**Introduction to Composites**
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**Lecture - 70**
**Quasi-Isotropic Laminates: Part-I**

Hello, welcome to Introduction to Composite Materials. Today is the 4th lecture of the ongoing week. And today we will introduce another lamination sequence and that particular lamination sequence is known as a sequence for Quasi-Isotropic Laminates. So, that is the theme of our discussion, quasi isotropic laminates is what we want to discuss today.

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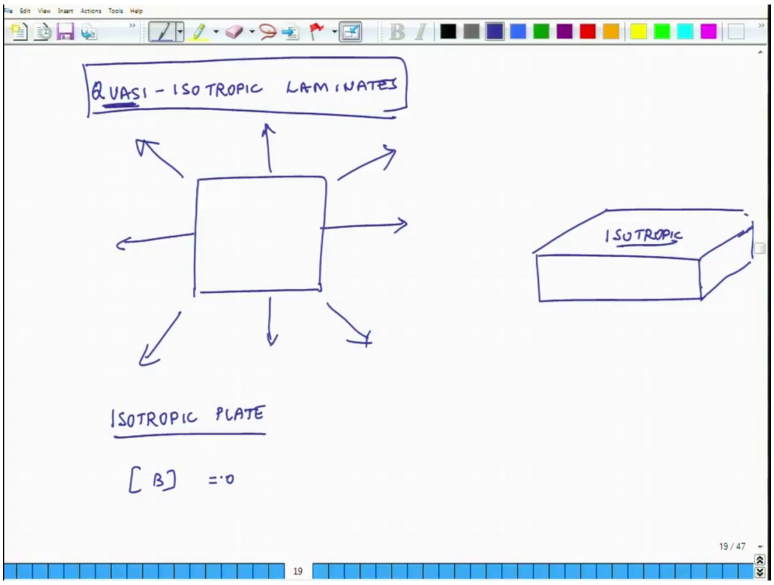


Now, here the purpose of the discussion or detailed analysis to somehow figure out a smart way such that the properties of the laminate not individual layers of the laminate are similar to that of isotropic materials in the in plane direction in the in plane direction.

So, what does that mean? What that means, is that if I have a laminate and if I pull the laminate in this direction or if I pull it in this direction or if I pull it in this direction or any direction for all like here about, then the behavior of the laminate should be same as that of an isotropic material, that is what we want to achieve. Because if that is the case then this material laminate will have equal stiffness in all directions and most likely if it will have equal strength in all directions. So, it will be reliable in terms of its performance regardless whatever direction of load it is facing.

Now, this discussion I wanted to again re-emphasize is in context of only in plane response, we are not talking about the plate is going to have similar response when it is being bend in different directions. So, when we are talking about quasi isotropy we are talking about the behavior of the plate in the in plane direction, regardless of whichever we direction we pull it it will extend by the same amount. So, that is what the in plane response implies. And that is why we are talking about quasi, and it is quasi because we are only worried about the in plane response we are not talking about the bending response of the system.

Now, if we look at an isotropic plate. Let us look at how it what kind of; so an isotropic plate will also have an A matrix, B matrix, D matrix ok. So, if you have a plate and it is made up of isotropic material, excuse me. So, this is, so let us say this is an isotropic plate you can also generate it is A, B and D matrices ok. If the plate is of uniform thickness then its B matrix an and the material is homogeneous throughout the thickness then because there is same material on top and bottom of the mid plane B matrix for this things is going to be 0, ok.

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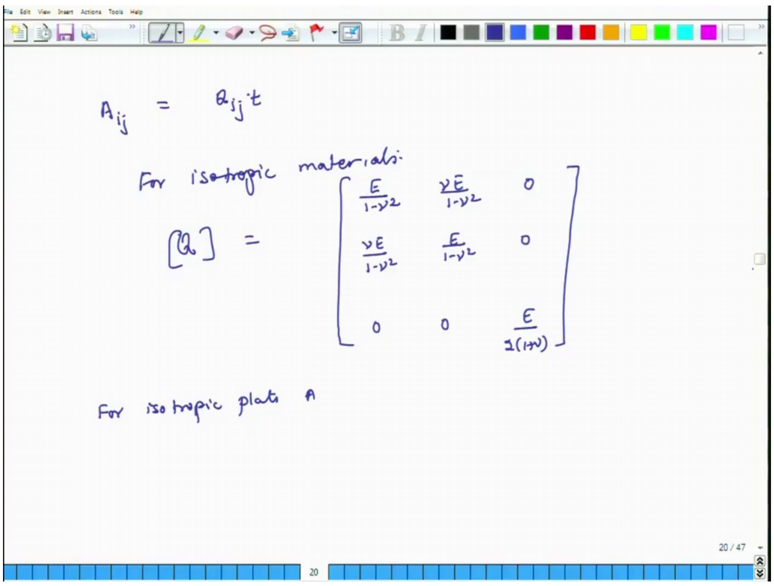


And then what about the A matrix? So, the A matrix elements can be defined as Q ij times its just single layer because its one single material, so thickness of the plate, right.

