**Introduction to Composites**
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**Lecture – 63**
**Resultant Forces and Moments**

Hello, welcome to Introduction to Composites. Today is the 3rd day of the ongoing week which is the 11th week of this course. Yesterday and day before that, we had developed relationships between strain and displacement functions, and also stress and strain for composite laminates.

Today in the beginning portion of this class we will explore these 2 issues a little bit further and we will try to understand how do these stresses and strains vary along the z direction that is for a given value of x and y, if we move in the z direction from top surface of the laminate to the bottom surface how do these stresses and strains vary.

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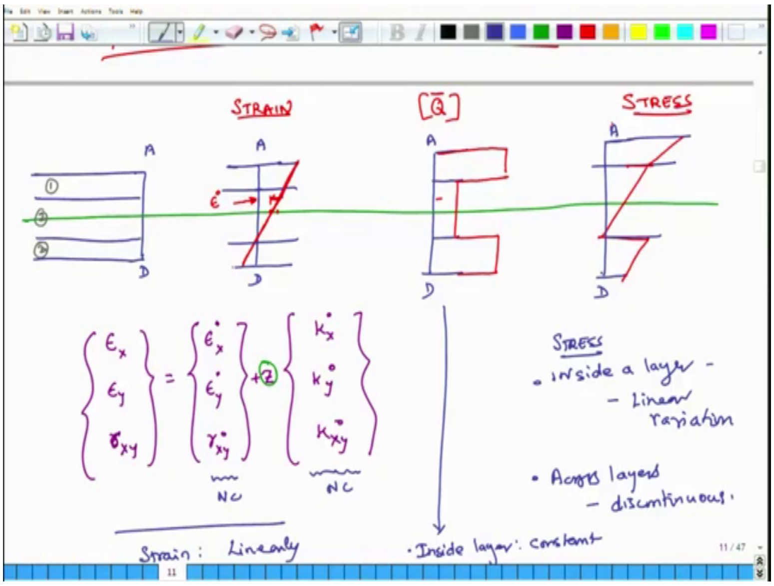


So, consider a laminate and we will consider a simple laminate which has let us say just 3 layers. So, this could be layer 1, this could be layer 2. So, this is 1, this is 2 and this is 3 and the midplane of the system is somewhere here, ok. So, this is, I am going to draw this again and one thing we have developed for strain is the relations which we have developed for strains are something like this that epsilon x, epsilon y, gamma xy equals epsilon x naught y naught gamma xy naught plus z times curvatures, ok.

So, along this line A D, this is my line A D this is again A D the value of x and the value of y do not change the only thing along line A D is changing is the coordinate z xy are not changing, which means this function does not change there is no change in this function because x is fixed and y is fixed. So, midplane strains are fixed similarly the curvatures are also not changing midplane curvatures are also not changing along line A D because what is curvature of the midplane it is second derivative of w with respect to x y or x and y. So, this is also not changing the only thing with changes is the z coordinate, the only thing which changes is the z coordinate.

So, and at the midplane the value of z is 0, at midplane the value of z is 0. So, at midplane what will be the strain? It will be the midplane strain. So, suppose we want to plot a long line A D epsilon x. So, strain will be midplane strain and then as I go up or down depending on whether curvature is positive or negative this strain overall strain will vary linearly because it is a constant plus z times another constant. So, it will be just a straight line ok.

So, the overall line may be something like this, where this represents the midplane strain this represents the midplane strain and the slope of this line represents the curvature and it just is a straight line. So, this is how a strain varies a along the thickness. Now, it can have a positive slope it can have a negative slope it all depends on the value of curvature, but the point is that a long line A D strain varies linearly across the entire thickness of the plate. So, this is something important to understand.

The second thing we have to understand is how does Q vary Q. So, so this is again A D and these are the 3 layers and here what we are going to see is how does Q, Q matrix vary and we will just consider 1 single element Q 11 bar or something, ok. So, the orientation in layer 1 can be some theta and then in layer 2 it can be some other theta. So, the change in angle is not sudden is not gradually it is sudden, ok.

So, if it is sudden it means the Q bar matrix for the first layer will be something and for the entire matrix say for the entire first layer it will be constant because it is same material and same angle, but for the second layer if theta changes then suddenly Q bar will change and then for the third layer it will again suddenly change. So, the Q bar matrix it changes suddenly. So, it can have any; so for a given layer it is constant because for a given layer theta is constant material same. So, for a given layer it is constant and as you move from 1 layer to other the Q bar matrix jumps. So, this is how Q bar changes. And finally, we look at how does stress change.

Now, we know we had developed this relation between stress and strain for the kth layer right. So, stress is equal to Q bar matrix times epsilon naught plus z times Q bar matrix times curvature, ok. So, what does that mean?

