involves.

In terms of the bond, in terms of the internal free flow, in terms of the overall heat loss coefficient to the ambient. So, F'is a quantity that gives you an overall effect on how the useful heat will be related to the fluid temperature and the ambient temperature.

So, now we have completely seen if we fix a particular value of y and look at the temperature distribution along the x-axis, what would be the form and what would be the contribution of that in the useful heat. Now, we have we recall that we have taken an assumption that in the x-direction and in the y-direction, the temperature distributions we have taken to be independent of each other.

So, what we can do now is that we take the information of the x-direction temperature distribution and use it and find out a temperature distribution along the y-direction. So, that is what we are going to do now.

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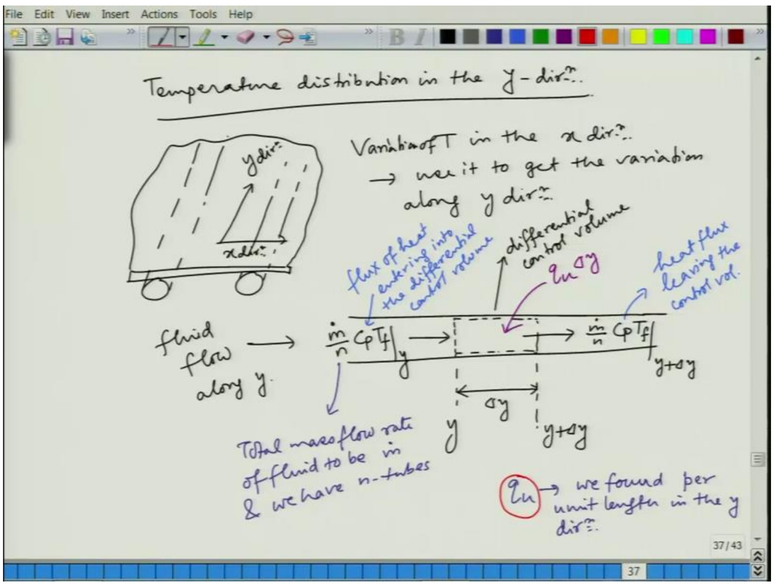

Now, you remember that what was the y-direction. This was the plate, and we had tubes running along the y-direction right. So, this was our y-direction, and this was our x-direction. Now, we know T in the or rather variation of T in the x-direction. So, now, we will use it to get the variation along the y-direction, and that we can do only when we can consider them to be independent.

Now, let us say along the y-direction, let us see say that this is the fluid flow direction along the y and along the y-direction. Now, if we take our differential element here inside the fluid, this is the differential control volume where you can see say that fluid is coming from this direction with certain specific heat C_p and with certain temperature T_f, which is at y. So, let us say this is y, and this is $y + \Delta y$, and this is the width Δy.

Now, one thing that is missing here is the mass flow rate. So, we need to multiply that mass flow rate which we write as $\frac{\dot{m}}{n}$. Why we write that write it? Because if we have a total mass flow rate of fluid to be $\dot{m}$ and we have n tubes. We have multiple tubes that are carrying $\dot{m}$ mass and each of them are of equal dimension and everything.

So, we can write for each individual tube the mass flow rate will be $\frac{\dot{m}}{n}$ and that is why we have written it. So, this particular term is giving you the flux of heat entering into the differential control volume.

Similarly, what can we do? We can write the flux that is going out again; the mass flow rate will be the same, C_p will be the same if it is not changing for a minuscule amount of heat addition, and this T_f will now we have to calculate it at $y + \Delta y$.

So, this is the heat flux leaving the control volume. Now, what else is happening here, we have this useful heat that is coming into the fluid, right, and that is coming from the x-direction distribution. So, what is coming here is this q_u which is the useful heat multiplied by this Δy because that is the length of the tube.

So, let me write it here that q_u we found per unit length in the y-direction, you remember that? So, all that this all the analysis that we have done so, where is that? Let me see. Yeah, so here we have written that we have done everything per unit depth in the y-direction. So, here we have that q_u will be per unit depth in the y-direction. So, the total heat that is coming in will be $q_u * \Delta y$.

