Elements of Solar Energy Conversion

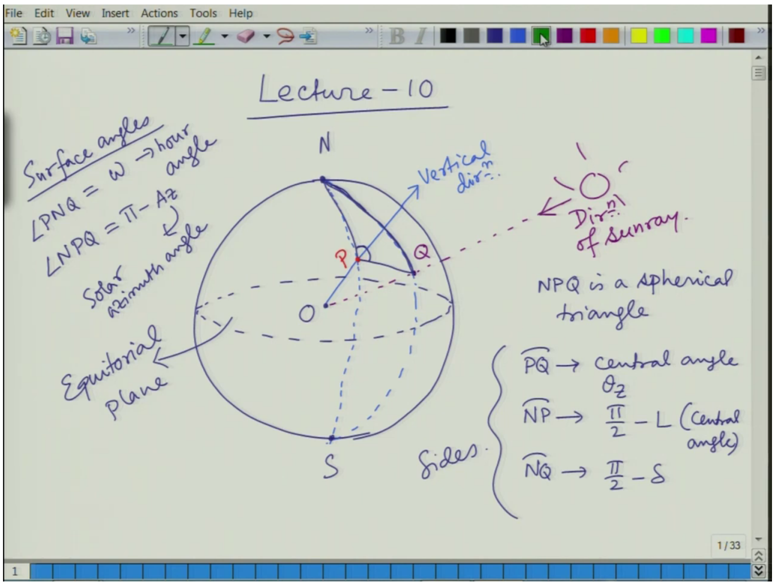
Prof. Jishnu Bhattacharya
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture – 10

Hello and welcome back to the series of lectures on Elements of Solar Energy Conversion. We are here at lecture number 10. So, in last 9 lectures, we have covered the basic concepts related to sun and earth relationship, the concept of time, then the role of atmosphere, how do we predict solar radiation availability, and what would be the component of diffuse and direct radiation all these things we covered.

Right now, we are looking at the important angles and their interrelationship, ok. So, we are halfway through it. Now, in this lecture, we are going to continue in that line and to derive few more very important relationships which will help us predict the angle of incidence on a tilted surface, ok.

(Refer Slide Time: 01:25)



So, let us start. We are here at lecture number 10, ok. So, last time where we stopped, there we have seen one spherical triangle where the sphere is our mother earth, ok. So, let me just do a little recap on that figure, ok. This is the center let us name it O and this is the equatorial plane. And here we have North Pole and here we have South Pole, ok.

Now, if we have a observer here then we can draw the meridian that passes through this observer location, ok. Let us name this point P, ok. And another where the sun ray reaches the cent or it the vector joining the sun and the center of the earth, ok. So, this is the direction of sun ray, direction of sun ray, ok. So, the point where it crosses the surface of the earth that vector, let us name that point Q. Again we can draw a meridian which passes through this point Q, ok.

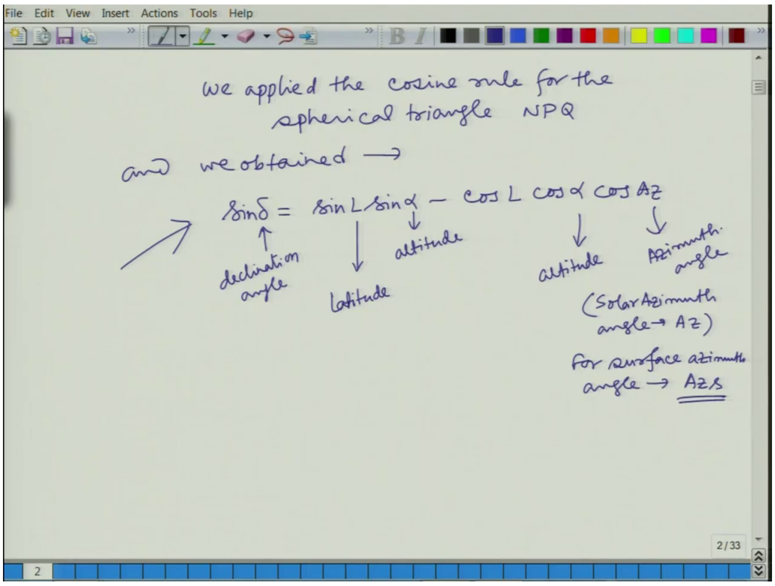
So, readily we have obtained one spherical triangle NPQ, ok. So, our NPQ is a spherical triangle, ok. So, what we have looked at is that the vector that goes from the center of the earth through the observer location that is the vertical direction, ok. So, here few important angles we already see, one is the curve arc on the surface of the sphere this PQ it represents the central angle theta Z, right. This is the zenith angle which is the angle between the vertical direction and the direction of sun rays, ok.

And what else do we see? The arc NP on the surface of the earth that is our that we can write to be pi by 2 minus latitude, ok. So, if you reach from N to the equatorial plane, that will be pi by 2, 90 degrees. And you are just subtracting L from it, so that NP is pi by 2 minus L that is also a central angle, ok. Now, this NQ similarly it is pi by 2 minus delta or the declination angle. So, from Q to the equatorial plane that will give you delta. So, NQ is nothing, but pi by 2 minus delta, ok.

So, all the sides of the triangle we have written. Now, we need to point out what are the surface angle. So, if we say surface angles, one is readily observable that is the surface angle of PNQ that is nothing, but our hour angle, ok. And the other angle that we see which is NPQ that is nothing, but pi minus A Z, A Z is our solar azimuth angle, ok.

So, how all these angles are we are getting that we have seen in the last lecture. So, if you are confused at this particular figure and the descriptions of the angles, please refer back to the lecture number 9 where all the explanations are given. But here just for a summarized description I am repeating it little bit, so that the continuity still remains, ok.