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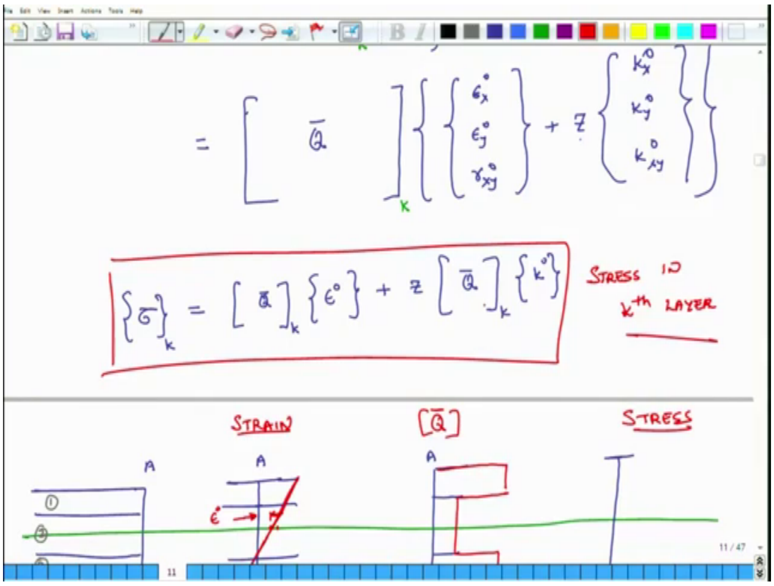


Strain is varying linearly Q bar is ma multiplied is jumping is jumping and it is discontinuous. So, what that means, is within a layer within a layer strain will vary linearly because Q bar is constant within a layer Q bar is constant. So, this is constant epsilon is constant k naught is constant. So, the only thing within a layer is varying is z, ok. So, within a layer strain change stress changes linearly, but as you move from once layer to other layer Q bar changes abruptly because we have shown it here and as a consequence stress jumps from 1 value to other value as you move from 1 layer to other layer.

So, the stress distribution or the stress function along the thickness of the plate. So, suppose these are 3 layers. So, the stress could vary something like this within a layer then because Q bar is changing and Q bar is becoming very small. So, it becomes very small and then again it changes linearly over the second layer and then again Q bar becomes large and again it changes linearly. So, within a layer, so stress inside layer it is linear and across layers it is discontinuous, ok.

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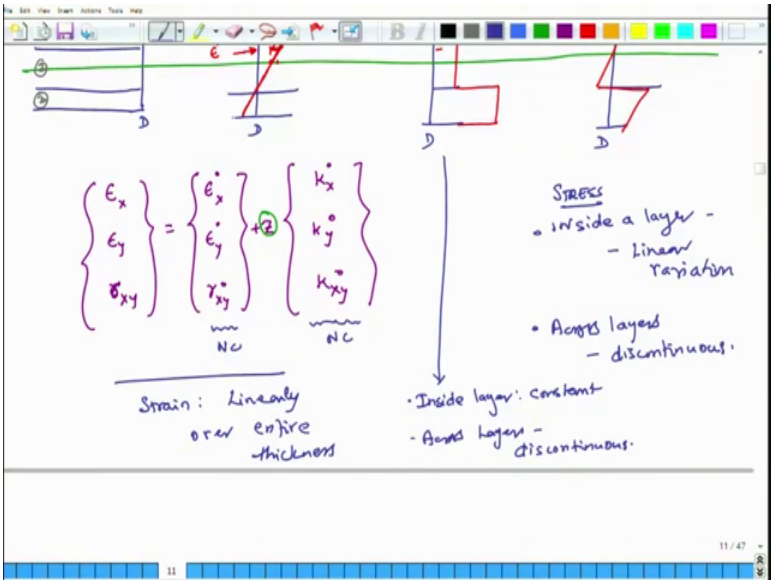


Across layers it is discontinuous within a layer it is changing linearly. Stress, a strain it changes linearly over entire thickness ok, and Q bar matrix it is discontinuous. What about Q bar matrix? It is inside layer constant and across layers discontinuous. So, in this case right, so this could be something like this ok. So, these are the 3 important observations strain varies linearly entire along the entire thickness of the composite the Q bar matrix and elastic properties are constant within a layer, but across layers they jump and they are discontinuous and stress is discontinuous across layers, but within a layer it varies linearly. So, this is something important to understand.

And with this we move to the next part of our discussion and we will introduce a new topic which is related to this and very useful and these are resultant, resultant forces and moments.