And what are the elements for an isotropic material for the Q matrix? So, for isotropy for isotropic materials let us look at whatever is what is the Q matrix. Q is equal to E divided by 1 minus nu square. So, this is something we have discussed earlier and then the second element is nu E divided by 1 minus nu square, third element is 0 nu E divided by 1 minus nu square E divided by 1 minus nu square 0, 0 0 and E divided by 2 times 1 plus Poisson's ratio. So, this is how the Q matrix looks for an isotropic material. And we can use these elements to compute elements of the A matrix for an isotropic plate. So, for isotropic plate A is nothing as, but we just multiply this by the thickness which is t. So, then we get. So, I will just write it down.

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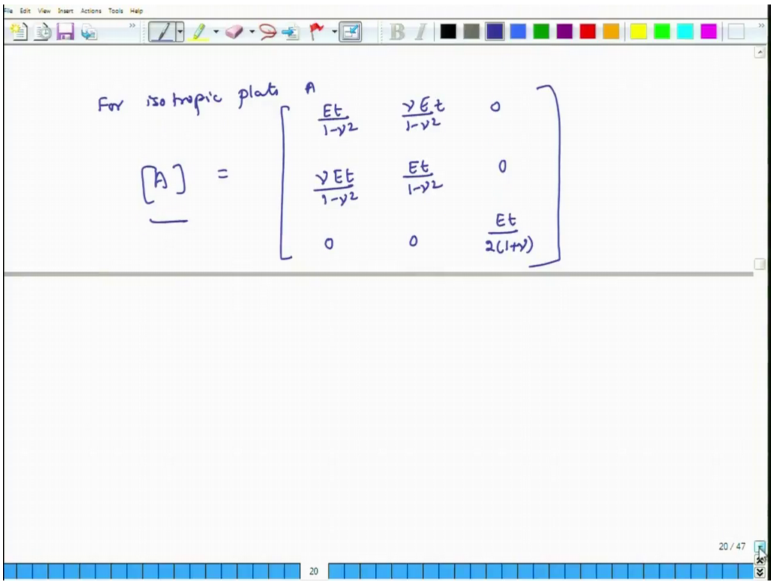


So, it is E times t divided by 1 minus Poisson square 1 minus Poisson square 0 0 there is a t here, Et divided by 1 minus Poisson square, 0 0 E t divided by 2 times 1 plus nu. So, this is the A matrix for an isotropic plate.

What we want is a lamination sequence where the matrix looks something like this where the matrix looks something like this, ok. So, suppose, this is an isotropic plate this is an isotropic plate and suppose there is a composite place; so let us say, let us say that a composite layer composite laminate not layer is quasi isotropic.

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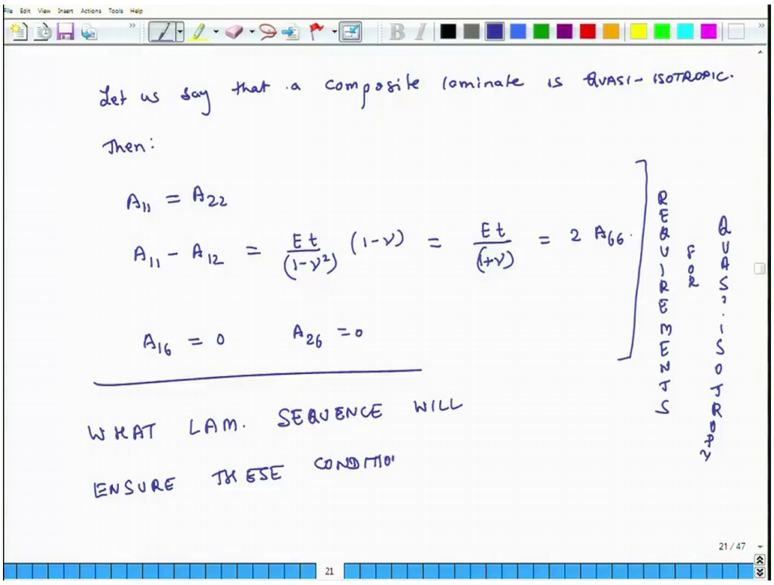