(Refer Slide Time: 07:46)



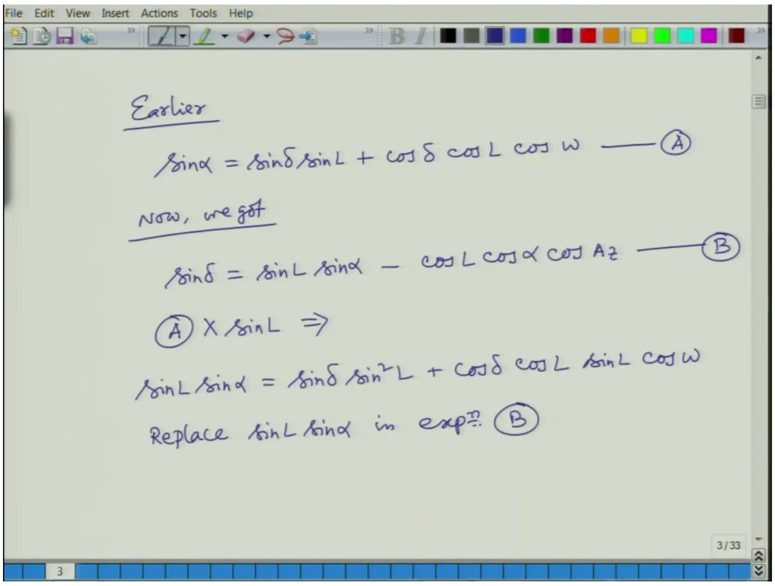
Now, with all these angles what we did we applied the cosine rule for the spherical triangle NPQ, and we obtained the following relationship. How we are getting it is covered in the lecture number 9. So, what we have done, let me write that only we obtained this particular relationship, ok. And again, I want to stress that all these angles they have a particular meaning, ok. And those designations you have to remember, so that you can see the whole picture clearly in the eye of your mind, ok.

So, let me just quickly tell you what are the angles this is the declination angle, ok, this is latitude, this is altitude. So, here you see that we have taken lot of care to find out this numeric or this later designations, like declination is delta. It makes sense, right. Latitude starts with L and the angle is referred as L. And alpha, alpha is the Greek letter for A and it represents the altitude angle, ok.

And here again this is altitude angle, and A Z is azimuth, A Z azimuth that also makes sense, right. Azimuth angle and here you have to remember for solar azimuth angle we do not use any subscript, ok, it is just A Z, ok. And for surface azimuth angle we use a subscript s, ok.

Now, S stands for, I mean it starts I mean it is at the start of solar as well as surface, but still we have to choose one, so we have chosen that S will be there for surface not for solar azimuth angle, ok. So, up to this or up to this relationship we have seen in the last class, ok.

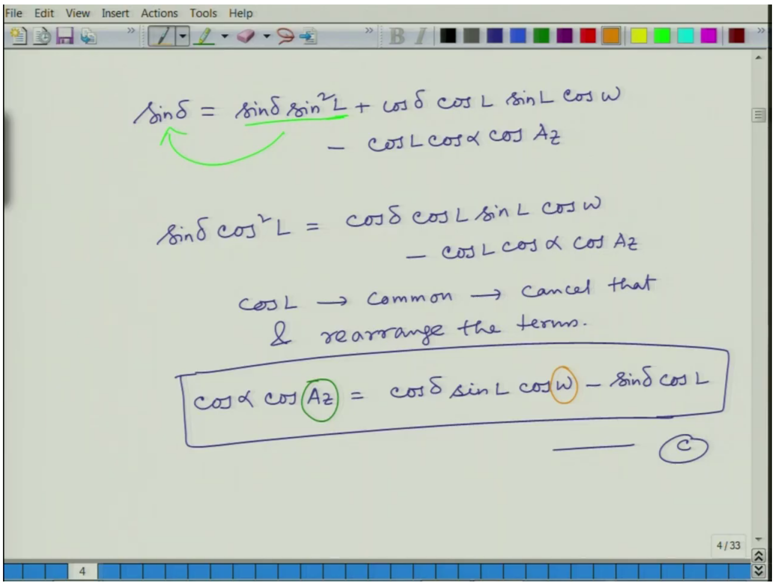
(Refer Slide Time: 10:57)



Now, earlier what we have obtained? If you look at the earlier notes what we obtained is this relationship. Cos delta, cos latitude, cos hour angle, this we have seen earlier. Let me name this a relationship A. And now, what we got? Sin declination is equal to sin latitude, sin altitude, minus cos latitude, cos altitude and cos of solar azimuth angle. Let me name this relationship B, ok.

Now, if you do a multiplication of A with sin L then that will give you sin L, sin alpha equal to sin delta, sin square L plus cos of declination, cos of latitude, sin of latitude and cos of hour angle, ok. So, now what we can do? We can replace this now replace sin L sin alpha in expression B, ok.

(Refer Slide Time: 12:58)



So, what we get is, sin delta is equal to sin of declination, sin squared of latitude plus cos of declination, cos of latitude, sin of latitude, cos of hour angle minus cos of latitude, cos of altitude and cos of solar azimuth angle, ok. Now, here you see, do you see some connection here? So, if you bring this particular term to the left hand side. What will you get?

We will get sin delta outside and within the parenthesis you will get 1 minus sin square L, right. So, if that is the case 1 minus sin square L will give you cos square L. So, this will give you that sin delta into cos square L will be equal to cos of declination cos of latitude, sin of latitude, cos of hour angle minus cos of latitude, cos of altitude and cos of solar azimuth angle, ok. This is simple.

