Introduction to Composites
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Lecture - 69
Simplification of Stiffness Matrices: Part-II

Hello, welcome to Introduction to Composites, today is the third day of the ongoing week; and today, we will continue our discussion which we had yesterday related to coupling responses in composites. And specifically what we have going to do today or discuss today is; the coupling between the bending response of a plate and the twisting response of plate. So, that is what we plan to discuss today.

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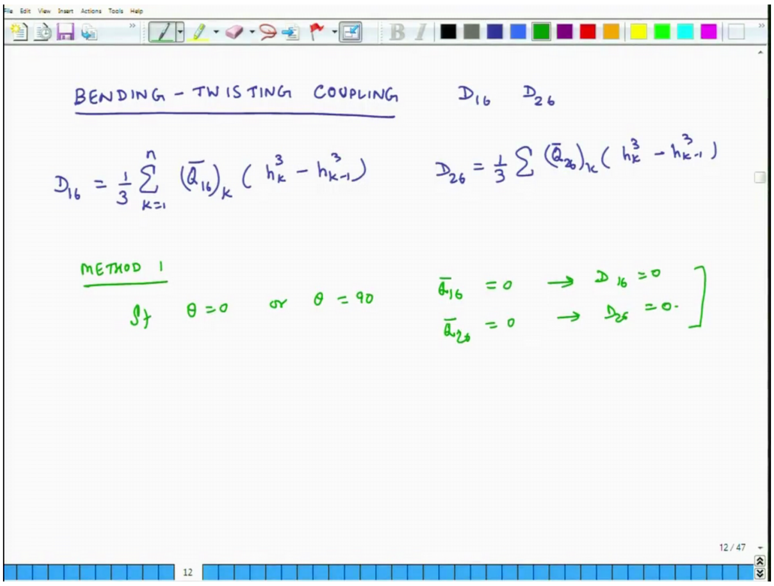

Bending Twisting Response and specifically, what we will explore is that how this kind of coupling. So, it I will call it coupling, how this coupling can be minimized. So, we had earlier explained that this is attributable to elements D 1 6 and element D 1 6 and D 2 6 in the D matrix; and let us look at the definition of these elements. So, D 16 equals sum of Q i j for the kth layer h k cube minus h k minus 1 cube. So, this is Q 1 6 and here this is entire thing is divided by 3 and this is summed over every single layer, if there are n layers in the system. Hm, and this is Q 1 6 bar and similarly D 2 6 equals 1 3rd of Q 2 6 bar for the kth layer h k cube minus h k minus 1 cube.

Now, how do we make sure that these numbers; these values D 1 6 and D 2 6 are eliminated. So, method 1, so, what is method 1? That if for all layers either theta is 0 or theta is 90. Then, Q 1 6 bar is equal to 0 and that means, D 1 6 equal 0; similarly, Q 2 6 bar is 0 and that implies D 2 6 is 0 ok; so, this is 1 approach. But, a limitation these types of laminates is that these laminates which are only oriented either in 0 degrees or 90 degrees the layers is that they are not strong in other directions.

So, these types of laminates cross ply laminates are not preferred whole lot in real systems, because they may be strong in 0 degree or 90 degree direction; but, in other direction they may not be strong. So, if I bend it some other direction they may easily crack or break. So, we do not like these types of laminates.