Let us assume that we have put layers in such a way that it is indeed quasi isotropic and if it is indeed quasi isotropic then A 11 for that material will be same as A 22. So, if it is indeed quasi isotropic then we can say A 11 will be same as A 22 because that is the condition in isotropy situation right, so A 11 is same as A 22. The other thing we see it is that if we look at A 11 minus A 12, if you take the difference A 11 minus A 12 what do we get? We get Et 1 minus nu square times 1 minus nu and this 1 minus mu cancels out from numerator and denominator. So, it is E t divided by 1 plus nu and this is nothing, but twice of A 66 it is twice of A 66.

So, if there is indeed a quasi isotropic plate what we have to ensure is that its A 11 should be same as A 22 and the difference between A 11 and A 12 should be twice of A 66. And the third thing we have to ensure is that A 16 should be 0, and A 26 should be 0 ok. So, these are the requirements. These are the requirements for quasi isotropy; these are the requirements for quasi isotropy.

So, the question is what kind of a lamination sequence I have to generate which ensures that A 11 is equal to A 12 the same lamination sequence should also give me a situation where A 11 minus A 12 should equal twice of A 66 and A 16 and A 26 should be 0, ok. So, the question is what lamination sequence will ensure these conditions. So, people have looked at this question and if you do some little bit of mathematical analysis you come up with the answer.

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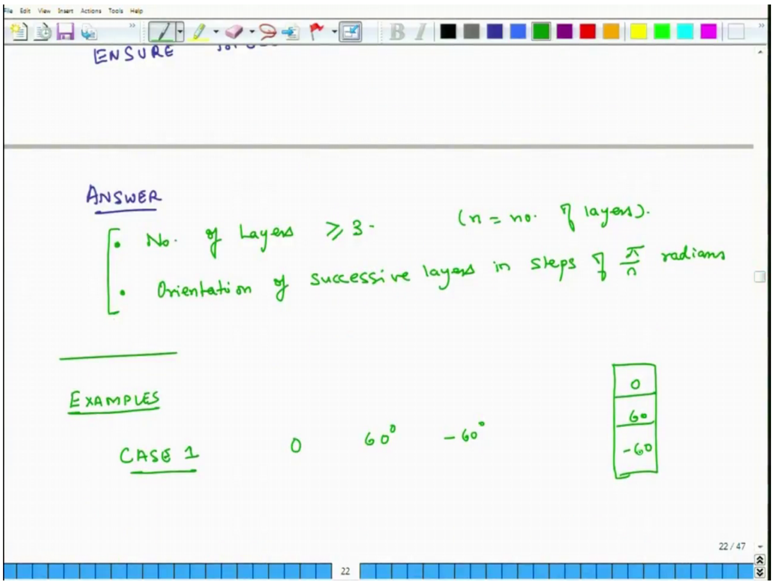


And the answer is having 3 parts ok, actually just 2 parts and both of them have to be satisfied. First one is number of layers should be more than or equal to 3 that is one, and the second one is that orientation of successive layers should be in steps of pi over n radians. So, number of layers, so n is number of layers n is number of layers ok. So, if you meet these two conditions you will ensure that there is quasi isotropy and the plate will behave as a quasi isotropic material in the in plane direction out of plane we still have to worry about it, but in plane it will ensure that; so examples, examples ok.

Case 1 you can have first layer at 0 degree, so minimum number of layer should be 3. So, it will have 0 then the second layer could be at 60 degrees and the third layer could be at 120 degrees or you can also call it as minus 60 degrees 120 is same as minus 60, ok. So, you can have a 3 layer laminate first layer 0, 60, minus 60 ok, first layer is 0, 60, minus 60 and when you do this it will ensure that. So, for this lamination sequence these conditions will be satisfied. A 16 and A 26 will be 0 because you have 60 and minus 60, and when you do all the calculations you will figure a find out that A 11 minus A 12 will be equal to twice of A 66 it will actually work out ok. But a problem with this lamination sequence is that it is not symmetric, so B will be nonzero. So, you can have another example.

