Introduction to Composites
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Lecture – 67
Calculation of A, B and D Matrices

Hello welcome to Introduction to Composites. Today is the start of the last week of this course and what we plan to cover over this week is some details about the classical lamination theory, which we developed in the last class and actually in the last week. So, we will develop some of the details of that even further, we will also learn the importance of specific types of lamination sequences and how these types of lamination sequences are advantageous from an engineering standpoint. And later if we have time we will try to have in a very brief format some discussion on predicting failure of composite laminates. So, this is what we plan to do over the next 6 lectures. Today what we will do is; we will start by doing an example.

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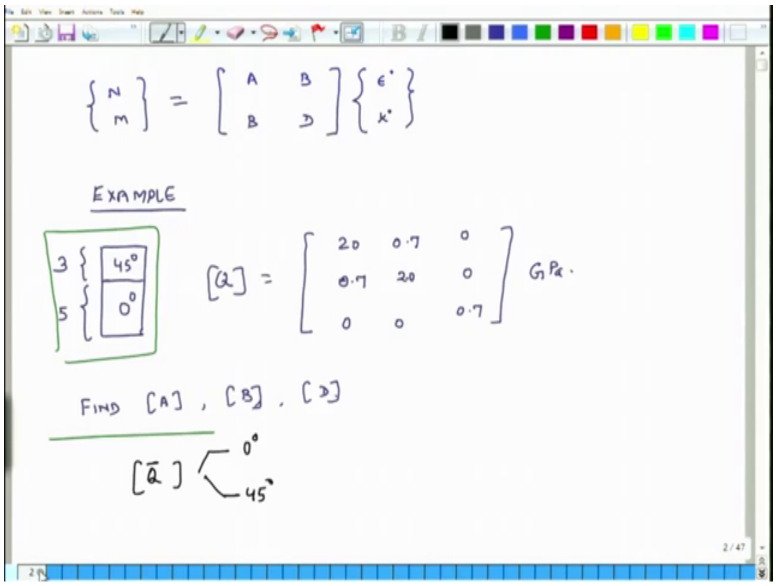

So, what we had shown was that; the stress the force resultant and the moment resultants can be expressed as products of elements of A matrix, B matrix, D matrix and mid plane strains and mid plane curvatures.

So, this is what we had shown. So, what we will do today; in this particular class is we will do an example, where we will actually calculate the elements of A matrix, B matrix, and D matrix for a relatively simple laminate. So, that is what we plan to do? So, here we are stating that there is a laminate, and it is having just two layers. So, the first where orientation of the top layer is 45 degrees, orientation of the bottom layer is 0 degrees, the thickness of the bottom layer is 5 millimeters and thickness of the top layer is 3 millimeters. And then we are stating that the material in both the layers is the same.

So, material properties are same, which means that the Q matrix for both the layers is the same and the values are given here. So, it is 20, 0.7, 20, excuse me; 2.0, 0, 0, 0.7, 0.7, 0, 0 this is in Giga Pascal's. So, given this what we have to find out is; find A, B and D matrices for this particular laminate; this is what our problem statement is. So, what do we do; we know that we have to find A, B and D matrices we have to find Q bar matrix for each layer.

Now, there are two layers, so we have to find Q bar matrix for 0 degree layer and Q bar matrix for 45 degree layer, and then using the relations which we developed earlier; we will be able to calculate A, B and D matrices. So, for the 0 degree layer Q bar matrix is same as Q ok.

