Introduction to Composites
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Lecture – 65
Physical Significance of Extensional Stiffness Matrix [A], Coupling Matrix [B] and Bending Stiffness Matrix [D] Matrices

Hello, welcome to Introduction to Composites. Today is the 5th day of the second last week yesterday we had defined 3 matrices, A matrix which is the extensional stiffness matrix, B which is the bend coupling matrix and D which is the a bending stiffness matrix. And these 3 matrices connect the force and moment resultants to midplane strains and midplane curvatures in a composite laminate.

Next what we will do today is we will discuss these matrices further and see what physical significance these matrices and specifically their individual elements carry.

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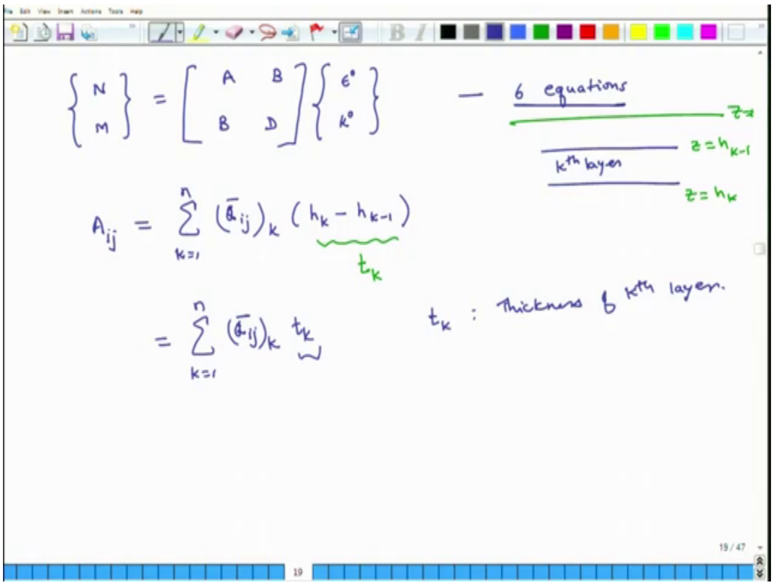

So, overall the relationship which we had developed was there is a n vector and there are moment resultant. So, they are related to this 6 by 6 matrix which is made up of A B and D sub matrices times midplane strains and midplane curvatures.

So, these are 6 equations, these are 6 equations. And the element; so we will consider each matrix term individually. So, first we will look at A ij which is the ij element of a matrix this was defined as Q ij for the kth layer times h k which is the coordinate of the bottom portion of that layer minus h k minus 1 and we do this integral or summation from first layer to all the n layers, ok.

Now, this is let us say the kth layer, and this is my midplane. So, midplane z is equal to 0. So, here z equals h k and this is z equals h k minus 1. So, this difference is nothing but the thickness of kth layer and thickness of kth layer is always a positive number it is always a positive number. So, I can also write this as Q ij for the kth layer times t k and this goes from 1 to n Q ij bar times t k, where t k is thickness of kth layer thickness of kth layer.

So, the point is that if Q ij is positive and typically Q 11, Q 22, Q 66 they are positive numbers always positive numbers, so in that case A 11, A 21, A 22, A 66 they will always be positive ok.

Same thing was also true for Q 12, Q 16 could be negative in that case A 16 and A 26 could be negative, but in all cases A 11, A 12, A 22 and A 66 will always be positive ok, they will always be positive because this t k is always remains positive and Q 11 Q 22 Q 16 Q 11 Q 22 Q 22 Q 12 and Q 66 are always positive. So, respective elements in the A matrix are always positive, this is one thing.

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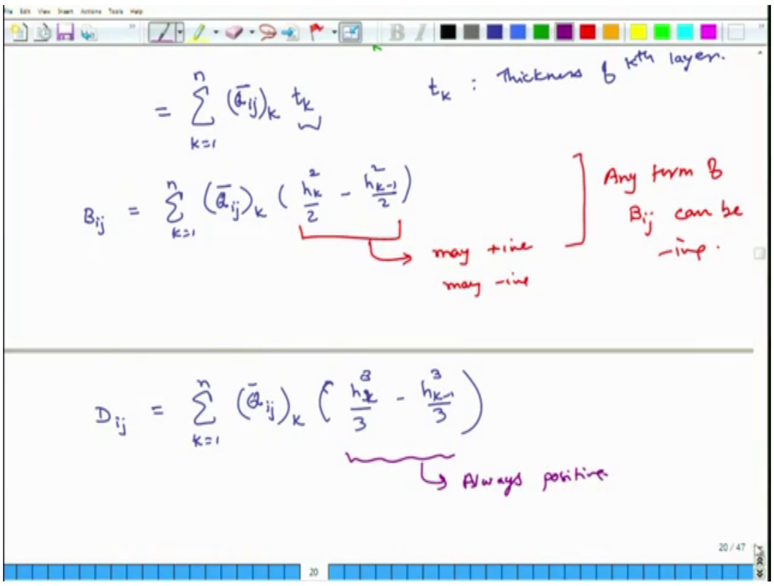