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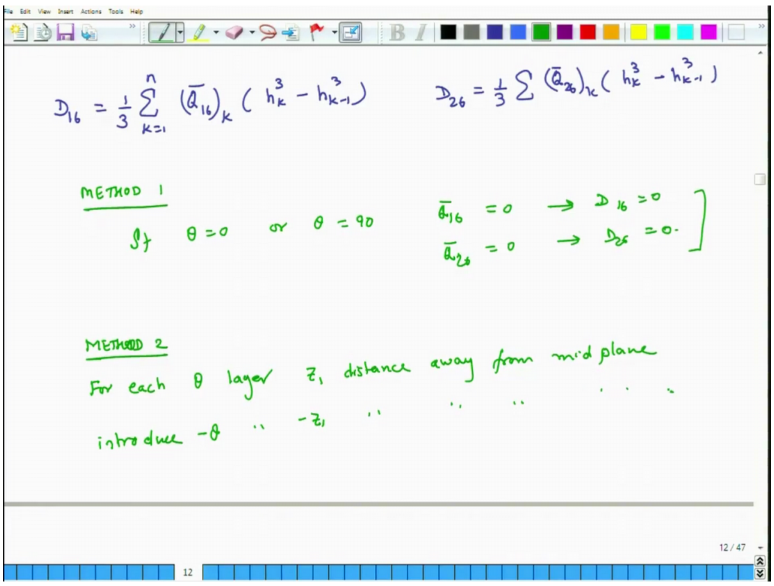

Then there is another method so, in the other method what we have is if we have a layer oriented at theta above the mid plane at a certain distance; and if we have another layer at an orientation negative theta below the mid plane at the same distance, then, their contributions will add and cancel out.

So, let us look out this method. So, here for each theta layer z 1 distance away from mid plane, we introduce negative theta layer minus z 1 distance away from mid plane. So, this is what we want to do. So, let us make a picture of it.

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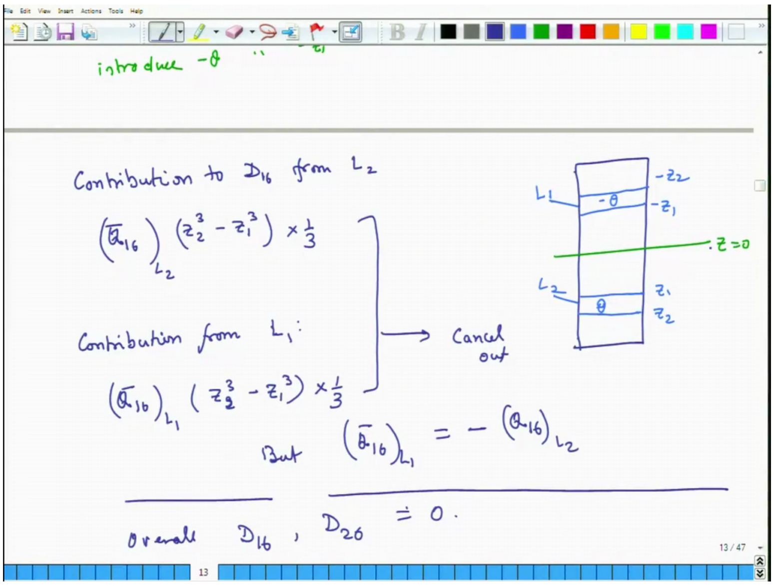

So, let us say this is the laminate, this is the mid plane. So, this is z is equal to 0, there could be 1 layer and this is oriented at theta degrees and this distance is z 1. So, this is coordinate is z 1 this coordinate is z 2 then there is another layer oriented at minus theta and here the coordinates are minus z 1 and minus z 2. So, if we look at the contributions of these two, so, let us say this is L 1 and this is L 2 contribution to D 1 6 from L 1. What is it? It is Q 1 6 bar Q 1 6 bar for the layer L 1.

So, first we will actually calculate for L 2 naught L 1. So, it is L 2, so, this is Q 1 6 bar for L 2 times z 2 square no z 2 cube minus z 1 cube into 1 by 3, ok. And contribution from d 2 contribution from L 2. What is it? It is Q 1 6 bar for L 1 am sorry f for L 1. So, this is L 1, and it is minus z 1 cube minus minus z 2 cube; so, that means, it is z 2 cube minus z 1 cube into 1 by 3, ok. But, Q 1 6 bar for L 1 is equal to negative of Q 1 6 bar for L 2. Because, the theta for L 1 is negative, ok; and Q 1 6 is an even function of odd function of theta. So, if this is the case then these 2 contributions add up and cancel out.