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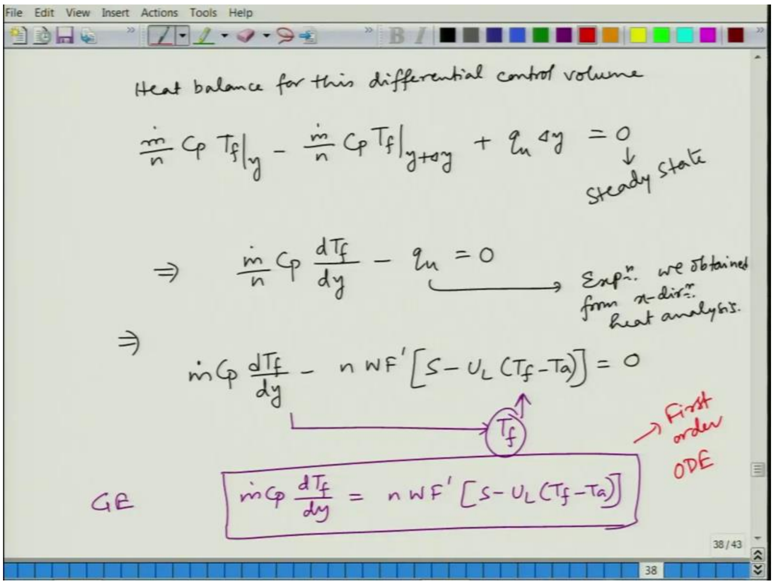

Now, if we want to do the heat balance for this differential control volume, then what can we write?

$$\frac{\dot{m}}{n} * C_p * T_{f_y} - \frac{\dot{m}}{n} * C_p * T_{f_{y+\Delta y}} + q_u * \Delta y = 0$$

Why 0? Because we have a steady-state, if it was not a steady-state problem, then this will not be 0, but the temperature will be a function of time as well.

So, if for a steady-state we can assume that, then this differential control volume equation can be simplified in a differential equation

$$\frac{\dot{m}}{n} * C_p * \frac{dT_f}{dy} - q_u = 0$$

Or, now, we will put the expression we obtained from x-direction heat analysis.

So, what expression did we get? Let me write that $\dot{m}$, we can take this n along with the q_u and write n *W * F,' which is the collector efficiency factor which we saw a couple of minutes ago and $\left(S - U_L(T_b - T_a)\right)$ and that is equal to 0.

So, now you will appreciate why we introduce this T_f in the first place in the end at the end of the x-direction temperature distribution or heat balance; what we did? We have converted the expression in terms of $T_f - T_a$ because then only you will be able to solve as a differential equation. So, both these T_fs will be giving you the differential equation dependent variable. That is why we are obsessed with taking it in terms of fluid temperature rather than plate temperature.

$$\frac{\dot{m}}{n} * C_p * \frac{dT_f}{dy} = n * W * F' * \left(S - U_L(T_b - T_a)\right)$$

That is our governing equation that is a first-order ordinary differential equation. So, how many boundary conditions will it need? It will need, so, first order so, it will need just a single boundary condition.

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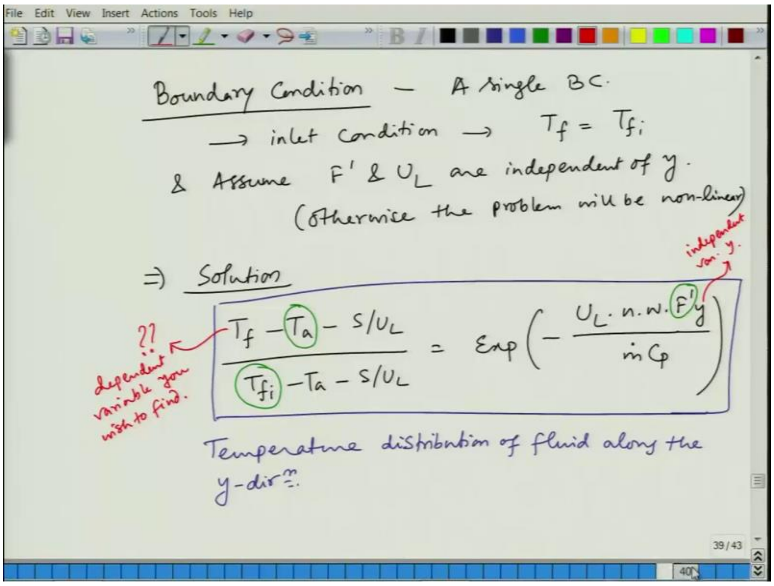

So, boundary condition will be a single one will be required single boundary condition will be required, and what is the most optimum or most easily obtainable boundary condition, that is the inlet condition. Now, the inlet condition, we can give that $T_f = T_{fi}$, which is nothing, but the inlet temperature of the fluid.

And, here we also assume that this F' and U_L if you see this governing equation we have two quantities here F' and U_L. For these two, we also need to assume that they are independent of y. So, F' and U_L are independent of y then only we can make it; otherwise it is a non-linear problem; let me write that as well.

So, that you can appreciate the importance of the assumption; otherwise, the problem will be non-linear, where the coefficients of the differential equation are also a function of the variable they are not constant. So, that is why the problem becomes non-linear.