Next look at a B ij; So, in case of B ij this is equal to k 12 n Q ij bar for the kth layer times h k square by 2 minus h k minus 1 by 2 square, ok. Now, in this case this term it may be positive or it may be negative also, it purely depends on the coordinates. Why because each of these terms is a positive number h k square is positive h k minus 1 square is a positive, but if h k minus 1 absolute value is much larger than that for h k then this can be negative, it can be negative.

So, the point is that because of this thing this term in parentheses this can be either positive or negative. So, then what that means, is that any term of B ij can be negative because consider at Q 11 the B 11, B 11 will depend on Q 11, Q 11 is always positive, but if the terms in the parentheses are negative then it can be negative and finally, we look at D ij. So, D ij is k is equal to 1 to n, Q ij for the kth of course, there is a bar times h k cube by 3 minus h k cube minus 1 by 3.

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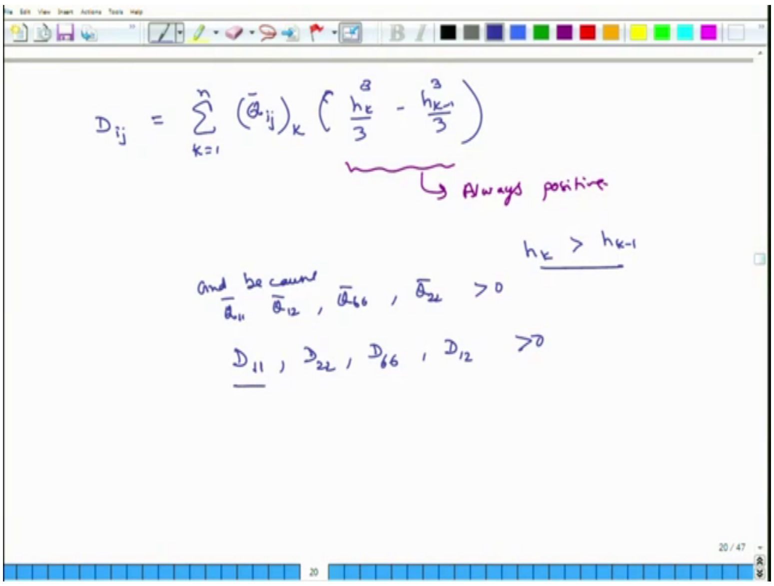

Now, this number is always positive. Why is it always positive? Because h k, h k is value because h k is always more than h k minus 1 ok, it is always more than h k minus 1. So, when I take the cube the difference will always be positive the same reason is for the same reason h k minus h k minus 1 is always positive, ok.

So, this is always positive. So, for that reason and because Q 11 bar, Q 12 bar, Q 66 bar and Q 22 bar are always positive we can also say that D 11, D 22, D 66 and D 12 are always positive. D 16 and D 26 can be negative because depending on the value of theta I can have negative values for those terms also.

So, the point tells that these terms are always positive, similar terms in the a matrix are also positive, but in terms of in context of B matrix any term can be positive or negative and it is given driven by the fact that h k square minus h k minus 1 square can be either positive or negative. So, this is one general observation I wanted to state.

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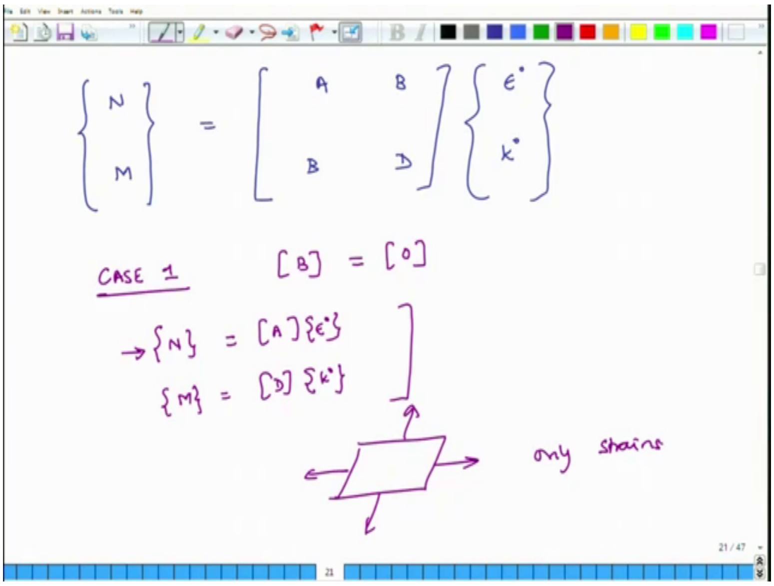