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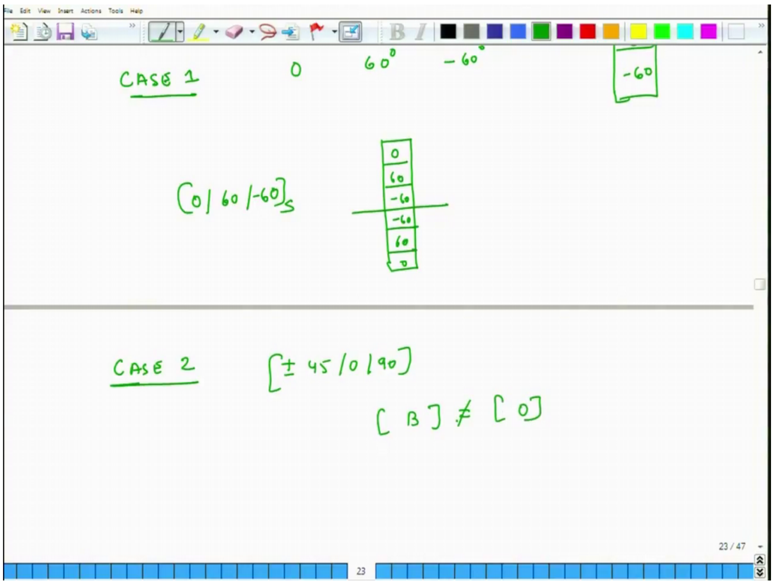


So, you can have 0, 60, minus 60, minus 60, 60, 0. So, this is essentially 0, 60, minus 60, symmetric and in this case B will be also 0 and it will still be quasi isotropic because A matrix only depends on the thicknesses A matrix only depends on thicknesses. So, the same sequence is being repeated. So, the quasi isotropic will be retained.

The other thing is that in this case because 60 and minus 60 are next to each other and minus 60 and 60, D 16 and D 26 will also be a little bit less we are talking about the group. So, the same group can be repeated another case. This is plus minus 45, 0 90 ok. So, you have 4 layers, 0 degree layer, 90 degree layer, plus 45 degree layer and minus 45 degree layer. So, here also we will get quasi isotropy, but B will not be 0. So, if you want a symmetric B to be also 0 then what you do is you go to plus minus 45, 0, 90 symmetry. And what that will do is that B will be 0 and also D will be somewhat less I mean B is less here also because there is a plus 45, sitting next to a minus 45, but the third thing you can do is.

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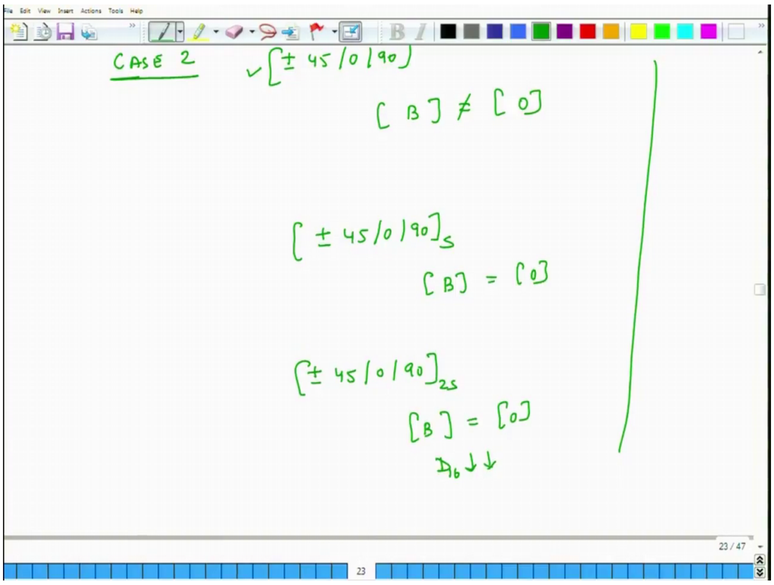


Student: B 0 B equal to (Refer Time: 15:50).

Sorry B is equal to 0. The third thing you can do is plus minus 45, 0, 90, 2 symmetric and here D will become even less and D 16 goes down further because it is twice symmetric ok. So, this is how you achieve quasi isotropy. But quasi isotropy does not have to be symmetric, but if you have symmetry then B also becomes 0 which is an added advantage.

So, this is what I wanted to discuss today in context of quasi isotropy. Tomorrow we will actually do an example for quasi isotropy, and in that example we will see how we can predict the Young's modulus of a randomly oriented fiber composite using this principle of quasi isotropy. So, that is something we will discuss tomorrow, and then we will also discuss some other topics as well. So, thank you and we will meet once again tomorrow.

Thanks.