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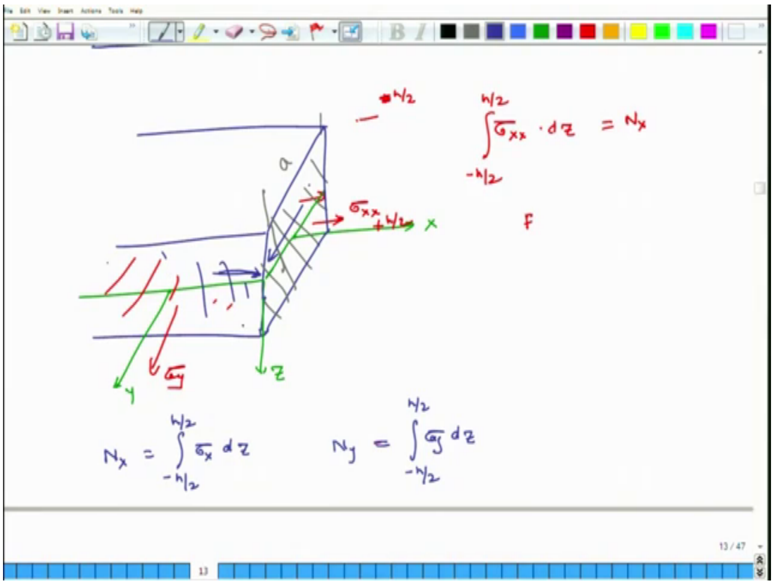


So, we define resultant forces as N’s and moments as M’s and these are resultant forces and moments per unit length they are not their units are not Newtons and Newton meter because they are per unit length ok. So, we will actually define that. So, first we will explain the concept.

So, suppose you have a composite sample this is my midplane ok, this is my z axis, this is let us say this is x, this is y, ok. So, the stress in the x direction on this face is sigma xx, it is sigma xx if I integrate this sigma xx in the z direction. So, I integrated in the z direction and I integrate it from, so let us say the total thickness of this composite laminate is h millimeters or h meters. So, I integrate it from plus h by 2 o minus h by 2 to plus h by 2 ok. So, my integration limits are minus h over 2 to h over 2, where h is the thickness then this is called an x, this is called an x.

And essentially it is the force, so if I integrate sigma x over this entire area you think about it if I integrate this sigma x over this entire area. Then what is it? It is the total force which is being exerted in the x direction and if I divide that total force by this distance a, then it is force per unit force per unit the length right. So, N x is basically force which is being applied in the x direction on the composite laminate in the x direction per unit length per unit length and that is sigma xx times d z integral of that, ok.

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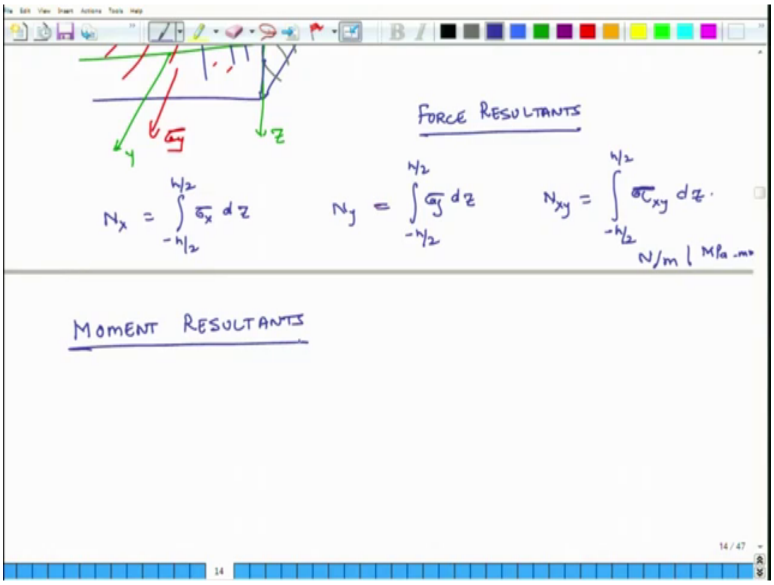


So, similarly we define other quantities. So, N x is equal to integral of minus h over 2 to h over 2 sigma x d z similarly N y. So, what is N y? In this direction this is sigma y. So, if I integrate sigma y on this entire surface and then I use it on upper unit length basis then then I get N y, so that is equal to sigma y d z.

And the last force resultant is called N xy and it is basically the integral of shear. So, this is sigma xy this is sigma xy if I integrate it on if I integrate sigma xy on this entire surface or I integrate sigma xy on this entire surface then I get the force resultant N xy. So, N xy is equal to minus h over 2 to h over 2 tau xy d z. So, and their units are Newtons per meter or Newtons per or new yeah Newtons per meter or sometimes we also use as MPa millimeter millimeter ok. If we want to use thickness only in millimeters then it becomes MPa millimeters otherwise to be internally consistent Newtons per meter is mathematically more consistent, ok. So, these are force resultants. So, these are not forces these are force resultants force resultants. Similarly we have moment resultant, ok.