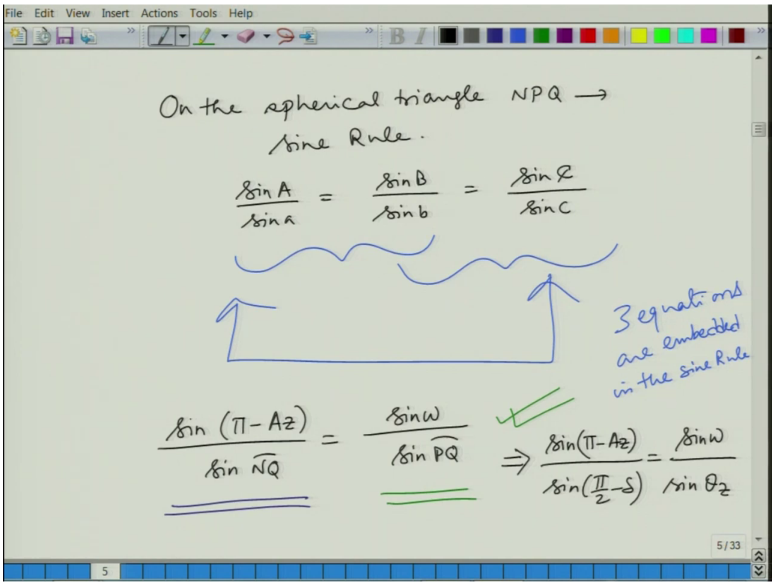
So, now what we can write is we can you can see that cos of latitude is common to all 3 terms. So, we can just get rid of that, ok. So, cos of L is common. We can get rid of that. And what we can do? We can bring or rearrange. So, you can cancel that and rearrange the terms.

What will you get? You get cos of altitude cos of azimuth or solar azimuth will be equal to cos of declination, sin of latitude, cos of hour angle minus sin of declination, cos of latitude, ok. That is another important angle relationship which will name at name as C, ok.

So, here you note that we what was our original motivation. We wanted to make the analysis completely general, right. And the hour angle is a local thing we wanted to connect it to the azimuth angle or solar azimuth angle. This is one relationship where you can actually see that is happening. This is the solar azimuth angle and this is the hour angle, right. So, this is a relationship where you can actually see the connection between them, ok. But we are not done yet, we will continue working on this relation, ok.

Now, let me go back to this figure again. This is the triangle or spherical triangle that is giving us all these fantastic relationships, ok. But this here we have only considered the cosine law, cosine rule. Now, we can use that is the sin rule, and that is what we are going to use now.

(Refer Slide Time: 17:10)



So, on the spherical triangle NPQ we will use the sine rule, ok. And sine rule you can remember that this will be sin A by sin small a sin capital B by sin small b and sin capital C by sin small c, right. So, either of that pair gives you a an equation. So, either this one or that one or between these two terms. So, 3 pair are embedded in the sine rule. Do you agree? Yes.

Now, so we are going to use just one in this case. So, let me write the equation first and then I will explain why we are writing it, ok. This is the relationship that we will obtain from the sine rule. So, first look at the left hand side term. What we are getting? This is sin pi minus A Z divided by sin of curve NQ, ok. So, if you go to the equation or this figure. So, what you can see here that pi minus A Z is this angle, right, on the surface and NQ is the opposite side, ok.

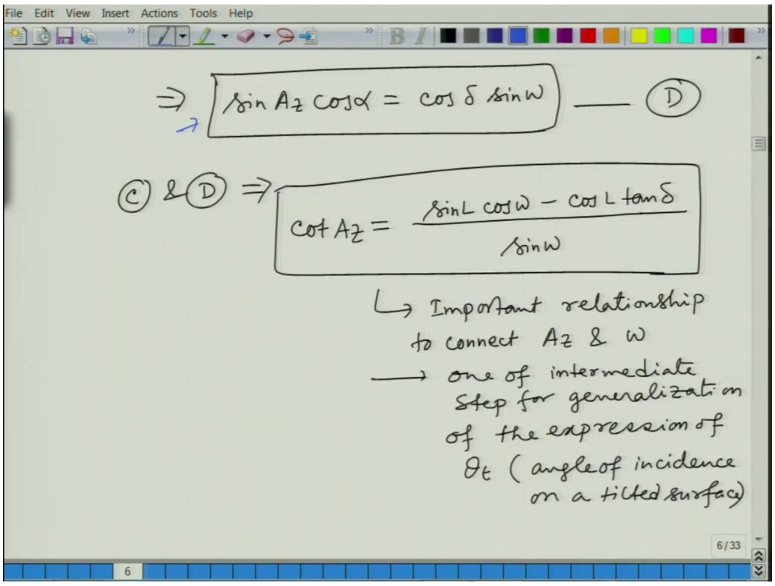
So, that is what sine rule says, right sin A by sin small a. And for the other one let me see what was the relation here the second term that we have we are equating to that is sin omega divided by sin PQ. So, if you go to that triangle omega is this angle, right, this is omega, and the opposite side to that is this PQ.

Again, that makes sense, right. So, please check whether you agree with me on this sine rule or not, ok. So, go back to the figure and look at the angles their nomenclature and ensure that you follow what I mean by this equation, ok.

So, if this is true then what we get here, we can simplify this or we can put the values pi minus A Z and NQ is nothing but pi by 2 minus delta. Again, you go back to the figure and ensure that you follow why I wrote NQ equal to pi by 2 minus delta, ok. And here we have sin omega divided by sin theta Z. So, PQ the curve PQ makes an angle theta Z or the zenith angle at the center of the earth or the center of the sphere on which we are considering the spherical triangle.

So, this is lot of visualization you have to do. So, I insist that you please close your eyes stop this video, close your eyes and think of all these spherical triangle angles, ok. It is very very important and that feeling has to be carried even after the course is over, ok. So, let me continue.