Next look at some more details about this these matrices. So, if we have n and n these are the vectors, so this is equal to A B B and D these are sub matrices epsilon k. So, these are midplane stresses, a midplane strains and midplane curvatures. So, we will consider several cases case 1.

We can have a lamination sequence we can have a laminate such that and we will discuss the conditions when that happens maybe later. So, there could be a situation that then B matrix all the elements in B matrix are 0, all the elements in the B matrix are 0. Then N equals A times epsilon and M equals D times curvatures ok.

So, what this tells us is that if for A laminate the B matrix is 0 then if I apply only forces on the edge of the plate if I apply only forces on the edge of the plate. So, I am only is stretching it suppose I am not applying any moments I am not trying to bend and twist it, then it will generate only strains it will not generate curvatures and if I apply only moments if I apply only moments then I generate only curvatures, ok.

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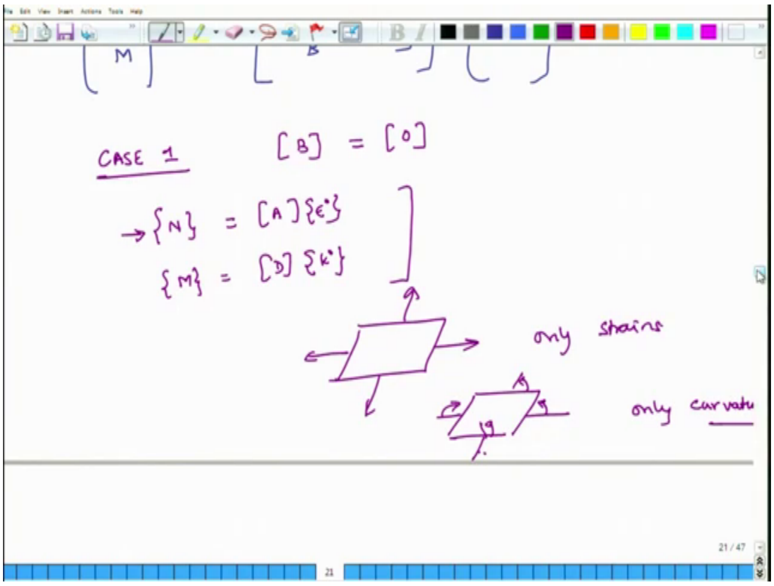

So, what this says is that if the B matrix for a laminate is 0, force resultants generate only midplane strains and they do not generate midplane curvatures and moment results only generate midplane curvatures and they do not generate any midplane strains this is what it means ok. It is very important to understand that.

In lot of isotropic materials the B matrix is not lot all isotropic materials made up of isotropic systems this is actually the case where you have in the lot of systems B matrix is 0 and so extensional forces only generate strains and moments only generate curvatures they do not generate midplane strains and vice versa. So, this is something important to understand.

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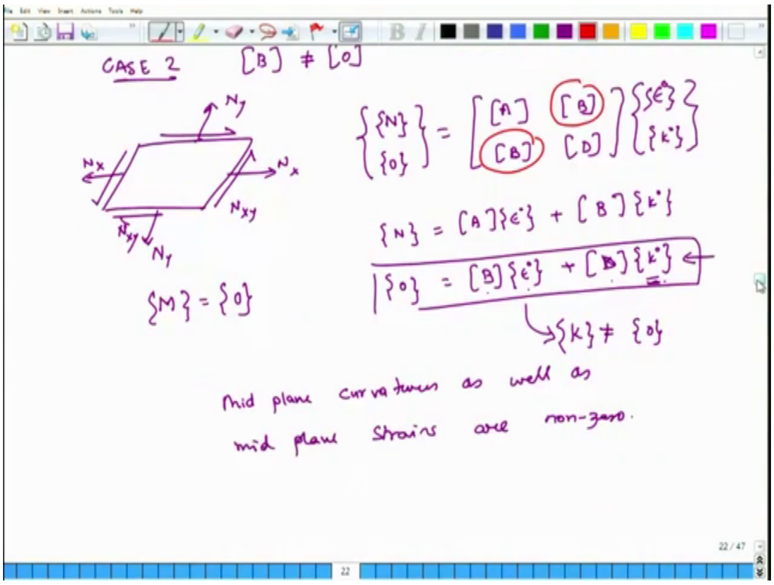