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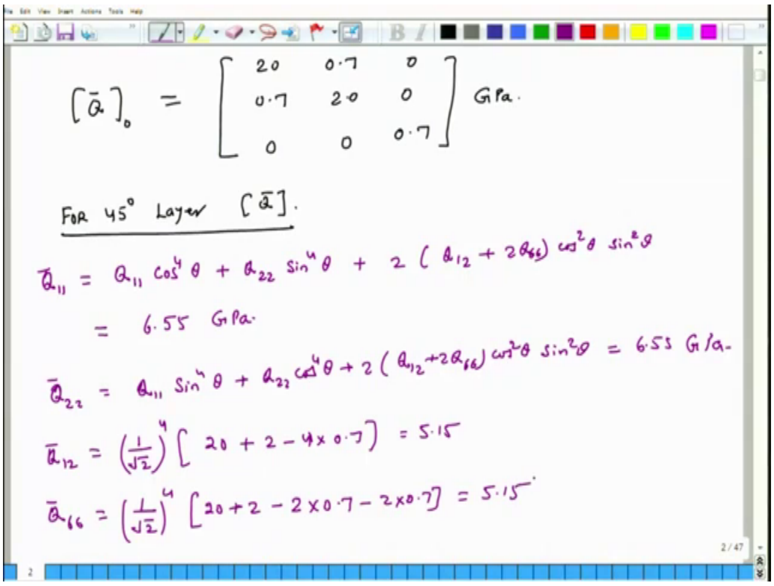

So, Q bar for 0 degree layer is same as Q. So, the number is given here 20, 0, 0.7, 0, 0.7 2.o, 0, 0, 0, 0.7 GPa. For 45 degree layer we actually calculate every single element. So, for 45 degree layer we are going to calculate the Q bar matrix.

So, first we calculate Q 1 1 bar. So, Q 1 1 bar is Q 1 1 cosine 4 theta plus Q 2 2 times sin 4 theta plus 2 into Q 1 2 plus 2 Q 6 6 cosine square theta sin square theta and theta is 45 degrees, we know the values of Q 1 1, Q 2 2, Q 1 2 and Q 6 6. So, if you plug in all the values we find that Q 1 1 bar it comes out to be 6.55 GPa. Similarly Q 2 2 bar is Q 1 1 sin 4 theta plus Q 2 2 cosine 4 theta plus 2 times Q 1 2 plus 2 Q 6 6 cosine square theta sin square theta and this is also same as 6.55 GPa we also find out Q 1 2.

So, I am going to directly plug in the numbers, so this is equal to 1 over square root of 2 to the power of 4 times 20 plus 2 minus 4 into 0.7. So, this works out to be 5.15; Q 6 6 bar is 1 over square root of 2 to the power of 4 times 20 plus 2 minus 2 into 0.7 minus 2 into 0.7, so this is also 5.15 ok.

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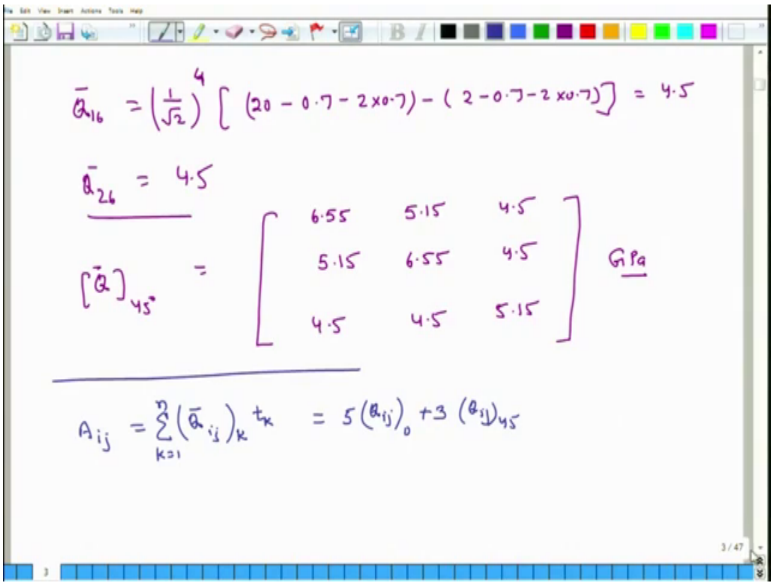

And Q 1 6 bar equals 1 over square root of 2 entire thing raised to the power of 4; 20 minus 0.78 minus 2 into 0.7 minus 2 minus 0.7 minus 2 into 0.7. So, this comes out to be 4.5 and Q 2 6 bar is also 4.5. So, the Q bar matrix for 45 degree layer is this thing 6.55, 5.15, 4.5, 6.55, 4.5, 5.15, 4.5, 4.5, and 5.15 GPa. So, that is my Q bar matrix for 45 degree layer and then we have another one for the 0 degree layer.