(Refer Slide Time: 21:37)



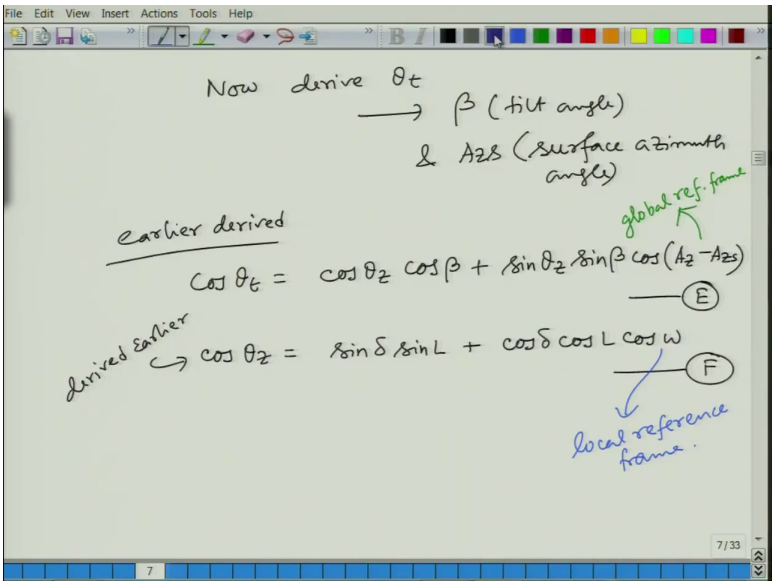
So, once we have that relationship what we can write the simplification of that will give us sin A Z cos alpha will be equal to cos delta into sin omega, ok. And that just gives you a easier form for the same relationship and let me name that D. So, we have 4 relationship developed A, B, C, D.

And if you combine C and D, you will get another important relationship which singles out the solar azimuth angle in cotangent form, and that is sin of latitude cos of hour angle minus cos of latitude, tangent of declination angle and divide whole divided by sin of hour angle.

So, that is another very important relationship, important relationship to connect the solar azimuth angle and the hour angle, ok. So, and it is one of the important intermediate step for generalization of the expression of theta t. Theta t is what? The angle of incidence on a tilted plane on a tilted surface, ok.

So, I am just reminding this, so that with all these different relationship coming from here and there and you may just get lost about where what we are heading to, ok. We are heading to a general expression for theta t or the angle of incidence on a tilted surface, ok.

(Refer Slide Time: 24:31)



So, let us continue that. Now we need to derive this expression of theta t. So, whenever we talk about a tilted surface we have to bring in beta, right, that is the tilt angle, tilt angle. And what else? The solar surface azimuth angle, ok, surface azimuth angle without them it cannot happen, right. So, these are basically specifications where or how the surface is lying, ok.

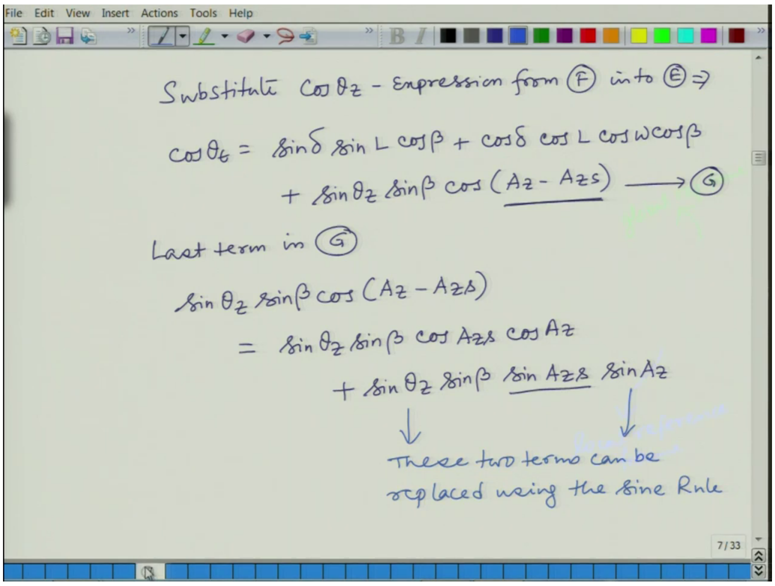
Now, earlier we have seen, earlier derived that cos of theta t is cos of the zenith angle cos of tilt angle plus sin of zenith angle sin of tilt angle and multiplied by cosine of this difference between the solar and the surface azimuth angle, ok. So, please look back at the earlier notes and ensure that you follow this relationship.

We have obtained this from the when we considered a spherical triangle where the center is at the observer location, ok. There we have obtained this. So, let me name this relationship as E, otherwise it will be very difficult to track which relationship is going where, ok.

And another relationship again we obtained earlier that is the cosine of zenith angle which is sin of declination angle multiplied by sin of latitude plus cos of declination angle, cos of latitude and multiplied by cos of hour angle, ok. So, again we have derived earlier this particular relationship, ok. And let us name them, name that relation to be F, ok.

Now, you see that here this solar and surface azimuth angle, they make it compatible with global reference frame, ok. And this hour angle, this makes compatible with the local reference frame, ok.

(Refer Slide Time: 27:55)



So, now what we will do? We will combine them two and we will substitute cos of theta Z expression from the relationship F into the relationship E, ok. If we do that, what will we get?

That cos of this angle of incidence on a tilted plane that will be equal to sin of declination multiplied by sin of latitude cos of tilt angle plus cos of declination, cos of latitude, cos of hour angle multiplied by cos of tilt angle plus sin of zenith angle multiplied by sin of tilt angle and multiplied by the difference between the solar and surface azimuth angle cosine of that, ok.