So, if for every theta layer I have another theta negative theta layer on the other side of the mid plane; then, the contributions are going to cancel out. Then, the contributions are going to cancel out. So, in this case so, in this case D 1 6 and D 2 6 will be 0. So, over all D 1 6 D 2 6 they can be made 0, but there is a problem with this solution also. Because, here D 1 6 and D 2 6 will be 0 also because there is also 1 theta layer and there is also equivalent negative theta layer A 1 6 and A 2 6 will be 0, but the laminate will not be symmetric. The laminate is not symmetric.

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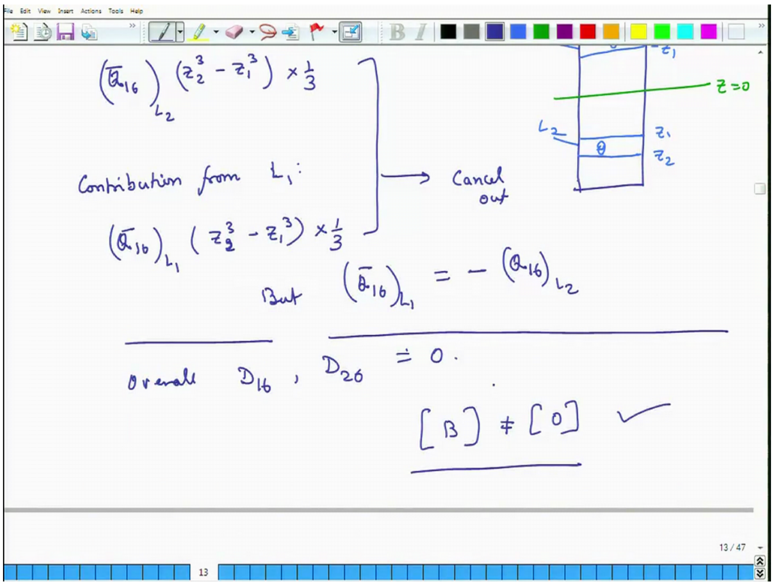

And because, laminate is not symmetric it means, that the b matrix will not be 0. So, we are able to decouple extension and shear risk response we are able to decouple bending and twisting coupling, but bending an extensional coupling is not decoupled because B matrix is not 0 and this is a much bigger problem than solving the other you know eliminating the other couplings.

So, again we do not like this solution also; theoretically we can design systems which have D 1 6 and D 2 6 to be 0; but, it comes at a cost that either it should be 0, you know cross ply laminate or it should be an non symmetric laminate, an empty symmetric laminate, ok. But that is something we really do not want, so, we are stuck.

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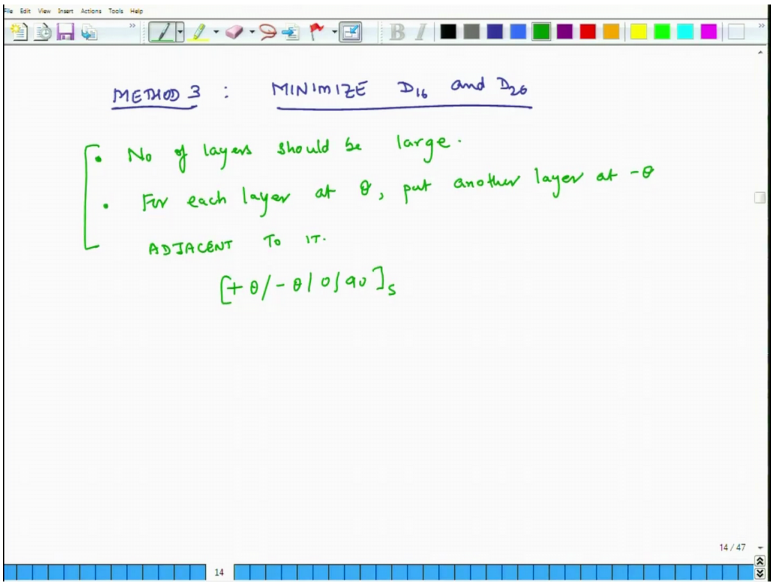

So, the third approach is that we do not like want to get maximum possible you know we do not want necessarily want exactly 0 values for D 1 6 and D 2 6, rather we will say that how much small we can make these values to be. So, method 3 method 3 is minimized. So, we do not try to make it 0. So, we minimize D 1 6 and D 2 6 we minimize D 1 6 and D 2 6. So, how do we do that? So, we do that using some guidelines.