So, next we start computing A, B and D matrices. So, Aij, so the ijth element of A matrix is what; sum of k is equal to 1 to n and we had discussed this earlier; so Q bar ij for the kth matrix for the k th layer times the thickness of the kth layer ok. It is actually hk minus hk minus 1, but hk minus hk minus 1 is thickness of the kth layer. Now, there are only two layers in this thing. So, this is what Qij for 0 degree layer and the thickness of 0 degree layer is; so it is 5 millimeters plus 3 times Qij for the 45 degree layer ok.

So, here I am putting values in millimeters, we can also put it in meters, but come numbers mathematics becomes a little simpler.

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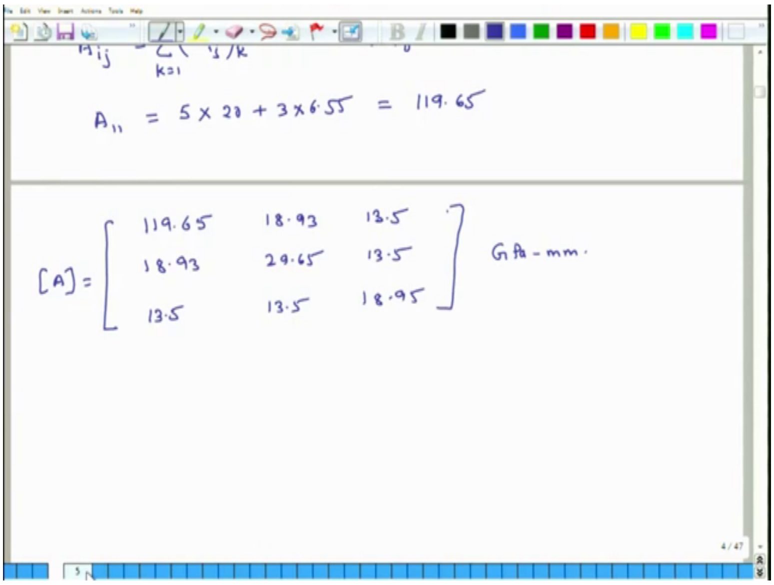

So, for instance A 1 1 will be what 5 times Q 1 1 bar; so 20 plus 3 times 6.15. So, if you do all the computation it comes to 119.65. So, likewise you can calculate all the elements of A matrix. So, your A matrix is this thing 119.65, 18.93, 13.5, 18.93, 29.65, 13.5, 13.5, 13.5, and 18.95 and because we have multiplied GPa with millimeters the units will be GPa millimeters had we multiplied thickness in terms of meters, then it would have been Newton's per meter.

But here we have to be exploit; so it is GPa millimeters. So, let us do the same computation for Bij.

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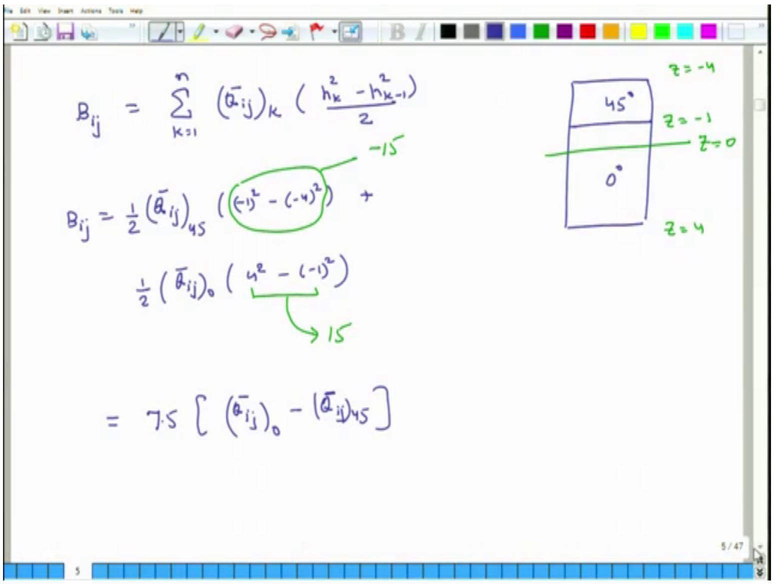