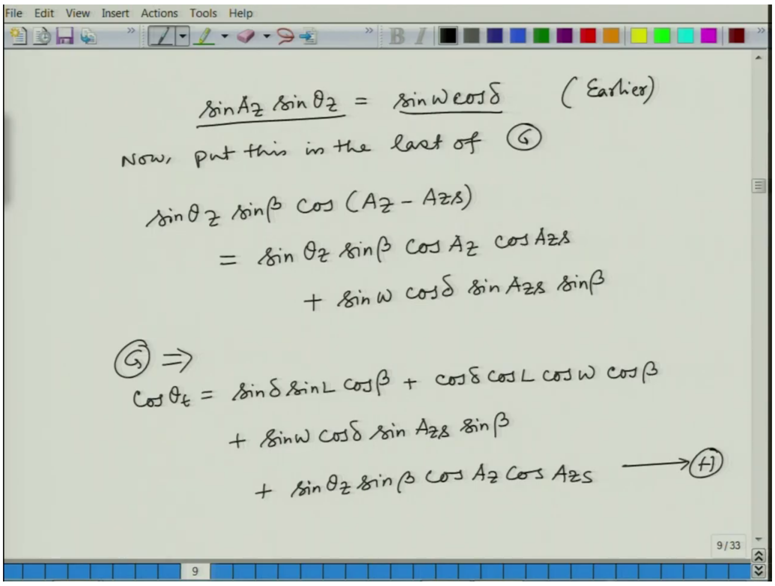
So, all of these steps I am writing it, but I insist that you do it on the notebook you have and ensure that all these steps you agree with it, ok. So, now this long relationship let me name this G, ok. Now, if we look at the last term in G this requires special attention because it has a difference term, right. So, this angle is difference between two terms. So, we need to expand that. So, that we get a general relationship, ok.

So, if we do that expansion what we can write? Sin of zenith angle multiplied by sin of tilt angle multiplied by cosine of difference between solar and surface azimuth angle. This is the last term for the expression G, right.

And this will be equal to sin of zenith angle, sin of tilt angle multiplied by cos of surface azimuth angle and cos of solar azimuth angle plus sin of zenith angle, sin of tilt angle multiplied by sin of surface azimuth angle and multiplied by sin of solar azimuth angle, ok. So, that is what we are getting now.

Now, you can see that these two equations. So, you can see that this term and this term are actually connected to each other through the sin rule, ok. So, these two terms can be replaced using the sine rule we just derived that expression. So, let me go back to that sine rule. So, here you see that the sin of A Z this expression can be taken from here, ok.

(Refer Slide Time: 32:17)



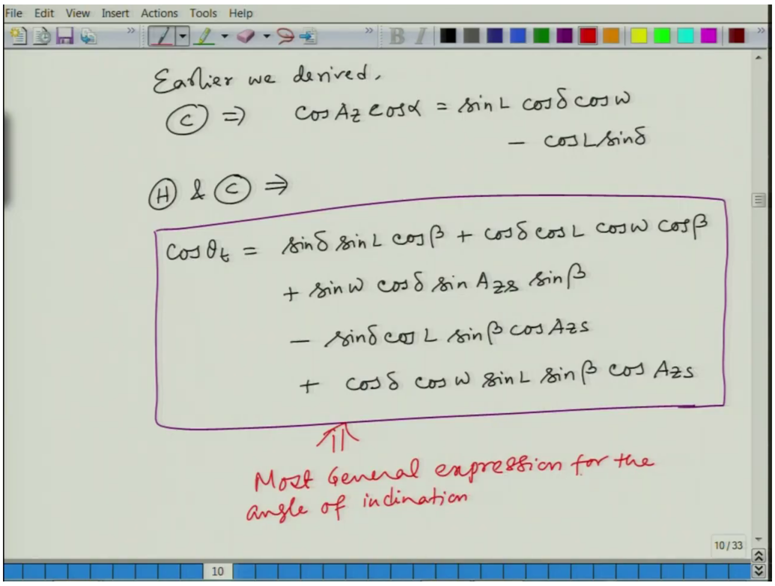
So, what expression we had? That sin of A Z multiplied by sin of theta Z this will be equal to sin of omega into cos of delta this we have derived earlier two lectures ago I think this we derived, ok. So, now put this in the last term of G, ok. So, what we can write?

The sin of zenith angle multiplied by sin of tilt angle, cos of solar minus surface azimuth angles this will be equal to sin of zenith angle, sin of tilt angle, cos of solar azimuth angle, cos of surface azimuth angle plus sin of hour angle cos of declination angle multiplied by sin of surface azimuth angle and sin of tilt angle, ok. So, we have just replaced this with this, ok.

So, basically the expression G is now converted into the following cos of theta t or the angle of incidence on the tilted plane. This will be sin of declination angle, sin of latitude angle, cos of tilt angle plus cos of declination angle, cos of latitude, cos of hour angle and cos of tilt angle plus sin of hour angle, cos of declination angle sin of surface azimuth angle multiplied by sin of tilt angle.

And the last term is sin of zenith angle sin of tilt angle, cos of solar azimuth angle into cos of surface azimuth angle, ok. So, a pretty long expression, but let us name this expression H, ok.