First thing is, number of layers should be large, ok. Second, for each layer at theta we put another layer at minus theta; but not on the other side of the mid plane, but rather what you do is adjacent to it adjacent to it, ok. So, these are the two guidelines. And if we do that in a smart way then, this helps reduce D 1 6 and D 2 6 significantly. So, what does this second condition mean? That, if suppose there is a theta layer then maybe then there should be another layer negative theta next to it, so, ok. And suppose this is 0, 90 and symmetric.

So, 0 degree will not contribute to D 1 6, 90 will not contribute to D 1 6, theta layer will contribute; but there is a negative theta layer next to it and it will cancels some of the effect. And the over laminate is still symmetric which ensures that B symmetric is 0, also because there is theta layer and negative theta layer D 1 6 and D 2 6 will be 0. D 1 6 and D 2 6 will not be exactly zero, but they will be reduced. So, we will what we will do is we will actually do couple of examples and see how this method works, ok.

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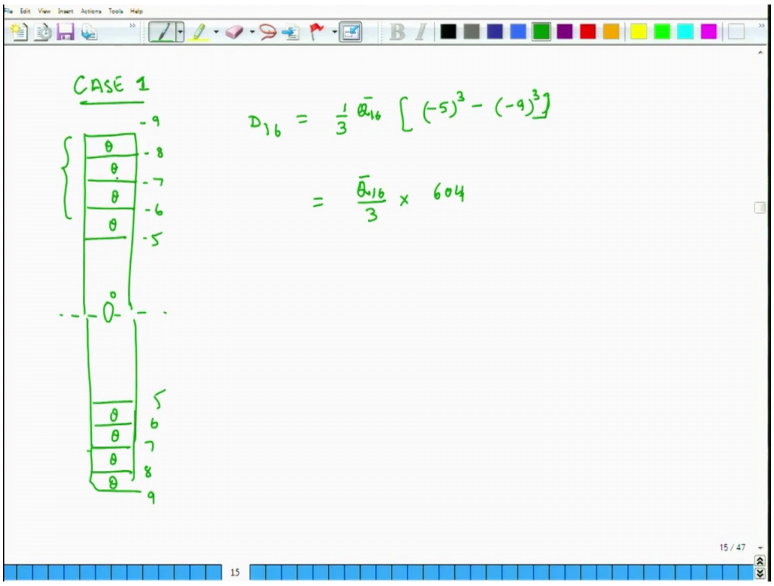

So, we will do 3 cases, so, case 1. So, suppose there is a stack up ok; and so, this is minus 9. So, these are the coordinates minus 8, minus 7, minus 6 and minus 5, ok; and these have theta, theta, theta, theta. And then let us say, this is a symmetric laminate suppose this is symmetric laminate. So, same thing on the other side also exists suppose this is a line of symmetry, ok. And let us say, so, it is like this. So, this will 9, 8, 7, 6, 5 and let us say between plus 5 and minus 5 all layers are 0 or 90. So, the only things which are generating D 1 6 are these theta layers only things. So, contribution of these 4 top layers will be what.

So, contribution from the top layers, so, all the middle layers are going to contribute nothing; because, all these are so, this is all 0 degree, ok. So, what will be the value of D 1 6, D 1 6 will be Q 1 6 bar times 1 by 3 and you can consider these 4 layers as 1 fat layer. So, this is minus 5 cube, minus minus 9 cube. Here, we can consider this as 1 single fat layer.