So, Bij equals sum of Qij for the kth layer times hk square minus hk minus 1 square by 2 and k is equal to 1 to n where, n is the number of layers. Now, here we have to calculate the coordinates of each layer. So, there are two layers; this is the mid plane. So, z is equal to 0 this is at 0; so this total thickness is 4 to minus 4. So, here z is equal to 4 millimeters, here z is equal to minus 4 millimeters, here z is equal to minus 1 millimeters ok.

So, let us; so in this case there are only two layers. So, this is the 45 degree layer and this is the 0 degree layer ok. So, there are only two layers involved in this. So, Bij equals Q ij for 45 degree layer times for 45 degree layer it is hk minus hk minus 1. So, it is minus 1 square minus minus 4 square into 1 by 2 plus 1 by 2 Qij 0 and so I am sorry Qij bar and for the 0 degree layer it is 4 square minus minus 1 square ok.

So, this becomes 1 minus 16 minus 15 right and this is how much; this is 15. So, it is basically when I divide it by 2 it is 7 and a half. So, this works out to be 7.5 and it is Qij 0 minus Qij 45 and then of course, there is a bar.

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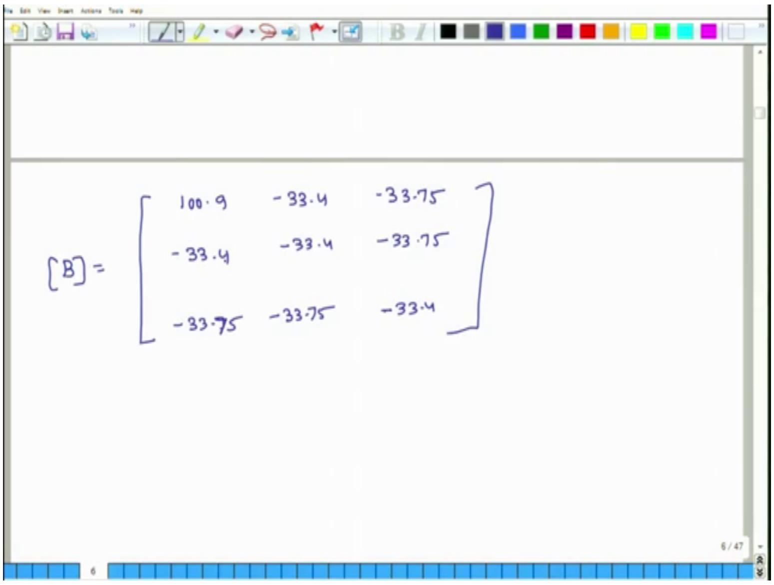

So, using this I can calculate different values of B matrix and if I do all the computations, what you find is that the B matrix looks something like this 100.9 minus 33.4 minus 33.75 minus 33.75 minus 33.4 minus 33.75 minus 33.75 minus 33.75 and the last element is minus 33.4 ok.

. So, B is nonzero and a why it is nonzero; because our material system the lamination sequence is not symmetric. The top layer is 45 degrees, bottom layer is 0 degrees, mid plane is passing through the 0 degree layer system is not symmetric, so B has to be nonzero ok. Now B will not be symmetric, because the lamination sequence is not symmetric; it is just two layers top layer is 45 degrees bottom layer is 0 degree layer.