(Refer Slide Time: 35:21)

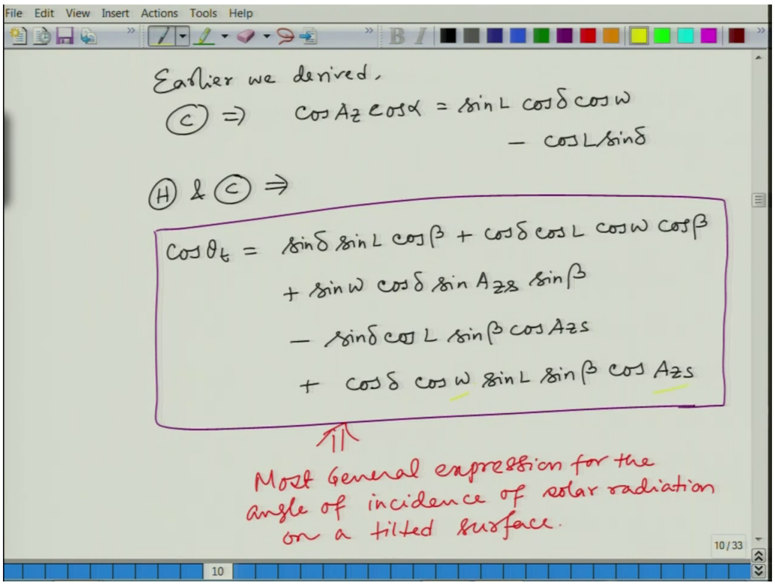


Now, earlier we derived and name it to be C. So, the expression C actually this gives us cos of solar azimuth angle into cos of altitude angle is equal to sin of latitude, cos of delta or declination, cos of hour angle minus cos of latitude sin of delta, or declination angle. This we have named to be the expression C. Now, if you combine H and C you will get the most general form of the expression of theta t.

So, let me write it cos theta t is equal to sin delta sin L cos beta plus cos delta cos L cos omega into cos beta plus sin omega cos delta sin A Zs which is surface azimuth angle multiplied by sin beta. And then minus of sin delta cos L sin beta cos of surface azimuth angle plus cos of delta cos of omega sin of L sin of beta into cos of surface azimuth angle, ok.

So, this is the expression of course. It is very long expression. I will say all kinds of angles are there and it is difficult to remember and I do not want you to remember either. Whatever you can look up you should not remember because your memory or your brain has better things to do. You understand and try to apply it in real problems, without trying to remember it in total.

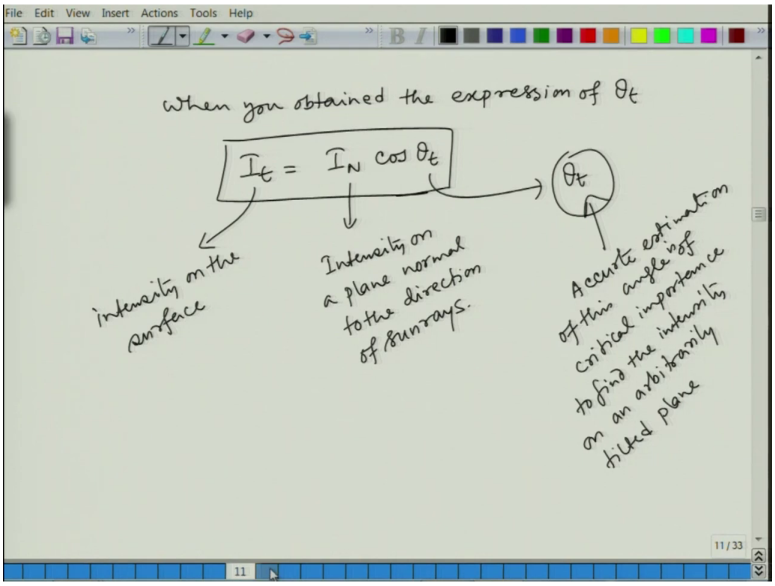
(Refer Slide Time: 38:08)



So, this is the most general expression for the angle of inclination, inclination or rather angle of incidence of solar radiation on a tilted surface, ok. Here you see that we have all kinds of angles. Particularly you notice that it does have the hour angle, and you have the surface azimuth angle. Usually, when we have an expression involving hour angle we do not need the involvement of the solar azimuth angle because they basically tell you the same information, ok.

So, but surface azimuth angle that is surface specific, so you have to have that. So, basically this is where we are looked at. Now, once you have obtained this theta t, why we are obsessed about deriving such a long expression? Ok.

(Refer Slide Time: 39:54)

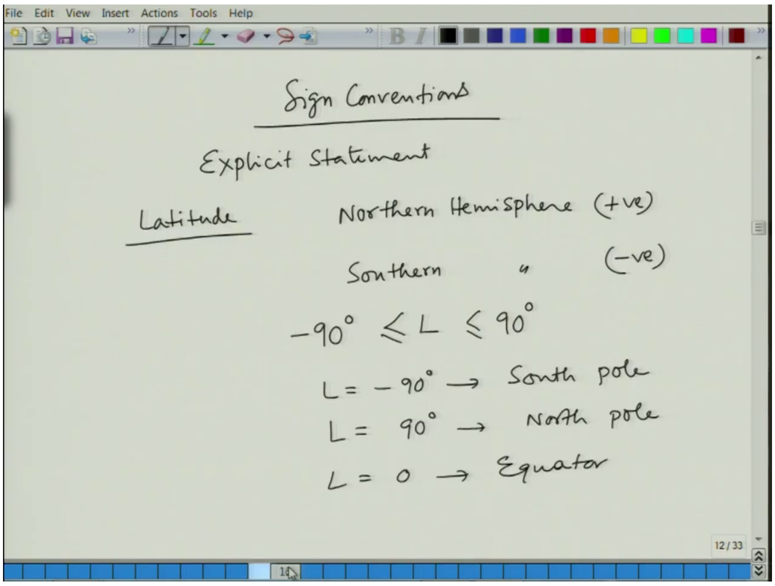


Because whenever when you obtained the expression of theta t you are at great that is that relationship is of great use because now you can find out the intensity on the tilted plane, right. So, it is just the normal intensity multiplied by cos of theta t, ok. So, that is such a useful relationship, right because this is the intensity on a plane which is normal to the direction of sun rays, ok. N stands for that. You have seen I O N was the extraterrestrial normal radiation intensity, ok.