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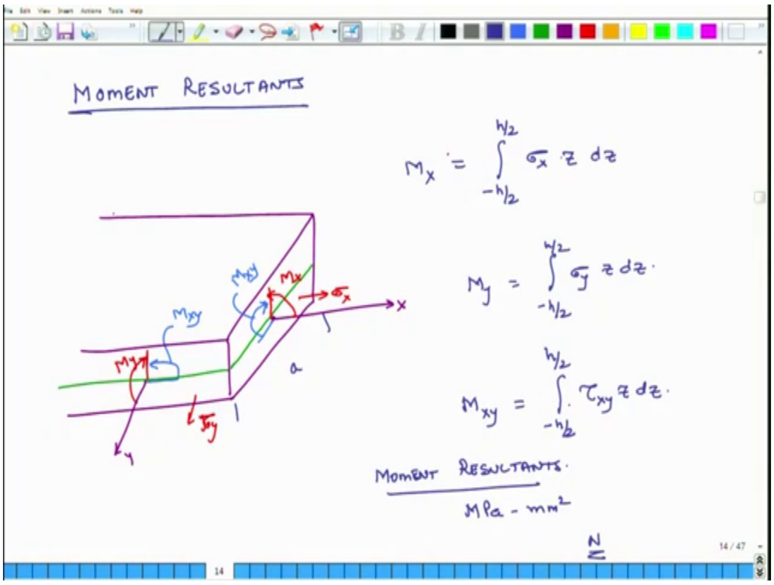


So, we will again draw pictures to make things clearer and then suppose I have a composite plate this is my midplane ok, and if I am having stresses in the x direction let us say this is sigma x. So, what will that stress do it will try to bend the plate up or down. So, it will it will not only try to pull it, but if it is off center then it will also try to bend it up and down suppose the stress is more on the lower half and less on the upper half then that plate will try to bend upwards if stress is more on the upper half and less on the lower half then plate will try to bend down. So, plate will not only stretch because of stress it will also try to bend. So, how do we figure out how much? And that bending will depend on the moment it will generate, ok.

So, we call this moment M x. So, suppose this is my x axis, if it tries to bend upwards we call it positive M x this is positive M x ok. And x, what does it represent? x represents the normal on the plane which it is acting. Similarly on the other surface, so this is my y axis and there is a M y and that my is a consequence of sigma y sigma y is uniformly spread above and below the surface then M y will be 0, if it is not uniformly then M y will be nonzero and then there is a twisting component. So, twisting component will be look something like this. So, this is M xy and similarly here it is like this M xy.

And how do we calculate the values of these M’s and M x, M y, M xy. So, again these are not moments these are moments per unit length moments per unit length. So, what is moment per unit length? It is basically integral, so we will write 1 relation M x is equal to integral of minus h by 2 to h by 2. So, if sigma x we integrate and we integrate not sigma x, but also sigma x times the distance times z d z then it is M x which is moment per unit length in the x direction ok, moment per unit length in the x direction.

Suppose I multiply it this by entire area a then it will be the actual moment the this entire length a then it will be similarly M y is integral of minus h by 2 to h by 2 sigma y z times d z and M xy is integral of minus h by 2 to h by 2 tau xy z times d z. So, essentially what we are calculating right now are forces acting on a plate per unit length on each edge, and moments acting per unit length on each edge. Forces per unit length are defined by N s which are integrals of sigma x times d z sigma y times d z and tau xy times d z and moments per unit length are sigma x times z times d z integral M y is sigma y z d z and M xy is tau xy z d z, ok.

So, these are moment result moment resultants. And what are their units? So, this will be MPa is the stress and times d z times z. So, then you can have millimeter square this is one, or if you are not integrating it in millimeters if you just use meters then it will be basically Newtons. So, I like the Newton unit better because it is mathematically more consistent, but lot of times people also use these mixed terms millim MPa times millimeter square. So, you should be aware of that, ok.

So, next what we will do is we will actually calculate M x, M y, M xy, N x, N y, N xy in terms of, so we already know how is these stresses sigma x sigma y tau xy are related to strains. So, we will put these equations for stress strain displacement, stress relations in these relations for moment and force resultant and then based on that we will develop a model for the behavior of a composite plate when it is subjected to externally known moment and force resultants.

So, that is what we plan to do in our next class. Until then I think it will be very important that you review all the material which we have covered particularly in this week because that will be very useful and important for you to know when we move on to this adjacent topic tomorrow.

Thank you and have a great night. Bye.