But we can have another case where B matrix is not of 0 matrix, B matrix is not a 0 matrix and suppose in this case you have let us say a plate and you apply some N x. you can also apply some N y and you can also apply some N xy.

And let us say that you do not apply any moments. So, you say that moment we do not apply any moments, but then what do the equations tell us the N matrix is nonzero right and in this plate I am applying 0 moments external moments. So, this is 0. So, this is equal to A B B and D midplane strains curvatures. So, we want to figure out what kind of strains and curvatures are in this plate.

Now, in this case because the B matrix is not 0 we cannot say for sure that midplane curvatures will be nonzero ok, because when I write 2 individual sets of equations what do I get N equals A epsilon plus B times curvature and for because moments are 0 external moment resultants are 0. So, this is 0 is equal to B times midplane strains plus D times midplane curvatures.

The second equation tells us that for this system curvatures are nonzero because the sum of B times epsilon plus D times curvatures is 0. So, second equation tells us that curvatures are not 0 and we have to solve all these 6 equations to find curvatures as well as midplane strains. So, in this case midplane curvatures as well as midplane strains are nonzero and this is all happening because of the B matrix.

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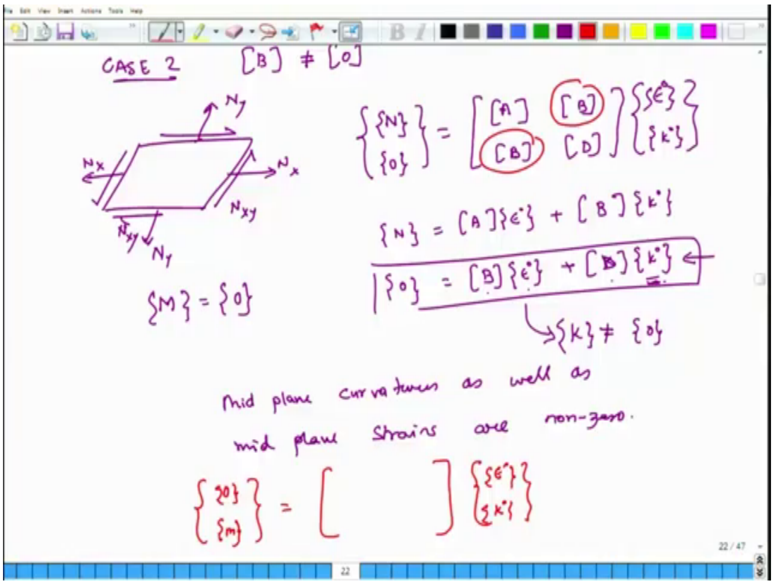

Similarly, we can have another situation where we have no force resultants, but we have only moment resultants and in this case also we will find that both midplane curvatures and midplane strains will exist.

So, what this tells us is that it is the B matrix if it is nonzero then it generates even though if we are just stretching if we are just pulling the plate in a straight direction even then it will not only expand and contract, but it will also bend and that is why it is called coupling matrix.

That is why it is called coupling matrix, because it couples the midplane response of the plate to the midplane curvature of the plate it couples that ok. If this coupling matrix is not there then the response of the plate to M is purely strain response of plate to M is purely curvature, but if B is nonzero then it couples and you have a more complicated situation. So, this is the second case, the second.