So, here O is not there because it is not extraterrestrial we are talking about a surface which is on the surface of the earth, ok. That is why now O is there. But N still remains because its normal to the direction of radiation and this is the intensity on the surface that may or may not be perpendicular, right. It can have any other orientation and the linkage between this I N and I t is coming through this theta t which is the angle of incidence, ok.

So, that is why we are obsessed about this and we have to find this very accurately. Accurate estimation of this angle is of critical importance to find the intensity on an arbitrarily tilted plane, ok. That is why we spend so much time on this.

(Refer Slide Time: 42:52)



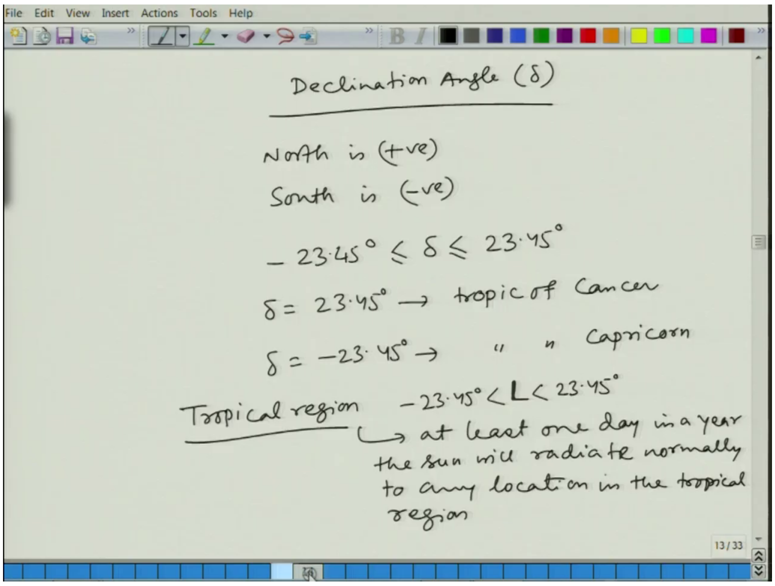
Now, let me look at few of the sign conventions of this different angles. So, far we did not talk about the sign conventions because we assume tacitly assume these sign conventions. So, now, will explicitly state it. Explicit statement is required because we will not be restricted to the northern hemisphere or some particular time of the day. So, all these things will be changing, right.

So, that is why we need to state them clearly where will we set the boundaries, which part will be positive, which part will be negative otherwise all these relationships will make you make the whole analysis a mess unless we designate that boundaries, ok. So, first is the simplest one the latitude. Latitude you know that northern hemisphere is taken positive, and southern hemisphere is taken negative.

This is just because the humanity the portion of us who live in the northern hemisphere they actually started looking at these issues that is why they have taken it to be positive, nothing else, ok. So, that is how the latitude is designated. Now, what can be the values? So, this L can be minus 90 degree and plus 90 degree, ok. So, when L is minus 90 degree, that means, you are on the South Pole, ok.

And L is positive 90 degree; that means, you are at North Pole, ok. And L 0 means what? You know this, right. This is equator, ok. So, that is the sign convention for latitude. I guess you were familiar with this particular sign convention.

(Refer Slide Time: 45:37)

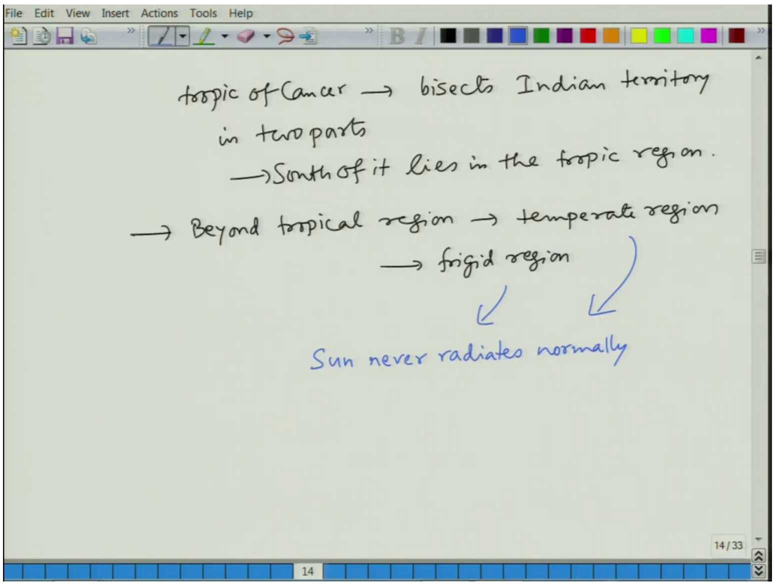


Now, we look at the declination, declination angle. This also we have talked about in details where when it is positive when it is negative and let me still say that again north is positive and south is negative. And what are the limiting values? Limiting values are 23.45 degree to; I mean negative 23.45 degree to positive 23.45 degrees, ok.

And when delta is 23.45 positive we call that line the tropic of Cancer and when it is negative 23.45 degrees we call it tropic of Capricorn, ok. And within these two tropics we call it tropical region is this 23.45 L. So, latitude is between positive 23.45 degrees and negative 23.45 degrees, that region we call tropical region, right. So, tropical region means what? At least on a single day of the year the sun will radiate normally to that particular location.

So, at least one day in a year the sun will radiate normally to any location on the tropical region or rather in the, ok. So, in our country the tropic of cancer actually bisects us in north and south, ok.