So, if you do this you see that it is equal to Q 1 6 bar divided by 3 times, and if you do all the math it comes out to be times 604, so, this is case 1. 604 times Q 1 6 bar divided by 3 this is case 1. Next what we do is, we have the same layer; but we change some theta as with minus thetas.

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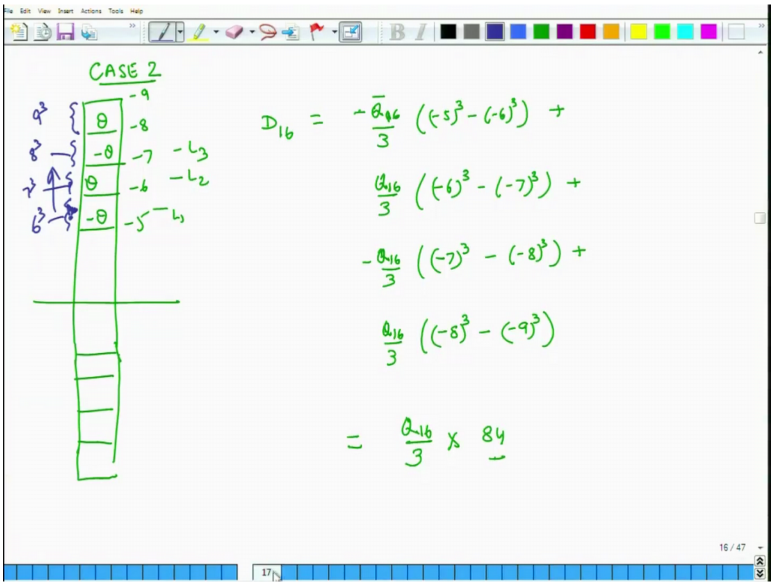

So, in the second case what we do is, so, this is case 2. So, we have theta, minus theta, theta and minus theta; and we are only going to compute the contribution from the top half. So, bottom half will just double, so, it does not really matter right. So, this is the overall layer, ok. So, again this is minus 9, minus 8, minus 7, minus 6, minus 5 these are the coordinates, and we are just seeing the contribution only from the top half. So, what will be D 1 6, D 1 6 will be; so, we will start from this. So, we will go like this hm. So, D 1 6 will be equal to. So, first we would do. So, so let us say this is layer a.

Now, we will. So, this is equal to. So, this is the layer which is closest to the mid plane is having an orientation of negative theta. So, let us say its Q 1 6 is minus Q 1 6 ha, because it is. So, minus Q 1 6 bar minus Q 1 6 bar divided by 3 times minus 5 cube minus minus 6 cube this is the contribution from L 1, plus contribution from L 2 this is L 2 hm. So, this is equal to Q 1 6 bar by 3. So, here it is positive Q 1 6 because theta is positive times minus 6 cube minus minus 7 cube ok, plus Q 1 6 bar by 3. So, this is L 3 this is minus 7 cube minus minus 8 cube, plus Q 1 6 bar by 3 and what you get is minus 8 cube minus minus 9 cube, ok.

So, if you do all the math and you add up together what you get is Q 1 6 bar divided by 3 times 84. So, here what we have done is we have added we have a stack up sequence theta minus theta theta minus theta. So, they are cancelling, here it was Q 1 6 bar divided by 3 times 604; but just because we have put theta minus theta theta minus theta this kind of a sequence that 604 becomes smaller and it becomes 84, there is even a better way and that is case 3.