Bottom layer is 0, if the lamination sequence across the mid plane is symmetric, then it will be it will not be you are taking, but Qij and Qij bar for 0 degrees and 45 degrees are not same Qij bar for 0 degrees, Qij what is the Q matrix for 0 degree.

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If they may be symmetric, but the matrix will not be 0.

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Oh yeah you are right. I this I wrote incorrectly. So, this should be minus 33.4 yes; you are right yes. So, this is the B matrix and finally, we compute elements of D matrix.

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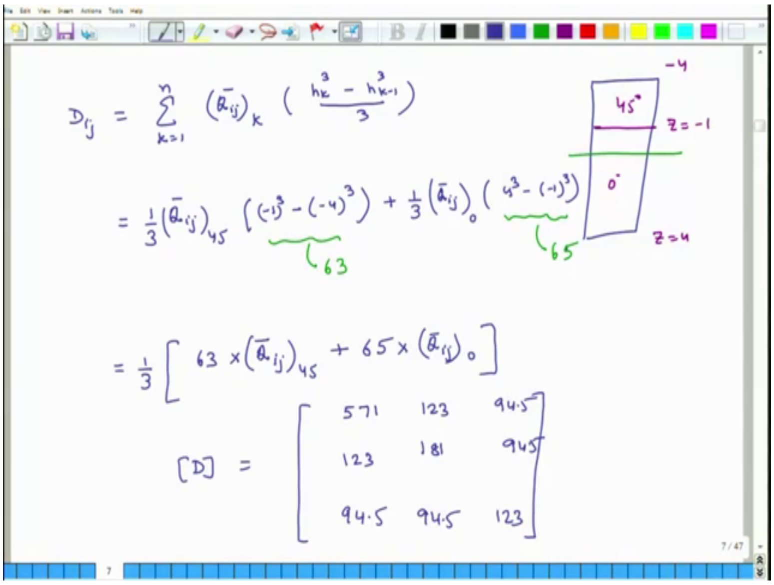

So, D ij equals k is equal to 1 to n Q ij for the k th layer Q ij bar for the k th layer times hk cube minus h k minus 1 cube divided by 3 ok. And once again we draw the laminate cross section, this is the mid plane, this is and the first layer; so this is 45 degree layer, this is 0 degree layer, here z is equal to minus 4 here z is equal to 4 here z is equal to minus 1.

. So, if I expand this relation for a two layer system I get Q i j for 45 degree layer times hk minus 1. So, it is minus 1 cube minus minus 4 cube time divided by 3. So, 1 by 3 plus 1 by 3 Qij bar for 0 degree layer times h k for the 0 degree layer is 4 millimeters. So, 4 cube minus minus 1 cube ok. So, what is this? This is 64 plus 1, so this is 67 ok. And and this is all this is how much this is 65 to 5 and then you divide it by by a factor of 3. So, basically what you end up getting is equal to 1 over 3 65 times Qij bar for 45 degree laminate plus 67 times Qij bar for 0 degree laminate oh I am sorry. So, this should also be 65.

So, 64 plus 1 65, 64 plus 1 yeah and this is and this is 63. So, this is 63 and this is 65. So, for each matrix for each element you can input the values of Qij bar and essentially; and you end up getting the D matrix and the D matrix if you do the computations correctly, it works out to be something like this, so 571, 123, 94.5, 181, 94.5, 123, 94.5, 94.5 and 123.

So, these are our three elements of three matrices A, B and D and I wanted you to see how these matrices are calculated. So, now, that you have the method and I have explained the method for any laminate, you can use this approach and create A, B and D matrices and using that those matrices, then you can do further computations. This is what I wanted to cover in today's discussion.

Tomorrow, what we will do is? We will start looking at specific lamination sequences and see what kind of advantages these different lamination sequences can provide us with. So, we will look at sequences, which help us achieve B matrix to be 0, we will look at sequences where which help us achieve A 1 6 and A 2 6 to be 0 and so on and so forth. So, that is what we plan to do tomorrow and till then have a great evening and I look forward to seeing you tomorrow.

Thank you.