(Refer Slide Time: 48:40)

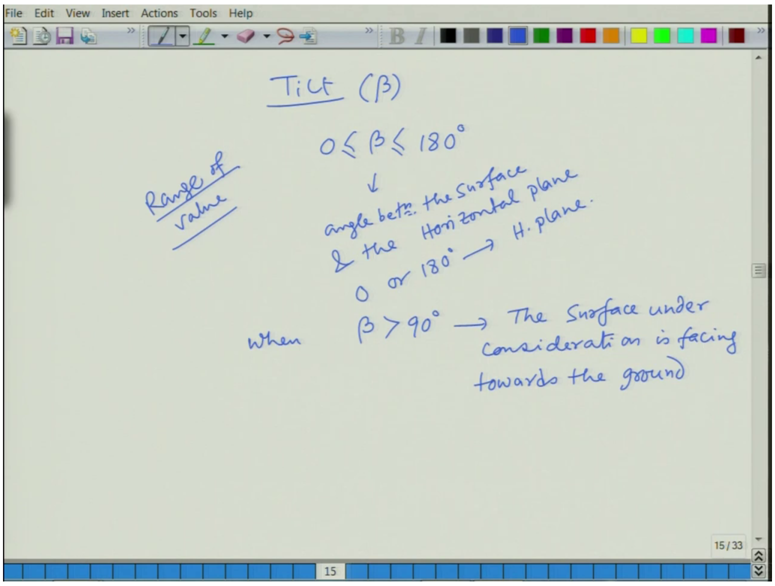


So, tropic of Cancer bisects Indian territory in two parts. So, south of it lies in the tropical region. So, I am standing here at Kanpur and Kanpur is just above the tropic of Cancer. So, below you can say below Kanpur or below Jhansi all the part of India that is in the tropical region, but above that like in Kanpur in Kashmir everything is in the temperate region, ok.

So, beyond tropics or beyond tropical region we have temperate region and above 66 degree north or below 66 degree south we have frigid region near the poles, ok. So, this is basically, gives you some idea about this declination angle. And so, for these two regions for temperate and frigid sun never radiates normally and that makes sense, right because whenever sun radiates normally you have the maximum intensity.

And those are the tropical regions are the hot part of the world and the cooler part of the world where the sun does not radiate normally any time of the year those are temperate and frigid regions, ok.

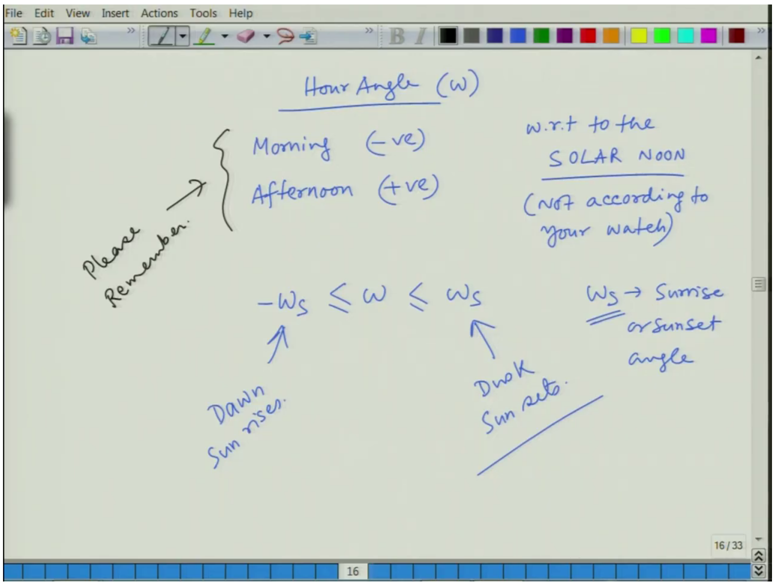
(Refer Slide Time: 51:04)



So, let us look at other angles. Other important angle is tilt angle which is designated as beta and it can have this particular range, right. So, when you have tilt angle tells you what? It tells you the angle between the surface and the horizontal plane, right. So, when it is 0 or 180 rather you can say it is horizontal plane itself, and in between it can go up to 90 degree and then it goes till 180 degree. So, when beta is greater than 90 degree, it means the surface under consideration is facing towards the ground, ok.

So, that does not make much sense because we want the collector surface to face the sun not the ground, ok. So, that is why usually the tilt angle is between 0 to 90 for most solar collector configurations, but for some particular typical optical arrangement you may have a situation where the surface is looking downwards against the sun, ok. So, that is where you need that is where you will get beta to be more than 90 degree. But it cannot go less than 0. So, rather the sign convention I would say the range of value, ok, fine.

(Refer Slide Time: 53:35)



What else do we have? We have the hour angle which is very important to have the sign convention ingrained into your head, ok. For hour angle we have morning to be negative, and afternoon to be positive. And whenever we call morning that means, with respect to the solar noon, not with respect to your watch, ok. So, all of these with respect to the solar noon not according to your watch, ok.

So, that is a very important point you have to note. With respect to the solar noon if you are before that particular time then your hour angle, hour angle is designated with omega, omega will be negative and if it crosses the solar known it will be positive. And what would be the value of that? That will be your sunrise or sunset angle, ok.

So, omega s is your sunrise or sunset angle. And in the morning, you can see that this is the limit this you can say the dawn or when the sun rises and this positive in the afternoon that is the dusk or when the sun sets, ok. So, that is a very important. So, please I usually do not tell you to remember much, but these things you need to remember to build your intuition that is why the hour angle morning is negative, important piece that you please remember, ok.

So, we will continue from here in the next class. Today, let us stop here. We have looked at the most general expression for the tilted angle incidence and then we have talked little bit about the sign conventions of different angles, ok.

Thank you very much for your attention.