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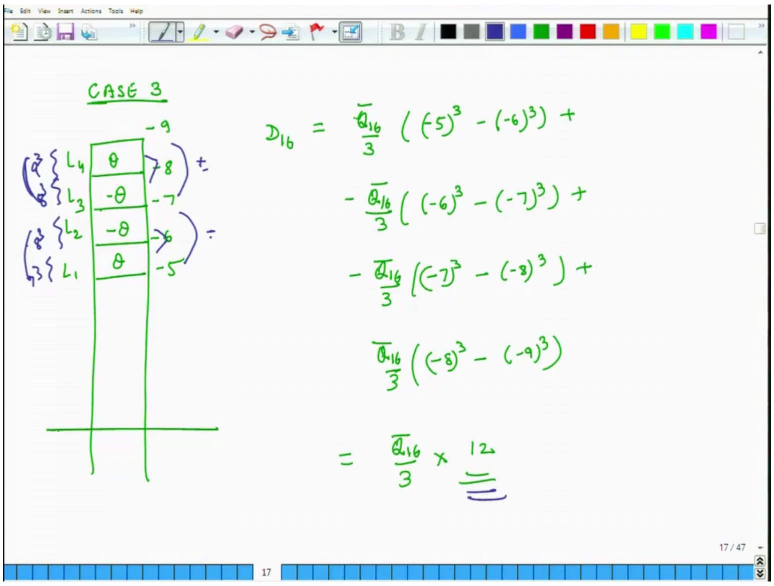

And here let us say this is the mid plane. So, this is theta this is minus theta this is minus theta and this is theta ok. So, coordinates are minus 9, minus 8, minus 7, minus 6, minus 5 ok. Here let us compute D 1 6. So, D 1 6; so, let us do for L 1 then we will do for L 2 then L 3 and L 4 hm. So, for L 1 it is Q 1 6 bar by 3 times minus 5 cube minus minus 6 cube plus now, L 2 and L 3 they have the same orientation. So, I can consider them as one single layer ok, I can consider them as 1 single layer or I will just to be consistent I will consider them as 2 layers does not matter I will get the same answer.

So, Q 1 6 will be negative of this times, so, for layer L 2 it is minus 6 cube minus minus 7 cube plus minus Q 1 6 bar by 3 minus 7 cube minus minus 6 cube. Oh, I am sorry. This is 8 plus Q 1 6 bar by 3 minus 8 cube minus minus 9 cube and here, if you add up everything together what you end up is Q 1 6 by three. So, this is all Q 1 6 bar times twelve times twelve.

So, in case 2 it was 84 in case 1 originally it was 604, and it has come down to a very small number twelve. And the reason this third case is better than second case is because, if you look in the second case here we have a 9 cube term and that is being reduced by a 8 cube term from here, ok. And then, so, so 9 cube is larger than 8 cube. So, it and then again we have a 7 cube term here and that is being reduced by a 6 cube term from the layer below.

In this case we have a 9 cube term and we have an 8 cube term. So, that gets reduced, but then here we have a 8 cube term which is of opposite sign and that is getting reduced by a 7 cube term. So, the; so, this total may be a positive number this total may be a negative number. So, these 2 branches also you know bundles also cancel out each other. So, contribution from for this and the contribution from this. So, again I will I will explain that contribution from this cancels contribution from this and we get some residue and let us say that is a positive number.

In the lower portion contribution from this and contribution from this cancels out, but it is not exact cancellation and the residue is negative number. And then, this positive and negative number also cancel out each other. So, because of this third stack it provides even a lower value of D 1 6. So, the point is that using these types of approaches we can reduce D 1 6, D 2 6 to a small number and for such stacks A 1 6 and A 2 6 will be anyway 0, because for each theta there is a negative theta layer and for such stacks b will also be 0 because the overall matrix is overall lamination sequence is symmetric.

So, in this way we are able to get exactly b is equal to 0 that condition we are also able to exactly ensure A 1 6 and A 2 6 is 0 and we can reduce D 1 6 and D 2 6 to fairly small number. So, that the coupling between bending and twisting is also minimized not exactly made 0 or eliminated be minimized. And in this way, we have a system which behaves in the way we just like us we would ourselves like it to behave.

So, this is what I wanted to discuss about bending extensional coupling and tomorrow we will talk about another lamination sequence in context of isotropy. So, that is what we plan to discuss tomorrow, until then have a great time look forward to seeing you tomorrow bye.