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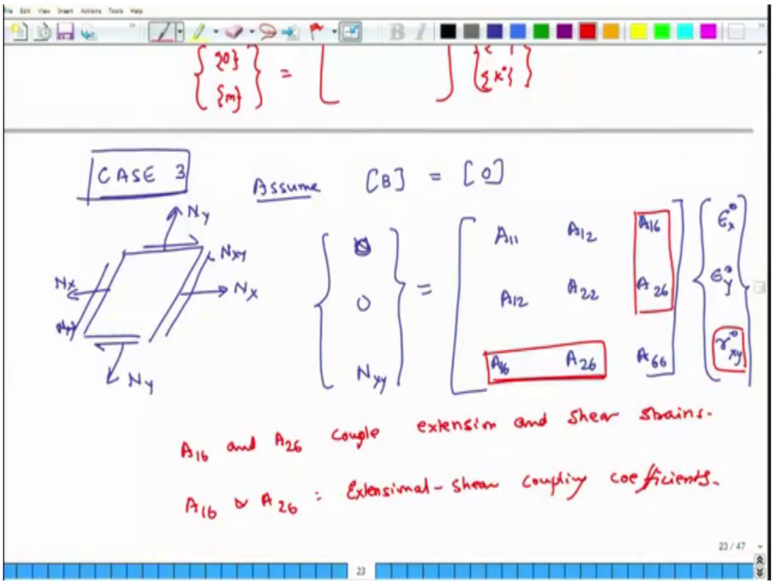

Let us look at some more cases. So, there is a third case ok, third case. And here we look at the a matrix in detail. So, we assume that we have designed the plate such that it is B 0, you have designed such that it is B 0 and we are loading the plate only in using N. So, the plate is experienced in N x and y and maybe also N xy, ok.

So, if B is 0 then the plate will not have any midplane curvatures because the bending the coupling matrix is 0, but then let us look at the equations. So, the equation will be N x, N y, N xy and that equals epsilon x naught, epsilon y naught, gamma xy naught. And here it will be A 11, A 12, A 16, A 22, A 26 and A 66, A 12, A 16, A 26.

Now, consider the case that we are applying only N x in the situa; in the on the plate suppose I apply only N x on the plate. So, if I apply only N x on the plate then N y and N xy is 0, ok. If A 16 and A 26 terms are not 0 then even though I am pulling the plate in the straight direction only in extensional mode the plate will also develop a shear strain because A 16 and A 26 are not 0, ok.

So, A 16 and A 26 couple extensional extension and shear strains, if A 16 was 0 and A 26 was 0 then if I pull it, it will only exhibit stretching and Poisson contraction, but if A 16 A 26 are not 0 and if I pull it will also generate shear strain.

Similarly, if I applied only shear stress you know N xy suppose I applied only N xy, if I applied only N xy and if A 16 and A 26 are not 0 then these terms because they appear in the matrix here they again couple epsilon x, epsilon y and gamma xy. So, even though I am applying pure shear forces on the plate it is not only experiencing shear strains, but it is also getting stressed and compressed ok.

So, that is why A 16 and A 26 are extensional shear coupling coefficients, the couple the shear response and the stray extensional response ok. And if they are 0 then the plate if it experiences purely tensional strain then it has only extensional strains if it experiences purely shear stresses then it exhibits only shear stresses.

Now, of course, this is in the context that B matrix is 0, ok. And we look at a 4th case.

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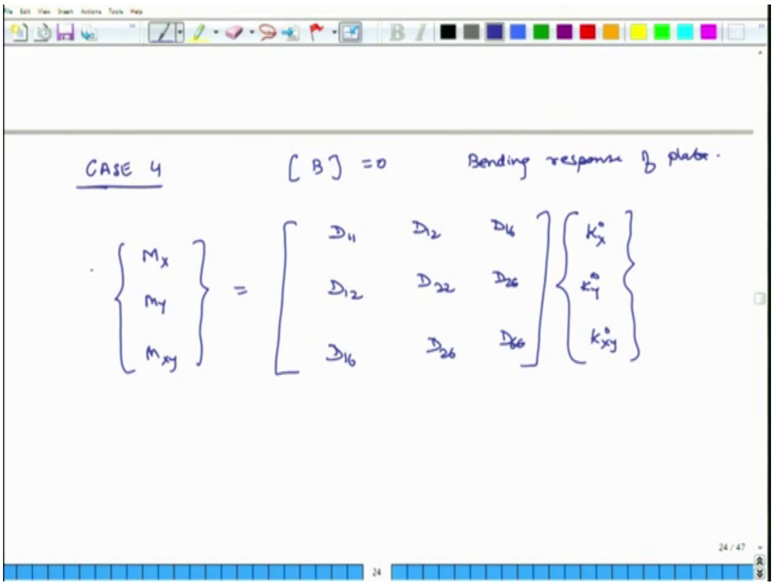

We look at a 4th case. So, again for simplification purposes we still assume that B is equal to 0 and I want to see the bending response of the plate. So, I am interested in seeing the bending response of the plate. So, what do I see? So, in that when B matrix is 0 then M x, M y, M xy equals D 11 D 12 D 16; D 12 D 22 D 26; D 16 D 26 D 66 this is K x, K y and K xy, ok. So, if B matrix is 0 then if I am applying only moments on the system then these are the equations. If N is 0 then the first 3 equations are irrelevant and if because only moments are being applied moment resultants are being applied.