So, if you find the solution again, I am not going to solve this. This is a very straightforward ODE with a first-order ODE with a known solution. So, I am just writing it here

$$exp(\frac{U_L * n * W * F'_y}{\dot{m} * c_p})$$

So, let me just put a box around it because this is again an important solution that will give you the temperature distribution along the y-direction for a fixed x. This is the temperature distribution of fluid along the y-direction. So, here you note that what important things you want to know, that this thing is the inlet condition which is the inlet temperature of the fluid.

And another thing you require is the boundary condition, or this is also a condition that is coming from the ambient, which is ambient temperature.

And, here from the x-direction, you have this quantity F' which is the collector efficiency factor, and you want to know that this is the variable that you want to know T_f, and this is the dependent variable, or this is the independent variable y, and this is the dependent variable that you want to know, you wish to find.

So, I again insist that you please go and do the actual derivation or actual solution of this differential equation, which will give you exposure to the methods, and also, you will recall all the techniques that you require to find the solution.

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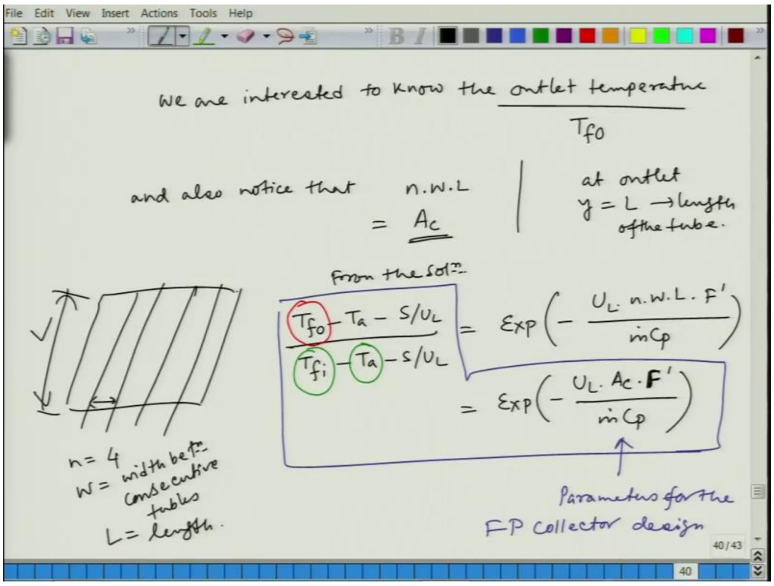

Now, we are interested in knowing the outlet temperature we are not really interested to know what is happening in between; rather we want to know I am giving you the fluid to get in getting entry to the flat plate collector at maybe ambient temperature, which is 30˚C and I want it out at 60 or 70˚C.

So, the outlet temperature is our target, and we need to know in terms of all the other quantities. So, this outlet temperature let us say it is T_{fo}; T_{fi} stands for the inlet temperature, and T_{fo} stands for the outlet temperature and also, notice that this $n * W * L$ that quantity we have it here in the expression itself.

So, at the outlet, our y will be equal to l, which is the length of the collector or length of the tube. So, if that is the case, then you will have here if you note here you will have this $n * W * L$, and what is that quantity $n * W * L$? That is nothing but your area of the collector.

So, suppose we have there are whatever four tubes. So, n is four, and this W is the width between them. So, W is the width between consecutive tubes and L is the length, L is the length. So, when you multiply $n * W * L$ that will give us the area of the collector, right. So, that gives quite a simplification here.

So, what can we write?

$$\frac{T_{f,0} - T_a - \frac{S}{U_L}}{T_{f,i} - T_a - \frac{S}{U_L}} = exp(-\frac{U_L * A_c * F'}{\dot{m} * c_p})$$

So, that is the expression we are after. So, here we have what we require, that is T_{f0} in terms of what is given T_{fi}, T_a, and all the right-hand side you have; those are parameters for that particular collector ok parameters for the collector or rather a flat plate collector design.

This is the overall loss coefficient, the area, mass flow rate, the specific volume, the specific heat of the fluid that you have chosen as the working fluid, and the collector efficiency factor F'.

So, in today's class, what we saw is we have completed the x-direction temperature distribution and connected it to the fluid temperature, and then we have taken a step to find out the y-direction or along the tube how the temperature will vary. And, by that, we have reached a point where we are in a position to find out the total temperature rise from the inlet to the outlet of the working fluid as a function of different parameters of the design of the flat plate collector.

So, this is an important juncture we will stop here for today, and in the next class, we will see that one more characteristic quantity like this fin efficiency f and this F' which is collector efficiency factor, we will introduce another quantity which is called collector heat removal factor and then we will go to this testing of these flat plate collectors.

So, I hope you enjoyed these connections between different parameters, and we will stop here and start in the next class.

Thank you.