Again we should understand what are K x K y and K xy. So, what I will do is I will again show you this book, and suppose this is the x axis suppose this is the x axis this is the y axis and downwards is the z axis if I bend the book like this then the plate or the book is exhibiting purely K x, it is exhibiting purely K x curvature in the x direction it does not have curvature in the y direction when I bend it like this because when you look at it it is still straight. So, it is not having any curvature in the y direction

If I bend it like this then it is having purely curvature in the y direction. So, if I bend it like this it is having pure K y, if I bend it like this it has pure K x, K y is 0 if I bend it like this it is having pure K y, K x is 0. And then there is something called K xy and what is that? So, this is bending this is also bending, but then I can also twist it like this.

In this case it is having curvature in this direction also, it is having curvature in that direction also it is having curving and it is also a cross components ok. So, you can have a plate which has curvature in this direction, that direction and also a cross component. So, so w the slope is changing, the w del, w del second derivative of w with respect to x and y is nonzero ok. So, that is called twisting that is called twisting. So, if I am trying to twist it or something like this. So, that is the twisting thing.

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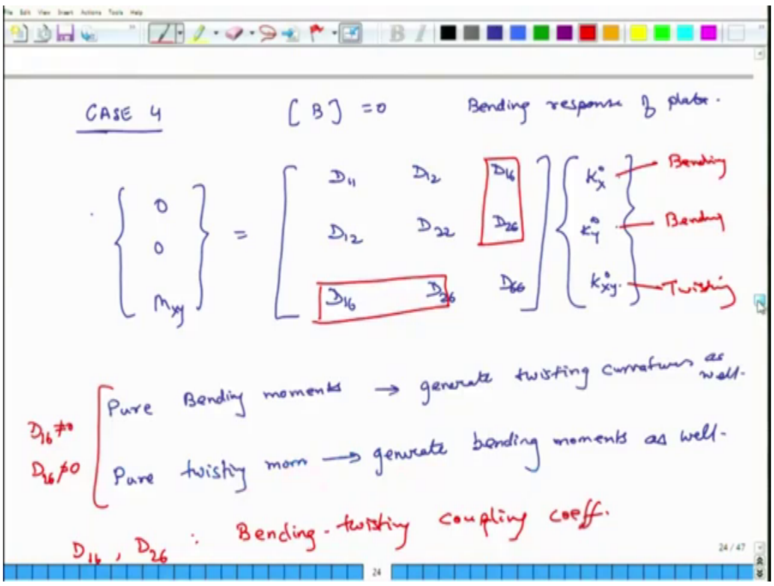

So, this is a bending curvature this is a bending curvature and this is a twisting curvature, is the twisting curvature. Now, consider a situation where I apply only M x, and I do not apply M y and M xy, so M y and M xy are 0. In that case if these terms are nonzero D 16 and D 26 then pure bending moments. So, M x is a purely bending moment they generate twisting curvatures as well ok, because D 16 and D 26 is not 0. Similarly, if I apply a pure twisting moment; So, what is the twisting moment? M xy, M xy is a pure twisting it is a twisting moment because I am trying to twist it.

So, pure twisting moments these guys they generate bending moments as well ok, because of if D 16 and D 26 are not 0. So, this happens when D 16 is not 0 or D 26 is not 0 and typically we do not want our structure to twist because it then tries to shear off easily. So, that is why these terms D 16 and D 26 couple the bending and twisting response of the plate and that is why D 16, D 26 they are called bending twisting coupling coefficients, ok.

So, we have discussed 4 cases the first case is when B is 0, and if B 0 then the coupling between the bending response and the midplane strain response extensional response and the bending response it gets eliminated that is why it is known as coupling matrix.

The second, so that was about case 1 and case 2 then we looked at the case when A 16 and A 26 are not 0, and in this case the extension and the shear strains they get coupled that is why A 16 and A 26 are known as extensional shear coupling coefficients. In the last case is when D 16 and D 26 are not 0 then bending and twisting response of the plate they get coupled. So, that is why it is known as bending twisting coupling coefficient.

So, this concludes our discussion. Tomorrow we will have another discussion on lamination sequence and how lamination sequence is written using standard lamination code. So, that is what we plan to do tomorrow until then have a great time.

Thank you.